Variational phase field model for dynamic brittle fracture

Bleyer J., Roux-Langlois C., Molinari J-F.

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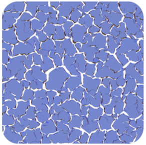

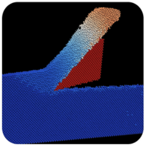

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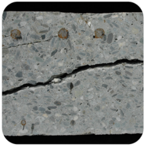

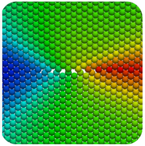

Outline

Mechanisms of dynamic fracture

Variational phase-field model of brittle fracture

Crack branching in homogeneous medium

Crack propagation in heterogeneous medium

Crack velocity

Limiting crack velocity : in theory, $v_{lim} = c_R$ for mode I
never attained in experiments, rarely exceed $0.4 - 0.7c_R$
seems to depend on experimental setup (geometry, loading conditions)

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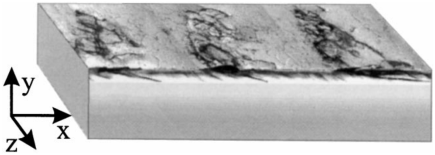
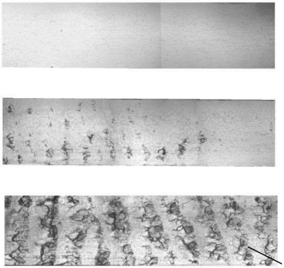
explained by **crack tip instabilities** [Sharon and Fineberg, 1996]:

- microbranching ($\sim 0.4c_R$) : small (1-100 μm in PMMA) short-lived micro-cracks, highly localized

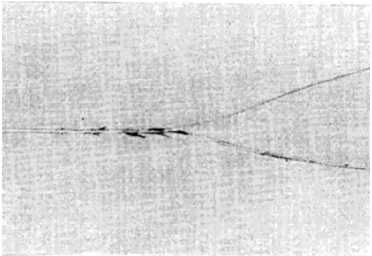




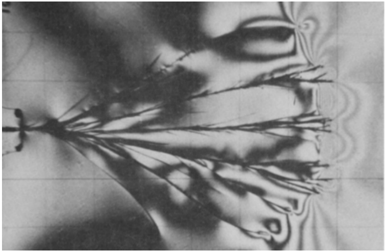
- mirror, mist, hackle patterns

Crack branching

Macroscopic branching at even higher velocities



[Ramulu and Kobayashi, 1984]



[Kobayashi and Mall, 1977]

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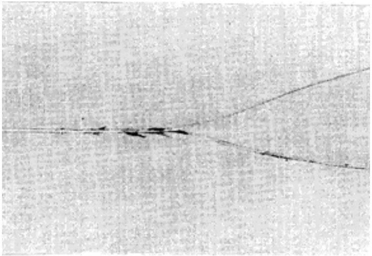

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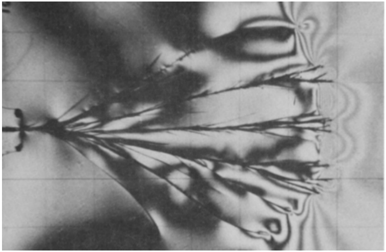


[Kobayashi and Mall, 1977]

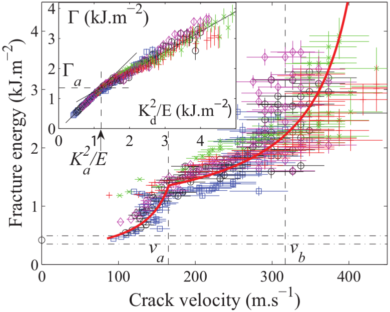
Criterion for branching ? question is still open...

- experiments and numerical simulations seem to exclude a criterion based (only) on crack tip velocity
- existence of a critical SIF or ERR ?

Velocity-toughening mechanism

Experiments on PMMA report a strong increase of apparent fracture energy with velocity : **velocity-toughening mechanism**

- a large part is attributed to an increase of created fracture surface due to microbranching
- recent experiments show an increase from 400 J/m^2 to 1 200 J/m^2 between $0.11c_R$ and $0.18c_R$ [Scheibert et al., 2010]



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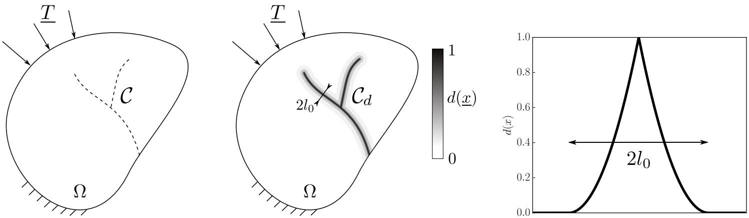
Phase-field approach

- alternative to cohesive elements or XFEM for simulating crack propagation
- non-local approach : continuous scalar field $d(\underline{x})$ representing the crack + a regularization length l_0 [Bourdin et al., 2000]
- can be formulated as a **damage gradient model**



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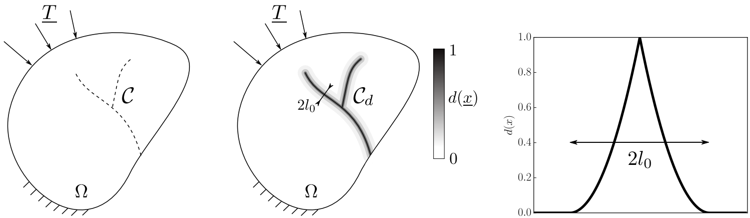

- convergence to Griffith theory when $l_0/L \to 0$, at least for quasi-static propagation

Phase-field approach

Many constitutive modeling choices are possible, we follow [Li et al., 2016]

- elastic strain energy density :

$$\psi(\underline{\underline{\varepsilon}}, d) = (1-d)^2 \left(\frac{\kappa}{2} \langle \mathrm{tr}\, \underline{\underline{\varepsilon}} \rangle_+ + \mu \underline{\underline{\varepsilon}}^d : \underline{\underline{\varepsilon}}^d \right) + \frac{\kappa}{2} \langle \mathrm{tr}\, \underline{\underline{\varepsilon}} \rangle_-$$

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- non-local fracture energy :

$$w_{frac}(d, \underline{\nabla d}) = \frac{3G_c}{8l_0} \left(d + l_0^2 \|\underline{\nabla d}\|^2 \right)$$

Remark : existence of an elastic phase for this model

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Numerical resolution using a staggered approach :

- minimization of total energy with respect to u : explicit dynamics
- minimization with respect to d : quadratic function with bound constraints ($d_n \leq d_{n+1} \leq 1$) to enforce damage irreversibility

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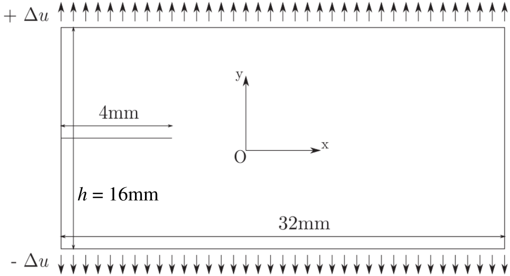
Prestrained plate geometry

Prestrained PMMA plate, fixed boundaries [Zhou, 1996]

$E = 3.09$ GPa, $\nu = 0.35$, $\rho = 1180$ kg/m^3, $G_c = 300$ J/m^2, $c_R = 906$ m/s



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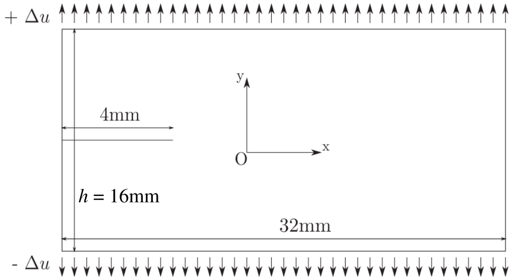
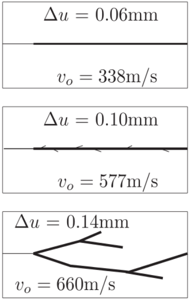
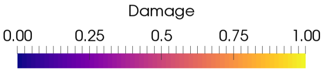


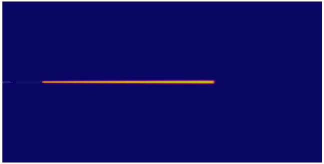
Crack patterns



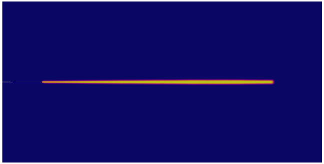
- strip geometry $\Gamma = 2E(\Delta U)^2/h \Rightarrow$ crack should accelerate to c_R
- transition from straight propagation to branched patterns
- apparent toughness increases with loading/crack velocity

Prestrained plate geometry

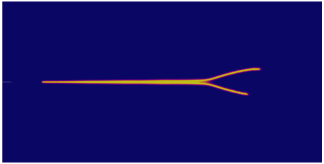




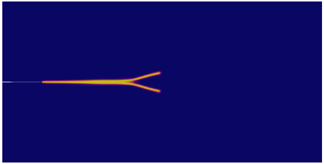
(a) $\Delta U = 0.035$ mm at $t = 40$ μs



(b) $\Delta U = 0.038$ mm at $t = 40$ μs



(c) $\Delta U = 0.040$ mm at $t = 40$ μs



(d) $\Delta U = 0.045$ mm at $t = 20$ μs

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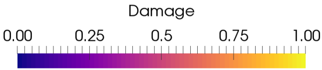

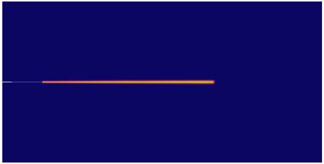

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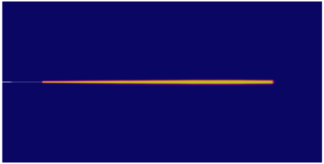

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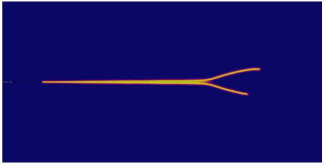

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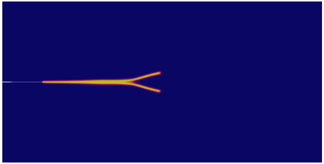
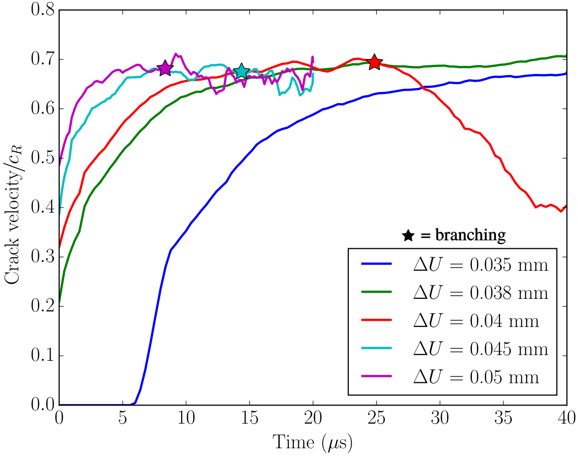


(d) $\Delta U = 0.045$ mm at $t = 20$ μs

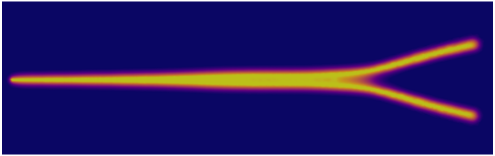
however : branching occurs at smaller load levels than in experiments, crack is too fast $\Rightarrow$ same problem with CZM, non-local integral approach

Crack velocities



no evident decrease of crack speed after branching
limiting velocity around $0.68c_R$

Damage zone thickening



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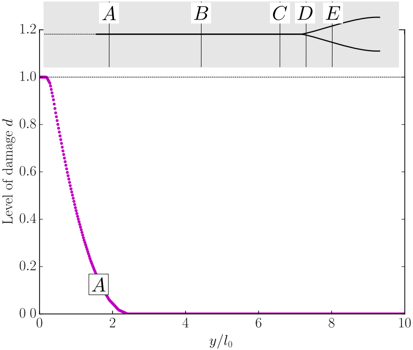

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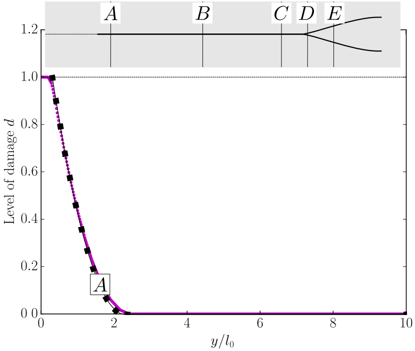

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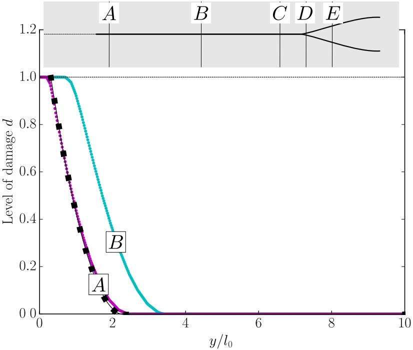

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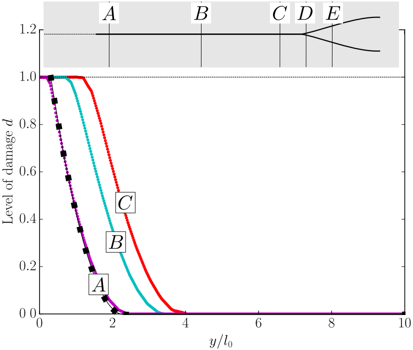

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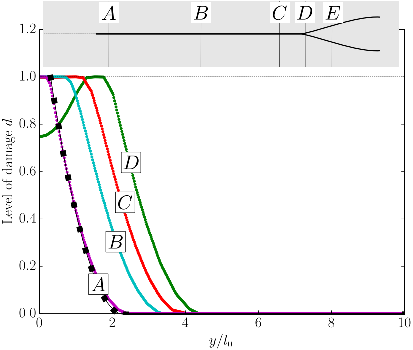

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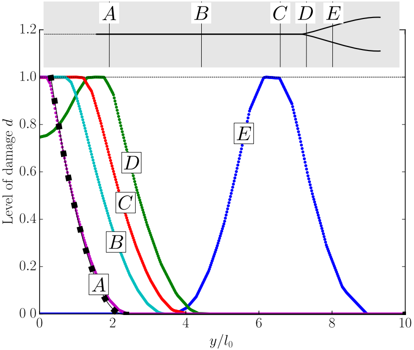

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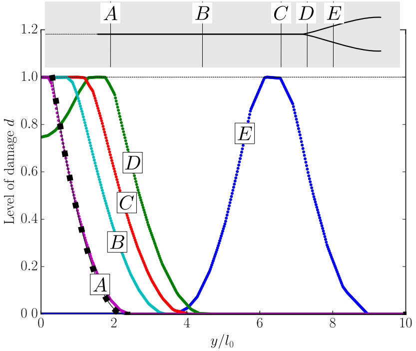
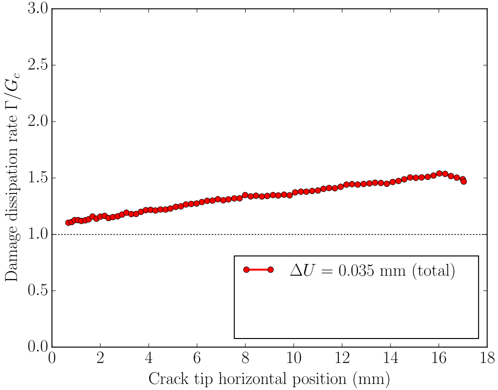


- progressive thickening of the damaged band before branching
- similar observation using peridynamics
- branching viewed as a progressive transition from a widening crack to two crack tips screening each other
- branching angle seems to depend on geometry

Apparent fracture energy

Damage dissipation rate $\Gamma = dE_{frac}/da$ interpreted as the apparent fracture energy



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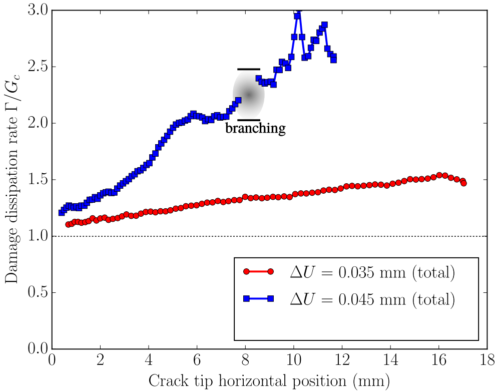

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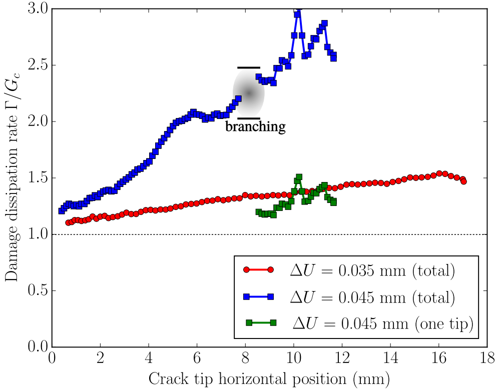

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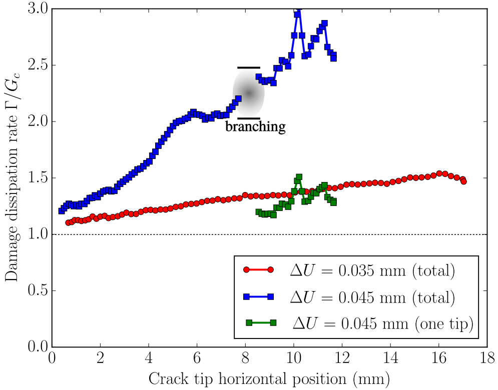
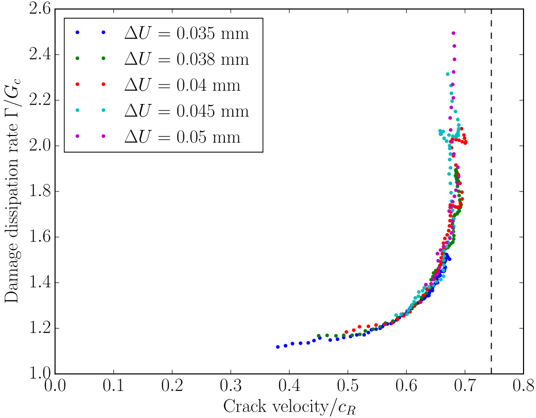


suggests a critical value of $\Gamma \approx 2G_c$ associated to branching

Velocity-toughening mechanism

during propagation and before macroscopic branching



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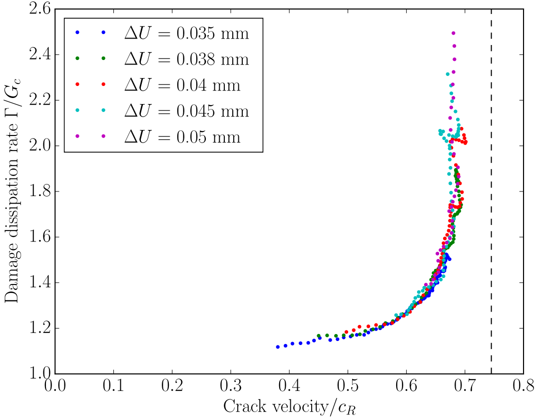
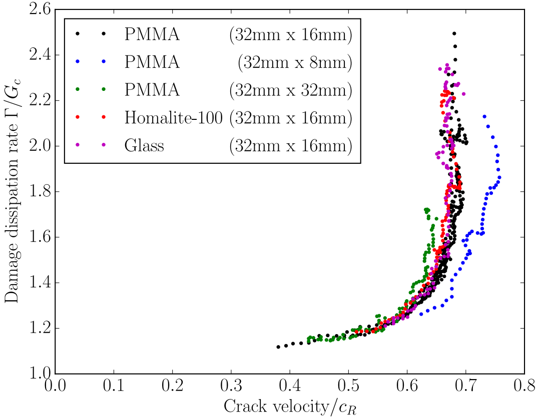


existence of a well-defined $\Gamma(v)$ relationship associated to a velocity-toughening mechanism

Velocity-toughening mechanism



the $\Gamma(v)$ relationship seems material-independent but geometry-dependent

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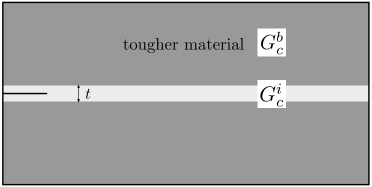
Propagation in constrained path

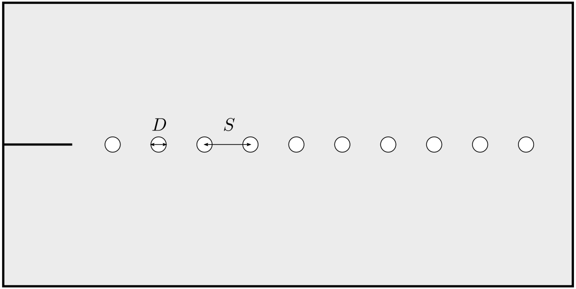
experiments report that crack can reach c_R if constrained in a weak plane [Washabaugh and Knauss, 1994]



Loading ΔU (mm)	Stored energy (N/m)	Crack velocity (c_R)
0.04	618	0.81
0.05	966	0.87
0.10	3,863	0.94
0.15	8,691	0.98

Propagation in constrained path

idem for a series of holes on crack path



$D = 0.4$ mm and $S = 0.9$ mm

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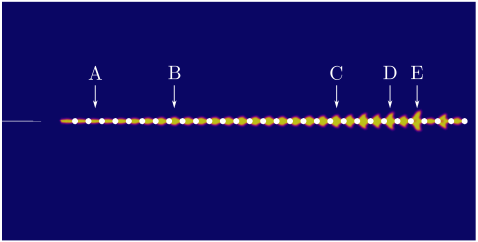

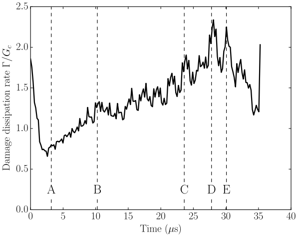

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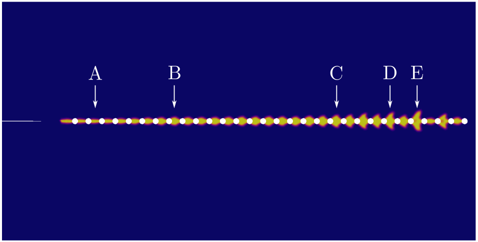
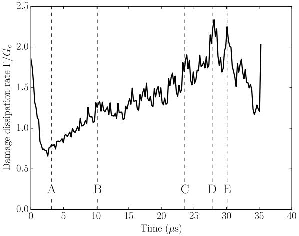
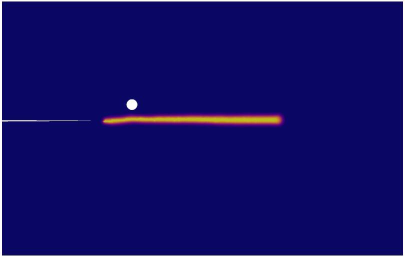




- velocity of $0.9c_R$ for $\Delta U = 0.05$ mm
- shares qualitative similarities the nucleation and growth of microcracks interacting with defects
- the apparent fracture energy is much higher than the average toughness $G_{c,weak} = (1 - D/S)G_c \approx 0.56G_c$

Interaction with distant heterogeneities

crack passing near a hole



1mm from notch

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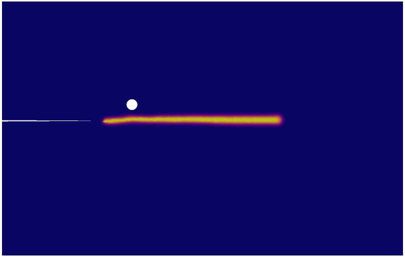

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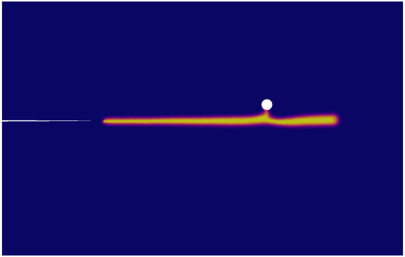

6mm from notch

Interaction with distant heterogeneities

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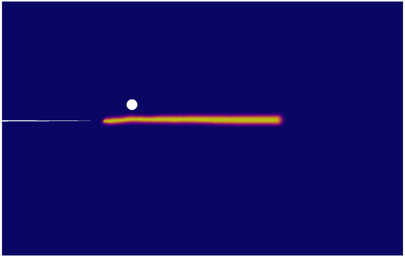

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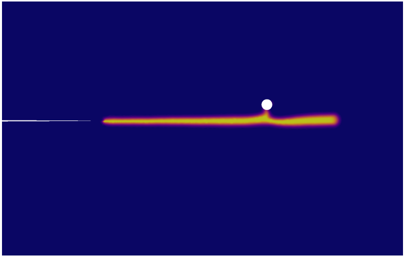
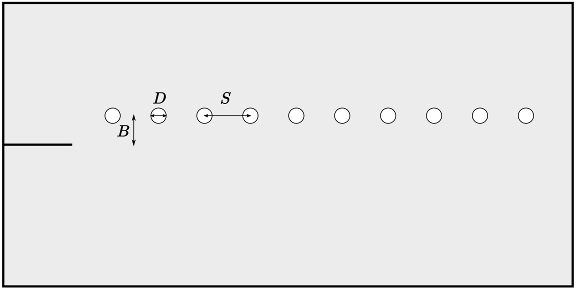


6mm from notch

- velocity of the crack tip is larger in the second case
- crack is more attracted : different near-tip stress fields ? faster crack looks for other ways of dissipating energy ?

Interaction with out-of-plane heterogeneities

Configuration with an array of holes located away from the middle plane



$B = 0.5$ mm offset, $\Delta U = 0.04$ mm

$B = 0.5$ mm offset, $\Delta U = 0.05$ mm

$B = 0.6$ mm offset, $\Delta U = 0.04$ mm

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Conclusions and perspectives

Conclusion : some physical aspects of dynamic fracture can be reproduced with the phase-field approach

- propagation characterized by a damage band widening
- widening associated to an increase of the apparent fracture energy
- existence of a well-defined $\Gamma(v)$ relationship
- macroscopic branching observed when $\Gamma \geq 2G_c$
- existence of a limiting velocity around $0.7c_R$
- c_R can be reached in constrained geometries
- strong influence of heterogeneities on branching process

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Open questions

- rate-dependent model for PMMA ?
- energy-based branching criterion ?
- better understanding of 3D effects and role of defects