A unified modeling and solution framework for stochastic routing problems

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Outline

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Problem description

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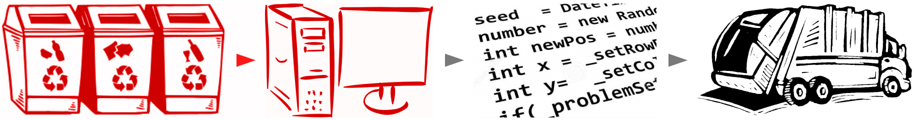

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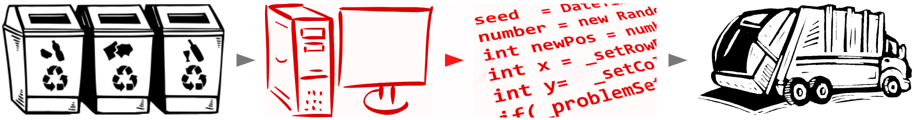

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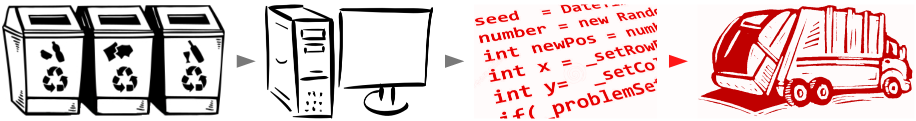
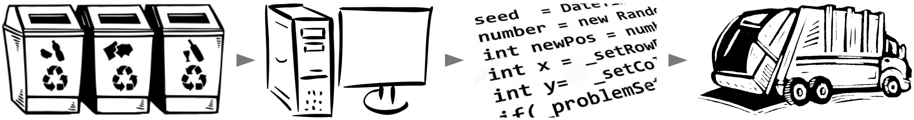


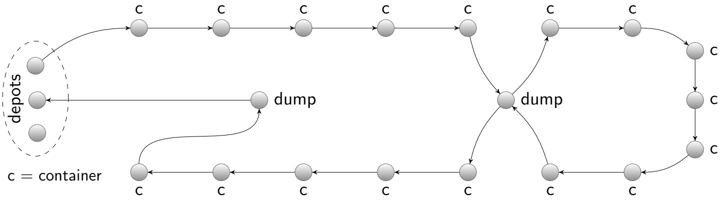
Problem description

- Sensorized containers transmit level data to the server.
- Level data is used for *demand forecasting* and *tour planning* over a finite planning horizon.
- Vehicles perform the resulting tours.
- Solving this *inventory routing problem* involves
  - deciding which containers to visit each day
  - and optimizing the collection tours.



Daily tour structure

Figure 1: Basic vehicle tour



Information uncertainty

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  3. roll over and solve for updated levels and forecasts
- *Solving the problem day by day in isolation leads to myopic decisions.*

Related literature and contributions

- Main approaches in the literature:
  - stochastic programming, MDP (Pillac et al., 2013)
  - approximate dynamic programming (Powell, 2011)
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- Characteristics of our approach:
  - unified approach with few distributional assumptions
  - explicit modeling of *undesirable events* and *recourse actions*
  - cost-oriented with priced risk
  - applicable to rich real-world problems

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Demand forecasting

Sets

- $\mathcal{K}$: set of vehicles
- $\mathcal{T}$: set of days in the planning horizon
- $\mathcal{P}$: set of containers

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Forecasting model

- stochastic non-stationary demand ρ_{it} for container $i \in \mathcal{P}$ on day $t \in \mathcal{T}$:

$$\rho_{it} = \mathbb{E}\left(\rho_{it}\right) + \varepsilon_{it} \tag{1}$$

- combine ε_{it} in a vector:

$$\boldsymbol{\varepsilon} = \left(\varepsilon_{11}, \ldots, \varepsilon_{1|\mathcal{T}|}, \varepsilon_{21}, \ldots, \varepsilon_{|\mathcal{P}||\mathcal{T}|}\right) \tag{2}$$

- let $\boldsymbol{\varepsilon} \sim \Phi$ with $\text{var}\left(\boldsymbol{\varepsilon}\right) = \mathrm{K}$ that can be simulated
- use any forecasting model that provides $\mathbb{E}(\rho_{it})$ and Φ

Inventory policy

Context

- Order-Up-to (OU) level policy (Bertazzi et al., 2002)
- Maximum Level (ML) policy (Archetti et al., 2011)

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Discretized ML policy

- for tractable pre-processing of stochastic information
- Λ_{it}: inventory after collection of container i on day t

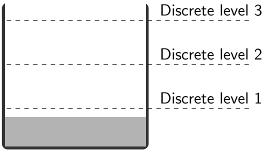

Figure 2: Discretized ML policy example

Undesirable events

Container overflows

- $\sigma_{it} = 1$ for overflow of container i on day t, 0 otherwise
- entails an overflow cost
- *recourse*: emergency collection with a cost

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Route failures

- inability to complete a depot-to-dump or dump-to-dump *trip* $\mathcal{S}$
- due to insufficient vehicle capacity
- *recourse*: detour to the nearest dump with a cost

Overflow probabilities

- Overflow probability of container i on day t:

$$\mathrm{p}_{it}^{\mathrm{DP}} = \mathbb{P}\left(\sigma_{it} = 1 \mid \Lambda_{im} \colon m = \max\left(0, g < t \colon \exists k \in \mathcal{K} \colon y_{ikg} = 1\right)\right) \tag{3}$$

where:

- y_{ikg} 1 if vehicle k visits container i on day g, 0 otherwise

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- The complexity is linear in the number of discrete levels.

Route failure probabilities

- Route failure probability of trip $\mathcal{S}$ performed by vehicle k:

$$\mathsf{p}^{\mathsf{RF}}_{\mathcal{S},k} = \mathbb{P}\left(\Gamma_{\mathcal{S}} > \Omega_k\right) \tag{4}$$

where:

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- Use the ECDFs at runtime to approximate route failure probabilities.

Outline

Principal cost components, I

Routing cost

- daily deployment cost
- travel distance related cost
- travel, service and waiting time related cost

Principal cost components, II

Expected overflow and emergency collection cost

$$\sum_{t \in \mathcal{T} \cup \mathcal{T}^+} \sum_{i \in \mathcal{P}} \left(\chi + \zeta - \zeta \sum_{k \in \mathcal{K}} y_{ikt} \right) \mathrm{p}_{it}^{\mathrm{DP}} \tag{5}$$

where:

- χ overflow cost
- ζ emergency collection cost

Principal cost components, III

Expected route failure cost

$$\sum_{t \in \mathcal{T} \setminus 0} \sum_{k \in \mathcal{K}} \sum_{\mathcal{S} \in \mathscr{S}_{kt}} \psi C_{\mathcal{S}} \mathsf{p}^{\mathsf{RF}}_{\mathcal{S},k} \tag{6}$$

where:

- $\mathscr{S}_{kt}$ set ot trips performed by vehicle k on day t
- $C_{\mathcal{S}}$ dump detour cost for trip $\mathcal{S}$
- ψ *route failure cost multiplier*

Objective function

- Components:
  - routing cost
  - expected overflow and emergency collection cost
  - expected route failure cost
  - various deterministic cost components
    (inventory holding, number of visits, workload balancing)

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  - various deterministic cost components
    (inventory holding, number of visits, workload balancing)
- Overestimates the real cost:
  - due to modeling simplifications
  - for tractability reasons
  - *do-nothing vs. optimal reaction policy*

Deterministic constraints

- accessibility restrictions
- vehicle capacity and dump visits
- time windows
- maximum tour duration
- periodicities and service choice
- inventory tracking and container capacity
- inventory policy definition
- etc...

Probabilistic constraints

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Applications

Stochastic demand problems

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- waste collection inventory routing
- supermarket delivery routing
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Probability-based routing problems

- e.g. facility maintenance
- facility breakdown probability grows with number of days since last visit

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- Rich operator pools:
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- Admits intermediate infeasibilities.
- Performance:
  - competitive on benchmarks (Archetti et al., 2007)
  - *stable*: 0-3% between best and worst over 10 runs
  - *fast*: 10-15 min. per instance; operational speed

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Waste collection case study

- Waste collection IRP instances:
  - 63 realistic instances from Geneva, Switzerland
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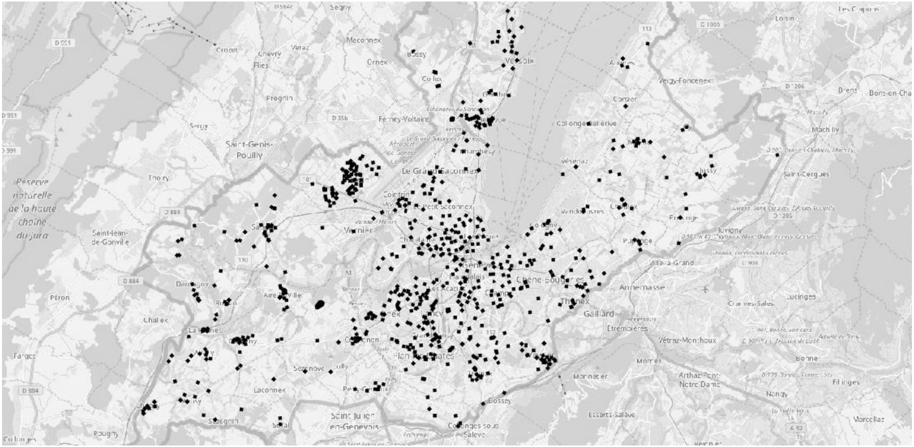
- Simulate undesirable events on final solution.

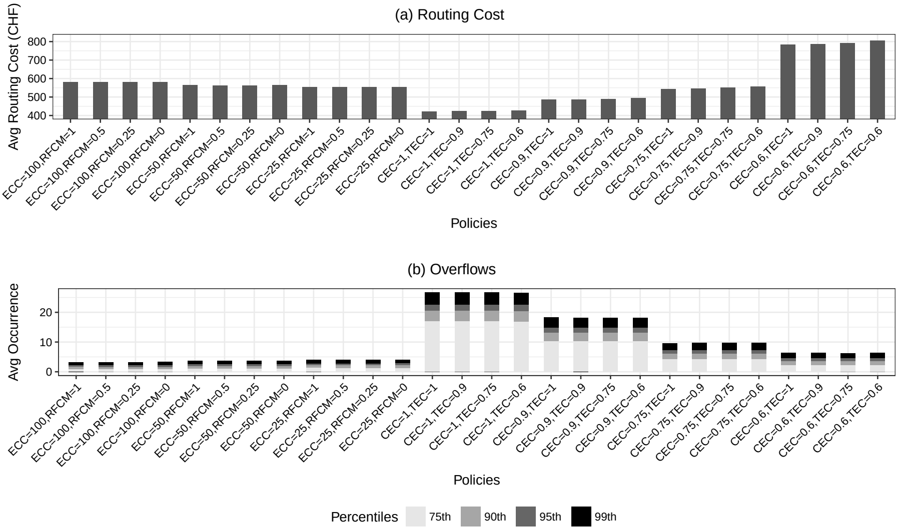
Waste collection: Service area

Figure 3: Geneva service area



Waste collection: Policy comparison

Figure 4: Routing cost and overflows for probabilistic and deterministic policies



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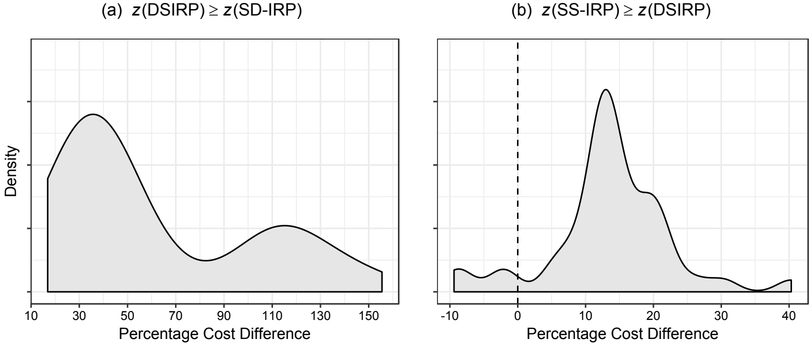
- Hypothesize:
  - $z(\text{SS-IRP}) \geqslant z(\text{DSIRP}) \geqslant z(\text{SD-IRP})$

Waste collection: Rolling horizon

Figure 5: Analysis of rolling horizon bounds



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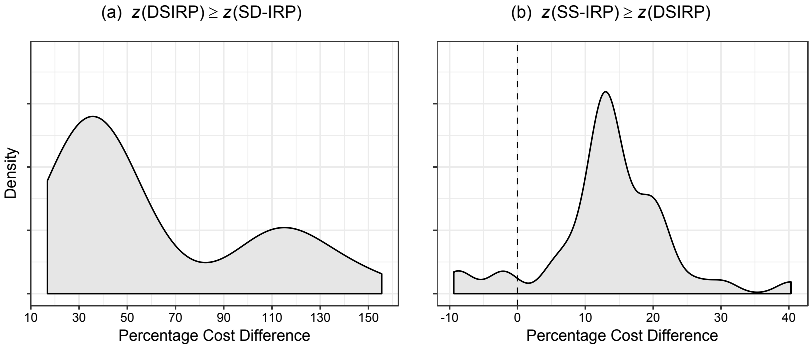

The rolling horizon approach is beneficial.

Waste collection: Impact of ECDFs

- Numerical approximation vs. ECDFs for route failure probabilities:
  - 100 bins: squared error of 10^{-6}
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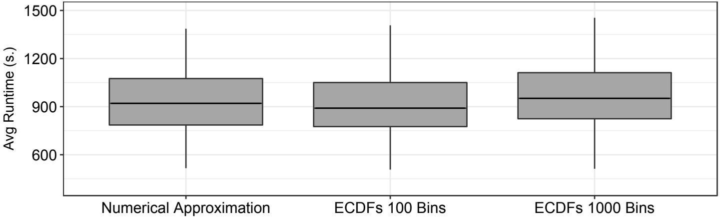
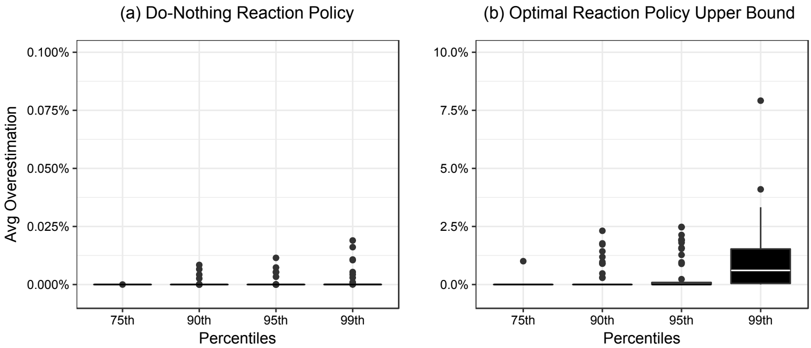
Figure 6: Runtimes of different configurations



Waste collection: Objective overestimation

Figure 7: Objective function's overestimation of the real cost for ECC = 100 CHF, RFCM = 1



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  - 93 instances derived partially from real data
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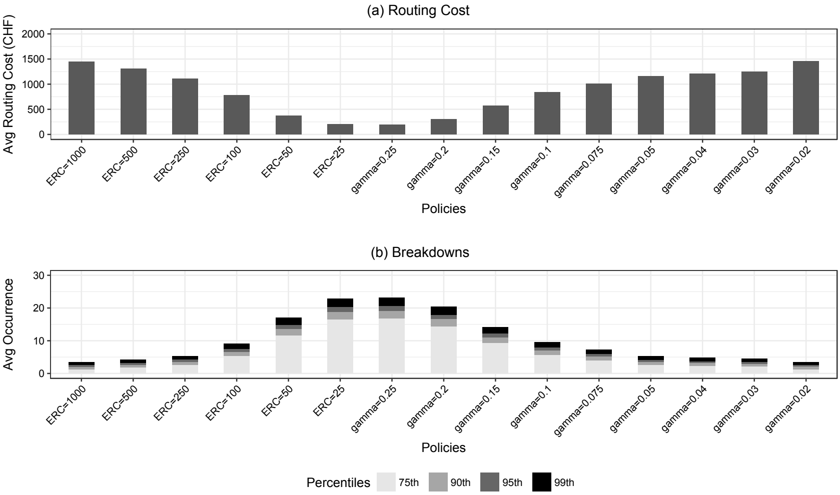
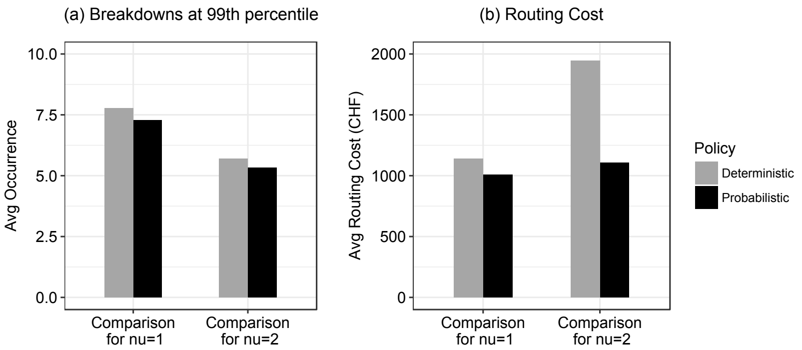
Facility maintenance: Policy comparison

Figure 8: Routing cost and breakdowns for probabilistic objective vs. probabilistic constraints model



Facility maintenance: Policy comparison

Figure 9: Probabilistic vs. deterministic policies



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- Few distributional assumptions.
- Negligible deviation of modeled from real cost.
- Efficient and competitive solution methodology.
- Tractability through the ability to pre-process.
- Clear-cut superiority of stochastic (rolling horizon) approach.

Future research

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- Richer objective: modeling realism vs. tractability.
- Column generation for lower bounds.

Thank you

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Figure 10: Container state probability tree