On Dynamical Systems for Sensorimotor Contingencies. A First Approach from Control Engineering

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Abstract. According to the sensorimotor approach, cognition is constituted by regularities in the perceptual experiences of an active and situated agent. This perspective rejects traditional inner representational models, stressing instead patterns of sensorimotor dependencies. Those relations are called sensorimotor contingencies (SMC). Many research areas and accounts are working on and related with it. In particular, four distinct kinds of SMCs have been previously introduced for environment, habitat, coordination and strategy using dynamical models from a psychological perspective. As dynamical systems, in this paper we analyze SMCs, for the very first time, from a modern control engineering perspective. We provide equations and block diagrams translating the psychological proposal to control engineering. We also analyze the original toy example proposed from the psychological domain into the modern control engineering point of view, as well as we propose a first approach to this toy example coming from the control engineering domain.

Keywords. sensorimotor contingencies, dynamical systems, modern control engineering

1. Introduction

The phenomenal character of a perceptual experience was stated in [1], namely that patterns of sensorimotor dependencies (or 'sensorimotor contingencies') can be defined as the regularities in how sensory stimulation depends on the activity of the perceiver. This perspective rejects traditional inner representational models [2], stressing instead patterns of sensorimotor dependencies. The sensorimotor approach to cognition has brought together research from several disciplines over the last decade such as ecological psychology [3,4], enactivism [5,6], cybernetics [7] or developmental robotics [8,9], among others.

In [10], attention is payed to the notion of SMCs itself, appealing to the few attempts to formally define it. Some of the few existing models are rather abstract and focus almost exclusively on the problem of extracting the proper dimensionality of the interaction space of an agent [11]. Others focus more directly on robotic applications and assume a probabilistic, discrete-time interpretation of SMCs [12].

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In the remaining of this work the notions of SMCs are treated as dynamical systems according to the psychological proposal in [10]. Taking advantage of the definition of SMCs as dynamical systems and for the very first time, we analyze SMCs from a modern control engineering perspective. We provide equations and block diagrams translating the psychological proposal to control engineering for the easiest assumptions, linear time-invariant single input single output (LTI SISO) systems. Then, we express the original toy example proposed from the psychological domain using the control engineering approach, as well as we propose a variation to this toy example fitting the LTI SISO assumptions. Further research and conclusions are finally displayed.

2. Notions of SMCs

According to the approached discipline, not a unique definition of the SMC term exists, providing a range of useful interpretations. Hence, different notions based on lawful structures can be identified in agent-environment interaction and other specific forms of sensorimotor coordination that are strategically deployed by the agent.

Choosing different lawful structures, four distinct kinds of SMCs are identified in [10]: *sensorimotor environment*, *sensorimotor habitat*, *sensorimotor coordination*, and *sensorimotor strategies*. The sensorimotor coupling of an agent with the environment is formalized using a dynamical systems approach. The coupled agent-environment world is partitioned into components describing the dynamics of the environment ($\mathbf{e}$) and the agent in bodily configuration ($\mathbf{p}$), sensory ($\mathbf{z}$) and motor instances ($\mathbf{m}$) as well as its internal state ($\mathbf{a}$). In general, such a system could be described by the set of equations:

Environment information. Corresponds to the environment evolution depending on the agent's pose and the past state of the environment,

$$\dot{\mathbf{e}} = E(\mathbf{e}, \mathbf{p}) \tag{1}$$

Agent contingencies. Corresponds to the dynamics of the agent,

- Sensory dynamics: agent's sensory depends on the environment and agent's internal state, but not on past values of the agent's sensory,

$$\dot{\mathbf{z}} = Z(\mathbf{e}, \mathbf{a}) \tag{2}$$

- Internal dynamics: agent's internal state evolves depending on the sensed environment and the past internal state; agent's motors work based only on agent's internal state,

$$\dot{\mathbf{a}} = A(\mathbf{a}, \mathbf{z}) \tag{3}$$

$$\dot{\mathbf{m}} = M(\mathbf{a}) \tag{4}$$

- Body dynamics: agent's pose changes accordingly to agent's motors and environment,

$$\dot{\mathbf{p}} = P(\mathbf{m}, \mathbf{e}) \tag{5}$$

Summing-up, only the environment and the agent's internal state introduces feedback, that is, they are storing self information.

3. The Control Engineering Approach

Our objective is to describe the coupled dynamical equations (1)-(5) using a modern control engineering approach. For the sake of clarity, we formulate equations for linear time-invariant (LTI) systems with a single input and a single output (SISO). We consider all initial conditions null. Further, we describe the kinds of SMCs using block diagrams from the original completed coupled agent-environment world. Several conclusions can be obtained from this new approach.

Starting by considering LTI systems with null initial conditions, equations (1)-(5) can be described following the standard space-state notation, $\dot{\mathbf{x}} = \mathbf{A} \cdot \mathbf{x} + \mathbf{B}\mathbf{u}$, for dynamical systems:

$$\dot{\mathbf{e}} = \mathbf{A}_E \cdot \mathbf{e} + \mathbf{B}_E \cdot \mathbf{p} \tag{6}$$

$$\dot{\mathbf{z}} = \mathbf{A}_Z \cdot \mathbf{0} + \begin{bmatrix}\mathbf{B}_{1Z}\mathbf{B}_{2Z}\end{bmatrix} \cdot \begin{bmatrix}\mathbf{e}\\ \mathbf{a}\end{bmatrix} \tag{7}$$

$$\dot{\mathbf{a}} = \mathbf{A}_A \cdot \mathbf{a} + \mathbf{B}_A \cdot \mathbf{z} \tag{8}$$

$$\dot{\mathbf{m}} = \mathbf{A}_M \cdot \mathbf{0} + \mathbf{B}_M \cdot \mathbf{a} \tag{9}$$

$$\dot{\mathbf{p}} = \mathbf{A}_P \cdot \mathbf{0} + \begin{bmatrix}\mathbf{B}_{1P}\mathbf{B}_{2P}\end{bmatrix} \cdot \begin{bmatrix}\mathbf{m}\\ \mathbf{e}\end{bmatrix} \tag{10}$$

For any 'agent-environment' coupled system some variables can be considered as describing the environment and some others as describing the agent. In the same sense, some of them can be considered as input to the system, but all of them can be also considered as well as system states, avoiding the inclusion of exogenous inputs, resulting in the alternative description:

$$\begin{bmatrix}\dot{\mathbf{e}}\\ \dot{\mathbf{z}}\\ \dot{\mathbf{a}}\\ \dot{\mathbf{m}}\\ \dot{\mathbf{p}}\end{bmatrix} = \begin{bmatrix}\mathbf{A}_E & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{B}_E\\ \mathbf{B}_{1Z} & \mathbf{0} & \mathbf{B}_{2Z} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{B}_A & \mathbf{A}_A & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{0} & \mathbf{B}_M & \mathbf{0} & \mathbf{0}\\ \mathbf{B}_{2P} & \mathbf{0} & \mathbf{0} & \mathbf{B}_{1P} & \mathbf{0}\end{bmatrix} \cdot \begin{bmatrix}\mathbf{e}\\ \mathbf{z}\\ \mathbf{a}\\ \mathbf{m}\\ \mathbf{p}\end{bmatrix} \tag{11}$$

Hence, with this formulation we are describing the agent-environment couple as a whole.

Simplifying now to the proposed SISO approach and using the Laplace transformation, equations of the coupled system result in:

- How the environment changes from a past state depending on the current agent's pose,

$$\frac{E(s)}{P(s)} = \frac{b_E}{s - a_E} \tag{12}$$

- What the agent senses depends on what is captured from the environment, but also on the agent's internal state,

$$sZ(s) = b_{1Z}E(s) + b_{2Z}A(s) \tag{13}$$

Recurrence is not considered in this approach for the agent's sensor.

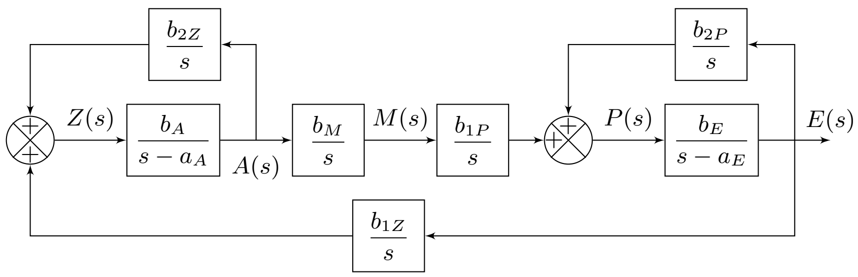

Figure 1. Block diagram representing the closed-loop sensorimotor trajectories linked to the *sensorimotor habitat*. Pose $P(s)$ and Environment $E(s)$ are the usual variables considered by an external observer.

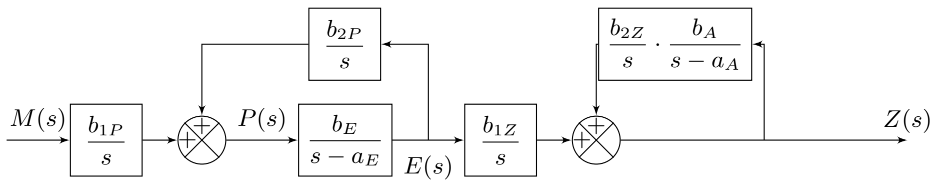

Figure 2. This block diagram represents the open-loop motor-induced sensory variations linked to the *sensorimotor environment*.

- How the agent's internal state evolves depending on the sensed environment and the past value of the internal sate,

$$\frac{A(s)}{Z(s)} = \frac{b_A}{s - a_A} \tag{14}$$

- How agent's motor works based on agent's internal states,

$$\frac{M(s)}{A(s)} = \frac{1}{s} b_M \tag{15}$$

- How agent's body changes according to agent's motor commands and environment evolution,

$$sP(s) = b_{1P}M(s) + b_{2P}E(s) \tag{16}$$

These equations lead to the block diagram in Figure 1. This block diagram represents the closed loop sensorimotor trajectories linked to the *sensorimotor habitat* according to its definition in [10].

4. Analyzing Two Kinds of SMCs

Depending on the degree of agent-centredness, four kinds of sensorimotor contingencies have been distinguished in Section 2. In this initial work we will focus on the two first levels of SMCs: SM Environment and SM Habitat.

4.1. *The Sensorimotor Environment,* $\mathbf{z} = g(\mathbf{m})$

This sensorimotor dependency describes how sensory input $\mathbf{z}$ changes with induced motor activity $\mathbf{m}$ in an open-loop form. This evolution depends on the embodiment of the agent $\mathbf{p}$ and the environment $\mathbf{e}$ only, not on the agent state $\mathbf{a}$. Unlike our approach, most examples of SMCs in the literature refer to the instantaneous sensory consequences of arbitrary changes in perspective or movements in general, without considering how the movements themselves are related to sensory feedback.

This kind of SMCs can be captured by considering how an agent's sensor values $\mathbf{z}$ change in relation to given motor states assuming that the motor command $\mathbf{m}$ varies freely – in other words, $\mathbf{m}$ is taken as an independent variable, decoupled from the agent's variables $\mathbf{a}$. The global sensorimotor loop depicted in Figure 1 is opened by removing (4), that is, using the modern control engineering approach, removing (9), or (15) for the SISO case.

From now, since motor commands $\mathbf{m}$ vary freely, they can be considered as inputs to the system, hence the resulting alternative description, after removing (4), is:

$$\begin{bmatrix} \dot{\mathbf{e}} \\ \dot{\mathbf{z}} \\ \dot{\mathbf{a}} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_E & \mathbf{0} & \mathbf{0} & \mathbf{B}_E \\ \mathbf{B}_{1Z} & \mathbf{0} & \mathbf{B}_{2Z} & \mathbf{0} \\ \mathbf{0} & \mathbf{B}_A & \mathbf{A}_A & \mathbf{0} \\ \mathbf{B}_{2P} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{e} \\ \mathbf{z} \\ \mathbf{a} \\ \mathbf{p} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{B}_{1P} \end{bmatrix} \cdot \mathbf{m} \tag{17}$$

Equivalently, we remove the associated component in the block diagram of Figure 1. Now, $M(s)$ is considered as input and $Z(s)$ as output, $z = (0\,1\,0\,0) \cdot (e\,z\,a\,p)^\top$, in the agent-environment coupled dynamical system. so we arrive to the block diagram in Figure 2.

Two main relationships can be distinguished in this open-loop system:

1. from the motor command $M(s)$ to the environment $E(s)$, where the body pose is intrinsically considered,

$$\frac{E(s)}{M(s)} = \frac{b_E b_{1P}}{s(s - a_E) - b_E b_{2P}} \tag{18}$$

   and,

2. from the environment $E(s)$ to the sensor values $Z(s)$, where agent's states are implicit,

$$\frac{Z(s)}{E(s)} = \frac{b_{1Z}(s - a_A)}{s(s - a_A) - b_{2Z} b_A} \tag{19}$$

4.2. *Some Remarks about the Pose in the Sensorimotor Environment*

It is claimed in [10] that relevant aspects of the relationship between motor commands and sensor variables could be captured, whenever possible and at least locally, by the partial derivative $\partial\mathbf{z}/\partial\mathbf{m}$, that is by analysing changes in sensor values $\mathbf{z}$ resulting from changes in the independent variable $\mathbf{m}$ while all the remaining variables are held constant. However, since

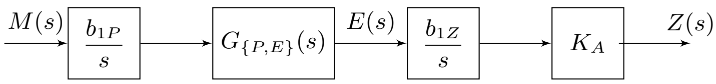

Figure 3. This block diagram represents the simplified open-loop motor-induced sensory variations linked to the *sensorimotor environment*.

$$\frac{\partial z}{\partial m} = \frac{dz/dt}{dm/dt} = \frac{b_{2Z}}{b_m} + \frac{b_{1Z}e(t)}{b_M a(t)} \tag{20}$$

a constant value can be considered for the agent and the environment variables to perform the local study, but this is not the case for the pose, which is not included in this expression. A study about pose variations for similar motor commands can be found in [13], which indicate that further work is needed about this claim on constant body pose.

Authors in [10], in order to justify the claimed simplification, consider 'constant' the feedback loop, that is, the same closed-loop, around the body pose for agents with the same bodies in a given environment. Our proposal, by analysing the latter equations, differs: the agent's feedback around $A(s)$ can be simplified to a constant K_A, usually easily skipped, $K_A = 1$. However, the feedback loop around the body pose can not so easily be defined like a constant. Hence, the complete system simplifies in the block diagram depicted in Figure 3. In an equivalent form, for the space states representation results in:

$$\begin{bmatrix} \dot{\mathbf{e}} \\ \dot{\mathbf{z}} \\ \dot{\mathbf{a}} \\ \dot{\mathbf{p}} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_E & \mathbf{0} & \mathbf{0} & \mathbf{B}_E \\ \mathbf{B}_{1Z} & \mathbf{0} & \mathbf{B}_{2Z} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{B}_{2P} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{e} \\ \mathbf{z} \\ \mathbf{a} \\ \mathbf{p} \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{B}_{1P} \end{bmatrix} \cdot \mathbf{m} \tag{21}$$

4.3. The Sensorimotor Habitat

The *sensorimotor habitat* is the set of all sensorimotor trajectories that can be generated by the closed-loop system in Figure 1. This relation looks at co-variations obtained once the loop is closed by taking into account the agent's internal activity and responsiveness to sensory changes, i.e. we close the loop again and take into account the agent's internal state and its influence on the effectors.

Information about the SM habitat, i.e. how the SM environment is 'inhabited', can be captured using the full set of equations, where $\mathbf{m}$ is not longer considered an independent variable (input). The "laws" of the SM environment constrain but do not fully determine the regularities of the SM habitat.

5. One Toy Example, Two Different Approaches

A minimal model is depicted in [10] to illustrate the different kinds of SMCs proposed. In this example, the task to be solved by the agent is a continuous version of the discrete task proposed in [12]: the agent, moving along a one-dimensional environment, will discriminate between two bell-shapes with wide and narrow widths. The agent can sense these shapes via a single distance sensor. The time-derivative of the sensor signal serves as input to a small dynamic neural network (CTRNN) that delivers continuous motor commands controlling the agent's velocity. The task to be solved by the agent is the discrimination between wide and narrow shapes, requiring it to move away from the former, and approach the peak of the latter.

5.1. The Original Approach

The proposed set of equations for the couple agent-environment system can be translated in this example to:

- The environment are two bell-shaped curves of different widths w, height h, and peak position c, defined as,

$$\dot{\mathbf{e}} = E(\mathbf{e},\mathbf{p}) \;\Rightarrow\; e = E(p) \;\Rightarrow\; e(t) = h \cdot \exp \frac{(p(t)-c)^2}{2w^2} \tag{22}$$

  that is, a static environment ($\dot{\mathbf{e}} = 0$) without recurrent behaviour; it only depends on the pose of the agent, $p(t)$.
- The sensor measures the proximity of the Gaussian shape to the agent's position,

$$\dot{\mathbf{z}} = Z(\mathbf{e},\mathbf{a}) \;\Rightarrow\; z = Z(e) \;\Rightarrow\; z(t) = -\frac{1}{d_{\max}} \cdot e(t) \tag{23}$$

  again defined in a static form. The approximation to the time derivative of the sensor signal, $\dot{z}(t) \approx \Delta z(t)$, will serve as input to the agent.
- The agent represents to control the whole system using a small (2-node) continuous-time recurrent neural network (CTRNN). Each node in this network is governed by the equation

$$\tau_i \dot{y}_i(t) = -y_i(t) + \sum_{j=1}^{n} w_{ji}\sigma(y_j(t)+\theta_j) \tag{24}$$

  where $y_i(t)$ is the activation node i, τ_i its time constant, w_{ji} the associated weights, θ_j a bias term, and $\sigma(\cdot)$ the logistic activation function. One of the nodes receives $\Delta z(t)$ as an additional term in the equation,

$$\tau_1 \dot{a}_1(t) = -a_1(t) + \sum_{j=1}^{2} w_{j1}\sigma(y_j(t)+\theta_j) + \Delta z(t) \tag{25}$$

  and the other node delivers continuous commands,

$$\tau_2 \dot{a}_2(t) = -a_2(t) + \sum_{j=1}^{2} w_{j2}\sigma(y_j(t)+\theta_j) \;\;;\;\; m(t) = a_2(t) \tag{26}$$

  Hence, the agent's internal state $\mathbf{a}$ corresponds to the vector representing the activity of the neural network's hidden node and motor neuron,

$$\dot{\mathbf{a}} = A(\mathbf{a},\mathbf{z}) \;\Rightarrow\; \dot{\mathbf{a}} = A(\mathbf{a},\Delta z) \tag{27}$$

$$\dot{\mathbf{m}} = M(\mathbf{a}) \;\Rightarrow\; m(t) = a_2(t) \tag{28}$$

- Finally, the motor command $m(t)$ is remapped to the range $\bar{m}(t) \in [-1, 1]$ and specifies directly the velocity of the agent,

$$\dot{\mathbf{p}} = P(\mathbf{m}, \mathbf{e}) \;\Rightarrow\; \dot{p} = P(\bar{m}) \;\Rightarrow\; p(t_k) = p(t_{k-1}) + \bar{m}(t_k) \cdot \Delta t \qquad (29)$$

As it can be checked, the example in the original approach is merging continuous-time with discrete-time equations, so discrete-time equations are used everywhere when simulating for the problem. Our approach will be constrained to the original form of the proposed equations and, initially, will consider only linear dynamical systems.

5.2. Our LTI SISO Approach

Since it is unable the use nonlinear dynamical systems from our introduced starting study, we will approach the toy example with simpler assumptions with respect to the original one. Hence, again, the agent is moving along a one-dimensional environment. However, it has an easier task, to move to the center of the environment. The agent can sense the distance to this center via a single distance sensor.

The set of equations defining the system are proposed as follows,

- The environment variable $e(t)$ is the same that the robot pose $p(t)$, the robot position (let us name it $x(t)$) and it is static ($\dot{\mathbf{e}} = 0$), that is,

$$\dot{\mathbf{e}} = E(\mathbf{e}, \mathbf{p}) \;\Rightarrow\; e = E(p) \;\Rightarrow\; e(t) = p(t) = x(t) \qquad (30)$$

In general, for the linear dynamical system case, since $\dot{\mathbf{e}} = 0$, it could be written,

$$\dot{\mathbf{e}} = \mathbf{A}_E \cdot \mathbf{e} + \mathbf{B}_E \cdot \mathbf{p} \;\Rightarrow\; e(t) = -\frac{b_E}{a_E} \cdot p(t) = x(t) \qquad (31)$$

with $b_E = -a_E$. Similarly to the original approach to the toy example, we are not expressing this equation in a dynamical form.

- The sensor measures the proximity of the robot position to the center. Now, we will consider the dynamic definition for this equation,

$$\dot{\mathbf{z}} = \mathbf{A}_Z \cdot \mathbf{0} + \begin{bmatrix}\mathbf{B}_{1Z}\mathbf{B}_{2Z}\end{bmatrix} \cdot \begin{bmatrix}\mathbf{e} \\ \mathbf{a}\end{bmatrix} \;\Rightarrow\; \dot{z}(t) = b_{1Z} \cdot e(t) + b_{2Z} \cdot a(t) \qquad (32)$$

but, like in the original approach, we will obviate the dependence on the agent's state $a(t)$, that is,

$$\dot{z}(t) = b_{1Z} \cdot e(t) = b_{1Z} \cdot p(t) = b_{1Z} \cdot x(t) \qquad (33)$$

which leads to the solution,

$$z(t) = \frac{b_{1Z}}{2} \cdot x^2(t) \qquad (34)$$

considering the constant term from the integral, $C = 0$. The sensor measure can be considered as a 2-norm distance to the center.

- The agent variables represent to control the whole system. In the original approach, a CTRNN is considered. In our case, we will start with an easier linear nonrecurrent dynamical law for the controller,

$$\dot{\mathbf{a}} = \mathbf{A}_A \cdot \mathbf{a} + \mathbf{B}_A \cdot \mathbf{z} \;\Rightarrow\; \dot{a}(t) = b_A \cdot z(t) \tag{35}$$

leading to the solution,

$$a(t) = \frac{b_{1Z} \cdot b_A}{6} \cdot x^3(t) \tag{36}$$

considering the constant term from the integral, $C = 0$. It can be observed that $b_{1Z} \cdot b_A < 0$ is not ensuring an stable control law.

- The motor command $m(t)$, following the same reasoning, can be defined

$$\dot{\mathbf{m}} = \mathbf{A}_M \cdot \mathbf{0} + \mathbf{B}_M \cdot a(t) \;\Rightarrow\; \dot{m}(t) = b_M \cdot a(t) \tag{37}$$

which corresponds to

$$m(t) = \frac{b_{1Z} \cdot b_A \cdot b_M}{24} \cdot x^4(t) \tag{38}$$

considering the constant term from the integral, $C = 0$. The motor command holds the same sign for positions on the left and on the right of the center. Hence, it gives an idea about power in the movement, but not about its direction.

- Finally, following the same simple reasoning about how variables are defined,

$$\dot{\mathbf{p}} = \mathbf{A}_P \cdot \mathbf{0} + \begin{bmatrix}\mathbf{B}_{1P}\mathbf{B}_{2P}\end{bmatrix} \cdot \begin{bmatrix}\mathbf{m} \\ \mathbf{e}\end{bmatrix} \;\Rightarrow\; \dot{p}(t) = b_{1P} \cdot m(t) + b_{2P} \cdot e(t) \tag{39}$$

but, like in the original approach, we will obviate the dependence on the environment, that is,

$$\dot{p}(t) = b_{1P} \cdot m(t) \tag{40}$$

which leads to the solution,

$$p(t) = \frac{b_{1Z} \cdot b_A \cdot b_M \cdot b_{1P}}{120} \cdot x^5(t) \tag{41}$$

considering the constant term from the integral, $C = 0$.

In matrix form, our system can be written as,

$$\begin{bmatrix}\dot{z} \\ \dot{a} \\ \dot{m} \\ \dot{p}\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & b_{1Z} \\ b_A & 0 & 0 & 0 \\ 0 & b_M & 0 & 0 \\ 0 & 0 & b_{1P} & 0\end{bmatrix} \cdot \begin{bmatrix}z \\ a \\ m \\ p\end{bmatrix} \quad ; \quad e = \begin{bmatrix}0\,0\,0\,1\end{bmatrix} \cdot \begin{bmatrix}z \\ a \\ m \\ p\end{bmatrix} \tag{42}$$

6. Conclusions

The notions of SMCs are firstly introduced according to the psychological proposal in [10]. As dynamical systems, we analyze SMCs, for the very first time, from a modern control engineering perspective. We provide equations and block diagrams translating the psychological proposal to control engineering for the easiest case, linear time-invariant single input single output (LTI SISO) systems. Then, we analyze the original toy example proposed from the psychological domain into the modern control engineering point of view, as well as we propose a first example coming from the control engineering domain.

Our study determines strengths and some weakness in the original notions of SMCs, related with some simplifications for the sensorimotor environment definition. Future research lines are pointed out. The provided original toy example is easily translated to the control engineering domain. As only the LTI SISO is considered in this work, many points of extension and improvement can be considered in the near future.

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