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THE HUBBLE PARAMETER IN A VOID UNIVERSE: EFFECT OF THE PECULIAR VELOCITY

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ABSTRACT

We investigate the distance-redshift relation in a simple void model. As discussed by Moffat and Tatarski, if an observer stays at the center of the void, the observed Hubble parameter is not so different from the background Hubble parameter. However, if the position of the observer is not at the center of the void, we must consider the correction to the redshift due to the peculiar velocity, which is determined by the observed dipole anisotropy of the cosmic microwave background. This correction of the redshift is crucial to determine the Hubble parameter, and we shall consider this effect. Further, the results of the *N*-body simulation of Turner et al. will be also discussed.

Subject headings: cosmology: theory — distance scale — large-scale structure of universe

1. INTRODUCTION

Recent observation suggests that the Hubble parameter is large, that is, 80 ± 17 km s^{-1} Mpc^{-1} (Freedman et al. 1994). A low Hubble universe, however, is favored since a small value of the Hubble parameter is consistent with almost all observations except for that of the Hubble parameter itself (Bartlett et al. 1994). One of the theoretical bases for the possibility of a smaller Hubble parameter than that determined by local observations is given by Turner, Cen, & Ostriker (1992). They performed very large scale *N*-body simulations and constructed an ensemble of a universe filled with galaxies that, roughly speaking, were defined by density peaks of collisionless particles. Then, one of those galaxies was identified as "our galaxy," and they investigated the relation between the distance from the other galaxies to ours and the relative velocity corrected for the peculiar velocity of our galaxy only. Their result suggests that the Hubble parameter determined by such observations has a scale-dependent variance. In order to obtain the correct Hubble parameter, we need the observation of galaxies over a very wide region.

On the other hand, Moffat and Tatarski considered a single void universe in which the observer was assumed to be at the center of the void and investigated the effect of the void on the Hubble parameter determined through the redshift-distance relation (Moffat & Tatarski 1994). Their result reveals that when the observer is at the center of the void, the Hubble parameter is not so different from the true value as long as the observed region is smaller than the curvature radius inside the void. This seems to contradict the results of Turner et al.

In this paper, we investigate a void universe but do not restrict the position of the observer to be at the center of the void. Our void model is more simplified than that of Moffat and Tatarski, but it will clarify the effect of the inhomogeneities, especially the peculiar velocity, on the determination of the Hubble parameter.

This paper is organized as follows. In § 2, we shall show the simple void model and introduce the cosmic microwave background radiation (CMB) rest frame inside the void. Here the CMB rest frame means the coordinate system in which the CMB is isotropic for an observer moving along the constant spatial coordinate curve. The peculiar velocity is defined in this CMB rest frame. In § 3, we investigate the distance-redshift relation with the correction for the peculiar velocity of a comoving observer and discuss the effect of that on the determination of the Hubble parameter. Further, we shall discuss here the relation between our result and that of Turner et al. In § 4, we consider the effect of the void on the anisotropy of the CMB through the Sachs-Wolfe effect and discuss the constraint on the scale of the void from the *COBE* result (Smoot et al. 1992). Section 5 is devoted to discussion.

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2. SIMPLE VOID MODEL AND CMB REST FRAME

We assume that the inside of the void is approximated by a Friedmann-Robertson-Walker (FRW) universe with the present density parameter $\Omega_0 < 1$, while the outside is also a FRW universe but with $\Omega_0 = 1$. The boundary of the void can be ignored as long as the observer is within the void and observes only the inside of that. Here we will assume such a situation. Further, we assume that the age of both the inside and outside of the void is the same, and hence the time coordinate is the common cosmic time t. This assumption corresponds to the fact that the void structure is generated from a purely growing mode of the initial density perturbation since the density contrast between the inside and outside of the void vanishes as $t \to 0$, i.e., at the initial singularity.

The metric within the void is written as

$$ds^2 = -dt^2 + \frac{a_v^2(t)}{1 + (R_v/R_c)^2}\, dR_v^2 + a_v^2(t) R_v^2\, dS^2 , \qquad (1)$$

where R_c is the comoving curvature radius and $dS^2 = d\theta^2 + \sin^2\theta\, d\varphi^2$ is the line element on the unit sphere. We should note that the center of the void agrees with the origin $R_v = 0$, and hence, as for the time coordinate t, dS^2 is common to the inside and outside of the void. As is well known, the scale factor a_v is given by the parametric form using the conformal time η:

$$\frac{a_v}{a_{v0}} = \frac{\Omega_{v0}}{2(1 - \Omega_{v0})} (\cosh \eta - 1) , \qquad (2)$$

$$H_{v0}t = \frac{\Omega_{v0}}{2(1-\Omega_{v0})^{3/2}}(\sinh\eta - \eta)\,, \tag{3}$$

where H_{v0}, a_{v0}, and Ω_{v0} are, respectively, the present Hubble parameter, the present scale factor, and the present value of the density parameter, within the void.

On the other hand, we assume that the outside of the void is a flat FRW universe, and hence its metric is given by

$$ds^2 = -dt^2 + a_b^2(t)(dR_b^2 + R_b^2\,dS^2)\,, \tag{4}$$

and the scale factor a_b is written as

$$\frac{a_b}{a_{b0}} = \left(\frac{9}{4}H_{b0}^2 t^2\right)^{1/3}, \tag{5}$$

where a_{b0} and H_{b0} are, respectively, the present scale factor and the present Hubble parameter, outside the void.

As discussed by Bartlett et al. (1994), the ratio H_{v0}/H_{b0} varies over the range 3/2 to 1 as Ω_{v0} varies from 0 to 1. Hence, the maximum Hubble parameter within the void is at most 3/2 times the background Hubble parameter H_{b0}. However, it should be noted that H_{v0} is not observed directly. The observed Hubble parameter is determined through the relation between the distance and redshift with the correction for the peculiar velocity of the observer.

Here we introduce the CMB rest frame to define the peculiar velocity which is crucial to estimating the true cosmological redshift. As mentioned in § 1, the CMB rest frame means the coordinate system in which the CMB is isotropic for an observer moving along a constant spatial coordinate curve. We assume here that the CMB is homogeneous and isotropic. This means that a comoving observer outside the void observes the isotropic CMB, while a comoving observer inside the void observes an anisotropic CMB. It should be noted, however, that even inside the void the CMB is isotropic for the observer along $\tilde{R} \equiv (a_v/a_b)R_v =$ constant curve since such a noncomoving observer moves in the same manner as the comoving observer outside the void. Hence we obtain the CMB rest frame by adopting $\tilde{R}$ as the new radial coordinate. The transformation matrix is given by

$$d\tilde{t} = dt\,, \tag{6}$$

$$d\tilde{R} = \frac{a_v}{a_b}(H_v - H_b)R_v\,dt + \frac{a_v}{a_b}dR_v\,, \tag{7}$$

$$d\tilde{S}^2 = dS^2\,. \tag{8}$$

In the original coordinate (1), the comoving observer and comoving observed source move along $R_v =$ constant lines and hence the components of those 4-velocities are given by the common $u^\mu = (1, 0, 0, 0)$. On the other hand, in the CMB rest frame, the components are given by

$$u^{\tilde{t}} = \frac{\partial\tilde{t}}{\partial t}u^t = 1\,, \tag{9}$$

$$u^{\tilde{R}} = \frac{\partial\tilde{R}}{\partial t}u^t = \frac{a_v}{a_b}(H_v - H_b)R_v\,, \tag{10}$$

$$u^{\tilde{\vartheta}} = 0 = u^{\tilde{\varphi}}\,. \tag{11}$$

The radial component $u^{\tilde{R}}$ corresponds to the peculiar velocity of the comoving observer inside the void.

3. THE HUBBLE PARAMETER BY THE DISTANCE-REDSHIFT RELATION

In order to obtain the relation between the distance and redshift, it is sufficient to approximate the light ray by a null geodesic, i.e., to treat the propagation of the light ray by the geometric optics (Misner, Thorne, & Wheeler 1973). By virtue of the spherical symmetry of this system, without loss of generality, we focus only on the null geodesic on the equatorial plane $\theta = \pi/2$. The solution for the null geodesic tangent k^μ is then given by

$$k^t = \frac{a_{v0}}{a_v(t)}\omega_{v0}\,, \tag{12}$$

$$k^{R_v} = \pm\frac{a_{v0}}{a_v^2(t)}\sqrt{\left[1+\left(\frac{R_v}{R_c}\right)^2\right]\left[\omega_{v0}^2 - \left(\frac{L_0}{R_v}\right)^2\right]}\,, \tag{13}$$

$$k^\varphi = \frac{a_{v0}L_0}{a_v^2(t)R_v^2}\,, \tag{14}$$

and $k^\theta = 0$. The radial trajectory of the null geodesic is obtained as

$$R_v = R_k(\eta) \equiv R_c\sqrt{F^2(\eta) - 1}\,, \tag{15}$$

with

$$F(\eta) = \sqrt{1 + (L_{v0}/\omega_{v0}R_c)^2}\cosh\{\cosh^{-1}[\sqrt{1+(R_{v0}/R_c)^2} \times \sqrt{1+(L_{v0}/\omega_{v0}R_c)^2}] \pm (\eta - \eta_0)\}^{-1}\,, \tag{16}$$

where R_{v0}, L_{v0}, and ω_{v0} are the integration constants and η_0 is the present conformal time. It should be noted that, *at* $\eta = \eta_0$, $(R_v, \varphi) = (R_{v0}, 0)$, and this corresponds to the position of the comoving observer. L_{v0} is the conserved angular momentum of the light ray, while ω_{v0} is the angular frequency measured by the comoving observer. Together with ω_{v0}, L_{v0} determines the angle θ_k between the radial direction toward the center from the observer and the propagation direction of the light ray as (see Fig. 1)

$$\cos\theta_k = \mp\sqrt{1 - \left(\frac{L_{v0}}{\omega_{v0}R_{v0}}\right)^2}\,. \tag{17}$$

Next, we consider the effect of the peculiar velocity on the angular frequency of the light ray. The comoving observer (comoving observed source) detects (emits) the light ray k^μ with the angular frequency, $\omega_v \equiv -k_\mu u^\mu = -k_t$. On the other hand, the observer and observed source moving along $\tilde{R} =$ constant curve have 4-velocity $w^{\tilde{\mu}} = (1, 0, 0, 0)$ in the CMB rest frame, and hence the angular frequency for those is given by

$$\begin{aligned}\omega_c &\equiv -k_{\tilde{\mu}}w^{\tilde{\mu}} = -k_{\tilde{t}} = \omega_v + k_{\tilde{R}}u^{\tilde{R}}\\ &= \omega_v + (H_v - H_b)R_v k_{R_v}\,.\end{aligned} \tag{18}$$

It should be noted that ω_c corresponds to the angular frequency with the correction for the peculiar velocity. Observationally, we can find the effect only of our own peculiar velocity, and hence hereafter we focus on the quantities with the correction for the peculiar velocity only of the observer and those without any corrections for the peculiar velocity. Then we define the following two kinds of redshift as

$$z = \frac{\omega_v}{\omega_{v0}} - 1\,, \quad \text{and} \quad z_{co} = \frac{\omega_v}{\omega_{co}} - 1\,, \tag{19}$$

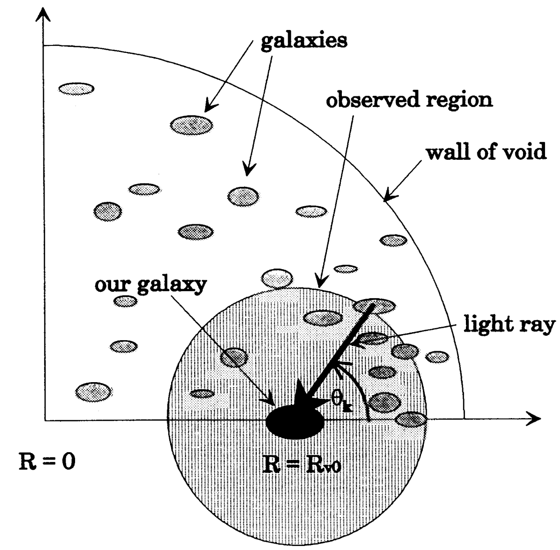

FIG. 1.—Schematic diagram of the position of the observed and the observed direction. The angle θ_k is defined in eq. (17).

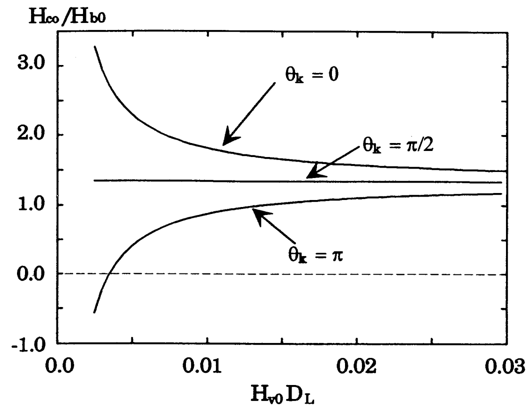

FIG. 2.—The Hubble parameter H_{co} with the correction for the peculiar velocity of the observer only is plotted against the luminosity distance D_L for various directions. The density parameter Ω_{v0} within the void is 0.1.

where ω_v is the angular frequency of the light ray at the observed source while ω_{co} is given by

$$\omega_{co} = \omega_{v0} + (H_{v0} - H_{b0}) R_{v0} k_{R_v}(\eta_0) . \tag{20}$$

Hence, z is the bare observed redshift, and z_{co} is the redshift with the correction for the peculiar velocity only of the observer.

We shall employ the luminosity distance D_L as the distance measure between the observer and observed source. Here, the luminosity distance D_L is given by the well-known relation in the FRW universe with equation (1) as

$$D_L = \frac{1}{H_{v0} q_{v0}^2} [z q_{v0} + (q_{v0} - 1)(-1 + \sqrt{2 q_{v0} z + 1})] . \tag{21}$$

where $q_{v0} = \Omega_{v0}/2$. It should be noted that the luminosity distance D_L is just the observed quantity that is determined by, for example, the Tully-Fisher relation. Then, using D_L, we define the observed Hubble parameter H_{co} with the correction for the peculiar velocity only of the observer, with the assumption that the observer regards his/her own universe as the flat FRW spacetime:

$$H_{co} = \frac{2}{D_L} (z_{co} + 1 - \sqrt{z_{co} + 1}) . \tag{22}$$

In fact, we can measure H_{co} instead of H_{b0} in the real observations. In Figure 2, H_{co} is depicted for $\theta_k = 0$, $\pi/2$, and π. In this figure, the density parameter inside the void, Ω_{v0}, is equal to 0.1 and the radial position of the observer is fixed as $a_{v0} R_{v0} = 1 \times 10^{-2} H_{b0}^{-1} \sim 30 h_b^{-1}$ Mpc.

We find that, for $H_{v0} D_L \ll 1$, H_{co} strongly depends on the observed direction along which the light ray propagates. This comes from the inadequate peculiar velocity correction. Here it should be noted that the Hubble parameter defined by Turner et al. is the volume average of just H_{co}.

To understand the directional dependence of H_{co}, we investigate the behavior for $H_{v0} D_L \ll 1$. In this case, $H_{co} \sim z_{co}/D_L \sim H_{v0}(z_{co}/z)$, and, assuming the case of $\Omega_{v0} = 0.1$, we obtain $H_{v0}/H_{b0} - 1 \sim 0.35$. Further, $a_{v0} R_{v0}$ is assumed to be less than about $100 h_b^{-1}$ Mpc, i.e., $a_{v0} H_{b0} < 3 \times 10^{-2} \ll 1$. Hence, we obtain

$$z_{co} \sim H_{v0} D_L - 10^{-2} \frac{1}{\omega_{v0}} (H_{v0} D_L + 1) k_{R_v}(\eta_0) \left(\frac{R_{v0}}{100 \text{ Mpc}} \right) . \tag{23}$$

Since $a_{v0} R_c = H_{v0}^{-1}(1 - \Omega_{v0})^{-1/2} \sim H_{v0}^{-1}$. R_{v0}/R_c is much less than unity, and hence we can see that $k_{R_v}(\eta_0) \sim -\omega_{v0} \cos \theta_k$. Then we get

$$\frac{H_{co}}{H_{b0}} \sim \frac{H_{v0}}{H_{b0}} + 10^{-2} \frac{1}{H_{v0} D_L} \cos \theta_k \left(\frac{R_{v0}}{100 \text{ Mpc}} \right) . \tag{24}$$

From the above equations, when the distance of the observer from the center of the void is $30 h_b^{-1}$ Mpc, when such an observer looks in the direction $\theta_k = 0$ and the observed distance is $D_L = 3 \times 10^{-3} H_{v0}^{-1} \sim 7 h_b^{-1}$ Mpc, the observer may estimate H_{co} to be 2 times larger than H_{b0}. On the other hand, if that observer looks in the opposite direction $\theta_k = \pi$, the observer may obtain almost vanishing H_{co}. This is just the dipole anisotropy due to the wrong correction for the peculiar velocity.

Here we shall consider the relation between our simple void model and the results of Turner et al. In our case, the averaged H_{co} agrees with H_{v0} as

$$\langle H_{co} \rangle = \frac{1}{\pi} \int_0^{\pi} d\theta_k H_{co} = H_{v0} . \tag{25}$$

It should be noted that we assume a uniform distribution of observed sources, i.e., galaxies, when we perform the above averaging. However, in the N-body simulation, the "galaxies" are not uniformly distributed in contrast with our model, and the integral of the second term on the right-hand side of equation (24) may remain. Figure 1 shows an example in which the number of galaxies in the direction $\theta_k = 0$ is larger than that in the $\theta_k = \pi$ direction. In such a case, the averaged H_{co} is greater than H_{v0}. Therefore, it may be a reason that the variance of the Hubble parameter depends on the scale of the observational regions and there appears a large variance of the Hubble parameter in the small-scale observation in the results of Turner et al. Of course, in order to confirm this expectation, detailed investigation by N-body simulations is needed (Gouda et al. 1995).

4. ANISOTROPY OF THE CMB BY SACHS-WOLFE EFFECT

Here, we should comment on the effect of the void on the anisotropy of the CMB. Here we shall assume that the CMB is completely isotropic at the last scattering surface and that the anisotropy is caused only by the effect of one void. The dipole anisotropy of the CMB is about v/c, where v is the peculiar velocity of the comoving observer in the void, and it is given roughly as $(H_{v0} - H_{b0})a_{v0} R_{v0}$ by equation (10). If the density parameter inside the void is nearly zero, we obtain $v \sim 1.5 \times 10^3 (a_{v0} R_{v0}/100 h_b^{-1}$ Mpc) km s^{-1}. Assuming that the observed dipole anisotropy comes from the peculiar velocity of our local group, it is estimated as about 600 km s^{-1} (Smoot et al. 1991). If we live in such a void, then our position is $10h_b^{-1}$ Mpc away from the center of the void. However, our void considered here is nothing but a toy model, and it should not be seriously considered.

The rather serious subject is the quadrupole or higher multipole anisotropies which come from the gravitational redshift. We consider the situation that the size of the void is much smaller than the horizon scale L of the background flat FRW universe, and hence the Newtonian approximation is applicable. In this case, the metric is written as

$$ds^2 = -(1 - 2U)dt^2 + a_b^2(t)(1 + 2U)(dR^2 + R^2 dS^2) , \quad (26)$$

where $|U| \ll 1$. Further we assume the following density configuration:

$$\rho = \begin{cases} \rho_v(t) & R < R_{\rm void} \\ \rho_b(t) & \text{otherwise} , \end{cases} \quad (27)$$

where ρ_b corresponds to the critical density. Then the Newton potential U inside the void $R < R_{\rm void}$ is obtained as

$$U = 2\pi \, \delta\rho l^2 - \frac{2\pi}{3} \, \delta\rho (a_b R)^2 , \quad (28)$$

where $l \equiv a_b R_{\rm void}$. Here, since we consider the case in which $\delta\rho \sim -\rho_b \sim -H_b^2 = -L^{-2}$, we see that $\delta\rho l^2 \sim \kappa^2 \equiv (l/L)^2 \ll 1$. Thus we can roughly estimate the Newtonian potential as $U \sim \kappa^2 - \kappa^2 (a_b R/l)^2$, $\partial_t U \sim H_b U$ and $\partial_r U \sim \kappa^2 (a_b/l)^2 R$.

Here we shall estimate the Sachs-Wolfe effect by the above Newtonian potential. The anisotropy of CMB is expressed by the integrated brightness temperature perturbation Θ, and the equation for Θ is written as

$$\frac{d}{dt}(\Theta - U) \equiv \left(\partial_t + \frac{\gamma^i}{a_b} \partial_i\right)(\Theta - U) = -2\partial_t U , \quad (29)$$

where γ^i is the direction cosine of the photon (Kodama & Sasaki 1986). Then, the difference between the two opposite radial directions is roughly estimated as

$$\frac{\Delta T}{T} \equiv \Theta = 2\left(\int dt \, \partial_t U \Big|_{\theta_k = 0} - \int dt \, \partial_t U \Big|_{\theta_k = \pi} \right) \sim \kappa^3 \left(\frac{R_o}{l} \right) , \quad (30)$$

where R_o denotes the radial position of the observer, and the integration is performed along the path of the light ray. In the above estimation, we have ignored the contribution of the peculiar velocity of the observer. It should be noted that the above result is consistent with the analysis by Thompson & Vishniac (1987) and Mészáros (1994) for the case that the position of the observer is outside of the void. From equation (30), if we live in the $100h_b^{-1}$ Mpc scale void, since $\kappa^3 \sim 4 \times 10^{-5}$, the higher multipole anisotropy of the CMB does not conflict with *COBE* results (Smoot et al. 1992). However, this estimate is so rough that we need more detailed investigation, and this is in progress.

5. DISCUSSION

In this paper, we investigated the effect of the void on the determination of the Hubble parameter and have shown the importance of the estimation of the peculiar velocity to obtain the true Hubble parameter. Inadequate correction for the peculiar velocity leads to the dipole anisotropy of the Hubble parameter determined from the distance-redshift relation.

From the observational point of view, if the variance of Hubble parameter comes from the dipole anisotropy as mentioned above, it is important to confirm the isotropy of the Hubble parameter. Lauer and Postman reported the highly isotropic Hubble parameter by a rather large scale observation $z \leq 0.05$ (Lauer & Postman 1992). Hence, even if we stay in the void considered here, we are near the center of that. In the case that we stay near the center of the void, the observed Hubble parameter is H_{v0}, and this varies the range H_{b0} to $1.5H_{b0}$. Since this variance is not so large, we can find almost the same Hubble parameter as the background one. Of course, our model is too simple, and more complicated situations may be imagined, which makes us fail to determine the true Hubble parameter. Hence, further theoretical investigation should be continued, and deeper observation over the whole sky is very important.

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