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HUMANISTIC MATHEMATICS NETWORK

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HARVEY
MUDD
COLLEGE
CLAREMONT,
CALIFORNIA
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Newsletter #1

August 3, 1987

DEPARTMENT
OF
MATHEMATICS

714-621-8023

Dear Colleague,

This newsletter follows a three-day **Conference to Examine Mathematics as a Humanistic Discipline** in Claremont 1986 supported by The Exxon Education Foundation, and a special session at the AMS-MAA meeting in San Antonio January 1987. A common response of the thirty-six mathematicians at the conference was, "I was startled to see so many who shared my feelings".

Two related themes that emerged from the conference were 1) teaching mathematics humanistically, and 2) teaching humanistic mathematics. The first theme sought to place the student more centrally in the position of inquirer than is generally the case, while at the same time acknowledging the emotional climate of the activity of learning mathematics. What students could learn from each other, and how they might better come to understand mathematics as a meaningful rather than an arbitrary discipline were among the idea of the first theme.

The second theme was focused less upon the nature of the teaching and learning environment and more upon the need to reconstruct the curriculum and the discipline of mathematics itself. The reconstruction would relate mathematical discoveries to personal courage, relate discovery to verification, mathematics to science, truth to utility, and in general, to relate mathematics to the culture in which it is embedded.

Humanistic dimensions of mathematics discussed at the conference included:

a) An appreciation of the role of intuition, not only in understanding, but in creating concepts that appear in their finished versions to be "merely technical".
b) An appreciation for the human dimensions that motivate discovery- competition, co-operation, the urge for holistic pictures.
c) An understanding of the value judgements implied in the growth of any discipline. Logic alone never completely accounts for *what* is investigated, *how* it is investigated, and *why* it is investigated.
d) There is a need for new teaching, learning formats that will help wean our students from a view of knowledge as certain, to-be-received.
e) The opportunity for students to think like a mathematician, including a chance to work on tasks of low definition, to generate new problems and to participate in controversy over mathematical issues.
f) Opportunities for faculty to do research on issues relating to teaching, and to be respected for that area of research.

This newsletter, also supported by Exxon, is part of an effort to fulfill the hopes of the participants. Others who have heard about the conferences have enthusiastically joined the effort. The newsletter will help create a network of mathematicians and others who are interested in sharing their ideas and experiences related to the conference themes. The network will be a community of support extending over many campuses that will end the isolation that individuals may feel. There are lots of good ideas, lots of experimentation, and lots of frustration because of isolation and lack of support. In addition to informally sharing bibliographic references, syllabi, accounts of successes and failures, . . ., the network might formally support writing, team-teaching, exchanges, conferences, . . .

Please send references, essays, half-baked ideas, proposals, suggestions, and whatever you think appropriate for this quarterly newsletter. Also send names of colleagues who should be added to the mailing list. All mail should be added to the mailing list. All mail should be addressed to

Alvin White
Department of Mathematics
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Claremont, CA 91711

This issue contains some papers and excerpts of papers that were presented at the conferences.

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Newsletter #2

DEPARTMENT
OF
MATHEMATICS
714-621-8023

March 1988

Dear Colleague;

Much has happened since the first issue. Even though the concepts of Mathematics as a Humanistic Discipline and Humanistic Mathematics Network are not well-defined, many have felt that the ideas have great potential, and have sought to be part of the Network by asking for and by contributing to the Newsletter. The number and diversity of responses and especially the enthusiasm have been very encouraging. Mathematicians seem to have an intuitive sense of the naturalness of the Network and begin to create for themselves and their colleagues a sharper definition or a set of examples and illustrations of humanistic aspects of mathematics.

A proposal to the Fund for the Improvement of Postsecondary Education (FIPSE) and the NSF to strengthen and sustain the network and its activities is being reviewed. Many mathematicians have contributed good ideas to the proposal. Among the central ideas of the proposal is the support of those who want to teach in non traditional ways and those who want to do non traditional research. Network support is needed to prevent professional isolation of such faculty.

The Southern California section meeting of the MAA November 12, 1988 will include a special session of contributed papers on Humanistic Mathematics. The national meeting of the MAA in January 1989 will probably have a similar session of contributed papers.

Jack Gray is organizing a special session in section A5.4 of ICME-6 on "revitalizing undergraduate mathematics." Three topics are (1) revitalizing the undergraduate curriculum/materials, particularly in calculus; (ii) alternative teaching/lecturing methods; (iii) rejuvenating faculty involved in undergraduate teaching. He encourages all interested colleagues to contact him and to come to ICME-6. His address is

Jack Gray
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The University of New South Wales
P.O. Box Box 1
Kensington, New South Wales
Australia 2033

If the FIPSE-NSF project is funded, your suggestions about conferences, workshops mini-courses, etc. are welcome. National and regional committees will be set up to coordinate network activities, including electronic bulletin boards.

Contributions to Newsletter #3 are invited.

Sincerely,

Al White

Alvin White

Please Carry Your Coals To Where They are Needed, Professor Stein

I read the well-written article *Gresham's law: algorithm drives out thought* by Sherman Stein in the first issue of the *Newsletter*. It documents once more that rote memorization of algorithms does not teach our students how to think, communicate, or solve problems. In it some authorities are quoted who rail against courses in mathematics that demand little more than that students reproduce procedures that differ only trivially from ones the student has seen before many times. He exhorts teachers of mathematics to change their ways and gives some constructive suggestions on how to prevent the dominance of algorithms from driving out thought. No reasonable mathematician could disagree with Professor Stein, so why does this problem persist?

I have been teaching college-level mathematics since 1948 in publically supported, as well as private institutions and have yet to meet any college teacher of mathematics who likes to teach by rote or who thinks this is good pedagogy. As we all know, most of our teaching efforts go into "service" courses that are taken by students who enroll in them because they are required or advised to do so by nonmathematicians. They are told to take these courses to learn how to solve certain kinds of problems that arise in the subjects in which they are really interested. They are told that they need to know how to do specific things such as solve annuity and compound interest problems, linear programming problems, calculus problems, solve linear partial differential equations with the aid of Fourier series, or calculate correlation coefficients. An assignment to teach a "service" course is usually accompanied by a densely packed course outline which allows little or not time to teach why the algorithms to which the students are being exposed work. If one persists in trying to teach "why" in addition to "how", most of the students resent it and feel you are adding an unnecessary burden for them to bear, and if they complain vigorously enough to their advisors, you may hear from your department chairman or dean that your students feel you are spending most of your time on "theory" instead of teaching them what they need to know. The faculty from department X are dissatisfied, for, after all, most of the students that come back to them from such "service" courses can't cope with the mathematical problems that arise in X-ology, so you must be spending your time teaching them irrelevancies; worse yet "pure" mathematics. Whatever the reason, I learned early in my teaching career to be leery of burdening the students in "service" courses with too many "whys" and that failing to "finish" a course outline was much more likely to promote dissatisfaction than turning out a class of students almost none of whom had a real understanding of the subject matter. In short, I learned how to "process" students so I could survive and spend my time on better things such as my own research or teaching students who were not hostile to mathematics.

Professor Stein is addressing the wrong audience. Our natural instincts and desires as college teachers of mathematics is to teach for understanding to help our students to solve problems in their areas of interest besides the illustrative examples in the text. It is harsh reality that forces us to accept impossibly crowded course outlines and anti-intellectualism.

Indeed reality gets harsher all the time as increases in tuition and fees make college administrators more and more nervous to the point that they know less and less that there is any possible difference between keeping students happy and having them learn the nonsuperficial. Any non-tenured faculty member who follows Professor Stein's advice may be playing Russian roulette with his career. As for the speeches by college presidents and other leaders of society that learning computational skills without understanding is of little value, quoting them never seems to help a mathematics department under attack for rebelling against teaching algorithms by rote. If we won't do it, the department of X-ology will drop the mathematics requirement and we will lose positions. This is not just a rationalization; I can cite chapter and verse on how many a mathematics department got reduced in size in this way. These lofty speeches on the importance of understanding remind me of the passages in William Whyte's *Organization Man* that described how many a president of an influential company would address college students urging them to get as broad and general an education as possible, while his own personnel department refused to interview any job candidate who didn't have a laundry list of highly specialized skills.

In summary, Professor Stein is carrying coals to Newcastle even as the bottom is falling out of the coal market. It is the "consumers" of mathematics he has to convince, not the teachers; a monumental task indeed.

Yet the situations is not hopeless. First of all, despite the odds and the difficulties, there always seem to be a small number of individuals (e.g., Professor Stein) who persist in trying to teach students registered in "service" courses what the need to know as opposed to what they want to know when they enter the classroom. Even though the best selling texts are those whose size approximate that of big city telephone directories, there are still a few that encourage their readers to think. The torch is kept burning even if only a few students benefit from its light and the teachers who keep it lit at the expense of an extraordinary expenditure of time get the usual reward for virtue. One experience I had many years ago may give a way to make a dent in the problem on a larger scale.

In the early 1960's, the mathematics department at Purdue University was moved for a couple of years into the School of Engineering. Its Dean, the late George Hawkins had grown weary of the bickering between the mathematics department and the various departments of engineering over the contents of mathematics service courses. He appointed a committee consisting of three mathematicians and a lot of mathematically knowledgeable engineers with the charge to decide what kinds of mathematics should be taught by mathematicians and what should be made a part of various engineering courses. It met weekly for an academic year, its proceedings did not always go smoothly, and we never really settled the problem posed in our charge. Yet serving on that committee taught all of us invaluable lessons. Engineers do want their students to understand mathematics, but they don't see how $\in$-δ proof techniques help and they know little about the real problems of mathematical pedagogy. I still remember a chemical engineer who made sophisticated use of partial differential equations in his research, but didn't know why we had to spend time teaching solid analytic geometry to students before teaching them how to evaluate multiple integrals. He accepted our explanation readily and my initial shock at such ignorance was replaced by a realization of my own naivete at assuming that professors that had never taught any mathematics would have any idea of the problems of mathematical pedagogy. I also learned that when a professor of engineering says, our student need to "know" how to solve linear differential equations of the second order, they mean something rather different from what a mathematician would mean by that assertion, and the differences are not easy to describe. What did emerge from our deliberations was an understanding of both the similarities and the differences in educational goals between the two groups, an increase in mutual respect, and direct communication between individuals instead of formal communications from faculty members in department X to his chairman to his dean to another dean to the chairman of the mathematics department to a faculty member in the mathematics department. I left Purdue a few years later, and the "era of good feeling" there lasted for 7 or 8 years. But personnel changes and a failure to renew the old efforts eroded away the good will that had made communication possible, and the old hostilities resumed.

I think it will take this kind of effort between departments to even begin to solve this problem, which, of course goes back to the students first introduction to mathematics in grade school. The impetus will have to come from the top; unless college administrators prove by their actions that they want mathematics courses for the bulk of the students to be more than a series of memorized rules. They will have to reward those who are willing to spend time on these problems and stop avoiding their real responsibility by pretending to a false neutrality in departmental disputes. I hope also, that "humanistic" mathematicians will spend less time exhorting the mathematical community and more time talking to people who might be able to do something about the problem. Remember also, that it is possible to overemphasize the "why" over the "how" as was pointed out years ago by Alfred North Whitehead when he said *It is a profoundly erroneous truism, repeated by all copy books and by eminent people when they are making speeches, that we should think about what we are doing. The precise opposite is the case. Civilization advances by extending the number of important operations we can perform without thinking about them.*

Melvin Henriksen
Mathematics Department
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Sherman Stein replies

I'LL CARRY MY COALS WHERE THEY'RE NEEDED, PROFESSOR HENRIKSEN

Yes, as Prof. Henriksen reminds us, there are external boundary conditions on what we can do. But I would not put the whole blame on those departments for which we offer service courses.

Even with service courses there may be more leeway than we imagine. The engineering faculty I've spoken with do want their students to understand what the derivative, definite integral, and Stokes' theorem mean, for these concepts are used not simply to aid computations but as words in a language. That suggests that we can and should assign conceptual exercises, even if they involve writing. However, it does not mean that we can turn calculus into a jungle of epsilons and deltas.

Perhaps the constraints that Prof. Henriksen refers to are most constricting in our freshman service courses, but even there we may assign a small project that requires exploration, thought, and writing. We should then be prepared to read the results and criticize them in detail, even reading revisions. The main constraint here is one of time--our own.

I suspect that the style in which we conduct our service courses may insidiously corrupt the way we teach even the courses for our majors: we tend to think mainly in terms of topics rather than in terms of changes we want to occur in the student's way of dealing with problems. For instance, in an upper-division algebra course we may state and prove that a subgroup of a cyclic group is cyclic instead of giving the students the time and opportunity to find on their own "all the subgroups of cyclic groups."

I confess that I have a tendency to teach as I was taught, to lose sight of the main goal in the hurly burly of lectures, office hours, committee meetings, and exams. For me, certainly, and maybe for others, some of the constraints are subliminal and internal. If so, we should every so often stop and think about what we are doing.

As an ancient Chinese sage, a master of the two-fold way, once observed, "Civilization advances in two ways: by extending the number of important operations that can be performed without thinking about them, and also by thinking about important operations we perform too often without thinking about them."

Sherman Stein

Allan Muir, The City University, London.

Foundational Studies and Mathematics Teaching

Due to sudden increase in governmental direction of education at all levels, universities in Britain are currently trying to respond to demands for a greater variety of teaching committments than is traditionally encountered. We expect to deal in the near future with a wider catchment area for our student intake and with demands for greater flexibility in form and content of courses. So far as mathematics is concerned, we need to encourage greater inter-relation with other departments and to respond to new applications of mathematics, arising from modern technologies and activities. - eg. computer science, actuarial science etc.

It remains, I believe, the main task of higher education to make links between advanced study, which I take to embrace research, and teaching. Part of this project is critical evaluation, which is, unhappily, almost totally absent from the concept of mathematics held by the majority both of its creators and its users. Critical evaluation depends upon a firm bedrock of foundational principles; I prefer to think of these as relatively, rather than absolutely, constant, since I cannot conceive of any real thing which is totally static. In the climate of change we are facing, to secure a critical position against the vagaries of fashion, we must dig deeper into our subject's foundations.

There are, thus, direct links between foundational studies and the practise of mathematics teaching. These, while present within our conventional presentations, have been rendered invisible by familiarity; our material abounds in ossified attitudes derived from old disputes - axiomatics from formalism, set-theoretic foundations from logicism, insistence on rigor and the demise of geometry from attempts to construct secure foundations.

The current inadequacy of established teaching styles and contents to satisfy contemporary users of mathematics, shows clearly that even the deepest foundational discussions can acheive, at best, only temporary stasis. But now, the role of foundational studies in permitting critique of un-reflective practise takes on a fresh urgency. There are more false ideas about mathematics, both within and without that community, than in any other subject. Without too much exaggeration, we may refer to these ideas as myths. Some of the more potent of these are:-

(i) The myth of individualism: maths is the product of individual minds and the history of mathematics is the story of those individuals.

(ii) The myth of elitism: maths is the product and ability of a special, even rare, kind of mind; mathematicians walk a lonely track.

(iii) The racial myth: maths, at least since the renaissance, is a history of white, western discovery, with the inevitable implication that prior mathematical history was a tale of error or inadequacy.

(iv) The sexist myth: women have contributed nothing to mathematics.

(v) The mechanistic myth: maths is cast-iron and irrevocable in its methods of proof.

(vi) The absolutist myth: maths deals with truths which stand secure above the flux of real things.

Of course, deliberately self-conscious mathematicians will not avow these views explicitly, but students of mathematics, if questioned, exhibit precisely these views, derived unconsciously from the material they are taught. The above myths need to be countered by new paradigms of mathematics which emphasise its human-ness, its normality, its rootedness in real, social practise. Something akin to an anthropology of mathematics - an examination of psychological and social behaviour of mathematicians - should precede, and be the true basis for, a new philosophical statement about foundations.

It is quite remarkable that less than a decade ago it would be almost impossible to find much discussion of the nature of mathematics which began from its status as a <u>human</u> activity. Abstract discussions of truth assignment, formalised symbol manipulation, platonic ontologies abounded. The only major exception - the work of Piaget - came from outside the field. Now things are freeing up; it is intriguing that articles on social paradigms of mathematical truth and social histories of mathematical discoveries are emerging at the same time as renewed efforts to popularise the practise of mathematics.

The impact of computers has been profound, through the revelation of impoverishment in much algorithmic manipulation which we teach as mathematics. More benignly, they have offered a tool which restores geometric insight, stimulates the study of discrete systems and algorithms and presents afresh an impetus to the constructive view of mathematics. The relation of these impacts to the task of "de-mythifying" needs study.

MATHEMATICS AND PHILOSOPHY*

D. Bushaw

It is the first premise of this discussion that mathematics is indeed a humanistic discipline. We should therefore be interested in the relations and possible interactions of mathematics with other humanistic disciplines. Many people are aware of some of the relations and interactions of history with literature, or of literature with philology; so why should they not be aware of some of the relations and interactions of mathematics with, say, philosophy, which is possibly the humanistic discipline <u>par excellence</u>?

Long ago I heard or read someone's quip that René Descartes was regarded as a great mathematician by philosophers and as a great philosopher by mathematicians. The quip is fair neither to philosophy, to mathematics, nor to Descartes; but it does remind us of the somewhat ambiguous historical relationship between mathematics and philosophy.

Without making a systematic search, I easily come up with the names of quite a few people who figure, in at least a minor way, in histories of both mathematics and philosophy. They include Pythagoras, Boethius, Nicolas of Cusa, Pascal, Leibniz, Wronski, Bolzano, C. S. Peirce, Bertrand Russell. The list could be much extended.

There have been other philosophers who, without exactly being mathematicians, knew a great deal of mathematics. Kant and Husserl were examples. (Husserl, the leading phenomenologist, was a student of Weierstrass.)

At the same time, many philosophers who never claimed any intimate technical knowledge of mathematics nevertheless did have penetrating things to say about our discipline. In this group my own favorite is George Santayana.

On the other hand, many mathematicians who would never claim to be philosophers have nevertheless been interested in philosophy, if only in the way that every person with a civilized outlook should have some interest in philosophy.

These affinities should not be surprising. Without committing

*Contribution to a panel discussion "Mathematics as a Humanistic Discipline," organized by Alvin White, at the Joint Mathematical Meetings, San Antonio, 1987.01.23.

ourselves to a definition of either mathematics or philosophy, we can probably agree that each can be described as an aggregate of systems of abstract ideas that are often very attractive in themselves, but also can provide wonderful frameworks within which to organise and analyze other aspects of human experience. Are there any other disciplines that can be so characterized?

Indeed, there have been those, like Russell and other logicists, who have claimed that mathematics is a part of philosophy--more precisely, of that part of philosophy called logic. On the other hand, there have been those, like Descartes and--in the twentieth century--Heinrich Scholz, who have argued that philosophy and a great many other subjects should aspire to the condition of mathematics and thereby, perhaps, become part of applied mathematics.

What do our students know of all this? Being our students, they study mathematics; some of them, but certainly not so many as we might like, study philosophy too; but it must be a rare undergraduate indeed who even begins to appreciate past, present, and latent relationships between the two subjects.

I would like to suggest again that it would be a very good thing to create opportunities for easy interaction between mathematical and philosophical ideas and ways of thought in the minds of undergraduates--and of their instructors.

I am not speaking here only of the philosophy of mathematics, especially when understood as the study of the foundations of mathematics, nor of those branches of philosophy, like "mathematical logic," that sometimes have the word "mathematical" in their names. I have in mind mathematics in general and philosophy in general.

Here are a few examples of questions that could be considered: What are some of the historical connections between mathematics and philosophy--in particular, how did some of the great mathematician-philosphers regard the relationship between the two subjects? What should we make of Pascal's rejection of mathematics--until he returned to it to take his mind off a toothache--as a dangerous distraction from the contemplative life? What are we to make of Wronski's use of his mathematics as a "guarantee" of his philosophy? What is the meaning of "existence" in mathematics?

A mathematician can hardly read the literature of the modern philosophical movement called "structuralism" without sensing its inherently mathematical character. How might this character be made more explicit? How, if at all, might lattice theory be applied to axiology, the theory of values? What is the relation, if any, between simplicity and validity in mathematical models?

Clearly, inquiries like these could be concerned primarily with study of existing sources; but they might also involve much original--or locally original, so to speak--speculation. They could be taken up in courses, or in tutorial arrangements of some kind, or just in independent study projects. Capstone courses in general education programs might offer ideal opportunities. Visitors between colleges, or at least between departments, might be enlisted. Team-teaching might be in order. Papers might be written, even for publication. I do not want to make my suggestions too concrete. It seems to me that we should try all sorts of things, then gather and disseminate information about successes and failures.

(Some parts of philosophy may be distant from mathematics, and some parts of mathematics may be remote from philosophy. But the traditional boundaries between philosophy and mathematics (not to mention other disciplines) are after all somewhat arbitrary, and may be of little significance except for purposes of academic convenience. If all went well, a major outcome of experiments of the kinds I have been suggesting could be the emergence of a group of people who would have learned that it is possible to ignore those boundaries, or at least to leap lightly back and forth over them. This in turn might have all sorts of marvelous consequences.)

Joint Math Meetings San Antonio, January, 1987.

Excerpts from and inserts into my January 23 talk at the Mathematics as a Humanistic Discipline session

Anneli Lax

When Alvin White invited me to join this panel, I felt very much as if I were joining a conversation whose beginning I had missed, and that I therefore was in danger of bringing in observations that had already been discussed, followed up, resolved, transformed.

On the other hand, I don't like those panel discussions in which each panelist presents his Weltanschau ng as if the other panelists didn't exist, and where the audience, directed to ask questions, has the job of making some coherent picture out of this performance, but where there is never enough time for discussion because the panelists feel they need every allotted minute to get their philosophies across.

Nor am I comfortable in a situation where I am expected to respond immediately to the remarks of preceding panelists, because I am a slow listener (and reader) in the sense that I need to mull over what I have just heard (or read), and my nervousness at having to address many people interferes with my attention to what is being said. My responses will not be ready for public consumption when my turn comes.

Alvin had asked me for a title, and I tentatively said Our Fragmented Curriculum. But I felt I needed a little more guidance, and so I wrote him the following letter:

Dear Profession White:

Just a note to let you know roughly what I plan to say at the Mathematics as a humanistic discipline session on Friday morning, January 23.

The title, our Fragmented Curriculum, is perhaps too specialized; for I want to talk about putting together fragments not only of the curriculum, but of many other things:

1. Human learning involves both, the learner's intellect and his emotional attitudes. Mathematics instruction should not be divided into therapy for the "math-anxious" and instruction in mathematical concepts and techniques; instead the human student should initiate and develop his subjective mathematical inquiry using his psychological traits as well as his personal mental powers. He or she must get acquainted with both.

2. Humans retain what they learn best if they can put new material into a human context, e.g. connect it to past experience, future aspiration, previously acquired mental structures.

3. Mathematical reasoning is a natural part of general reasoning. Yet it gets pushed out of other school subjects (like social science, literature, even the natural sciences) into mathematics classes, which then become cluttered with bits and pieces of tools whose use is not made clear. The early elimination of reasoning from the humanities, separation of reading from writing, listening from speaking, are instances of harmful fragmentation of human activities.

4. The isolation of mathematics from the rest of life causes further fragmentation of mathematics itself into arithmetic, algebra, geometry, etc., and these subdisciplines get cut up into sections or modules or skills to be mastered, tested and forgotten.

It is not surprising that adults, schooled in this manner, remain dependent on "experts" (accountants, lawyers, ...) in their private and community life, and that they see little relation between their jobs in the work place and the general purpose of the enterprise which employs them. Unfortunately, our education enterprise, at all levels, is no exception.

These things probably need not be said to the people who are meeting in San Antonio. Indeed, members of this group are doing great things to counteract the robotization of students. There are, moreover, lots of excellent recommendations for educational reforms made by various prestigious committees composed of knowledgeable dedicated people. What needs to be discussed are perhaps the difficulties in implementing good ideas.

I could describe my recent experience in urban public schools, the unreasonable pressures on teachers, the contradictory messages received by teaching and administrative staffs and the social problems compounding the difficulties. But I can also describe a few instances where teachers have successfully involved students in meaningful responsible learning, through innovative use of writing, reading, respectful listening and thoughtful guiding of student explorations.

I would observe that we, who teach, are also victims of this fragmentation; we are experts in this or that, we care about our own learning and that of our students. Few of us are renaissance people or generalists. But we have observed or experienced how people in different fields with different personalities occasionally supplement one another's strengths and interests, become intrigued with one another's work and begin a stimulating and fruitful collaboration which leads to promising educational programs. I don't know any recipes that lead to the building of such communities of collaborators and I don't believe such activities can be mandated. But I do believe they are most likely to happen in the same kind of learning environment as the one we strive to create for our students. We need to find out how such collaborations get started and what makes them thrive. I don't have much faith in replicating any particular "program" or "project", but I do have great hopes for replicating the spirit that often inspires and initiates it.

I should appreciate your opinion on what needs emphasis in San Antonio.

Best regards,

Anneli

Anneli Lax

Alvin responded. He expressed the hope that this San Antonio meeting would encourage that community and collaboration by making more people aware of the widespread interest and support for the concept of Mathematics as a Humanistic Discipline.

He also suggested that I illustrate the points in my letter with some specific examples from my recent experiences in some New York City public schools. I am sorry I did not manage to get these into my talk.

I was gratified to see, in the program of these San Antonio meetings, an extraordinarily large number of sessions devoted to educational concerns, with many thoughtful, concerned and experienced participants; in fact so many that it was difficult to attend any one of these sessions without missing some of the others. In spite of the small sample that I heard, let me use it as supplement to some other evidence I have come across in my reading to make a strong point for the pedagogical aspects of Mathematics as a Humanistic Discipline.

There are descriptions of programs - often targeted at special student populations - e.g. women, minorities, talented, learning disabled - which record with great insight how students have been turned off, and how the interests of many of them can be and have been engaged and maintained. In these readings, I have been particularly impressed and stimulated by specific anecdotal and nuts-and-bolts-y accounts. And in almost all cases, my reaction was that these little success stories were based on a spirit which was quite independent of the particular sub-population of students at which the effort was aimed. So I tried to formulate what that common underlying spirit was and came to the conclusion that the authors of all these vignettes had gone to considerable lengths to find out who the students in these programs were, where they came from what went on in their heads and their hearts when they worked on math and how well and by what means they could cope with problems in their out-of-school life. At Tuesday's MAA/NCTM panel, for example, there was a presentation by Carol Greenes which debunked the claim that learning disabled students could learn no mathematics except computation. She gave evidence that, on the contrary, these students had considerable mathematical skills in attacking real world problems from various fields, that computation per se was not their forte, but that they were able to maintain their computational skills, since they were using them in a meaningful context.

The point of this and other efforts referred to is that we need to listen to students. This is one of the first steps in getting them to become responsible for their own learning.

Now my experienced, seasoned colleagues who teach elementary mathematics courses know exactly which errors their students will probably make, are unhappy about not being able to prevent these standard erros and cheer one another up by joking about this predicament. Many have experienced utter shock when they asked, for example calculus students at the end of one semester, to tell (or to write) what a derivative is. The results were so painful that most will never ask again.

But some have discovered, by listening to students, that interesting things are in fact going on in students' heads. Students are thinking, seeing patterns and analogies, but not necessarily the ones the teacher had in mind. Moreover, these instructors find it much more stimulating to learn how various students explore a mathematical idea than they find lecturing, year after year, about that same mathematical idea or technique in vacuo, with the predictable result of being ineffective.

Learning to listen to students involves teaching students to express themselves so that their classmates and teachers can understand. It has been our experience that when students see that somebody is interested in what they have to say, they make a serious effort to communicate their ideas, and this alone is important training in the use of language skills.

I am convinced that the use of language - reading, writing, listening and speaking - is an essential part of learning anything, and especially mathematics. I am also convinced that having students talk and write about how and what they are learning naturally makes them connect new material with past experience and future goals, so that what they learn becomes part of them. Furthermore, this practice would give schools the opportunity to take advantage of the enormous heterogeneity of the students, especially in large cities. At present the cultural and linguistic diversity of student populations is regarded as an enormous obstacle because in spite of lip service to respect for the individual, we keep trying to impose crippling uniformity.

I am gratified that some of these humanistic aspects of mathematics instruction are being advocated by an increasing number of commissions and task forces. Although it is probably a bad idea to close on a negative note, let me list a few things that still need to be attended to.

The two attributes of mathematics usually cited by those who want to sell mathematics are: (1) its tremendous utility and almost universal applicability on the one hand, and (2) its playful, puzzle like, fun aspects. I have no quarrel with that. But I think an important third aspect is being neglected, namely the feeling of mental power, or at least mental fitness (a term coined by JoAnne Growney) on the part of those who use mathematical reasoning when appropriate. Just as having physical control and coordination over one's body adds to one's sense of well being and confidence, so does having mental control over one's thought processes. I believe most normal human beings would work quite hard to achieve both. Both help people gain control over their lives.

There is much criticism of our outdated curriculum and lively discussion about

1. How to up-date it for the 20th century (or the year 2061)
2. What to eliminate in order to make room for all the new stuff brought about by recent advances and future needs.

Even if there were an updated agreed upon curriculum, and even if it could be mandated throughout the school system, it would freeze and become obsolete, especially if a corresponding set of multiple choice tests became associated with it.

(I think of curriculum reform as a continuous process which goes hand in hand with keeping in touch with the real world. This means encouraging students to solve real world problems and to give teachers the opportunity to grow in their fields and the autonomy to guide their students.)

I have looked at the 6, 7, 8 grades syllabi in a New York middle school and the New York State mandated 9th, 10th and 11th grade syllabi known as the "integrated math sequence". Indeed, the topics to be covered in any particular year seem overwhelmingly many; but the (overlap is great) The (pressure on teachers to cover material) on which students will be tested virtually prevents them from even considering any realistically paced, in-depth inquiries.

It is difficult and will require long-term collaborative efforts to influence the present school pressures and customs. I am cheered by reports on many grass roots projects at all levels that are making their little dents. I consider today's accounts of my co-panelists important implementations, mainly on the university level, of (stimulating instructional principles which stress mathematics as a humanistic discipline.)

THE BASIS FOR THE SUCCESS OF THE POTSDAM PROGRAM

Rick Luttmann

Based on visit to Potsdam College, 13-15 April 87

Much of what distinguishes the program at Potsdam is not what the department "does" so much as the way it "thinks" — it is a matter of attitude. I believe that these points are the fundamentals of the Potsdam model:

1. The department members adopt the view that — contrary to the prevailing belief in this country, but consonant with that in most other industrialized countries — success in mathematics is due much more to hard work than to innate talent. Many can achieve success in mathematics by persevering — it is not limited to an elite class of geniuses. Faculty must personally accept this view, as well as press it on students in both formal and informal ways, e.g., advising, pamphlets, bulletin boards.

2. The faculty must also be willing to "suspend disbelief" with respect to students whose past records have been undistinguished. There have been many success stories by those whose early records were abysmal, starting with Albert Einstein. Who is to say which of those students who have not yet shown promise are incapable of blossoming later? Don't wait for the "good" students. Again, advising, pamphlets, bulletin boards, can press this point of view on students. Case histories can be assembled to prove this point — preferably alumni of the institution, but others would do also.

3. The secret of getting students to succeed is to keep up morale. Therefore **students must constantly be given things they can do. They should be** challenged, but each challenge should be at the appropriate level. Once the student's confidence is shot, he's lost to the discipline. The teacher who presides over failure excuses himself by saying the students "didn't work hard enough." But they didn't because he didn't inspire them to. Avoid all temptation to "inspire" by threats, abuse, competition, impossible problems, guilt trips, invidious comparisons, anything negative — it won't work.

   Thus there should be nothing called "remediation" and no placement tests. No one is ever "ready." Let everyone feel the pride of trying a high-prestige subject. Throw them in and let them learn to swim. Believe in them, and they will probably do it. They can be given simple problems at first, so they will succeed and gain confidence, and they can be led on to greater and greater levels of achievement with problems of constantly increasing difficulty.

   Every success should be recognized. Every formal and informal method should be employed to see that achievements are publicized and publicly recognized and appreciated.

4. Abandon the traditional lecture format of teaching. It rarely works. In our own experience as students, we didn't learn from listening. We learn by explaining, or otherwise getting actively involved. Students should be learning in the classroom, which means not just listening passively. They should be solving problems then and there. Helping each other — good for both helper and helpee. There should be formal ways for students to help each other — such as a Math Lab.

Everybody knows the professor can do the proof. No one benefits from him rehearsing it, no one needs to see him do it to believe he can. Students benefit from discovering it themselves and explaining it to others. An instructor must learn to "bite his tongue." The "lecturer" never penetrates the student's mind, never shares his confusion. And the student is quickly left in the dust.

Some teachers say "I taught them — but they didn't learn it." Imagine a car salesman telling his boss — "I sold it to them — but they didn't buy it."

Some teachers expect the students to learn how they teach. The teacher should teach the way the students learn. (This isn't always the same. It may never be the same. Good teachers are above all flexible.)

5. The important thing in the math curriculum is not racing through a long syllabus that students are largely not going to absorb anyway and leaving them panting and breathless and overwhelmed and discouraged after the final exam; but learning enough of the subject and learning it well enough to understand the point of it, the philosophy, the general strategy, the essential idea. Emphasis should be on maturity and technique, not merely content. Students will enjoy math when they can say "I understand." They can read and learn on their own after that. They will become life-long learners. The mental skills they learn will transfer to any subject they want to learn. They can even become teachers of others. The math major curriculum is an eight-semester mega-course in independent learning, in thinking, in conceptualizing, in intellectualizing. In later life, this sort of skill will be much more valuable than specific knowledge of specific mathematics. If and when the time comes that they need to know some particular mathematics, they will have the capacity to learn it on their own if they have been properly trained to it. The old proverb "Catch fish for a man and he will eat today; teach him to fish and he will eat all his life" has some application here. At Ats

There can be honors sections for large enrollment courses to challenge those who can learn faster or have stronger backgrounds. But they should be allowed to take standard tests so they are not penalized for trying the honors level.

For courses which are prerequisites to later courses, a certain minimal syllabus should be established and agreed on; but the emphasis should be on minimal. Some flexibility and good will is necessary between instructors.

6. All courses should be oriented toward pure mathematics, and the joy of doing it and understanding it. There is plenty of chance elsewhere and elsewhen to apply what one learns if and when it is necessary — other courses, or later work experience. This even applies to others than math majors. There should be no "service courses." A good course for a math major is a good course for anybody else, and vice versa.

7. Most important of all, an atmosphere must be engendered of total support for the student. The function of the educator is to serve the student, to meet him wherever he is and help him grow, help him achieve his goals, help him prepare to flourish in later life however he defines this. The educator must be deeply committed to this task and must constantly convey to the student his direct personal concern for the student's welfare. There must be a loving, supportive, almost familial atmosphere in the department, a sense of community, of mutual support, everyone helping everyone else, everyone proud of everyone else's achievement. What benefits one benefits all; what one achieves is an achievement for all. There is no place for competitiveness, except to the extent that every faculty and student in the institution are on the same team. (This is very much the way an Eskimo village operates to succeed against the elements.)

Presented to MAA, January 1988, Atlanta

See "A Modern Fairy Tale?" Amer Math Monthly Vol. 94 (1987) pp 291-295.

Two poems by a modern scientist-poet exemplify a play of wit and insight not easily held within disciplinary bounds.

Two Poems

Miroslav Holub
translated by Jarmila and Ian Milner

Brief Thoughts on the Theory of Relativity

Albert Einstein, discussing—
    (knowledge is discovering
    what to say)—discussing
    with Paul Valéry,
    was asked:

Mr. Einstein, what do you do
    with your thoughts? Write them down
    immediately they come to you? Or wait
    till evening? Or morning?

Albert Einstein responded:
    Monsieur Valéry, in our craft
    thoughts are so rare
    that when you have one
    you certainly won't forget it

Even a year after.

A. White (Ed.). *New Directions for Teaching and Learning: Interdisciplinary Teaching,* no. 8.
San Francisco: Jossey-Bass, December 1981

Brief Thoughts on Exactness

Fish
    move exactly there and exactly then,
just as
    birds have their inbuilt exact measure of time and place.

But mankind,
    deprived of instinct, is aided
    by scientific research, the essence of which
    this story shows.

A certain soldier
    had to fire a gun every evening exactly at six.
    He did it like a soldier. When his exactness
    was checked, he stated:

I follow
    an absolutely precise chronometer in the shop window
    of the clockmaker downtown. Every day at seventeen
    forty-five I set my watch by it and
    proceed up the hill where the gun stands ready.
    At seventeen fifty-nine exactly I reach the gun
    and exactly at eighteen hours I fire.

It was found
    that this method of firing was absolutely exact.
    There was only the chronometer to be checked.
    The clockmaker downtown was asked about its exactness.

Oh, said the clockmaker,
    this instrument is one of the most exact. Imagine,
    for years a gun has been fired here at six exactly.
    And every day I look at the chronometer
    and it always shows exactly six.

So much for exactness.
    And the fish move in the waters and the heavens are filled
    with the murmur of wings, while

The chronometers tick and the guns thunder.

Miroslav Holub is chief research immunologist of the Institute for Clinical and Experimental Medicine in Prague. He has published thirteen books of poetry and four books of prose and essays. During spring 1979, Holub was guest writer-in-residence at Oberlin College.

STUDENTS BECOME DATA,
STATISTICS COMES ALIVE.

Every teacher looks for ways to convince students that course material is useful, interesting, capable of being mastered, and, well, relevant. I've found a neat strategy for doing this in statistics, as difficult as that may seem to those who have struggled with the subject.

Early in the introductory course -- during the first day or two, if possible -- I stage a short correlation exercise. I ask for several volunteers who are not overly sensitive about their height, weight or grade-point averages. (Six, nine or 12 are convenient numbers to work with because they simplify calculations, and a mix of women and men serves a useful purpose.)

I ask the volunteers to come forward and to arrange themselves in a line according to height. They do, and other students verify that the arrangement is reasonably accurate. I give each of the volunteers a small blue card on which is a number that indicates the rank in height, number one being the tallest.

Then I ask the volunteers to confer with each other and to rearrange themselves according to weight. They do, and I pass out green cards to indicate rank. I ask the observers whether they think there is any relationship between height and weight.

"Yes, it seems so," they say.

"How strong?" I ask.

"Well, moderate or pretty good," they might say.

I ask, "If someone wanted to grow taller in order to compete better in basketball, should they just eat more and more?"

They quickly point out the fallacy and venture that weight and height are not in a cause-effect relationship. By now, the more astute observers, scanning the group again, might point out that we could call the relationship stronger if there were not a mix of sexes.

Then the volunteers are asked to arrange themselves according to grade-point average. They do (amidst lots of chuckling). The observers might suggest that the relationship between height and GPA is weak or perhaps "sort of turned around a little." We recognize a need to quantify our statements.

That's when I pull out the short-cut formula for Spearman's rank-order correlation coefficient. I ask the volunteers to subtract their green number (rank in weight) from the blue number (rank in height), then square the difference. I add up the results, introducing the concept of summation and the symbol Σ as I do. We use the formula $r = 1 - \frac{6 \Sigma d_i^2}{n(n^2 - 1)}$ (where d_i represents the differences whose squares are to be summed and n is the number of volunteers) to get an index. Perhaps it's about +.60 if the sexes are mixed, perhaps it's about +.75 if the volunteers are of one sex. We examine the formula and note that if there were a "perfect" positive relationship all the differences would be zero, giving $r = +1.00$.

Can we expect the same relationship in the population in general, based on our results? I project a simple table onto a screen and ask students to accept it on faith (a good thing to get used to early in statistics, I find). We note that +.60 might not be statistically significant unless we have more data -- say, over nine volunteers.

We do similar calculations, using height and GPA ranks as shown by the blue and red cards. Often the correlation coefficient is about -.40 and we laugh about an unwarranted conclusion that eating less shortens students and thus improves grades. We speculate what a perfect negative relationship produces and we verify that it would be r = -.1.00.

Thus, in a matter of 15 minutes or so, a number of statistical concepts are introduced and worked with. Students are doing statistics and finding it's not too hard. Never mind that the topic of correlation doesn't really come until the end of the course, never mind that students might not yet know the distinction between mean and median or how to compute standard deviations. They are turned on, learning easily and enjoying it. Best of all, both volunteers and observers are actively involved in the subject from the start.

I have only one problem with the exercise: students want to exp~~p~~erience something like this every day, and it's hard to meet that expectation! But I try.

Linc. Fisch
Lexington, KY
December 28, 1987

A Reply to the Question "Why Math?"

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In [2], R. P. Driver's book Why Math? [1] is reviewed. In that review, the reviewer alleges that he gives "the right answer" to the question "Why Math?" (which he states in full as: "Why should students who are going to be neither mathematicians, scientists, nor engineers study some topics in precalculus mathematics?"). Let us agree to call such students non-technical. Before giving "the right answer", the reviewer disposes of (or, at least, says that he disposes of) the way in which the question has been answered in the past, the way in which it is answered now, and the way in which Professor Driver answers it. This note is a reply to the reviewer's remarks.

The reviewer's "right answer" is: "[The value of mathematics]...is exactly the same as the value of any other game. ...for most students it will never be more than a game. ... Let us present mathematics, as mathematics. Why math? Because."

The reviewer's "right answer" isn't. (The other answers he discusses are all better answers than his--though none of them is the right answer, either.) That we should present mathematics as a game, and as such of intrinsic interest to human beings, can be suggested only from the myopic viewpoint of the specialist. That specialist, although he asserts the utility of mathematics, has no first-hand knowledge of that utility; he is ignorant of the historical and cultural context which affect the development of mathematics and have in turn been affected thereby.

But beyond the specialist's ignorance, the reviewer's "right answer" displays the specialist's arrogance as well. His "right answer" may well be the reason why many colleges and universities no longer impose universal mathematics requirements: For too long, we have arrogantly asserted the value of our discipline for non-technical students, arrogantly accepted their enrollments in our courses, and then arrogantly refused to deliver anything of value--even the value of a game. That we ourselves find the game interesting has no bearing on other people's findings. We should recall that games (of the non-circus variety) are of compelling interest only to players who are gifted with high "limits imposed by talent and application" (the reviewer's phrase).

Scholarship is always a game to the scholar. Our students are not yet scholars, and some--most?--never will be. To them, mathematics is not "only a game". They very clearly do "mind...the unreward of losing since it is, after all" a grade they are thinking about. And it is frequently their advisee's grades that our colleagues in other disciplines are thinking about when they advise their non-technical students and when they weigh potential general education requirements before voting thereon. And recall that the reviewer purports to justify, both to us and to those colleagues, a universal mathematics requirement. But the "right answer" he gives serves just as well to justify universal college requirements in bridge and chess.

If the reviewer's answer is wrong, so is his question. To most professional mathematicians, the phrase "precalculus mathematics" means not just mathematics that requires no background in the calculus, but a specific body of mathematics intended to provide the tools the student will need in the study of calculus. If one takes the question in this sense, and if one does not believe that calculus is appropriate for all undergraduate students, then the answer to the question is probably, at best, "Why, indeed?". Let us ask instead whether non-technical students should be required to study mathematics at some level below that of calculus, but not necessarily directed at preparation for a calculus course. The answer to this question is "Certainly!". And the reviewer's reason is the least of the reasons why.

Consider Driver's book and the approach it represents. I have no quarrel with the reviewer's analysis of this work--which I have not actually seen. If the remarks the reviewer directs at the book are correct, then I can safely say that I have seen many books of its ilk. These books reflect the current standard approach to General Education Mathematics courses.

The standard approach to General Education Mathematics courses is characterized by three conditions. The first of these conditions is that the mathematics must be trivial; if it isn't, then our audience--of whose stupidity we are firmly convinced--won't understand it. The second condition is that the mathematics must be of "intrinsic" interest to mathematicians; if it isn't, we won't be able to get anybody to agree to teach the courses. The third boundary condition is that the mathematics must be "applicable". Now, given the first pair of boundary conditions, it isn't very surprising that the third condition is almost impossible to satisfy. But we don't let that stop us. If we can't find real applications, we just <u>contrive</u> some. The reviewer's criticisms of the problems in Driver's book make this very point: Driver's (<u>i</u>.<u>e</u>., the standard) approach advertises these problems as demonstrating the applicability of the mathematics that he has presented. The point is not, as the reviewer seems to think, that the problems are worthless. It is that the advertising is false.

One thing is certain about the standard approach: We aren't fooling anyone. Except possibly ourselves. We assuredly aren't fooling our students--who aren't anywhere nearly as stupid as we think they are. Nor have the defects of this approach gone unnoticed until now. See Chapter 6 of [4] for a tirade on these matters.

That is not to say that G. Chrystal, M.A., whose nineteenth-century algebra book the reviewer cites approvingly, had the right approach either. Chrystal's approach ("Here it is. Take it or leave it.") was acceptable a century ago for a number of reasons that no longer obtain. (We are back to the matter of historical and cultural context here.) I will mention only that modern pedagogy recognizes, as last century's did not, the futility of an appeal to authority in support of an effort to inculcate the habit of critical thinking.

The reviewer's justification of General Education Mathematics courses reflects the specialist's ignorance and arrogance. It merits little attention outside of the mathematical community, and that is precisely what it gets. The current standard justifications (Beauty and Utility) for these courses are misleading. Each of them is in fact a good reason to study mathematics. But both together do not justify a universal requirement for the study of mathematics. After all, chess is beautiful. Auto mechanics is utilitarian. And bookbinding, rug-weaving, and a thousand other crafts are both.

The old standard justification (that one learns transferable skills) for the study of mathematics is, as the reviewer grudgingly admits, unproven. Now, as mathematicians we all know very well that <u>unproven</u> and <u>false</u> are quite different things, and it may well be that we wrongly ask for "scientific" evidence in this complicated arena of human capabilities. (The quotation marks are the reviewer's; they betray his agreement here.) At the very least, it is certainly also unproven that the skills do <u>not</u> transfer. And if not the skills, what of the <u>habits</u> of precision and skepticism that one learns to practice in mathematics? One could argue, at some risk, that we have in these unproven possibilities already better reason to study mathematics than any we have considered so far. Fortunately, we need not take this route; for there are much better reasons than these for the study of mathematics.

The reviewer's "<u>right</u> answer" begs the question "Why must we justify the study of mathematics?", which we must now consider. As I hinted earlier, we must convince our non-technical colleagues, because they, ultimately, are the ones who decide which students will and which students will not undertake the study. Any justifications we give them must be extrinsic ones, and not the intrinsic ones that suffice for us. We are thus led back to the utility of mathematics--which we have already discarded as justification for a universal mathematics requirement.

But we were then speaking of the trivial utility that we commonly see in today's General Education Mathematics courses. There is a great deal more to the utility of mathematics--at all levels--than this trivial utility. There is in fact an essentiality to mathematics that it shares with language. Mark

Van Doren [8] has written:

> "'Language and mathematics are the mother tongues of our rational selves'--that is, of the human race--and no student should be permitted to be speechless in either tongue, whatever value he sets upon his special gifts, and however sure he may be at sixteen or eighteen that he knows the uses to which his mind will eventually be put. This would be like amputating his left hand because he did not seem to be ambidextrous. It is crippling to be illiterate in either, and the natural curriculum does not choose between them. They are two ways in which the student will have to express himself; they are two ways in which the truth gets known."

Other authors (see, e.g., [5]) have written of language and of mathematics each as "a calculus of thought". (It is a telling comment on the coequality of language and mathematics that the metaphors these authors chose to describe their mutual essentiality are those of "mother tongue" and "calculus".)

We ourselves appear to be re-awakening to the value of our discipline. See [3] for a refreshing new approach to General Education Mathematics that implicitly recognizes the tremendous essentiality of mathematics by giving real applications of mathematics below the level of calculus. It is to be hoped that others will follow the trail Prof. Growney has blazed.

If this were all that one could say in support of mathematics for non-technical students, it would surely be enough. But it isn't all. Consider the famous quotation of Arnold Toynbee, taken from [7], that appears in [6]:

> "...I chose to give up mathematics, and I have lived to regret this keenly after it has become too late to repair my mistake. The calculus, even a taste of it, would have given me an important and illuminating

> additional outlook on the Universe... ...the rudiments, at least, of the calculus ought to have been compulsory for me. One ought, after all, to be initiated into the life of the world in which one is going to have to live. I was going to have to live in the Western World at its transition from the modern to the post-modern chapter of its history; and the calculus, like the full-rigged sailing ship, is...one of the characteristic expressions of the modern Western genius."

Toynbee keenly regretted the amputation of which Van Doren wrote and to which he had submitted himself. He explicitly mentioned the "...outlook on the Universe" that mathematics provides. And then he went further: "One ought, after all, to be initiated into the life of the world in which one is going to have to live." Here is compelling justification for the study of meaningful mathematics, and not least because it has been given by a man whose accomplishments gave him the literary, historical, and cultural perspectives that our accomplishments tend to deny us.

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Educational Studies in Mathematics

Journal of Research in Mathematics Education

For the Learning of Mathematics (Journal)

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University of Illinois

COGNITIVE SCIENCE studies of mathematical thought:

a. The most complete report of all of our work is contained in:

Davis, Robert B., Learning Mathematics: the Cognitive Science Approach to Mathematics Education. [1984] Available in the U.S.A. from Ablex Publishing Corporation, 355 Chestnut Street, Norwood, New Jersey 07648; available in Europe from: Croom Helm Ltd., Provident House, Burrell Row, Beckenham, Kent BR3 1AT, England; in Australia, Croom Helm Australia Pty Ltd., First Floor, 139 King Street, Sydney, NSW 2001, Australia.

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