ASSESSMENT OF THE PROBABILITY OF COLLAPSE OF STRUCTURES DURING EARTHQUAKES

E. Miranda[(1)], L. Eads[(2)] , H. Davalos[(3)] and D. Lignos[(4)]

[(1)] *Associate Professor, Dept. of Civil and Environmental Engineering, Stanford University, USA, emiranda@stanford.edu*
[(2)] *Risk modeler, Risk Risk Management Solutions, London EC3R 8NB, UK, laura.eads@rms.com*
[(3)] *Ph.D. Candidate, Dept. of Civil and Environmental Engineering, Stanford University, USA, hdavalos@stanford.edu*
[(4)] *Associate Professor, Ecole Polythecnique Federale de Lausanne, Switzerland, dimitrios.lignos@epfl.ch*

Abstract

Avoidance of collapse is the most important objective in earthquake resistant design, but assessing the probability of collapse of a structure during an earthquake is technically challenging and it is computationally demanding. This paper summarizes recent investigations conducted by the authors at Stanford University aimed at improving the assessment of the probability of collapse of structures. It is shown that methodologies based on estimating the probability of collapse at a single ground motion intensity, such as those that only evaluate the level of safety at the so called maximum considered earthquake are inadequate. Results are presented showing that estimating the probability of collapse using incremental dynamic analyses in which a model of the structure is subjected to ground motions scaled at increasing levels of intensity until collapse is produced may introduce significant bias in the results. An improved procedure, referred to as *Enhanced Two Stripe Analysis, E2SA* is presented which provides not only more accurate results compared to those obtained with an incremental dynamic analysis but can be obtained at a small fraction of the computational effort. The new approach is based on careful observation of the deaggregation the mean annual frequency of collapse that reveals that is typically dominated by earthquake ground motion intensities corresponding to the lower half of the collapse fragility curve. Therefore, rather than focusing on the estimation of the median collapse intensity as proposed in previous studies, the new method proposes to focus on intensities in the lower tail of the collapse fragility function. Furthermore, it is shown that the uncertainty in the collapse fragility curve and on the mean annual frequency of collapse is significantly reduced by increasing the number of ground motions used in the analysis, so instead of using a small ground motion set scaled at many increasing levels of intensity, the proposed method recommends conducting nonlinear response history analyses with a larger number of ground motions but only at two levels of intensity. Results indicate that using a larger number of ground motions at two intensity levels provides improved results. Finally, recent investigations conducted by the authors show that rather than selecting and scaling ground motions based on spectral accelerations at a period equal to the fundamental period of the structure alone or in combination with epsilon, a much better approach is to use an averaged spectral acceleration over a wide range of periods extending to periods shorter than the fundamental period to significantly longer than the fundamental period. Extensive studies conducted by the authors over a wide range of structures indicate that this new intensity measure is significantly better correlated with collapse and therefore leads to a more reliable estimation of the mean annual frequency of collapse while at the same time reducing the computational effort involved since the number of ground motions to be used in the analysis can be reduced.

Keywords: structural collapse; collapse intensity; ground motion intensity; enhanced two stripe analysis, spectral shape.

1. Introduction

Avoidance of collapse has been explicitly stated as the most important and often as the only objective of building codes for many years. However, at present time typical design procedure for new structures to be built in seismic regions do not require an explicit estimation of the probability that the structure being designed collapses during a future earthquake. Recent advances in the development of nonlinear analytical models that are capable of explicitly incorporating the effects of degrading phenomena in materials, structural members and connections such as yielding, softening, hardening, local and global buckling, crushing, fracture, bond slippage, cyclic degradation etc. together with increasing computational power in modern multi-core parallel processors now allow the engineers to conduct analyses that would have been impossible just a decade ago. However, assessing the probability of collapse remains a technically challenging and computationally demanding task for earthquake engineers.

Early studies on the collapse of structures focused on collapse of single-degree-of-freedom (SDOF) systems or multiple-degree-of-freedom (MDOF) systems primarily due to *P-Δ* effects in combination with simplified hysteretic models which typically did not incorporate cyclic and in-cycle deterioration and degradation (e.g., [1-4]). Furthermore, these studies would typically evaluate the collapse under a single ground motion, for example when investigating the collapse of specific structures in past earthquakes, and there was no attempt to estimate record-to-record variability nor the probability of collapse over the life of the structure. Miranda and Akkar [5] identified relationships between characteristics of the force-deformation relationships and the peak response of structures and computed probabilistic estimates of collapse for simple SDOF systems by using a relative intensity measure defined as the ratio of the lateral strength required for the system to remain elastic to the minimum lateral strength required to avoid dynamic instability under a given record. That study was subsequently expanded to systems with more complex hysteretic models and evolved into methods to obtain estimates of the probability of collapse of MDOF structures by using results from nonlinear static analyses [6].

A significant contribution to collapse assessment was presented by Ibarra and Krawinkler [7] who analyzed SDOF and MDOF models that combined *P-Δ* effects and a new hysteretic model which incorporated various kinds of strength and stiffness deterioration. In particular, that study provided the first examination on the effect of both record-to-record variability and modeling uncertainty on collapse risk and developed a rational procedure to compute collapse fragility curves and to combine them with the seismic hazard curve at the site to compute the mean annual frequency of collapse. In their procedure the collapse fragility function is obtained by conducting incremental dynamic analyses [8] of the model of the structure by subjecting it to a set of ground motions which are then scaled at increasing levels of intensity until the model of the structure collapses. Based on studies by Cornell and his research associates they proposed scaling the ground motions to all having the same 5% spectral ordinate at the fundamental period of vibration of the structure. Their proposed procedure was subsequently used in several studies at Stanford's John A. Blume Earthquake Engineering Center aimed at assessing the collapse of old and new reinforced concrete structures [9-11]. While the approach proposed by Ibarra and Krawinkler to estimate the mean annual frequency of exceedance is a rational procedure, it is computationally very demanding because it requires using 30 to 40 ground motions scaled at many increasing levels of intensity. Furthermore, the procedure requires the determination of the specific value of the ground motion intensity that produces the collapse of the structure. This typically requires combining root searching numerical analysis techniques with more nonlinear response history analyses (NRHA) requiring many hundreds and in some cases thousands of NRHAs.

Estimating the mean annual frequency of collapse of a structure requires not only a detailed model of the structure but the engineer conducting the analysis must have a procedure to conduct the assessment. The most demanding computational effort in such analysis is the estimation of the collapse fragility which requires knowing how many ground motions to use, to have a criteria to select and scale the ground motions, to know at what level of intensity and how many levels of ground motion intensity to use, etc. The purpose of this paper is summarize recent studies conducted by the authors aimed at developing improved methods and recommendations to assess the probability of collapse of structures subjected to earthquake ground motions. In particular, this paper summarizes an improved and efficient method of assessing the mean annual frequency of collapse referred to as *Enhanced Two Stripe Analysis*, ***E2SA***.

2. Mean Annual Frequency of Collapse

Using the probability of collapse at a single ground motion intensity, for example at the maximum considered earthquake (e.g., as done in the revised version of chapter 12 of ASCE 7-16 [12]), is an incomplete and therefore inadequate measure of the level of safety of a structure because it fails to recognize that the structure could also collapse under ground motions with smaller intensity levels which have a higher probability of occurrence or under ground motions with higher intensity levels which, even though have smaller probabilities of occurrence, they nevertheless can produce the collapse of a structure. An example that illustrates why evaluation of the level of safety at a single level of ground motion intensity is inadequate is to consider two structures with the same probability of collapse at a certain level of ground motion intensity (e.g., at MCE level) but with different collapse fragility curves, that is, the collapse fragility curves "cross" at the point in which the analysis is being conducted. If they have the same probability of collapse at that intensity one could erroneously conclude that the level of safety against collapse of both structures is the same or similar. However, clearly the level of safety against collapse can be significantly different in the two structures despite having the same probability of collapse at a single level of intensity. Similarly, evaluating the median collapse intensity, that is the calculation of the ground motion intensity at which half of the records in the ground motion set produce collapse or its relative value to a reference collapse intensity, such as the collapse margin ratio as implemented in FEMA P695 [13], although it is a better measure it still does not capture the possible difference in the level of safety in structures with the same median collapse capacity (or same collapse margin ratio) but with different record-to-records variabilities.

A better and more rational way to evaluate the level of safety of a structure against collapse is by using the mean annual frequency of collapse (λ_c) for collapse risk assessment. Two components are needed to calculate λ_c: the seismic hazard curve, which gives information on the mean annual frequency of exceeding different ground motion intensities at the site, and the structure's collapse fragility curve, which describes the structure's probability of collapse conditioned on the intensity of the ground motion. The intensity of the ground motion is quantified by an intensity measure (*IM*) which can be the 5%-damped spectral acceleration at the first mode period of the structure $S_a(T_1, 5\%)$ or improved measures of ground motion intensity that are better correlated with collapse. The mean annual frequency of collapse (λ_c) is computed by integrating the collapse fragility curve of the structure over the seismic hazard curve at the site using the following equation [14]

$$\lambda_c = \int_0^\infty P(C \mid im) \cdot \left| \frac{d\lambda_{IM}(im)}{d(im)} \right| d(im) \tag{1}$$

where $P(C \mid im)$ is the probability that the structure will collapse when subjected to an earthquake with ground motion intensity level *im*, and $d\lambda_{IM}(im)/d(im)$ is the slope of the seismic hazard curve at the site at intensity level *im*. In general, there is no closed-form solution to the integral in Eq. (1), and therefore this integral is typically solved using numerical integration. This is achieved by computing the product of the probability of collapse conditioned on *IM* and the slope of the seismic hazard curve at discrete *IM*s, multiplying by the increment in *IM* (Δim), and adding the results from all *IM*s using the following equation.

$$\lambda_c = \sum_{i=1}^{\infty} P(C \mid im_i) \cdot \left| \frac{d\lambda_{IM}(im_i)}{d(im)} \right| \cdot \Delta im \tag{2}$$

The integrand in Eq. (2) provides information about the deaggregation of $\boldsymbol{\lambda_c}$ which provides a way of identifying the ground motion intensities primarily contributing to the collapse of a structure. Eads et al. [14] proposed plotting the integrand in Eq. (2) as a function of *IM*. An example is shown in Fig. 1 which presents the deaggregation plot of a four-story steel moment frame building located in Los Angeles, California and Memphis, Tennessee in the United States where it can be seen that despite the large difference in slopes in the seismic hazard curves, λ_c is dominated by intensities corresponding to the lower portion of the collapse fragility curve.

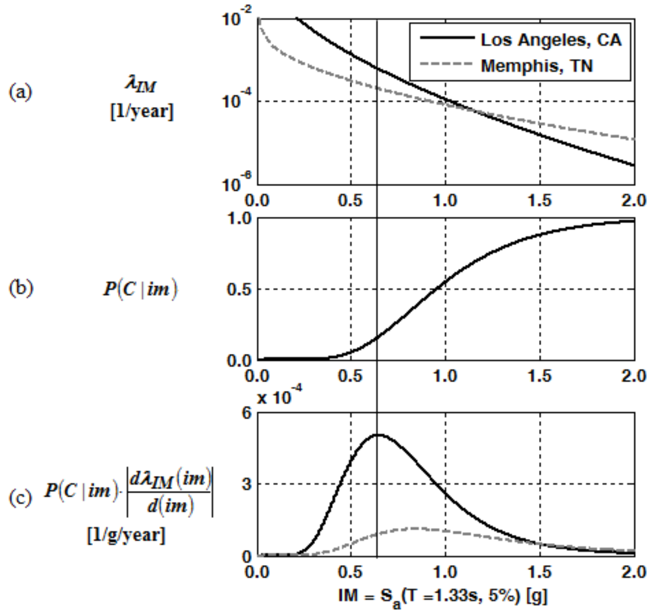

Fig. 1. Example of collapse risk assessment: (a) seismic hazard curves, (b) collapse fragility curve and (c) λ_c deaggregation curves.

3. Selection and Scaling of Ground Motions Records

Prof. Cornell and his students (e.g. [15-17]) recommended selecting and scaling records based on the five percent damped spectral acceleration ordinate at the fundamental period of the structure, $Sa(T_1)$, because it is the same ground motion intensity used in probabilistic seismic hazard analyses but also because they noted that this method offered a reduction in record-to-record variability in the response relative to a selection based on magnitude and distance pairs, and therefore reduced the required number of ground motions to achieve a certain level of error in the estimate of the response. For example, when using three different sets of records Shome and Cornell [15] noted that scaling records to $Sa(T_1)$ lead to an average reduction of 40% in the dispersion of peak interstory drift ratios of the structures they analyzed and therefore a smaller number of ground motions could be used.

While $Sa(T_1)$ provides an exact measure of intensity of the peak deformation of an elastic SDOF system, its efficiency to estimate seismic behavior of structures rapidly diminishes with increasing level of nonlinearity and it leads to a large record-to-record variability when used to estimate large nonlinear deformations in MDOF structures. Fig. 2 shows the spectral acceleration $Sa(T_1)$ by which 274 earthquake ground motions need to be scaled to in order to produce the collapse of a post-Northridge 4-story steel moment resisting steel building [14]. The ground motions were recorded in earthquakes with moment magnitudes between 6.9 and 7.6 and Joyner-Boore distances (horizontal distance between the site and the projection of the fault rupture onto the surface) between 0 and 27 km and on sites classified as NEHRP site classes C or D. It can be seen that the ground motions intensities, when characterized by $Sa(T_1)$, exhibit a very large record-to-record variability with some ground motions producing the collapse of the structure when the record is scaled to a spectral ordinate of 0.48g at T_1=1.33s while others need to be scaled to spectral ordinates as large as 3.27g to produce collapse. Also shown in the figure is the median collapse intensity which for this structure is 1.03g, the 5 percentile (ground motion intensity at which only 5% of the ground motions produce collapse in the structure) and 95 percentile (ground motion intensity at which 95% of the ground motions produce collapse). In this example, the intensity corresponding to the 95 percentile (2.11g) is 3.64 times larger than the intensity corresponding to the 5 percentile (0.58g) indicating a large variability of the ground motion intensity required to produce collapse. The corresponding logarithmic standard deviation is 0.39, which is very large.

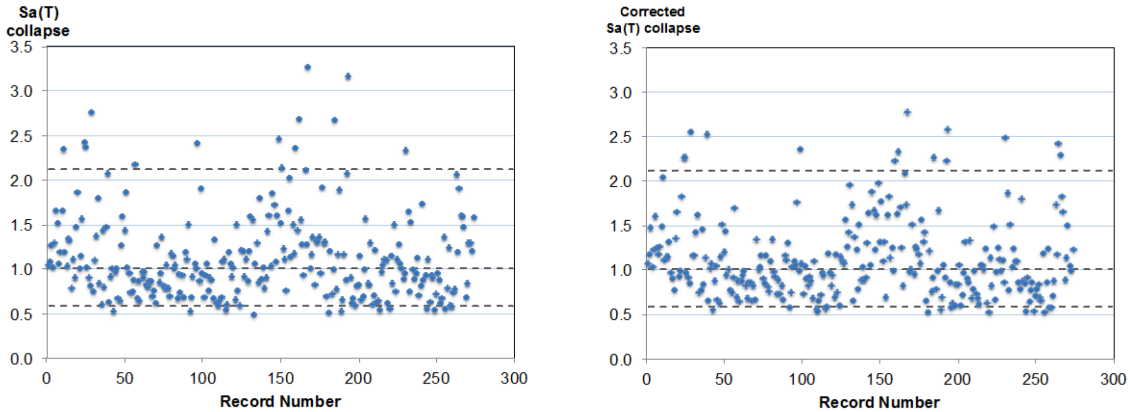

Fig. 2. Spectral accelerations at the fundamental period of vibration by which 274 earthquake recorded ground motions need to be scaled to in order to produce collapse of a 4-story steel moment resisting steel building [14] before (left) and after (right) applying the correction proposed in [19].

More recently, Baker and Cornell proposed using a vector *IM* that consists of the five percent damped spectral ordinate at the fundamental period of vibration of the structure $Sa(T_1)$ and the ground motion parameter ε [18]. The ground motion parameter ε is a measure of the difference between a record's spectral acceleration ordinate at a given period and the median predicted by a ground motion prediction equation (GMPE). They observed that ε could be used as a *proxy* to the spectral shape and when used together with $Sa(T_1)$ it could lead to an improved estimate of the seismic response of a structure. Furthermore, they noted that neglecting the spectral shape could lead to introducing a bias in the results. In particular, they noted that as epsilon increased, that is, as the spectral ordinate at the fundamental period of the structure became larger with respect to the value estimated by a ground motion prediction equation, the record was more benign, meaning it had to be scaled by a larger factor in order to induce a certain level of response or collapse of a structure. As an example, Fig. 3 shows the natural logarithm of the $Sa(T_1)$ by which 274 earthquake ground motions need to be scaled to in order to produce the collapse of a post-Northridge 4-story steel moment resisting steel building plotted as a function of ε [14]. Also shown in the figure is a linear fit regressed to the data. As illustrated in the figure, and as previously noted by Baker and Cornell, there is a tendency to increase the collapse intensity as epsilon increases.

In order to approximately account for the spectral shape when evaluating structures Haselton et al. [19] proposed a simplified procedure for correcting the collapse capacity of a structure when the spectral shape is not considered in the selection of the records by applying a correction factor whose amplitude is a function of ε of each record. Their method uses a general ground-motion set, selected without regard to ε values, and then corrects the calculated structural response distribution to account for the mean ε expected for the specific site and hazard level.

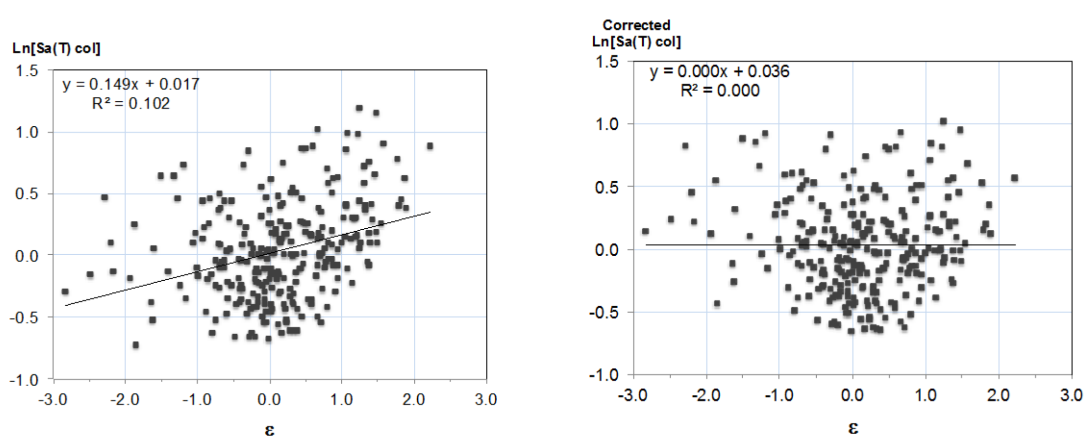

Fig. 3. Spectral accelerations necessary to produce collapse in 4-story steel moment resisting building [14] as a function of the ε of each record before (left) and after (right) applying the correction proposed in [19].

This procedure, which has been incorporated into the FEMA P-695 document [13], focuses on correcting the bias and pays no attention to the correlation of the *IM* with collapse or in the reduction of the variability/dispersion. As a matter of fact, considering ε does very little in terms of reducing the record-to-record variability and therefore the vector *IM* consisting on $Sa(T_1)$ and ε remains *a relatively inefficient intensity measure*, meaning it does not lead to a significant reduction in dispersion and hence, it still requires a large number of response history analyses in order to estimate the response of the structure with an acceptable level of confidence. Fig. 3 also shows the coefficient of determination (R^2) computed from the linear fit on the data, which is only 0.1 indicating a relatively poor measure of fit and of correlation of the collapse intensity with ε.

To illustrate this important, and often overlooked aspect of this recently proposed vector *IM*, the procedure proposed by Haselton et al. [19] to account for the effect of ε was applied to the results of the four-story building by decreasing the intensity producing collapse for records with epsilons larger than the mean epsilon in the record set and by increasing the intensity producing collapse for records with epsilons smaller than the mean epsilon in the record set. Please note that instead of using a generic slope recommended in their paper that is based on their buildings, here the best correction possible was applied by using the slope that is specific to this structure and this set of records. The corrected natural logarithms of the collapse intensities as a function of ε are presented on the right of Fig. 3. As expected, the bias (the slope of the linear trend) has now been fully eliminated, but a large dispersion remains. To get further understanding on this important result, the corrected collapse intensities for each record are plotted on the right of Fig. 3 for each ground motion in the same manner as the uncorrected collapse intensities were plotted on the left. Again 5, 50 and 95 percentiles, which are 0.58, 1.01 and 2.13, respectively, are also plotted in the figure with horizontal dashed lines. By comparing these figures it can be seen that, as previously mentioned, while considering ε corrects the bias, it does not lead to a significant reduction in dispersion. As a matter of fact, for this structure the ratio of corrected collapse intensities corresponding to 95 percentile to 5 percentile actually has increased to 3.66 which is slightly larger than the ratio of the two percentiles prior to correction for epsilon which was 3.64. The corresponding logarithmic standard deviation does reduce after the correction is applied to consider the effect of ε but the reduction is minimal, it only reduces from 0.39 to 0.37, which is only a reduction of approximately 5%.

The reason why consideration of ε does not lead to a significant reduction in dispersion is because ε is not a direct measure of spectral shape but only a proxy since a single spectral ordinate relative to the intensity measured by a GMPE cannot by itself provide a measure of spectral shape. Contrary to popular belief based on misleading information, ε alone does not provide information on whether the spectral ordinate is in a peak or a valley, just like providing the altitude of a point on Earth (height relative to sea level) cannot by itself provide an indication whether such point is in a peak or a valley with exception of extreme altitudes (e.g. above 8,000m or extreme bathymetries (e.g. -7,000m). For example, one could be in a relatively low altitude such as 200 meters above sea level and still be in the peak of a 200 meter tall mountain. One could be in a high elevation such as 2,400 meters above sea level and still be in a valley such as a location like Mexico City. Similarly, saying that a spectral ordinate has a negative epsilon, such as -1 does not necessarily imply that such spectral ordinate corresponds to a spectral valley nor a spectral ordinate that has a positive epsilon, such as 1.0 or 1.5 necessarily implies that such spectral ordinate corresponds to a spectral peak, and therefore ε is not a good measure of the intensity of a ground motion. Furthermore, several studies have shown that ε is ineffective in accounting for spectral shape in the case of near-fault pulse-like ground motions [18]. For example, Haselton et al. [19] when proposing their approximate method to consider the effect of ε explicitly wrote: "*the approach proposed in this paper should not be applied to near-fault motions with large forward-directivity velocity pulses*". This is very important because this type of ground motions is precisely the one that is more likely to produce the collapse of structures.

More recently, Eads et al. [20] proposed selecting and scaling records using an improved ground motion intensity measure Sa_{avg} which is computed as the geometric mean of spectral acceleration values between periods c_1T_1 and c_NT_1

$$Sa_{avg}\left(c_1T_1, \dots, c_NT_1\right) = \left(\prod_{i=1}^{N} Sa\left(c_iT_1\right)\right)^{1/N} \tag{3}$$

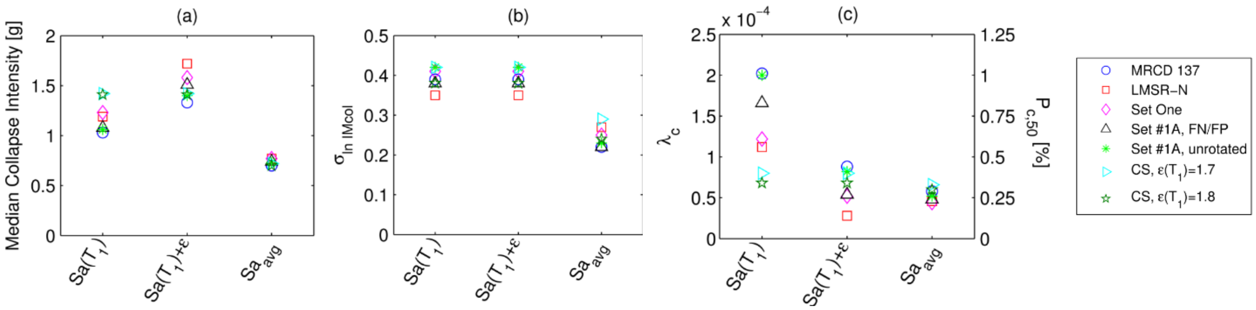

Fig. 4. Summary of collapse risk results for different IMs and different ground motion sets: (a) median collapse intensity; (b) standard deviation of collapse intensities, $\sigma_{lnIMcol}$; (c) mean annual frequency of collapse, λ_c, and probability of collapse in 50 years, $P_{c,50}$.

where N is the number of periods used to compute Sa_{avg} and the c_i terms are non-negative values. They proposed computing Sa_{avg} using a period range between $0.2{\cdot}T_1$ and $3{\cdot}T_1$. Moreover, they evaluated the efficiency and sufficiency of Sa_{avg} and compared it to the commonly used $Sa(T_1)$ and a vector IM consisting of $Sa(T_1)+\varepsilon$ for collapse prediction using nearly 700 moment-resisting frame and shear wall structures of various heights. Results of this comparison are presented in Fig. 4, which shows the median collapse capacity and dispersion of the collapse fragility function along with the mean annual frequency of collapse using the three different intensity measures when used in combination with 7 different ground motion sets. They found that, when compared to the two other *IM*s, using Sa_{avg} lead systematically in all cases to much smaller variabilities in collapse intensities while at the same time achieving more consistent and reliable estimates of the probability of collapse. In a more recent study, Eads et al. [21] concluded that one of the reasons why Sa_{avg} is a better measure of ground motion intensity is because it inherently contains information on *SaRatio*, defined as the ratio between $Sa(T_1)$ and the average spectral value over a period range, which is a direct measure of spectral shape. On the other hand, $Sa(T_1)$ or a vector IM consisting of $Sa(T_1)+\varepsilon$ are not efficient *IM*s because $Sa(T_1)$ has no information on spectral shape and ε provides spectral shape information only for very extreme values, such as those smaller than -2 or those larger than 2, and not for most typical intermediate values, making the vector IM a poor indicator of spectral shape. Fig. 5 presents a comparison of collapse intensities for the 4-story structure studied in [14] using $Sa(T_1)$ and Sa_{avg}. It is evident that a large reduction in scatter around the median value is produced when using Sa_{avg}. The figure on the left shows that the scatter when using $Sa(T_1)$ is due to differences in spectral shape and the figure of the right clearly shows that Sa_{avg} accounts for spectral shape.

4. Bias Introduced from Scaling of Records

As mentioned in the introduction, previous studies have typically estimated the collapse fragility function by using Incremental Dynamic Analysis (IDA) [8] in which the model of a structure is subjected to a set of ground motions scaled to a common *IM* and at increasing levels of intensity. In a typical IDA, the intensity of the records is varied by more than a factor of ten. Furthermore, the scarcity of recorded ground motions with high intensities leads to having to scale up many records in order to produce the collapse of the structure. The validity of scaling records is a controversial topic in both the seismological and engineering communities. While early

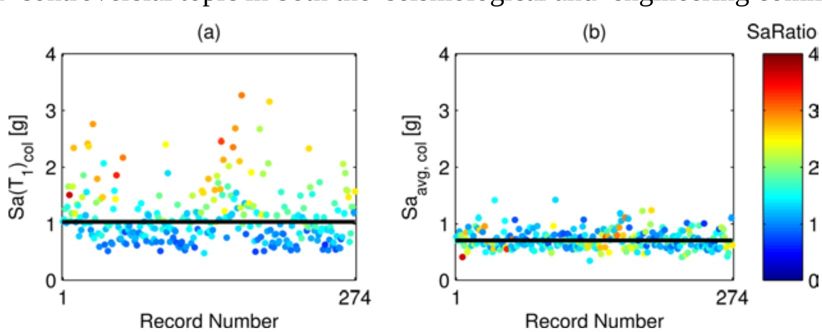

Fig. 5 Collapse intensity for individual records for (a) IM = $Sa(T_1)$; (b) IM = Sa_{avg}.

studies by Cornell and his associates (e.g., [8, 15-17]) reported that no bias was introduced in the seismic results by the scaling of the records, several recent studies have reported biases in the results (e.g., [22-24]). In current practice, no specific limits to scaling exists, and for example in [18] Baker and Cornell mention that there is no need to pay attention to the magnitude, distance or scaling of the record provided that records are selected based on ε. In general, when using IDA currently most people believe that, if records are selected adequately then the scaling of the records in the analysis will not induce bias in displacement demands.

The first and third authors of this paper recently conducted an investigation to further evaluate possible biases in seismic response introduced by record scaling. These studies included both SDOF and MDOF structures over a wide range of periods and types of hysteretic response. Figure 5 presents results on two SDOF systems subjected to a set of sixty ground motions scaled to reach the maximum considered earthquake, MCE, intensity at a site in downtown Palo Alto, California in the United States. The ground motions used consist of two sets of thirty records. The first set, Set A, had a mean scaling factor of 1.0, in which either no scaling was needed or minimal scaling, up or down, was needed to have a spectral ordinate equal to the target intensity. The other set, Set B, had a mean scaling factor of 10, in which records had to be strongly scaled in order to reach a spectral ordinate equal to the target intensity. This figure shows results for systems having a period of vibration of 0.25s and 1.5s and both of them having a 5% damping ratio. Each system has a bilinear hysteretic behavior with a yielding strength corresponding to a strength reduction factor of five, that is, they have a lateral strength of one fifth of the one required to maintain the systems elastic (R=5). Fig. 5. shows the displacements of the two systems subjected two both sets against the corresponding scale factor. Subpanels (a) and (b) correspond to displacement estimates of the systems with a 3% (positive) post-elastic stiffness. We can observe there that when subjected to Set B, the short period system has a mean displacement that is 1.61 times greater than the one computed using Set A, clearly indicating that an important bias is introduced when large scale factors are used despite the intensity of the records being the same in both sets once they are scaled. Moreover, the low p-value indicates a statistically significant correlation between the scale factor and the lateral displacement demand indicating that the bias cannot be neglected. On the other hand, interestingly, only a very small bias, which is not statistically significant, occurs in the longer period system. Subpanels (c) and (d) in Fig. 5 correspond to displacement estimates of SDOF systems with the same periods of vibration, damping and lateral strengths as those previously discussed but now the systems have a bilinear system with a negative 3% post-elastic stiffness. As indicated in the figure, the negative post-elastic system introduced a large bias in the number of collapses. For example, for the system with T=1.5s none of the 30 records in Set A (with mean scale factor of 1.0) produce collapse whereas five out of the thirty records (17% of the records) in Set B (with mean scale factor of 10) produce collapse. The bias is also very strong in the case of the short period system where 43% of the records in Set A produce collapse and 77% of those in Set B produce the collapse of the system. It should be pointed out that no consideration regarding spectral shape was made when assembling the record set, however, similar results were found when the conditional spectrum method was used for record selection, indicating that using the conditional mean spectrum alone does not preclude the introduction of bias in the results and that limits on scaling factors are necessary.

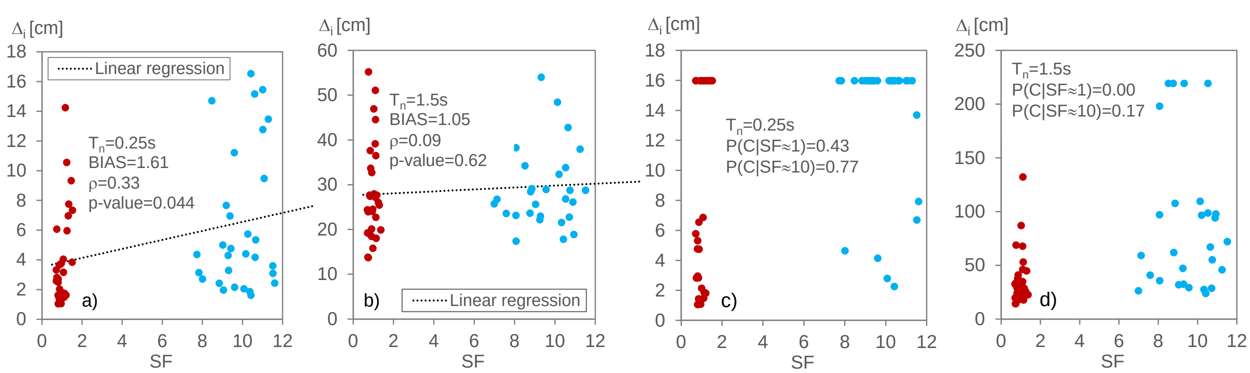

Fig. 5 – Influence of scaling factor on structural response. Subpanel (a) and (b) bilinear behavior with 3% post-elastic stiffness and R=5; (c) and (d) bilinear behavior with -3% post-elastic stiffness and R=5.

5. Enhanced Two Stripe Analysis

Assessment of the probability of collapse is a computationally demanding task. The computational effort involved in the numerical integration in Eq. (2) is minimal, so the computational demands are associated with the NRHA necessary to estimate the collapse fragility function. That is why any method that reduces the required NRHAs is of great interest to practicing structural engineers. Eads et al. [14] proposed an efficient method to compute the mean annual frequency of collapse. Efficiency is achieved by reducing the total NRHA that are required to obtain a good estimate of the collapse fragility function. By noting that the fragility function could be approximated relatively well by a cumulative lognormal distribution function which is fully defined by only two parameters, they proposed conducting NRHA at only two ground motion intensity levels, which in turn consisted in finding two points of the collapse fragility function and solving a system of two equations with two unknowns to find the parameters of the lognormal distribution. They highlighted that ordinates of the collapse fragility function have a binomial distribution and therefore it was more convenient to use more ground motions at two intensity levels rather than using a relatively small set of ground motions but scaled at many intensity levels in an IDA as had been done in previous studies. They demonstrated that the larger number of ground motions leads to smaller epistemic uncertainty on the ordinates of the collapse fragility function and that therefore a more reliable estimate of the mean annual frequency could be obtained for a given model of a structure while at the same time reducing the computational effort.

More recently, further study by the first and third authors of the various decisions involved in the two-stripe analysis has been conducted leading to an *Enhanced Two Stripe Analysis, E2SA* [25]. The improved method is based on further studies such as: (1) if 2 stripes is better than 3 stripes; (2) if selection of the intensity levels at which stripe analyses should be selected based on specific ordinates of the estimate of the probability of collapse or if it is better to base the selection on deaggregation results; (3) number of analysis to conduct on each stripe; and (4) sequence of the stripe analyses and updating of the location of the second stripe based on the results obtained in the first stripe for situations in which initial estimates of the parameters are not adequate.

A small number of representative results are presented here. Fig. 6 compares the results of a 2SA using a total of only 60 NRHA, 30 at each stripe, versus a 3SA with 20 NRHA at each stripe. The total number of NRHA in both cases is the same. A total of 10,000 estimations (realizations) of λ_c were computed by considering three different sites in the U.S. with very different seismic hazard curves. The median collapse capacity of the structure at each site was defined as the one that produced a λ_c equal to 1.75E-04, while σ_{lnSa} was kept equal to 0.4. Two metrics were used to evaluate the different types of analyses: the median ratio of the approximate to exact λ_c at each site and the width of the 95% confidence intervals in the 10,000 individual estimates of λ_c. The former parameter is a measure of the possible bias introduced by the method and the latter is a measure of the variability around the mean value. It can be seen that both the two- and the three stripe analyses provide mean ratios very close to one, meaning no significant bias is introduced when using a small number of stripes to estimate the collapse fragility function, but that confidence intervals shown in the right side are consistently smaller for the two-stripe analyses with 30 NRHA at each stripe relative to the three-stripe analysis with 20 NRHA at each stripe.

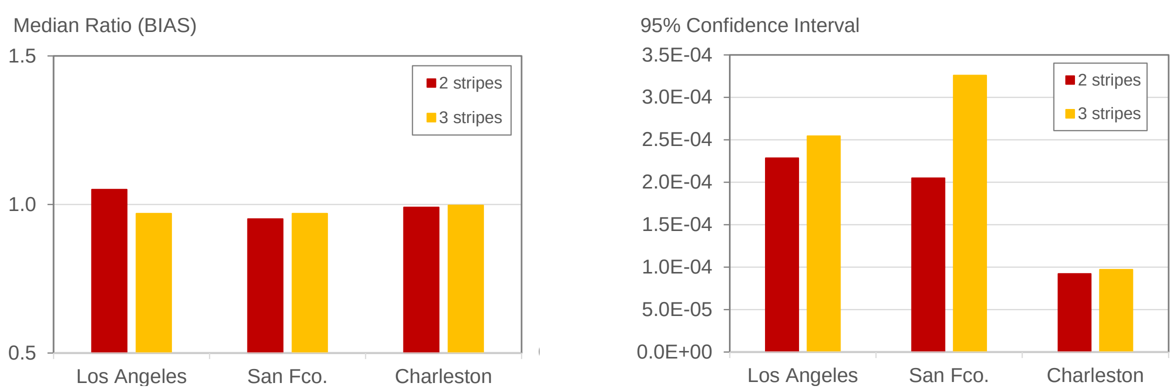

Fig. 6 – Study on alternative number of stripes (number of intensity levels).

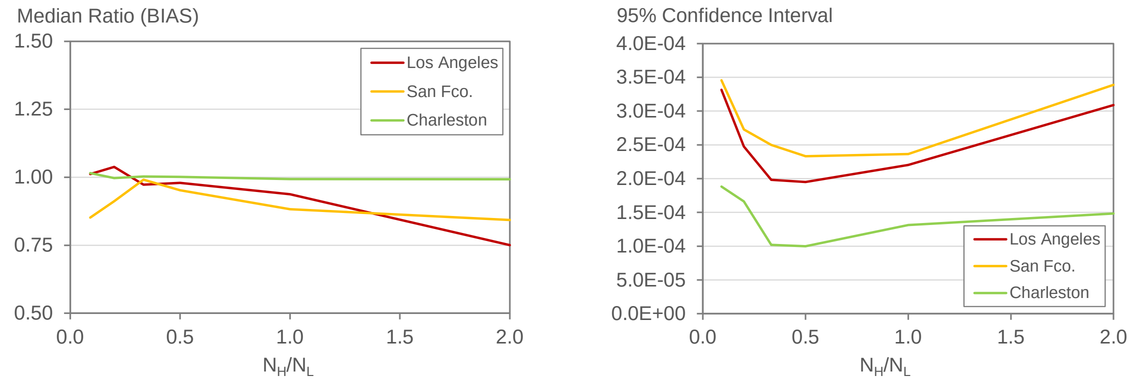

Fig. 7 – Optimum number of analyses at each ground motion intensity level.

Fig. 7 compares median bias factors and 95% confidence intervals computed with 10,000 estimations of mean annual frequency of collapse λ_c computed with different number of analysis in the higher intensity stripe, N_H and in the lower intensity stripe N_L. Again, 60 analyses were used at each N_H/N_L value. As shown in these figures, using N_H/N_L ratios in the order of 0.4 to 0.5, that is, to use 2 to 2.5 times more ground motions in the lower intensity level than in the higher intensity levels leads to median ratios that are very close to one while at the same time minimizes the variability in the estimate. This is expected as Eads et al. in [14] previously observed that the lower tail of the collapse fragility curve is a major contributor to λ_c. Results N_H/N_L =1.0 in this case correspond to the ratio proposed by Eads et al. in [14]. It can be seen that the proposed E2SA leads to smaller variabilities by conducting more NRHAs where larger contribution to λ_c is expected.

Summarizing, the *E2SA* method consists of conducting NRHA at two ground motion intensity levels that correspond to a 35% and 70% of the cumulative contribution to λ_c (IM_L and IM_H respectively). Better results are obtained if an improved ground motion intensity measure *IM* is used such as Sa_{avg} but the method can be used in other *IMs* such as $Sa(T_1)$. The intensity at which the high intensity analyses are to be conducted, IM_H, is determined by conducting a deaggregation of λ_c assuming an initial collapse fragility curve (CFC). Then, N_H analyses should be conducted at IM_H and the proportion of collapses is used to compute an updated estimate of the CFC. Using this new estimate of the CFC, a second deaggregation is conducted and IM_L is found. Then, N_L NRHA are conducted at IM_L and the proportion of collapses is used to determine the final estimate of the CFC and, finally λ_c is computed using Eq. 2, with the collapse fragility curve and the slope of the seismic hazard curve at the site.

Recently, Gokkaya et al. [26] adopted the 2SA proposed by Eads et al. in [14] however they proposed a Bayesian updating approach to obtain a posteriori (updated) version of the CFC by combining an apriori estimate

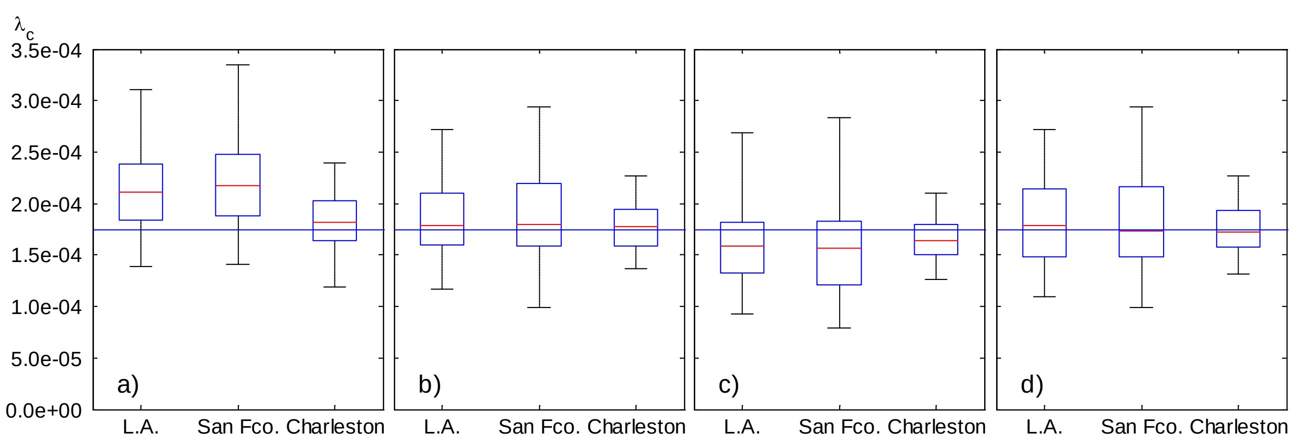

Fig. 8 – Comparison of the distribution of λ_c estimates obtained using the Bayesian Method (BM) and the E2SA. a) -30% initial BIAS and BM; b) -30% initial BIAS and E2SA; c) 30% initial BIAS and BM; d) 30% initial BIAS and E2SA. The real λ_c equals to 1.75E-4 and it is indicated with a continuous blue line.

of the CFC with results from the 2SA. Their method provides unbiased results when the apriori CFC is unbiased, however, where the initial estimate of the median collapse capacity underestimates or overestimates the actual value, the method produces biased estimates of λ_c. To illustrate the benefits of the proposed *E2SA* Fig. 8 shows the distribution of λ_c estimates computed using the Bayesian Method proposed in [26] to those computed with the E2SA. Results presented in figures 8a and 8b corresponds to cases where the initial estimates of the median collapse capacity of the structure underestimates by 30% the actual median collapse capacity, while 8c and 8c corresponds to cases where the initial estimates of the median collapse capacity of the structure overestimates by 30% the actual median collapse capacity. Results clearly illustrate that the proposed *E2SA* method generates practically unbiased median estimates while maintaining similar dispersion.

6. Summary and Conclusions

Avoidance of collapse is the most important objective of earthquake resistant design. However, current building codes do not include methods to verify that the structure has adequate safety against collapse. Computing the mean annual frequency of collapse or the probability of collapse during the life of the structure is a technically challenging and computationally demanding task. It requires having adequate methods to select ground motions, knowing how many records to use, the best measures of ground motion intensity to use and the number and location of intensity levels at which to conduct nonlinear response history analyses.

It has been shown that methods based on the probability of collapse at a single level of intensity or based on the estimation of the median collapse intensity are not good methods. In particular, the authors have proposed methods that are computationally more efficient and produce more reliable results such as those in FEMA P695.

Results have been presented that show that selecting records based on ε or using this parameter to correct collapse intensities, while it may reduce the bias in the results, still leads to a record-to-record variability which is not much better than the one obtained using the spectral ordinate at the fundamental period as the ground motion intensity measure. This is because ε is not a good measure of spectral shape and therefore fails to reduce record to record variability and hence requires a larger number of ground motions to produce good results. Furthermore, selection of records based on ε can bias the results depending on the ground motions that are selected. Meanwhile using improved *IM* such as Sa_{avg} leads to systematic reductions in variability while at the same time producing more consistent estimates of the mean annual frequency of collapse for different ground motion sets.

An improved method to estimate the mean annual frequency of collapse, referred to as Enhanced Two Stripe Analysis, E2SA, has been presented and results compared to other methods have been summarized.

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8. References

[1] Jennings, PC, Husid, R. (1968). Collapse of yielding structures during earthquakes. *Journal of Engineering Mechanics Division (ASCE)* 1968; **94**(5): 1045-1065.

[2] Ger, J-F, Cheng, FY, Lu, L-W. (1993). Collapse behavior of Pino Suarez building during 1985 Mexico City earthquake. *Journal of Structural Engineering (ASCE)* 1993; **119**(3): 852-870.

[3] Challa, VRM, Hall, JF. (1994). Earthquake collapse analysis of steel frames. *Earthquake Engineering and Structural Dynamics* 1994; **23**(11): 1199-1218.

[4] Gupta, A, Krawinkler, H. (1999). Seismic demands for performance evaluation of steel moment resisting frame structures. *Report No. 132*, The John A. Blume Earthquake Engineering Center, Department of Civil and Environmental Engineering, Stanford University, Stanford, CA.

[5] Miranda, E, Akkar, SD. (2003). Dynamic instability of simple structural systems. *Journal of Structural Engineering (ASCE)* 2003; **129**(12): 1722-1726.

[6] Vamvatsikos, D., Miranda, E. and Akkar, S. (2009). Estimating the dynamic instability of oscillators with non-trivial backbones. In *Proc. 2nd International Conference on Computational Methods in Structural Dynamics and Earthquake Engineering (COMPDYN 2009)(Papadrakakis M, Lagaros ND, Fragiadakis M, eds).*

[7] Ibarra, LF, Krawinkler, H. (2005). Global collapse of frame structures under seismic excitations. *Report No. 152*, The John A. Blume Earthquake Engineering Center, Stanford University, Stanford, CA.

[8] Vamvatsikos, D, Cornell, CA. (2002). Incremental dynamic analysis. *Earthquake Engineering and Structural Dynamics*; **31**(3): 491-514.

[9] Zareian, F, Krawinkler, H. (2007). Assessment of probability of collapse and design for collapse safety. *Earthquake Engineering and Structural Dynamics*; **36**(13): 1901-1914.

[10] Haselton, CB, Deierlein, GG. (2008). Assessing seismic collapse safety of modern reinforced concrete moment-frame buildings. *PEER Report 2007/08*, Pacific Earthquake Engineering Research Center, Univ. of California, Berkeley, CA.

[11] Liel, AB, Deierlein, GG. (2008). Assessing the collapse risk of California's existing reinforced concrete frame structures: Metrics for seismic safety decisions. *Report No. 166*, The John A. Blume Earthquake Engrg. Center, Stanford University, Stanford, CA.

[12] ASCE (2017). Minimum Design Loads for Buildings and Other Structures, American Society of Civil Engineers ASCE/SEI 7-16, Reston, VA

[13] FEMA (2009). Quantification of Building Seismic Performance Factors (FEMA P695 Report), prepared by Applied Technology Council for the Federal Emergency Management Agency. Washington, D.C.: FEMA

[14] Eads, L., Miranda, E., Krawinkler, H. and Lignos, D.G., (2013). An efficient method for estimating the collapse risk of structures in seismic regions. *Earthquake Engineering & Structural Dynamics*, *42*(1), pp.25-41.

[15] Shome, N., & Cornell, C. A. (1998). Normalization and scaling accelerograms for nonlinear structural analysis. In *Sixth US Nat. Conf. on Earthquake Engineering, Earthquake Egnineering Research Institute, Seattle, WA.*

[16] Shome, N., Cornell, C. A., Bazzurro, P., & Carballo, J. E. (1998). Earthquakes, records, and nonlinear responses. *Earthquake Spectra*, *14*(3), 469-500.

[17] Bazzurro, P., & Cornell, C. A. (2002). Vector-valued probabilistic seismic hazard analysis (VPSHA). In *Proceedings of the 7th US national conference on earthquake engineering* (pp. 21-25).

[18] Baker, J.W. and Cornell, C.A., (2006). *Vector-valued ground motion intensity measures for probabilistic seismic demand analysis*. Pacific Earthquake Engineering Research Center, University of California, Berkeley.

[19] Haselton, C. B., Baker, J. W., Liel, A. B., & Deierlein, G. G. (2009). Accounting for ground-motion spectral shape characteristics in structural collapse assessment through an adjustment for epsilon. *Journal of Structural Engineering*, *137*(3), 332-344.

[20] Eads, L., Miranda, E. and Lignos, D.G., (2015). Average spectral acceleration as an intensity measure for collapse risk assessment. *Earthquake Engineering & Structural Dynamics*, *44*(12), pp.2057-2073.

[21] Eads, L., Miranda, E. and Lignos, D., (2016). Spectral shape metrics and structural collapse potential. *Earthquake Engineering & Structural Dynamics*.

[22] Luco N, Bazzurro P (2007): Does amplitude scaling of ground motion records result in biased nonlinear structural drift responses? *Earthquake Engineering and Structural Dynamics*, **36** (13), 1813-1836.

[23] Zacharenaki A, Fragiadakis M, Assimaki D, Papadrakakis M (2014): Bias assessment in Incremental Dynamic Analysis due to record scaling. *Soil Dynamics and Earthquake Engineering*, **67**, 158-168.

[24] Davalos H, Miranda E (2016): Evaluation of the bias in computed seismic response due to record scaling. *Earthquake Engineering and Structural Dynamics* (under review).

[25] Davalos H, Miranda E (2016): Enhanced two stripe analysis for efficient estimation of the probability of collapse. *Earthquake Engineering and Structural Dynamics* (under review).

[26] Gokkaya BU, Baker JW, Deierlein GG (2015): Illustrating a Bayesian approach to seismic collapse risk assessment. *12th International Conference on Applications of Statistics and Probability in Civil Engineering, ICASP12*, Vancouver, Canada.