Optimal Load Sharing of Parallel Compressors via Modifier Adaptation

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***Abstract*— We discuss optimal load sharing of parallel gas compressors in the presence of plant-model mismatch. We formulate this problem as a static real-time optimization task and propose to tackle it by means of modifier adaptation. Under mild assumptions, the chosen approach guarantees optimal operating conditions upon convergence. Furthermore, we discuss how the specific problem structure can be exploited to estimate the plant gradients efficiently during process operation. Finally, we draw upon simulated case studies for two and six compressors to demonstrate the applicability and effectiveness of the proposed real-time optimization approach.**

***Index Terms*— real-time optimization, modifier adaptation, gas compressors, load sharing**

I. INTRODUCTION

Compressors are necessary components in many industrial processes. They provide air for combustion, recirculate fluids through processes, and transport gas through pipes and pipelines. The provision of compressed air and gas is considered to be one of the most expensive utilities in many industries. Furthermore, compressors are assumed to be among the major energy consumers in many industrial processes such as air separation [1]. Consequently, optimal operation of compressors is of considerable interest [2], [3], [4]. The available studies have considered different applications, such as natural and supply gas systems, and different system setups, such as fixed-speed drivers, variable-speed dual-shaft gas turbines and variable-speed electrical drivers [2]. In the present paper, we investigate the problem of optimal load sharing of parallel gas compressors in the presence of plant-model mismatch. A load-sharing strategy based on on-line estimation of compressor performance followed by the solution of the load-sharing optimization problem is proposed in [2]. Similarly, in [4], static regression models of compressors are built and then used to compute the best load distribution.

The aforementioned results fall into the class of so-called two-step approaches for real-time optimization (RTO) of uncertain processes [5], [6]. The two-step approach works well provided that structural plant-model mismatch is not significant and that there is sufficient process excitation for estimating the uncertain model parameters. It is well understood in the RTO community that, due to structural plant-model mismatch, two-step approaches do not necessarily converge to the plant optimum [7]. In other words, applying a two-step approach, it is usually not possible to give guarantees about optimal plant operation, see [8] for a tutorial example. Hence, other RTO approaches—such as modifier adaptation (MA) [7], [9], or the sufficient conditions for feasibility and optimality (SCFO) [10]—have been designed to enforce plant optimality upon convergence.

Herein, we rely on MA to tackle the problem of optimal load sharing of parallel compressors using inaccurate models. We will add first-order correction terms to the cost and constraint functions of the model-based optimization problem. We show that the case of parallel compressors allows exploiting the problem structure for the sake of efficient gradient estimation. The proposed RTO method addresses the problem with a fixed number of active compressors. Note, however, that the complexity of the algorithm is mostly independent of the number of compressors. We draw upon a simulated case study with six parallel compressors to demonstrate the effectiveness of the proposed approach.

The remainder of the paper is organized as follows. In Section II, we recall preliminary material, namely, the MA approach to RTO, the modeling of single compressors, and the load-sharing optimization problem. Section III proposes a MA solution to the optimal load-sharing problem. We also discuss the integration of the RTO algorithm into the existing control scheme. A case study for a compressor station is presented in Section IV, and final conclusions are provided in Section V.

II. PRELIMINARIES

A. Real-Time Optimization via Modifier Adaptation

The steady-state plant optimization problem is typically expressed as the following nonlinear program:

$$\mathbf{u}_p^\star = \arg\min_{\mathbf{u}} \quad \Phi_p(\mathbf{u}) \qquad \text{s.t.} \quad \mathbf{G}_p(\mathbf{u}) \leq \mathbf{0}, \tag{1}$$

where $\mathbf{u}$ is the n_u-dimensional vector of decision (or input) variables, Φ_p the cost function, and $\mathbf{G}_p$ the n_g-dimensional vector of constraints. The notation $(\cdot)_p$ is used to denote variables associated with the plant.

In any real world application, the functions Φ_p and $\mathbf{G}_p$ are not known accurately. Instead, the approximate models

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$\Phi(\mathbf{u},\boldsymbol{\theta})$ and $\mathbf{G}(\mathbf{u},\boldsymbol{\theta})$ are available, where $\boldsymbol{\theta}$ is a set of model parameters. Consequently, the solution to the original problem (1), $\mathbf{u}_p^\star$, can be approximated by solving the following model-based optimization problem:

$$\mathbf{u}^\star(\boldsymbol{\theta}) = \arg\min_{\mathbf{u}} \quad \Phi(\mathbf{u},\boldsymbol{\theta}) \qquad \text{s.t.} \quad \mathbf{G}(\mathbf{u},\boldsymbol{\theta}) \le \mathbf{0}. \tag{2}$$

In the presence of plant-model mismatch and changing operating conditions, since the solution $\mathbf{u}^\star$ obtained with the model does not generally match the plant optimum $\mathbf{u}_p^\star$, some adaptation strategy is needed.

In order to converge to the plant optimum, Problem (2) should be adapted in such a way that, upon convergence, the necessary conditions of optimality (NCO) of the modified problem match those of Problem (1) [11]. For instance, this can be achieved by modifying the model so as to enforce matching of the values of the constraints and the gradients of the cost and constraint functions at each RTO iteration by means of first-order correction terms. The resulting RTO scheme is referred to as MA in the literature [7], [12]. At the kth RTO iteration, the next input values are computed by solving the following modified optimization problem:

$$\mathbf{u}_{k+1}^\star = \arg\min_{\mathbf{u}} \ \Phi^{\text{mod}}(\mathbf{u},\boldsymbol{\theta}_0) := \Phi(\mathbf{u},\boldsymbol{\theta}_0) + (\boldsymbol{\lambda}_k^\Phi)^\mathsf{T}\mathbf{u} \tag{3}$$
$$\text{s.t. } \mathbf{G}^{\text{mod}}(\mathbf{u},\boldsymbol{\theta}_0) := \mathbf{G}(\mathbf{u},\boldsymbol{\theta}_0) + \boldsymbol{\varepsilon}_k^\mathbf{G} + (\boldsymbol{\lambda}_k^\mathbf{G})^\mathsf{T}(\mathbf{u}-\mathbf{u}_k) \le \mathbf{0},$$

where $\boldsymbol{\theta}_0$ are the *nominal* model parameters, $\boldsymbol{\varepsilon}_k^\mathbf{G} \in \mathbb{R}^{n_g}$ are the constraint value modifiers, $\boldsymbol{\lambda}_k^\Phi \in \mathbb{R}^{n_u}$ are the cost gradient modifiers, and $\boldsymbol{\lambda}_k^\mathbf{G} \in \mathbb{R}^{n_u \times n_g}$ are the constraint gradient modifiers. These modifiers represent the differences between plant and model quantities (namely, the values of the constraints as well as the cost and constraint gradients) at the current operating point $\mathbf{u}_k$:

$$\boldsymbol{\varepsilon}_k^\mathbf{G} = \mathbf{G}_p(\mathbf{u}_k) - \mathbf{G}(\mathbf{u}_k,\boldsymbol{\theta}_0), \tag{4a}$$
$$(\boldsymbol{\lambda}_k^\Phi)^\mathsf{T} = \frac{\partial \Phi_p}{\partial \mathbf{u}}(\mathbf{u}_k) - \frac{\partial \Phi}{\partial \mathbf{u}}(\mathbf{u}_k,\boldsymbol{\theta}_0), \tag{4b}$$
$$(\boldsymbol{\lambda}_k^\mathbf{G})^\mathsf{T} = \frac{\partial \mathbf{G}_p}{\partial \mathbf{u}}(\mathbf{u}_k) - \frac{\partial \mathbf{G}}{\partial \mathbf{u}}(\mathbf{u}_k,\boldsymbol{\theta}_0). \tag{4c}$$

The next operating point is obtained by filtering the inputs as follows:

$$\mathbf{u}_{k+1} = \mathsf{K}\mathbf{u}_{k+1}^\star + (\mathbb{I}-\mathsf{K})\mathbf{u}_k, \tag{5}$$

where K is a diagonal filter matrix of dimension $(n_u \times n_u)$ with eigenvalues in the interval $(0,1]$. Alternatively, it is possible to filter the modifiers [7]. The main advantage of MA is that, upon convergence and in the absence of measurement noise, the operating point $\mathbf{u}_\infty$ satisfies the NCO of Problem (1) [7].

B. Modeling a Single Compressor

The compressor model used in this study is based on the work of [3]. As depicted in Fig. 1, the main element is the compressor, which is surrounded by piping and valves. The main line starts with a suction side valve $('in')$ that is used to regulate the suction pressure. A recycle line connects the compressor outlet with the compressor inlet, and the flow

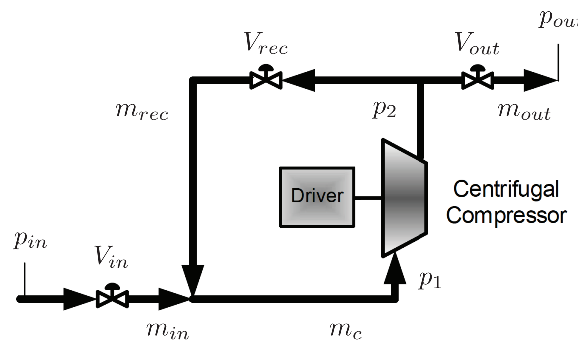

Fig. 1. Diagram of a single compressor.

through the recycle line is regulated by the opening of the anti-surge valve $('rec')$. The discharge side valve $('out')$ is used to regulate discharge pressure. The dynamic compressor model can be written as [3]:

$$\frac{dp_1}{dt} = C_1(m_{in} + m_{rec} - m_c) \tag{6a}$$
$$\frac{dp_2}{dt} = C_2(m_c - m_{rec} - m_{out}) \tag{6b}$$
$$\frac{dm_c}{dt} = C_3(p_1\Pi - p_2) \tag{6c}$$
$$\frac{d\omega}{dt} = C_4(\tau - \tau_{comp}) \tag{6d}$$
$$\frac{dm_{rec}}{dt} = C_5(m_{rec,ss} - m_{rec}), \tag{6e}$$

where m_c, m_{rec}, m_{in} and m_{out} denote the compressor, recycle, inlet, and outlet flows, respectively; p_1 is the suction pressure, p_2 the head pressure, Π the pressure ratio, ω the rotational speed of the shaft, τ the applied torque, while $C_i, i = 1, ..., 5$, are constant parameters. Furthermore, one can write:

$$m_{in} = k_{in}V_{in}\sqrt{|p_{in} - p_1|} \tag{6f}$$
$$m_{out} = k_{out}V_{out}\sqrt{|p_2 - p_{out}|} \tag{6g}$$
$$m_{rec,ss} = k_{rec}V_{rec}\sqrt{|p_2 - p_1|} \tag{6h}$$
$$\tau_{comp} = \sigma r \omega m_c, \tag{6i}$$

where p_{in} is the inlet pressure and p_{out} the outlet pressure. V_{in}, V_{out} and V_{rec} are the openings of the inlet, outlet, and recycle valves, respectively, and k_{in}, k_{out}, and k_{rec} are the corresponding valve gains. Eq. (6i) relates the compressor torque to the flow and the compressor speed, where σ is the slip factor and r the radius of the shaft.

The pressure ratio Π is modeled as a polynomial function of ω and m_c. Likewise, the polytropic efficiency η_p is modeled as a polynomial function of ω and Π [2]:

$$\Pi = \alpha_1 + \alpha_2\omega + \alpha_3 m_c + \alpha_4\omega m_c + \alpha_5\omega^2 + \alpha_6 m_c^2 \tag{7a}$$
$$\eta_p = \beta_1 + \beta_2\omega + \beta_3\Pi + \beta_4\omega\Pi + \beta_5\omega^2 + \beta_6\Pi^2. \tag{7b}$$

These compressor maps are typically provided by the manufacturer or they can be identified based on historical data [2]. The shaft power is calculated as

$$P_s = \frac{y_p}{\eta_p} m_c, \tag{8}$$

where y_p is the polytropic head

$$y_p = \frac{Z_{in} R T_{in}}{M_W} \frac{n_\nu}{n_\nu - 1} \left[\left(\frac{p_2}{p_1} \right)^{\frac{n_\nu - 1}{n_\nu}} - 1 \right]. \tag{9}$$

Here, Z_{in} is the inlet compressibility factor, T_{in} the suction temperature, M_W the molecular weight of the gas mixture, which is assumed not to change, and n_ν the polytropic exponent.

C. Load-Sharing Optimization Problem

Consider a station with N parallel compressors, and let $\mathcal{N} = \{1, \ldots, N\}$ be the corresponding index set. The optimal setpoints for the torque τ_i and the recycle valve opening $V_{rec,i}$ can be obtained by minimizing the sum of power consumption of all compressors. Under normal process operation, the power consumption is always minimized by keeping the inlet and outlet valves completely open, that is, $V_{in} = V_{out} = 1$. Mathematically, the problem can be stated as follows:

$$\min_{\tau_i, V_{rec,i}} \quad \sum_{i=1}^{N} P_{s,i} \tag{10a}$$

s.t.

steady-state model equations

$$\sum_{i=1}^{N} m_{c,i} = m_{tot}^{\text{sp}} \tag{10b}$$

$$s_{0,i} - s_{1,i} m_{c,i} + \frac{p_{2,i}}{p_{1,i}} \leq 0, \quad i \in \mathcal{N} \tag{10c}$$

$$c_{0,i} + c_{1,i} m_{c,i} - \frac{p_{2,i}}{p_{1,i}} \leq 0, \quad i \in \mathcal{N} \tag{10d}$$

$$m_{c,i}^{\text{L}} \leq m_{c,i} \leq m_{c,i}^{\text{U}}, \quad i \in \mathcal{N} \tag{10e}$$

$$0 \leq \tau_i \omega_i \leq P_i^{\text{U}}(\omega_i), \quad i \in \mathcal{N} \tag{10f}$$

$$0 \leq V_{rec,i} \leq V_{rec,i}^{\text{U}}, \quad i \in \mathcal{N}, \tag{10g}$$

where m_{tot}^{sp} is the required mass flow for the whole station, and $s_{0,i}$, $s_{1,i}$, $c_{0,i}$ and $c_{1,i}$ are positive constants. The feasible region for the compressors is defined by the following constraints:

- Surge constraint (10c): This constraint represents a lower limit on the flow that can be generated by a compressor for a given head or pressure ratio. If this limit is violated, then a flow instability called surge occurs, which can cause thermal and mechanical stress to compressor blades, potentially leading to damages and eventually also to machine failure.
- Choke constraint (10d): This corresponds to the maximum flow that can be generated by a compressor depending on the aerodynamic characteristics of the compressor and the discharge piping.
- Mechanical limits: Lower and upper bounds for the operating flows (10e), applied power (10f) and recyle valve openings (10g).

At the optimum, it turns out that, for each compressor, either the surge constraint (10c) is active or the recycle valve is completely closed, that is, $V_{rec,i} = 0$.

III. MODIFIER ADAPTATION FOR PARALLEL COMPRESSORS

In parallel compressor stations, the outputs of each compressor depend only on the inputs to that compressor. As we will see later, this feature confers a special structure to the load-sharing problem that can be exploited for the implementation of MA. For simplicity, we restrict our analysis to station flow setpoints m_{tot}^{sp} for which all compressors operate far from the surge line. In other words, we assume $V_{rec,i} = 0$, for $i = 1, \ldots, N$, which leaves the torques as the only decision variables. At the kth RTO iteration, the MA problem can be formulated as follows:

$$\boldsymbol{\tau}_{k+1}^{\star} = \arg\min_{\boldsymbol{\tau}} \quad \sum_{i=1}^{N} P_{s,i}^{\text{mod}}(\tau_i) \tag{11a}$$

s.t.

steady-state model equations

$$\left(\sum_{i=1}^{N} m_{c,i}^{\text{mod}}(\tau_i) \right) + \varepsilon_k = m_{tot}^{\text{sp}} \tag{11b}$$

inequality constraints (10c)-(10f).

The modified cost and constraints read

$$P_{s,i}^{\text{mod}}(\tau_i) = P_{s,i}(\tau_i) + (\lambda_k^{P_i})^{\mathsf{T}} (\tau_i - \tau_{i,k}), \tag{12a}$$

$$m_{c,i}^{\text{mod}}(\tau_i) = m_{c,i}(\tau_i) + (\lambda_k^{m_i})^{\mathsf{T}} (\tau_i - \tau_{i,k}), \tag{12b}$$

and the modifiers are computed as follows

$$\varepsilon_k = m_{tot}^{\text{meas}}(\boldsymbol{\tau}_k) - \sum_{i=1}^{N} m_{c,i}(\tau_{i,k}), \tag{13a}$$

$$(\lambda_k^{P_i})^{\mathsf{T}} = \hat{\nabla}_{\tau_i} P_{s,i}(\tau_{i,k}) - \frac{\partial P_{s,i}}{\partial \tau_i}(\tau_{i,k}), \ i \in \mathcal{N} \tag{13b}$$

$$(\lambda_k^{m_i})^{\mathsf{T}} = \hat{\nabla}_{\tau_i} m_{c,i}(\tau_{i,k}) - \frac{\partial m_{c,i}}{\partial \tau_i}(\tau_{i,k}), \ i \in \mathcal{N}. \tag{13c}$$

Here $m_{tot}^{\text{meas}}(\boldsymbol{\tau}_k)$ is the measured value of the total mass flow at the kth iteration, and for each compressor, $\hat{\nabla}_{\tau_i} P_{s,i}(\tau_{i,k})$ and $\hat{\nabla}_{\tau_i} m_{c,i}(\tau_{i,k})$ are the estimated values of the derivatives with respect to τ_i of $P_{s,i}$ and $m_{c,i}$, respectively, evaluated at $\tau_{i,k}$. Note that (i) the gradient modifiers $\lambda_k^{P_i}$ and $\lambda_k^{m_i}$ are scalars, and (ii) to estimate the plant derivatives $\hat{\nabla}_{\tau_i} P_{s,i}(\tau_{i,k})$ and $\hat{\nabla}_{\tau_i} m_{c,i}(\tau_{i,k})$, a single perturbation of the torque τ_i is required for each compressor. Furthermore, it is possible to perturb all the torques simultaneously in such a way that the disturbance to the total mass flow is negligible, as will be explained in the next subsection.

If MA is applied in an open-loop manner, the optimal torques $\boldsymbol{\tau}_{k+1}^{\star}$ computed from Problem (11) (or their filtered values) are applied directly to the compressors. However, in this case, the desired total station flow is only guaranteed to be reached upon convergence of the RTO iterations [7].

A. Closed-Loop Implementation of MA

In practice, a PID controller is in charge of tracking the total mass-flow setpoint m_{tot}^{sp} by computing the "controller" torque $\tau_k^{\text{c}}(t)$ that, if applied to all compressors, would result

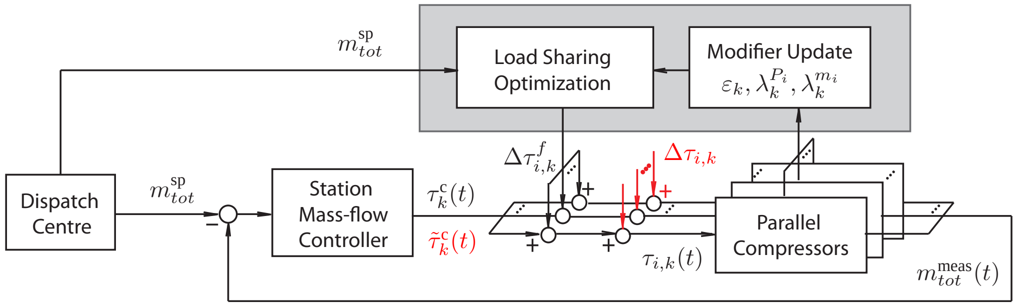

Fig. 2. Load-sharing optimization of a controlled compressor station. The control level is executed in continuous time t, while the optimization part is executed only once the system has reached steady state. $\Delta\tau_{i,k}^{\mathrm{f}}$ is the contribution of the optimization layer, while $\Delta\tau_{i,k}$ is the torque perturbation that is added to compute the experimental gradients. τ_k^{c} is the controller torque reached at the kth RTO iteration without the perturbations $\Delta\tau_{i,k}$, while $\tilde{\tau}_k^{\mathrm{c}}$ is the controller torque reached at steady state with the perturbations $\Delta\tau_{i,k}$.

in the desired station flow. Load-sharing optimization is added as a static layer that generates asymmetries in the load distribution of the parallel units. The proposed closed-loop implementation of MA is illustrated in Figure 2.

Consider the steady state reached at the kth RTO iteration, with the "controller" torque value τ_k^{c} and the torque applied to each compressor $\tau_{i,k}$. The RTO algorithm suggests applying next $\tau_{i,k+1}^{\mathrm{f}}$, which is the filtered value of the optimal torque computed from Problem (11), that is,

$$\boldsymbol{\tau}_{k+1}^{\mathrm{f}} = \mathsf{K}\boldsymbol{\tau}_{k+1}^{\star} + (\mathbf{I} - \mathsf{K})\boldsymbol{\tau}_k. \tag{14}$$

It follows that the contribution of the optimization for the next RTO iteration is:

$$\Delta\tau_{i,k+1}^{\mathrm{f}} := \tau_{i,k+1}^{\mathrm{f}} - \tau_k^{\mathrm{c}}. \tag{15}$$

This closed-loop implementation of MA maintains the attractive property of converging to a Karush-Kuhn-Tucker (KKT) point for the plant, as stated in the following Proposition.

Proposition 1 (Optimality upon Convergence): Consider the MA scheme (11)-(13) implemented in closed-loop by applying the feedforward torque differences (15) using the filter (14). Let the following assumptions hold:

a) The functions $P_{s,i}(\tau_i)$ and $m_{c,i}(\tau_i)$, $i = 1, \dots, N$, are continuously differentiable for the plant and the model.
b) m_{tot}^{meas} is measured without noise, and the derivatives $\hat{\nabla}_{\tau_i} P_{s,i}$ and $\hat{\nabla}_{\tau_i} m_{c,i}$ are perfectly estimated.
c) The constraints (10c)-(10f) do not become active.

Then, if the MA scheme converges, the converged point $\boldsymbol{\tau}_\infty = \lim_{k\to\infty} \boldsymbol{\tau}_k$ is a KKT point for the plant.

Proof: Note that

$$\tau_{i,k} = \tau_k^{\mathrm{c}} + \Delta\tau_{i,k}^{\mathrm{f}}, \tag{16}$$

which, upon convergence, can be written in vector form as

$$\boldsymbol{\tau}_\infty = \boldsymbol{\tau}_\infty^{\mathrm{c}} + \Delta\boldsymbol{\tau}_\infty^{\mathrm{f}}. \tag{17}$$

Using (15) in vector form and (14) gives:

$$\boldsymbol{\tau}_\infty = \boldsymbol{\tau}_\infty^{\mathrm{c}} + (\mathbf{I} - \mathsf{K})\boldsymbol{\tau}_\infty + \mathsf{K}\boldsymbol{\tau}_\infty^{\star} - \boldsymbol{\tau}_\infty^{\mathrm{c}},$$

which reduces to $\boldsymbol{\tau}_\infty = \boldsymbol{\tau}_\infty^{\star}$. The KKT conditions being necessary conditions, it follows that $\boldsymbol{\tau}_\infty^{\star}$ is a KKT point for Problem (11). MA ensures that $\boldsymbol{\tau}_\infty^{\star}$ is also a KKT point for the plant (see Theorem 1 in [7]). ■

B. Gradient Estimation Exploiting the Problem Structure

Perhaps, the most challenging part of MA lies in the fact that gradients of the plant outputs, with respect to the inputs, must be available. In general, steady-state perturbation methods require at least $(n_u + 1)$ steady-state operating points to estimate the gradients, and one has to wait for steady state each time the inputs are changed. In the case of the load-sharing problem (11), we have $n_u = N$, and we need to estimate the plant derivatives $\hat{\nabla}_{\tau_i} P_{s,i}$ and $\hat{\nabla}_{\tau_i} m_{c,i}$ for each compressor. As mentioned previously, a characteristic of parallel compressor stations is that the outputs of each compressor depend only on the torque (input) applied to that compressor. As a consequence, it is possible to perturb the torques of all compressors at the same time to estimate the derivatives. This has the advantage that the complete gradients can be estimated using only two steady-state operating points, regardless of the number of compressors. These steady states are the current RTO operating point $\boldsymbol{\tau}_k$ given by (16) and the perturbed operating point $\tilde{\boldsymbol{\tau}}_k$. The perturbed point is obtained by adding the torque perturbations $\Delta\tau_{i,k}$ to each compressor (see Figure 2):

$$\tilde{\tau}_{i,k} = \tilde{\tau}_k^{\mathrm{c}} + \Delta\tau_{i,k}^{\mathrm{f}} + \Delta\tau_{i,k}, \quad i = 1, \dots, N, \tag{18}$$

where $\tilde{\tau}_k^{\mathrm{c}}$ is the controller torque reached at steady state for the perturbed operating point. The derivatives of the power consumption are estimated by finite difference as follows :

$$\hat{\nabla}_{\tau_i} P_{s,i}(\tau_{i,k}) = \frac{P_{s,i}^{\mathrm{meas}}(\tilde{\tau}_{i,k}) - P_{s,i}^{\mathrm{meas}}(\tau_{i,k})}{\tilde{\tau}_{i,k} - \tau_{i,k}}. \tag{19}$$

The mass-flow derivatives $\hat{\nabla}_{\tau_i} m_{c,i}$ can be estimated using the same approach. The perturbations $\Delta\tau_{i,k}$ should be selected such that (i) the gradients are estimated with sufficient accuracy, and (ii) the perturbation of the total mass flow is negligible. With respect to (i), the error in the derivative estimates due to measurement noise will be large if the step $h = |\tilde{\tau}_{i,k} - \tau_{i,k}|$ is too small, while if h is too large, the error due to the finite-difference approximation of the gradient will be too large [12]. In general, good estimates can be obtained

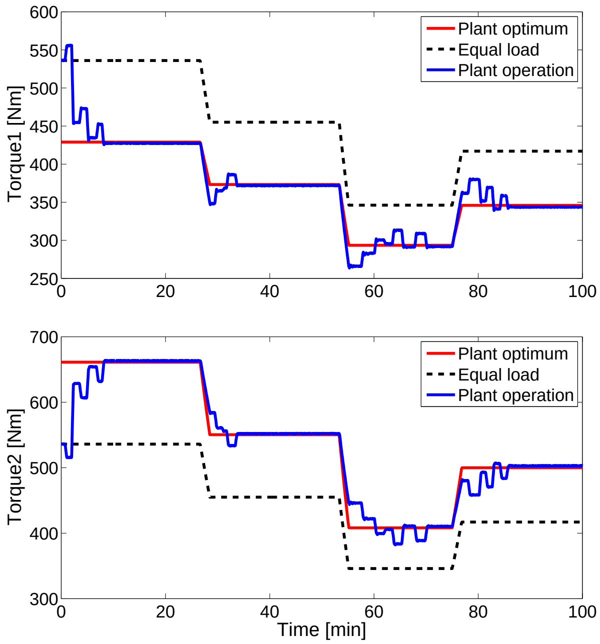

Fig. 3. Convergence of the torques to the optimal values of the two-compressor station.

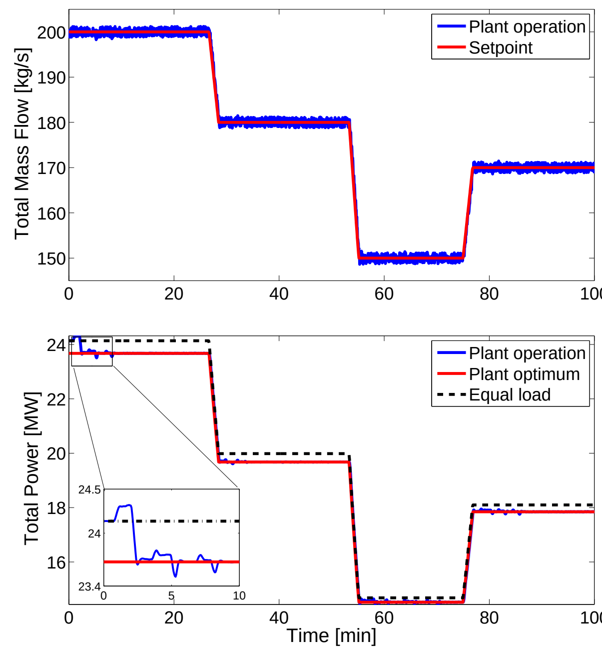

Fig. 4. Change in the total mass flow of the two-compressor station (top). Evolution of the power consumption of the station using the MA scheme, compared to the power consumption obtained via equal load distribution (bottom). For the sake of comparison, the power consumption for the plant is plotted without noise.

if h belongs to a certain interval $[h^{\rm L}, h^{\rm U}]$. With respect to (ii), one option is to formulate an optimization problem to compute the torque perturbations $\Delta\tau_{i,k}$ subject to (11b) and $\Delta\tau_{i,k} \in [h^{\rm L}, h^{\rm U}]$. In this paper, we choose $\Delta\tau_{i,k}$ such that $\sum_{i=1}^{N} \Delta\tau_{i,k} = 0$, which is a simple but very effective approach. For example, if the number of compressors N is even, one can select $\Delta\tau_{i,k} = \Delta > 0$ for $N/2$ compressors, and $\Delta\tau_{i,k} = -\Delta$ for the other $N/2$ compressors.

IV. SIMULATION RESULTS

In this section, the MA scheme described in Section III is tested in simulation. Two scenarios involving gas boosting stations composed of two and six parallel compressors, respectively, will be considered. Based on experimental data from real compressors, the measurement noises for the power consumption and the mass flows are selected as Gaussian noises with standard deviations of 0.33%. Once the controlled plant satisfies near steady-state conditions, the measurements are averaged over a moving time window of 10 sec. The torque perturbation $\Delta\tau_{i,k}$ used to estimate the gradients is a step of size 15 Nm. Instead of exciting at every RTO iteration, the perturbations are stopped once the cost improvement becomes negligible. For instance, at the kth RTO iteration, excitation is not carried out if, for a given $\varepsilon > 0$, the following condition is met:

$$\frac{\left|\sum_{i=1}^{N} P_{s,i}^{\rm meas}(\tau_{i,k}) - \sum_{i=1}^{N} P_{s,i}^{\rm meas}(\tau_{i,k-1})\right|}{\sum_{i=1}^{N} P_{s,i}^{\rm meas}(\tau_{i,k-1})} \le \varepsilon, \qquad (20)$$

in which case, the gradient modifiers are no longer updated. The gradient estimation is restarted every time the total mass flow setpoint is varied. In order to obtain smooth transitions between steady states, the changes in both the mass-flow setpoint and the feedforward contribution to the torques are implemented using ramps. The filter (14) is applied with $\mathsf{K} = 0.8\mathbb{I}$.

Plant-model mismatch is introduced by using different compressor maps (7) for the model and the plant. The efficiency maps for three different plants (A-C) and the model are shown in Figure 6. The model assumes that all the compressors have the same map. It follows that the *model* optimum corresponds to equal load distribution.

1) Two-compressor station: The plant compressors use the maps A and B in Figure 6. The initial setpoint for the total mass flow is 200 kg/s, with setpoint changes to 180 kg/s, 150 kg/s and 170 kg/s after 26 min, 53 min and 75 min, respectively. The initial inputs corresponds to the model optimum, that is, equal load distribution. The evolution of the torques applied to both compressors is shown in Figure 3. The evolution of the station mass flow, shown in Figure 4, follows the desired setpoint value even when the torques are perturbed for the purpose of gradient estimation. Using $\varepsilon = 10^{-3}$ in (20), the perturbations are stopped after only 3-4 RTO iterations. The improvement in power consumption with respect to equal load distribution varies from 1.6% for 200 kg/s to 1.1% for 150 kg/s, which is economically significant. The zoomed-in window in Figure 4 shows that the initial optimality loss of about 1.6% obtained for equal load distribution is almost completely suppressed after the first RTO iteration.

2) Six-compressor station: The plant compressors 1 and 2 use the map A, the compressors 3 and 4 the map B,

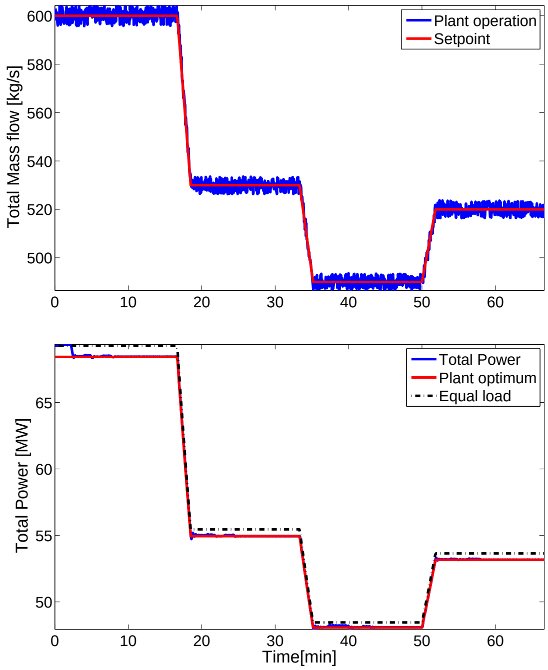

Fig. 5. Change in the total mass flow of the six-compressor station (top). Evolution of power consumption of the station using the MA scheme, compared to the power consumption obtained via equal load distribution (bottom). The power consumption for the plant is plotted without noise.

and the compressors 5 and 6 the map C in Figure 6. As shown in the top plot of Figure 5, the performance of MA is tested for several setpoint changes of the total mass flow. The bottom plot of Figure 5 shows that, although the number of compressors is increased, the scheme is very efficient and reaches the plant optimum in 3-4 iterations, just like in the case of two compressors. The improvement in the power consumption with respect to equal load distribution varies from 1.3% for 600 kg/s to 0.8% for 490 kg/s.

V. CONCLUSIONS

This paper has proposed a MA strategy for real-time load-sharing optimization of compressor stations involving multiple industrial compressors operating in parallel. The simulation results show that MA is able to quickly converge to the plant optimum without updating the model parameters or the compressor maps, even when the same model is used for all compressors. The approach relies on estimating the derivatives of the power consumption and the mass flows by perturbing the torques applied to the individual compressors. An interesting feature that makes MA particularly well suited for this problem is that, since the compressors operate in parallel, it is possible to excite all torques simultaneously without perturbing significantly the total mass flow. In the simulation results, plant-model mismatch was introduced as differences in the compressor maps. Note also that MA allows handling parametric as well as structural plant-model mismatch.

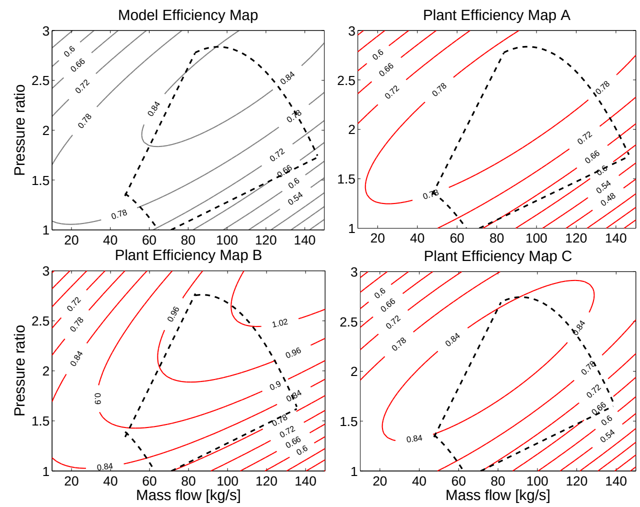

Fig. 6. Comparison between the model efficiency map (grey) and the plant efficiency maps (red) for three different compressors. The maps are shown as contour plots, whereas the dashed black lines represent the operating range of the gas compressor.

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