FEM modeling of energy dissipation in SMRs

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Abstract

- Suspended Microchannel Resonator [1], [2], [3]
- Development of a 3D coupled fluid-structure interaction model to extract Quality Factor as function of fluid dynamic viscosity
- Comparison between numerical, theoretical [1] and experimental [2] results
- Good match between experimental and numerical Q for first two modes
- Decreasing Q for increasing viscosity in contrast with theory

(1)

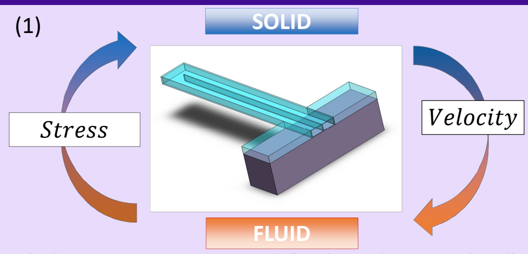

Fig. 1: Fluid-structure interaction is defined on the internal walls of the channel; a fixed constraint is imposed to the rigid channel (x<0). The cantilever is let free to vibrate (x>0). Linearized Navier-Stokes and Solid Mechanics equations are solved in COMSOL. The solid transfers momentum to the fluid, which sends back stresses to the cantilever, affecting its motion.

FEM model

- 3D eigenfrequency study in COMSOL Multiphysics®
- Device symmetry is exploited (fig.2a)
- Both 1-way-coupling and 2-way-coupling simulations are performed
- The quality factor is extracted as:

$$Q_{Comsol} = \frac{Im[\lambda]}{2Re[\lambda]}$$

where λ is the complex eigenvalue.

- The quality factor is scaled according to the analytical model proposed by Sader in [1], in function of the Reynolds number β:

$$Q = F(\beta)\frac{\rho_c}{\rho_f}\left(\frac{h_c}{h_f}\right)\left(\frac{b_c}{b_f}\right)\left(\frac{L}{h_f}\right)^2, \qquad \beta = \frac{\rho_f \omega h_f^2}{\mu}$$

- Parameters studied: compressibility ($\gamma = \left(\frac{\omega L}{c}\right)^2$ is the normalized acoustic wavenumber), dynamic viscosity, off-axis placement Z_0, Poisson ratio, mode number.

(2a) (2b)

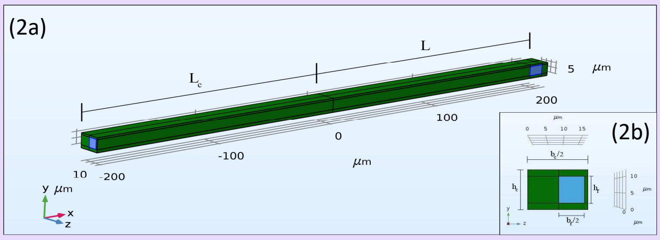

Fig.2a: COMSOL Model (half geometry) of Device A [1]: h_f=8 µm, h_c=12 µm, b_f=16 µm, b_c=33 µm, L=204 µm, L_c=210 µm, cantilever length= 210 µm, Z_0=0.06.
In green the elastic domain, in blue the fluid domain. Z_0 is the off-axis placement of the fluidic channel with respect to the beam neutral axis.
Fig.2b: Cross-section of half geometry of Device A [1] (symmetry boundary condition is exploited).

Theoretical model [1]

- 2D theoretical model is only due to fluid motion and viscous forces, through the rate-of-strain tensor **e**, defined as: $E_{ij} = \frac{1}{2}(\partial_j v_i + \partial_i v_j)$
- Quality factor is computed as:

$$Q = 2\pi \left.\frac{E_{stored}}{E_{diss/cycle}}\right|_{\omega_R}$$

- Strong effect of:
  - compressibility [1]
  - channel off-placement Z_0 [1]
  - Poisson's ratio [3]
  - mode number [2]

(3)

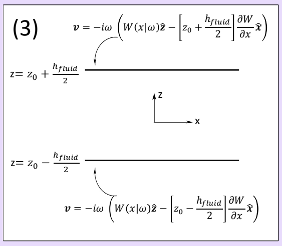

Fig. 3: 2D theoretical model; Euler-Bernoulli beam equations imposed as boundary conditions on the top and bottom wall; x is the coordinate along the length of the beam, z_0 is the off-placement of the channel with respect to the beam neutral axis [1].

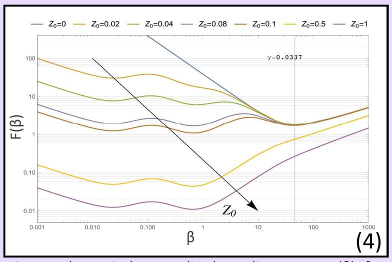

Fig. 4: Theoretical Normalized Quality Factor F(β) for various normalized off-placements Z_0 of the channel in the compressible case (γ=0.0337) for L_c=L; the theoretical model predicts an increasing F(β) for increasing viscosity (decreasing β) and lower F(β) for higher off-placements of the channel with respect to the beam neutral axis. For Z_0<0.2 this effect is stronger for β<10.

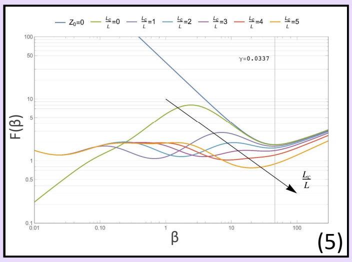

Fig. 5: Theoretical Normalized Quality Factor F(β) for various rigid lead channel lengths L_c in the compressible case (γ=0.0337) and Z_0=0.1; the theoretical model predicts a surprisingly different behavior when L_c=0. The local maxima and minima of F(β) are strongly affected by L_c.

Results

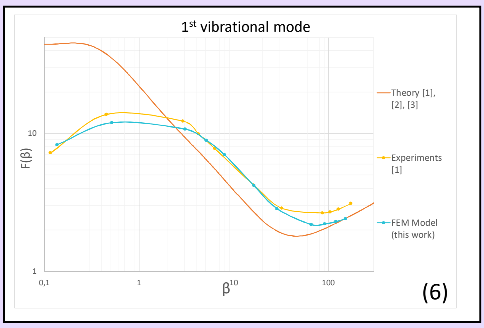

Fig. 6: Comparison of Normalized Quality Factor F(β) as a function of Reynolds Number between theoretical [1], [2], [3], experimental [1], [2] and numerical results for Device A (h_f=8 µm, h_c=12 µm, b_f=16 µm, b_c=33 µm, L=204 µm, L_c=210 µm, cantilever length=210 µm, Z_0=0.06, normalized wavenumber γ=0.12, Poisson's ratio=0.25) for Mode 1. Viscosity spans from to 1 mPa·s to 1000 mPa·s and is inversely proportional to Reynolds Number.

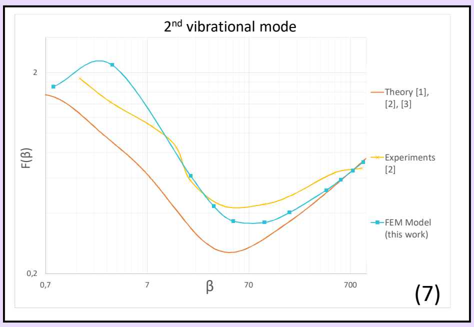

Fig. 7: Comparison of Normalized Quality Factor F(β) as a function of Reynolds Number between theoretical [1], [2], [3], experimental [1], [2] and numerical results for Device A (h_f=8 µm, h_c=12 µm, b_f= 16 µm, b_c=33 µm, L=204 µm, L_c=210 µm, cantilever length=210 µm, Z_0=0.06, normalized wavenumberγ=0.12, Poisson's ratio=0.25) for Mode 2. Viscosity spans from to 1 mPa·s to 1000 mPa·s and is inversely proportional to Reynolds Number.

Conclusions

- Good agreement between experimental and numerical results for β∈(1,1000) for first two modes
- Contrasting behavior at high viscosities between theoretical and numerical results (β<1)
- Dependence of Q on Z_0, L_c, compressibility, Poisson's ratio and mode number for β<1.
- Need of improvement of 2-way-coupling modelling

References

[1] E. Sader et al., "Energy dissipation in microfluidic beam resonators", J. Fluid Mech., 650, 215, 2010
[2] E. Sader et al., "Energy dissipation in microfluidic beam resonators: Dependence on mode number", J. Fluid Mech., 108, 114507, 2010
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