

Real-Time Control-Oriented Quality Modelling in Combined Urban Drainage Networks [★]

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Abstract: Urban drainage networks (UDN) carry urban wastewater to wastewater treatment plants (WWTP) in order to regenerate it before releasing it to the environment. Combined UDN (CUDN) carry both rain and wastewater together, which can overload the UDN and produce combined sewer overflows (CSO) that pollute the environment. Management of CUDN is receiving increasing attention from both researchers and water managers, in order to meet the high quality standards required for water and environment according to EU Water Framework Directive. Due to the complex dynamics of water quality, integrated control of CUDN and WWTP considering both flows and quality of the conveyed wastewater is a difficult problem. In order to design a real-time control (RTC) taking into account hydraulic and quality variables, the use of conceptual quality models is considered as a suitable option. This paper mainly presents a simplified conceptual quality modelling approach to represent the dynamics of suspended solid in sewers of CUDN oriented to real-time control. A sewer simulator implemented in SWMM (Storm Water Management Model) integrated with a lumped conceptual model for total suspended solid (TSS) is used for calibration and validation. A real example of Perinot sewer network is used as a case study. Discussions about RTC implementation in CUDN are also provided in this paper, where Model Predictive Control (MPC) is proposed as the suitable method to control the integrated water and quality models in CUDN as future motivation.

Keywords: CUDN, WWTP, CSO, RTC, conceptual quality model, SWMM, MPC.

1. INTRODUCTION

Combined urban drainage networks (CUDN) are generally designed to convey wastewater flows to treatment facilities in dry weather. During heavy-rain events, as soon as capacities of urban drainage networks (UDN) and Wastewater Treatment Plant (WWTP) are exceeded, mixed water is by-passed to receiving bodies producing combined sewer overflows (CSO), which are identified as polluting hazardous for biological species and ecological status as explained in Becouze et al. (2009); Gasperi et al. (2008). In order to optimize overall objectives of the complete urban drainage system, including the CUDN, the WWTP and the receiving environment, and prevent pollution of the receiving waters, an integrated control of both UDN and WWTP systems is required.

Since 1970s, the potential of using real-time control (RTC) within CUDNs has been discussed in Butler et al. (2005); Schilling (1989). References have proved that RTC is a reliable and cost-effective solution of CUDNs, which can improve the performance of CUDNs minimizing flooding and CSO volumes, thus protecting the environment as presented in Fu et al. (2010); Bulter et al. (2010); Butler et al. (2005); Cembrano

et al. (2004); Xu et al. (2013); Beeneken et al. (2013); García et al. (2015). Among real-time control methods, model predictive control (MPC), which can compute optimal control actions taking into account not only the current measurements but also predictive behaviors in a certain horizon, has been successfully tested in water supply and also in the context of advanced urban drainage networks by Cembrano et al. (2004); Puig et al. (2009); Butler et al. (2005); Pleau et al. (2005).

Nevertheless, the majority of RTC applied in CUDN have only focused on hydraulic model and control objectives without considering the polluting quality load inside the carried water, e.g. Cembrano et al. (2004). The few references which consider quality models during the control process mostly use simulation tools to overcome dealing with the complexity of modelling water quality directly. In Rathnayake (2015), control of CUDNs minimizing CSO is achieved by using a non-sorted genetic algorithm (NSGA) linked with storm water management model (SWMM) 5.0. Butler et al. (2005) optimizes integrated modelling of the operation and control of an integrated urban drainage system by developing a simulation package SYNOPSIS which integrates the sewer system (simulated by KOSIM), treatment plant (simulated by IAWPRC) and river model (simulated by DUFLOW) using different softwares.

[★] This research is funded by EU funding for the project LIFE EFFIDRAIN LIFE14 ENV/ES/00080

For the purpose of operating RTC of integrated urban drainage systems taking into account CSO pollution, appropriate quality models are needed. Such models allow for evaluation of waters according to water quality criteria as in Ahyerre et al. (1998). Because of the input data uncertainty and difficulty in calibration, modelling the generation and transportation of pollution in CUDN during a storm event is complex. There indeed exist some physically-based models which can present quality dynamics in CUDN, but the mathematical equations are difficult to be implemented within RTC as shown in Rouse (1937); Van Rijn (1984); Macke (1980); Ackers and White (1973).

In order to confront these challenges, a simplified conceptual quality modelling approach oriented to RTC for dynamics of total suspended solid (TSS) is presented. The SWMM software (Rossman (2015)) integrated with a lumped conceptual TSS model which depends indirectly on physical parameters through flow and computation of Saint-Venant model, is used to calibrate and validate the modelling approach. A real sewer in Bordeaux, Perinot sewer network is used as a case study.

The remainder of this paper is organized as follows: Section 2 presents the SWMM simulator integrated with a lumped conceptual TSS model, which represents characteristics of solid dynamics in the sewer. Section 3 presents the simplified conceptual quality modelling approach of TSS in sewers, a discussion about RTC applied in CUDN is also provided in the end of Section 3 as motivation of future work. In Section 4, calibration and validation for the proposed simplified modelling approach using SWMM-TSS and the real-life case Perinot sewer network is presented. Section 5 presents the conclusion of this paper.

2. SWMM-TSS BASED ON LUMPED CONCEPTUAL MODEL

As a representative example of sewer pollutants, models for solid concentration and load are presented in this paper, which generally present three dynamic behaviors by Bertrand-Krajewski (2006); Rossman (2015):

- Accumulation of solid sediments over urban catchment;
- Washoff of solid sediments by rainfall;
- Transfer, erosion, deposition of solids in sewer networks and retention tanks.

2.1 Concept of SWMM-TSS

Since solids in sewer are highly driven by hydraulics, a new quality model has been developed based on the SWMM 5 simulator (version SWMM5.1.011) that already includes a detailed description of hydraulics using full Barré de Saint Venant equations. The model library has been modified by adding equations describing solids behavior for the following phenomena:

- **Hydrology:** The exponential build-up model on the catchment was modified to take into account only the impervious area rather than the total area. The washoff model was replaced by the one proposed by Métadier (2011);
- **Sewer:** Accumulation and erosion phenomena are described using Wiuff (1985) energy balance based model used by Bertrand-Krajewski (1993);
- **Storage unit:** Mixing and settling processes are inspired from the work of Briat (1995); Maruéjols et al. (2012).

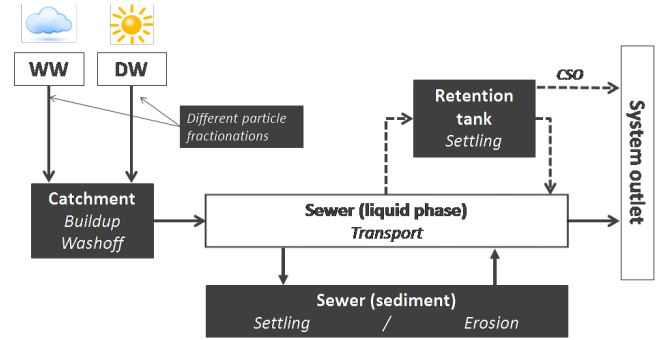


Fig. 1. Modifications made in the SWMM 5 library model

Figure 1 is the scheme illustrating the modifications made in the SWMM 5 library model. White boxes are for existing modules in SWMM 5 and grey boxes are for added quality module.

2.2 Sewer Equations in SWMM-TSS

As previously explained, the main addition in the model library are regarding the sewer accumulation and erosion model. This model is based on Wiuff (1985) that calculates a transport capacity of the flow CT depending on the water ρ_e and particle densities ρ_n , the water velocity U_m and pipe slope I , the particles' settling velocities W_n according to particles' fractions F_{p_n} for each particle classes and the yield coefficient η_n directly dependent on hydraulics characteristics.

$$CT = \frac{\rho_e U_m I}{\sum_{n=1}^N \left(\frac{F_{p_n} (\rho_n - \rho_e) W_n}{\rho_n \eta_n} \right)} \quad (1)$$

The CT is compared with the TSS at the pipe inlet to calculate the quantity of TSS that can settle. The erosion model is based on a first order equation dependent on the mass of sediment and a specific erosion coefficient for each particle classes.

3. SIMPLIFIED CONCEPTUAL MODELLING APPROACH

Lumped complex hydrology models work well for simulating real dynamics of TSS in sewer networks, but for the RTC purpose of CUDN, a simple model structure should be presented according to the following principles presented by Norreys and Cluckie (1997); Cluckie et al. (1999); Puig et al. (2009):

- Representativeness of the main dynamics;
- Simplicity, flexibility, expendability and speed;
- Availability of on-line calibration and optimization.

The modelling approaches presented in this paper are designed to be used by RTC to predict the evolution of TSS parameters over a short horizon, which prevent the limitation of classical modelling approaches for temporal measurement campaign. In order to evaluate TSS discharges to the receiving water, performing measurements at the outlet of sewer is the most suitable strategy. The simplified dynamic model of TSS in CUDNs will consider TSS in sewers and mass balance equation in the junctions. TSS behaviors in detention tanks, weirs and also WWTP will be part of future research. The least square function is used to measure performance of these approaches.

3.1 Simplified Modelling Approaches for Sewer

Originating from the hydraulic model of CUDNs as in Cembrano et al. (2004); Puig et al. (2009), a sewer trunk in a CUDN can be assumed as a water tank container with capacity to collect water based on the different between upstream (Q_{in} : m^3/s) and downstream (Q_{out} : m^3/s) flows as in Figure 2. The transport model of water volume (V) inspired from linear tank model for hydraulic transport will be expressed as:

$$V(k+1) = V(k) + \Delta t(Q_{in}(k) - cV(k)) \quad (2a)$$

$$Q_{out}(k) = cV(k) \quad (2b)$$

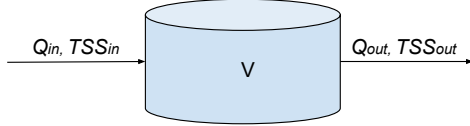


Fig. 2. Tank model for a sewer

In a sewer trunk, mass conservation does not necessarily apply to the solids because of solid settlement and erosion. In order to generalize transport model of TSS, an intermediate variable X which has no direct physical meanings is used and the in-out relation for TSS can be defined as:

$$X(k+1) = X(k) + \Delta t(TSS_{in}(k) - cX(k)) \quad (3a)$$

$$TSS_{out}(k) = cX(k) \quad (3b)$$

where

TSS_{in} : TSS (g/m^3) enter into a sewer

TSS_{out} : TSS (g/m^3) out of a sewer

k : the current time

Δt : sampling time

c : parameters need calibration

After combining the equation (3a) and (3b), the following dynamic model for TSS in a sewer is produced:

Model 1

$$TSS_{out}(k+1) = (1-c)TSS_{out}(k) + cTSS_{in}(k) \quad (4)$$

By letting the two coefficients of TSS_{out} and TSS_{in} be independent, a generalization of Model 1 readily comes to mind, leading to:

Model 2

$$TSS_{out}(k+1) = c_1TSS_{out}(k) + c_2TSS_{in}(k) \quad (5)$$

Furthermore, from the physical characteristics, in a sewer, the dynamic of TSS is affected by the TSS sediment, erosion and also time delays (Figure 3). After that, a new linear delayed expressions of TSS and water volume can be defined as:

Model 3

$$TSS_{out}(k) = c_{vc}TSS_{in}(k-d) + e_p \quad (6a)$$

$$Q_{out}(k) = t_{vc}Q_{in}(k-d') \quad (6b)$$

where

d : delay of TSS inside a sewer

d' : delay of flow inside a sewer

c_{vc}, t_{vc}, e_p : parameters need calibration

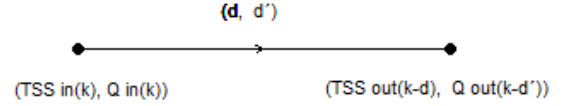


Fig. 3. TSS model in a sewer

Using these modelling methodologies, simplified models of TSS inside a sewer are created, which allow on-line model calibrations and real-time control of sewer networks. For each sewer, the parameters c , c_1 , c_2 , c_{vc} , d , t_{vc} and e_p need to be calibrated by GAMS using historic or real-time data from telemetry systems. Emphasized on TSS modelling approach, c , c_1 , c_2 , c_{vc} , d and e_p are will be calibrated.

3.2 Simplified Modelling Approaches for Junction

Junctions in a sewer network correspond to the points where water flows and also TSS are merged or split. These elements fit mass balance relations. When TSS comes into a junction in the sewer network, the dynamics of TSS is modelled as equality constraints related to upstream (TSS mass enters into this junction) and downstream (TSS mass exits from this junction). For different junctions, there could exist different numbers of branches (more than two). Take a three branches junction as an example (as shown in Figure 4), the expression of the mass conservation for TSS can be written as (it is considered that the junction does not add a significant delay):

$$TSS_{in3}(k)Q_{in3}(k) = TSS_{out1}(k)Q_{out1}(k) + TSS_{out2}(k)Q_{out2}(k) \quad (7)$$

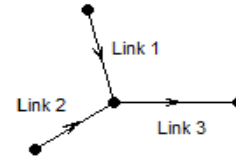


Fig. 4. TSS model in a junction

3.3 Model Performance Evaluation

In order to validate and compare implementations of these proposed modelling approaches, the least square function is used to evaluate model performance of calibrated value using these simplified model comparing with simulated results produced by lumped model of SWMM-TSS.

$$FC = \sum_{k=1}^K (V(k) - R(k))^2 \quad (8)$$

where

V : calibrated value using simplified model

R : simulated value using SWMM-TSS

K : time steps

Besides that, the fitting rate between the calibrated value with simplified model and the simulated value with SWMM-TSS is defined using Nash Sutcliffe model efficiency coefficient as explained in Nash (1970):

$$\bar{V} = \frac{\sum_{k=1}^K V(k)}{\sum_{k=1}^K k} \quad (9a)$$

$$FC_{TSS} = 100 \left(1 - \frac{\sqrt{\sum_{k=1}^K (R(k) - V(k))^2}}{\sqrt{\sum_{k=1}^K (R(k) - \bar{V}(k))^2}} \right) \quad (9b)$$

3.4 Real-time control of CUDN

While state-of-the-art implementations of RTC in UDN are based only on hydraulic variables, the integrated control of CUDNs and WWTP must take into account quality variables in order to minimize the overall polluting load to the receiving environment. The main ideas behind this integrated control are:

- controlling the water detention and diversion in the urban drainage network, so as to minimize TSS in the effluents of the systems;
- taking into account the variability in WWTP capacity, according to the influent flow and TSS concentration.

The process of CUDN control is also highly dependent on rainfall scenarios which require rain predictions. Considering these characteristic of CUDN control, MPC has been accepted to have advanced advantages in controlling urban drainage water systems as in Cembrano et al. (2004); Xu et al. (2013). MPC can generate optimal control actions by optimizing the objective function at every control step. Besides that, MPC can predict the future behavior of the system through using an internal model over a finite prediction horizon. In CUDN, state of space of MPC can be presented as in Xu et al. (2013):

$$x(k+1) = f(x(k), u(k), d(k)) \quad (10)$$

where x is a vector of network states (e.g. water volume and TSS mass in a tank); u is a vector of control variables such as flow across a commanded gate; d is a vector of disturbances related to rain intensity and runoff.

Continued efforts are focused on the integrated control of CUDN which involves water quantity and quality using feasible modelling approaches for improving performance of CUDN like in Butler et al. (2005); Vanrolleghem et al. (2005).

4. MODEL CALIBRATION AND VALIDATION

The real case study of the sewer network of Perinot in Louis Fargue catchment of Bordeaux Metropole covers a total area of 260 ha that is mainly residential. The sewer length is 3 km with an average slope of 0.007, quite constant over the whole catchment. It includes a retention tank separated in three hydraulically connected bodies for a total storage volume of 35000 m^3 . Even if the slope is generally low, there is no sediment issues on the sewer reported from the operators.

Rain scenarios used for calibration and validation come from the real rainfall measured at France in the year of 2007 as show in Figure 6. Rain scenarios $T1$, $T2$ in different time stage of the year 2007 which represent different rain densities are selected to produce TSS training data using SWMM-TSS in order to calibrate the proposed simplified models. After achieving calibrated TSS models, another two different rain scenarios $V1$, $V2$ are applied in SWMM-TSS to produce TSS data for validating the working of these produced simplified models.

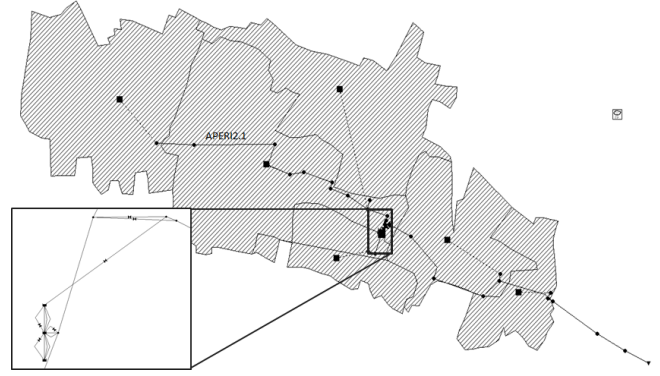


Fig. 5. SWMM configuration of the Perinot case study

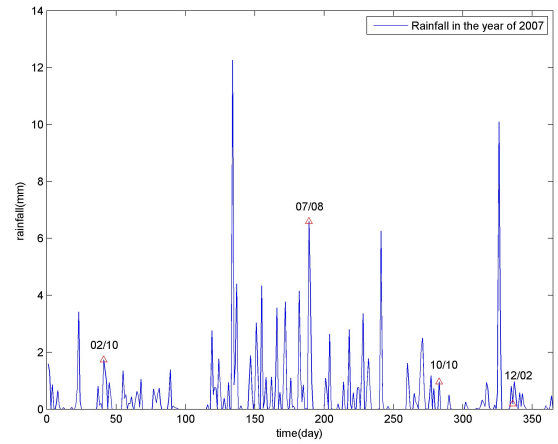


Fig. 6. Rain Scenario of Perinot in the year of 2007

The selected rain scenarios are the rainfall in the following time stages (where the time use format $MM/DD/YYYY HH : MM : SS$):

$T1$: 10/10/2007 00:00:00-10/11/2007 00:00:00

$T2$: 12/02/2007 00:00:00-12/03/2007 00:00:00

$V1$: 02/10/2007 00:00:00-02/11/2007 00:00:00

$V2$: 07/08/2007 00:00:00-07/09/2007 00:00:00

4.1 Model Calibration

For each elements in CUDN, on-line calibration is needed. Set the sampling and report time of SWMM-TSS as 5 minutes, antecedent dry days as 10. After applying $T1$, $T2$ rain scenarios into Perinot case, the TSS behaviors during these rainfall are produced, which is the training data to calibrate the conceptual models.

Taken conduit $APERI2.1$ (as shown in Figure 5) in the upstream of Perinot network as an example, where TSS_{in} represents TSS enters into conduit $APERI2.1$, TSS_{out} represents TSS out of this element. Table 1 provides detail parameters calibrated for

Table 1. Calibrated Parameters

	c	c_1	c_2	c_{vc}	e_p	d
$T1$	0.26	0.73	0.26	0.87	45.11	2
$T2$	0.20	0.80	0.20	0.80	72.77	2

the three models of conduit *APERI2.1* in $T1$ and $T2$ rain scenarios. Figure 7 and Figure 8 show the performances comparisons between calibrated models (TSS_{out} Model in the figures) and simulated value using SWMM-TSS (TSS_{out} SWMM in the figures) in $T1$ and $T2$ rainfall scenarios. These results confirm that, all the models have considerable performance. Model 2 works better than other two models. Model 3 has low fitting performance but the changing trends of TSS is similar with that in SWMM-TSS, which has meaning during the control process.

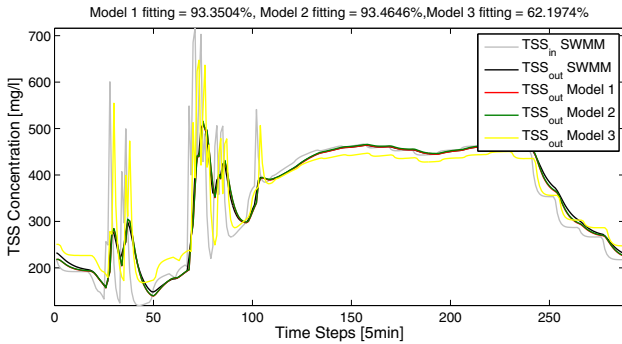


Fig. 7. Calibration performance for $T1$

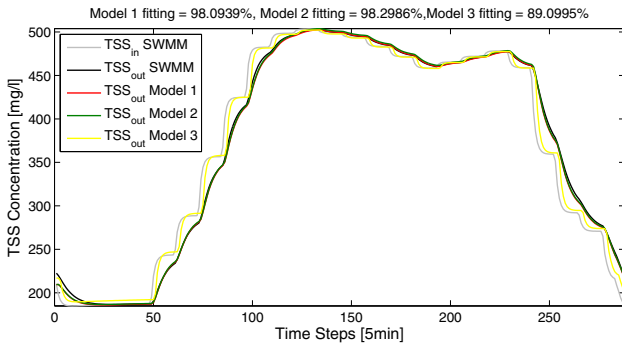


Fig. 8. Calibration performance for $T2$

4.2 Model Validation

In order to validate further the proposed calibrated models, rain scenarios $V1$ and $V2$ are applied to the models calibrated in $T1$ and $T2$ separately. Figure 9, 10, 11 and 12 provide the validation performance using $V1$ and $V2$ rain scenarios for the produced models by $T1$ and $T2$. The fitting comparisons provided from these figures shows that, comparing with simulation results from SWMM-TSS, all the proposed models are working well using the calibrated models.

Table 2 with detailed fitting performances for both the calibration and validation scenarios confirm these conclusions.

5. CONCLUSIONS

This paper proposed simplified conceptual modelling approaches for representing dynamic behaviors of TSS in sewer networks, which are oriented to be involved into the integrated

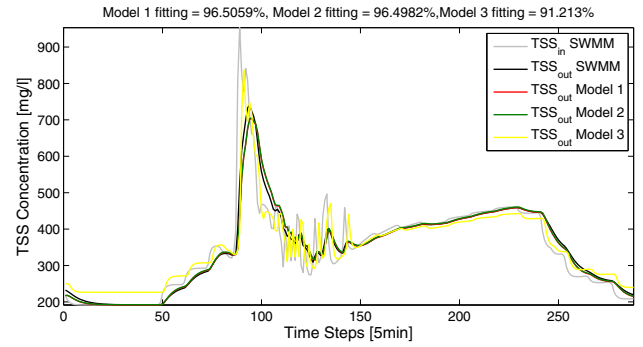


Fig. 9. $V1$ validation performance for $T1$

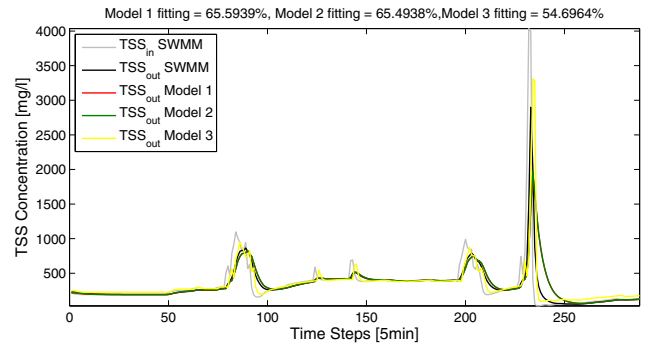


Fig. 10. $V2$ validation performance for $T1$

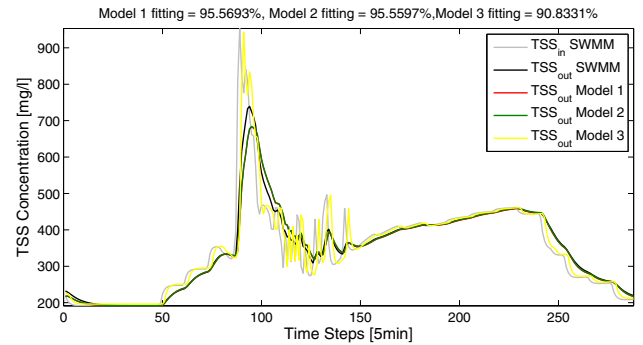


Fig. 11. $V1$ validation performance for $T2$

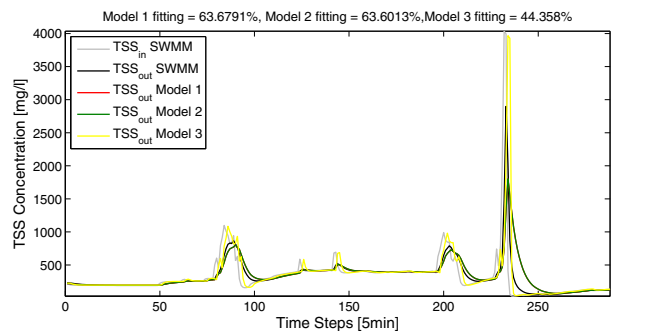


Fig. 12. $V2$ validation performance for $T2$

RTC controller of CUDN and WWTP for environmental protection. The SWMM is used to produce realistic simulated data for calibration and validation. The Perinot sewer network, which is a real life example, is used as a case study. The application and validation results of the proposed modelling approaches have proved that, the simplified conceptual model

Table 2. Performance Comparisons

Test/Validate Scenarios	Model 1	Model 2	Model 3
T1	93.35%	93.46%	62.20%
V1	96.51%	96.50%	91.21%
V2	65.59%	65.49%	54.70%
T2	98.09%	98.30%	89.10%
V1	95.57%	95.56%	90.83%
V2	63.68%	63.60%	44.36%

can capture the main characteristics of TSS evolution in sewer networks with equations that are simple enough to be used for integrated RTC control to CUDN. This work is part of a research project aimed to design Model Predictive Control for both water quantity and quality models in CUDN.

ACKNOWLEDGEMENTS

The authors wish to thank the support received by the European Commission research grant of project LIFE EFFIDRAIN (LIFE14 ENV/ES/000860), thank Aigües de Barcelona for the financial and technical support. Authors also thank Bordeaux Metropole and SGAC for technical support and financial support through the research convention signed with the LyRE.

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