Maximal Islanding Time For Microgrids via Distributed Predictive Control

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***Abstract*— Motivated by a specific application in electricity distribution networks, we present a hierarchical model predictive control algorithm for scheduling energy storage devices. We demonstrate that, for the proposed optimization problem, the alternating direction method of multipliers can be implemented in a distributed fashion. Numerical experiments supporting the theoretical results are provided.**

I. INTRODUCTION

A major challenge in decarbonizing energy generation is the integration of small-scale renewable generation and storage devices into existing distribution grids. In this context, the term *microgrids* is frequently used; c.f. [1], [2], [3], [4], [5]. One example of a microgrid is a residential neighborhood with a single point of common coupling between the neighborhood and the main grid. For our purposes, each residence in the neighborhood consists of a residential load, generation (e.g., in the form of solar photovoltaics), and storage (e.g., in the form of a battery). We refer to this collection of residential load, generation, and storage as a Residential Energy System (RES) [6], [7], [8], [9].

One benefit of an electricity distribution network containing microgrids is the potential to disconnect a microgrid from the main distribution network. This is referred to as *islanding*, whereby the microgrid maintains normal operation using only the locally stored energy, energy locally generated by renewable generation units (photovoltaic panels, wind power, etc.), and, if present, conventional generators [1]. An important question in this context is whether or not microgrids should be operated by a central entity, e.g. a microgrid operator. While many works suggest decentralized or distributed approaches to frequency and voltage stabilization [10], [11], scheduling of available storage devices, which is also important for reliable operation of microgrids, is often done in a centralized fashion [12], [13].

In previous papers [7], [9], we proposed a distributed model predictive control scheme minimizing variations in the vertical grid load, i.e. the load transferred at the point of common coupling. In the present paper, we extend these results to cover another important issue related to islanded operation with limited or even no conventional generation. At the start of and during islanded operation it is vital to know the maximal allowable time window for which the microgrid is able to locally maintain supply on its own without any need for conventional generation or connection to the grid. Herein, we propose an optimization problem to compute the maximal time interval (starting from a specific point) for which the microgrid can be disconnected without experiencing a shortage of energy.

In principle, such an optimization problem could be solved by a central entity—like the microgrid operator—provided knowledge of parameters and current state of charge of all storage devices as well as future net consumption profiles of each RES in the microgrid. This, however, would lead to an inflexible and communication intensive setup, since every change in an RES as well as the network topology would have to be monitored by the central entity. Moreover, privacy considerations may prevent the individual RESs from sharing all this information with a central entity. For this reason, we propose a hierarchical distributed solution strategy with limited information exchange based on the alternating direction method of multipliers (ADMM) [14].

The paper is structured as follows: In Section II the model of the microgrid is introduced and the problem of islanded operation is formulated. In Section III the hierarchical ADMM algorithm is developed and embedded in a receding horizon formulation in Section IV. In Section V we propose an optimization problem returning the maximal grid disconnection window in the optimal solution. The paper closes with numerical simulations in Section VII and conclusions in Section VIII.

II. MODEL AND PROBLEM DESCRIPTION

Firstly, in Subsection II-A, a model for a microgrid of RESs introduced in [6], [7] is recalled and extended. Secondly, in Subsection II-B, the general optimization problem,

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which will be tackled by means of a distributed setup, is described.

A. Distributed renewable energy systems

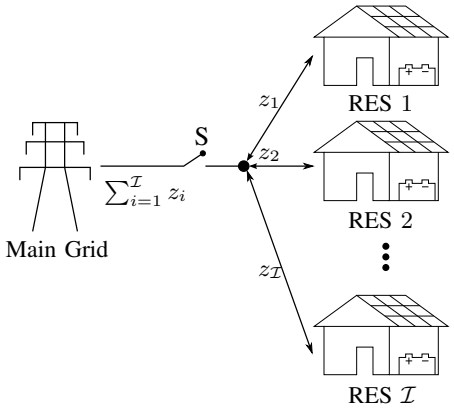

Fig. 1. Visualization of a microgrid. A number of $\mathcal{I} \in \mathbb{N}$ RESs connected to the main grid through a point of common coupling. We assume that the RESs can be disconnected from the main grid through the switch S.

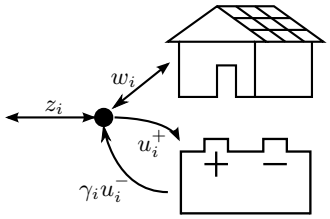

Fig. 2. Visualization of Equation (2) for a single RES. The power demand z_i depends on the power consumption and power generation w_i and can be manipulated by charging/discharging the battery.

We consider a network, visualized in Figure 1, of $\mathcal{I}$ RESs, $\mathcal{I} \in \mathbb{N}$ connected to a main grid through the individual power demand z_i, $i = 1, \dots, \mathcal{I}$. We assume that the RESs are physically decoupled and that the system dynamics of the i-th RES, $i \in \{1, 2, \dots, \mathcal{I}\}$, is defined as

$$x_i(k+1) = \alpha_i x_i(k) + T\left(\beta_i u_i^+(k) + u_i^-(k)\right) \quad (1)$$

$$z_i(k) = w_i(k) + u_i^+(k) + \gamma_i u_i^-(k) \quad (2)$$

with $(\alpha_i, \beta_i, \gamma_i) \in [0,1]^3$.

The variables $z_i(k)$ [kW], representing the power demand drawn from/supplied to the grid by each RES and given by Equation (2), depend on the net consumptions $w_i(k)$ [kW], i.e., the power generation of solar photovoltaic panels minus the energy demand of the residents, and the battery usage. The dynamics of the battery, Equation (1), are defined through the state of charge $x_i(k)$ in [kWh] and the input variables $u_i^+(k)$ and $u_i^-(k)$ in [kW]. The input $u_i^+(k)$ represents the power drawn from the grid to increase the state of charge of the battery and the power demand $z_i(k)$, while $u_i^-(k)$ can be used to discharge the battery and decrease the power demand of the i-th RES. The parameter α_i models losses due to self-discharge, β_i and γ_i model losses due to energy conversion, and T (in hours [h]) represents the discretization in time. Additionally, the storage device of each RES is constrained by:

$$0 \leq x_i(k) \leq C_i \quad (3a)$$

$$\underline{u}_i \leq u_i^-(k) \leq 0 \quad (3b)$$

$$0 \leq u_i^+(k) \leq \overline{u}_i \quad (3c)$$

$$0 \leq \frac{u_i^-(k)}{\underline{u}_i} + \frac{u_i^+(k)}{\overline{u}_i} \leq 1 \quad (3d)$$

If $\max\{\beta_i, \gamma_i\} < 1$ holds, energy can be *wasted* meaning that an RES can get rid of an energy surplus by continuously charging and discharging its battery. Note that this possibility is limited due to the battery constraint (3d). Additionally, it is possible to change from charging to discharging (and vice versa) between consecutive time steps k and $k+1$. Since in this case only a fraction of the time interval of length T is used for charging and discharging, the upper and lower bounds for charging and discharging have to be adapted, which is captured by the constraint (3d). The dynamics (1)–(2) and the constraints (3) extend the models introduced in [6], [7]. Similar models of energy networks are for example described in [15] and [16].

B. Problem formulation

In this section we define the optimization problem which we will apply later in this paper to the islanded operation of the microgrid model described in the previous section. For a given time k and a prediction horizon $N \in \mathbb{N}$ the notation $\mathbf{z} = (\mathbf{z}_1^T, \mathbf{z}_2^T, \dots, \mathbf{z}_{\mathcal{I}}^T)^T$ is defined componentwise by

$$\mathbf{z}_i = (z_i(k), \dots, z_i(k+N-1))^T;$$

i.e., $\mathbf{z} \in \mathbb{R}^{N\mathcal{I}}$. Moreover, we use $\bar{\cdot}$ to denote the average of a vector, e.g.

$$\overline{\mathbf{z}} := \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \mathbf{z}_i, \quad \overline{\mathbf{z}} \in \mathbb{R}^N, \quad (4)$$

defines the average power demand drawn from the grid. The same notation is used for the other variables, e.g. $\mathbf{x}$, $\mathbf{w}$, $\mathbf{u}^+$ and $\mathbf{u}^-$. $\mathbb{1}$ denotes the vector of appropriate dimension with all entries equal to one, and I denotes the identity matrix of appropriate dimension. For fixed initial values $x_i^0 = x_i(k)$ and given net consumptions $\mathbf{w}_i$, the variables $\mathbf{z}_i$, $i \in \{1, 2, \dots, \mathcal{I}\}$, are confined to the compact and convex (polytope) set

$$D_i = \left\{ \mathbf{z}_i \in \mathbb{R}^N \;\middle|\; \begin{array}{l} \exists\, \mathbf{x}_i, \mathbf{u}_i \text{ satisfying } x_i(k) = x_i^0, \\ (1)-(3)\ \forall\ j \in \{k, \dots, k+N-1\} \end{array} \right\}.$$

Let the cost function $F : \mathbb{R}^{N\mathcal{I}} \times \mathbb{R}^M \to \mathbb{R}$ be of the form

$$F(\mathbf{z}, \mathbf{s}) := \sum_{i=1}^{\mathcal{I}} f_i(\mathbf{z}_i) + g\left(\overline{\mathbf{z}}\right) + h(\mathbf{s})$$

where $f_i : \mathbb{R}^N \to \mathbb{R}$ are local cost functions of the individual RESs, $g : \mathbb{R}^N \to \mathbb{R}$ is a cost function chosen by the grid operator coupling the individual RESs, and $h : \mathbb{R}^M \to \mathbb{R}$ is another cost function defined by the grid operator in the auxiliary variable $\mathbf{s} \in \mathcal{S} \subseteq \mathbb{R}^M$, which will be used to identify the maximal time the RESs can be disconnected

from the grid operator (i.e., the maximal time the microgrid can be islanded) in Section V. Here, $\mathcal{S} \subseteq \mathbb{R}^M$ is a polytope, which equals $\mathbb{R}^M_{\geq 0}$ in our application. At every time step k, we consider a minimization problem of the form

$$\begin{aligned}(\mathbf{z}^\star, \mathbf{s}^\star) := \underset{\mathbf{z},\mathbf{s}}{\operatorname{argmin}} \quad & F(\mathbf{z}, \mathbf{s}) \\ \text{s.t.} \quad & A\textstyle\sum_{i=1}^{\mathcal{I}} \mathbf{z}_i + B\mathbf{s} - b \leq 0 \\ & \mathbf{z}_i \in D_i \quad \forall\, i \in \{1, 2, \ldots, \mathcal{I}\} \\ & \mathbf{s} \in \mathcal{S}\end{aligned} \tag{5}$$

where $A \in \mathbb{R}^{m \times N}$, $B \in \mathbb{R}^{M \times m}$, $b \in \mathbb{R}^m$ define polyhedral coupling constraints between the systems and the artificial variable $\mathbf{s}$. Recall that we have assumed that the RESs are physically decoupled. Nevertheless, in the minimization problem (5), the RESs are coupled through the objective function and the inequality constraints, and hence a centralized controller is necessary to solve the optimization problem to compute the individual charging and discharging strategies. In the following section we will present a method to rewrite the optimization problem in such a way that distributed optimization on the local level is possible without losing optimality with respect to the original formulation (5).

III. THE ALTERNATING DIRECTION METHOD OF MULTIPLIERS

The alternating direction method of multipliers (ADMM) is an iterative algorithm for solving the optimization problem (5). In our presentation we focus on a hierarchical form of the method which is suited to our problem and refer to [14] and [17] for more general forms of ADMM algorithms and dual decomposition methods.

A. The basic ADMM formulation and convergence results

To obtain a hierarchical algorithm, we introduce the variables $\mathbf{a}_i \in \mathbb{R}^N$ $(i = 1, \ldots, \mathcal{I})$, $\mathbf{a} = (\mathbf{a}_1^T, \mathbf{a}_2^T, \ldots, \mathbf{a}_{\mathcal{I}}^T)^T$, and rewrite the minimization problem (5) in the form

$$\begin{aligned}(\mathbf{z}^\star, \mathbf{a}^\star, \mathbf{s}^\star) := \underset{\mathbf{z},\mathbf{a},\mathbf{s}}{\operatorname{argmin}} \; & \sum_{i=1}^{\mathcal{I}} f_i(\mathbf{z}_i) + g\left(\overline{\mathbf{a}}\right) + h(\mathbf{s}) \\ \text{s.t.} \quad & (\overline{\mathbf{a}}, \mathbf{s}) \in P \\ & \mathbf{z}_i \in D_i \qquad \forall\, i \in \{1, 2, \ldots, \mathcal{I}\} \\ & \mathbf{z}_i - \mathbf{a}_i = 0 \quad \forall\, i \in \{1, 2, \ldots, \mathcal{I}\}\end{aligned}$$

where the polytope P is defined as

$$P = \left\{ (\overline{\mathbf{a}}, \mathbf{s}) \in \mathbb{R}^N \times \mathbb{R}^M \;\middle|\; \begin{array}{l} \mathcal{I} \cdot A\overline{\mathbf{a}} + B\mathbf{s} - b \leq 0 \\ \mathbf{s} \in \mathcal{S} \end{array} \right\}. \tag{6}$$

Observe that the optimization problem implicitly contains the constraints $\overline{\mathbf{a}} = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \mathbf{a}_i$. To simplify the notation, this constraint is not given in the problem formulation. The augmented Lagrangian $\mathcal{L}_\rho(\mathbf{z}, \mathbf{a}, \mathbf{s}, \lambda)$ of the minimization problem is given by

$$\sum_{i=1}^{\mathcal{I}} f_i(\mathbf{z}_i) + g\left(\overline{\mathbf{a}}\right) + h(\mathbf{s}) + \frac{\rho}{2} \left\|\mathbf{z} - \mathbf{a}\right\|^2 + \sum_{i=1}^{\mathcal{I}} \lambda_i^T (\mathbf{z}_i - \mathbf{a}_i).$$

for Lagrange multipliers

$$\lambda = (\lambda_1^T, \lambda_2^T, \ldots, \lambda_{\mathcal{I}}^T)^T \in \mathbb{R}^{\mathcal{I}N}$$

and a positive constant $\rho \in \mathbb{R}_{>0}$. For $\rho = 0$, we obtain the usual definition of the Lagrangian.

The idea of ADMM is to iteratively find a solution of the minimization problem (5) by repeatedly performing the following sequence of update steps:

$$\mathbf{z}_i^{\ell+1} := \underset{\mathbf{z}_i \in D_i}{\operatorname{argmin}} \; \mathcal{L}_\rho\left(\mathbf{z}, \mathbf{a}^\ell, \mathbf{s}^\ell, \lambda^\ell\right) \tag{7a}$$

$$(\mathbf{a}^{\ell+1}, \mathbf{s}^{\ell+1}) := \underset{(\overline{\mathbf{a}},\mathbf{s}) \in P}{\operatorname{argmin}} \; \mathcal{L}_\rho\left(\mathbf{z}^{\ell+1}, \mathbf{a}, \mathbf{s}, \lambda^\ell\right) \tag{7b}$$

$$\lambda^{\ell+1} := \lambda^\ell + \rho\left(\mathbf{z}^{\ell+1} - \mathbf{a}^{\ell+1}\right) \tag{7c}$$

for $\ell \in \mathbb{N}$. Convergence of the sequence $(\mathbf{z}^\ell, \mathbf{a}^\ell, \mathbf{s}^\ell, \lambda^\ell)_{\ell \in \mathbb{N}}$ can be shown under certain convexity assumptions on the objective function F. We use the assumptions and results given in [14, Chapter 3.2] which are directly applicable to the objective function proposed in the following section.

Assumption 3.1: Suppose that the following holds:

(i) The (extended-real-valued) functions f_i, $i = 1, \ldots, \mathcal{I}$, g and h are closed, proper, and convex.

(ii) The unaugmented Lagrangian $\mathcal{L}_0$ has a saddle point, i.e., there exists $(\mathbf{z}^\star, \mathbf{a}^\star, \mathbf{s}^\star, \lambda^\star)$ such that

$$\begin{aligned}\mathcal{L}_0(\mathbf{z}^\star, \mathbf{a}^\star, \mathbf{s}^\star, \lambda) &\leq \mathcal{L}_0(\mathbf{z}^\star, \mathbf{a}^\star, \mathbf{s}^\star, \lambda^\star) \\ &\leq \mathcal{L}_0(\mathbf{z}, \mathbf{a}, \mathbf{s}, \lambda^\star)\end{aligned}$$

holds for all $\lambda \in \mathbb{R}^{\mathcal{I}N}$ and $(\mathbf{z}, \mathbf{a}, \mathbf{s}) \in (\mathbb{R}^{N\mathcal{I}})^2 \times \mathbb{R}^M$.

Remark 3.2: One class of problems satisfying Assumption 3.1 are convex functions subject to nonempty, convex, and compact constraints. Since convex functions attain their minimum on compact sets, there exists a (possibly non unique) primal optimal solution $(\mathbf{z}^\star, \mathbf{a}^\star, \mathbf{s}^\star)$. This implies the existence of an optimal dual solution $\lambda^\star$ satisfying the saddle point condition (ii). For the results showing these properties we refer to [18, Appendix C].

Theorem 3.3: If Assumption 3.1 holds, the iterates of (7) satisfy the following convergence properties:

(i) The residuals $r^\ell := \mathbf{z}^\ell - \mathbf{a}^\ell$ converge to zero for $\ell \to \infty$.

(ii) The sequence $(F(\mathbf{z}^\ell, \mathbf{s}^\ell))_{\ell \in \mathbb{N}}$ converges to the optimal value $F^\star$ of Problem (5) for $\ell \to \infty$.

(iii) The dual variables λ^ℓ converge to the optimal dual point $\lambda^\star$ for $\ell \to \infty$.

A proof of this result, which is sufficient for our application, is given in [14, Appendix A]. For a more detailed analysis of the convergence properties of the ADMM scheme (7) we refer to the references in [14].

B. Simplification of the ADMM formulation

The ADMM algorithm (7) can be solved in a distributed manner because Equation (7a) splits into $\mathcal{I}$ separable optimization problems

$$\mathbf{z}_i^{\ell+1} = \underset{\mathbf{z}_i \in D_i}{\operatorname{argmin}} \; f_i(\mathbf{z}_i) + \frac{\rho}{2} \left\|\mathbf{z}_i - \mathbf{a}_i\right\|^2 + \lambda_i^T \left(\mathbf{z}_i - \mathbf{a}_i\right),$$

which can be solved in parallel by every RES individually. The optimization problem (7b) cannot be separated due to the coupling in the function g. However, it is possible to make the number of unknowns in this problem independent of the number of RESs $\mathcal{I}$ by using the average variables

$\overline{\mathbf{a}} \in \mathbb{R}^N$ instead of $\mathbf{a} \in \mathbb{R}^{\mathcal{I}N}$. In order to show this, we first rewrite the minimization problem (7b) as

$$\begin{aligned}(\mathbf{a}^{\ell+1}, \mathbf{s}^{\ell+1}) &= \operatorname*{argmin}_{(\overline{\mathbf{a}},\mathbf{s})\in P} \mathcal{L}_\rho\left(\mathbf{z}^{\ell+1}, \mathbf{a}, \mathbf{s}, \lambda^\ell\right) \\ &= \operatorname*{argmin}_{(\overline{\mathbf{a}},\mathbf{s})\in P} g\left(\overline{\mathbf{a}}\right) + h(\mathbf{s}) + \frac{\rho}{2}\left\|\mathbf{z}^{\ell+1} - \mathbf{a}\right\|^2 + \sum_{i=1}^{\mathcal{I}} \lambda_i^{\ell^T}\left(\mathbf{z}_i^{\ell+1} - \mathbf{a}_i\right) \\ &= \operatorname*{argmin}_{(\overline{\mathbf{a}},\mathbf{s})\in P} g\left(\overline{\mathbf{a}}\right) + h(\mathbf{s}) + \sum_{i=1}^{\mathcal{I}} \frac{\rho}{2}\left\|\mathbf{z}_i^{\ell+1} - \mathbf{a}_i\right\|^2 + \lambda_i^{\ell^T}\left(\mathbf{z}_i^{\ell+1} - \mathbf{a}_i\right) \\ &= \operatorname*{argmin}_{(\overline{\mathbf{a}},\mathbf{s})\in P} g\left(\overline{\mathbf{a}}\right) + h(\mathbf{s}) + \frac{\rho}{2}\sum_{i=1}^{\mathcal{I}}\left\|\frac{\lambda_i^\ell}{\rho} + \mathbf{z}_i^{\ell+1} - \mathbf{a}_i\right\|^2. \end{aligned} \tag{8}$$

We then require the following lemma.

Lemma 3.4: For $\mathbf{c}, \mathbf{y}_i \in \mathbb{R}^N$ $(i = 1, \ldots, \mathcal{I})$ the minimizer of

$$\min_{\mathbf{v}_i \in \mathbb{R}^N} \sum_{i=1}^{\mathcal{I}} \|\mathbf{v}_i - \mathbf{y}_i\| \quad \text{s.t.} \quad \frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}} \mathbf{v}_i = \mathbf{c}$$

is given by $\mathbf{v}_i = \mathbf{y}_i + \mathbf{c} - \overline{\mathbf{y}}$ for all $i \in \{1, \ldots, \mathcal{I}\}$ where $\overline{\mathbf{y}} = (1/\mathcal{I})\sum_{i=1}^{\mathcal{I}} \mathbf{y}_i$.

Proof: For $\mathbf{y}_i = 0$ for all $i \in \{1, \ldots, \mathcal{I}\}$ the triangular inequality implies

$$\begin{aligned}\|\mathcal{I} \cdot \mathbf{c}\| &= \min_{\mathbf{v}_i, \frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}} \mathbf{v}_i = \mathbf{c}} \left\|\sum_{i=1}^{\mathcal{I}} \mathbf{v}_i\right\| \\ &\leq \min_{\mathbf{v}_i, \frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}} \mathbf{v}_i = \mathbf{c}} \sum_{i=1}^{\mathcal{I}} \|\mathbf{v}_i\|\end{aligned}$$

and equality is obtained for $\mathbf{v}_i = \mathbf{c}$ for all $i \in \{1, \ldots, \mathcal{I}\}$. For the general case we use the coordinate transformation $\tilde{\mathbf{v}}_i = \mathbf{v}_i - \mathbf{y}_i$. Then the equality constraint reads

$$\frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}} \tilde{\mathbf{v}}_i = \frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}} \mathbf{v}_i - \frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}} \mathbf{y}_i = \mathbf{c} - \overline{\mathbf{y}}$$

which shows the assertion. ∎

Applying this result to the minimization problem (8) and fixing the variables $(\overline{\mathbf{a}}, \mathbf{s})$, we see that the optimal solution satisfies

$$\frac{\overline{\lambda}^\ell}{\rho} + \overline{\mathbf{z}}^{\ell+1} - \overline{\mathbf{a}} = \frac{\lambda_i^\ell}{\rho} + \mathbf{z}_i^{\ell+1} - \mathbf{a}_i \tag{9}$$

for all $i \in \{1, \ldots, \mathcal{I}\}$ according to Lemma 3.4 with the definition $\overline{\lambda} = \frac{1}{\mathcal{I}}\sum_{i=1}^{\mathcal{I}} \lambda_i$. Hence the minimization problem (8) is equivalent to

$$\begin{aligned}\min \quad & g\left(\overline{\mathbf{a}}\right) + h(\mathbf{s}) + \tfrac{\rho}{2}\textstyle\sum_{i=1}^{\mathcal{I}}\left\|\frac{\overline{\lambda}^\ell}{\rho} + \overline{\mathbf{z}}^{\ell+1} - \overline{\mathbf{a}}\right\|^2 \\ \text{s.t.} \quad & (\overline{\mathbf{a}}, \mathbf{s}) \in P\end{aligned} \tag{10}$$

where the number of optimization variables is independent of the number of RESs.

Additionally, it can be shown that $\lambda_i = \lambda_j$ holds for all $i, j \in \{1, \ldots, \mathcal{I}\}$ after the first iteration and hence, the Lagrange multiplier $\lambda \in \mathbb{R}^{N\mathcal{I}}$ can be replaced by a Lagrange multiplier $\overline{\lambda} \in \mathbb{R}^N$. The update of the Lagrange multipliers in Equation (7c) simplifies to

$$\begin{aligned}\lambda_i^{\ell+1} &= \lambda_i^\ell + \rho\left(\mathbf{z}_i^{\ell+1} - \mathbf{a}_i^{\ell+1}\right) \\ &= \overline{\lambda}^\ell + \rho(\overline{\mathbf{z}}^{\ell+1} - \overline{\mathbf{a}}^{\ell+1}),\end{aligned}$$

i.e., $\lambda_i^{\ell+1} = \lambda_j^{\ell+1}$ for all $i, j \in \{1, \ldots, \mathcal{I}\}$. Hence also the update of the dual variables is independent of the number of RESs.

With these considerations, which follow the arguments given in [14, Chapter 7.3], the updates of Equation (7b) and (7c) reduce to the minimization problem

$$\operatorname*{argmin}_{(\overline{\mathbf{a}},\mathbf{s})\in P} g\left(\overline{\mathbf{a}}\right) + h(\mathbf{s}) + \frac{\rho \cdot \mathcal{I}}{2}\left\|\frac{\overline{\lambda}^\ell}{\rho} + \overline{\mathbf{z}}^{\ell+1} - \overline{\mathbf{a}}\right\|^2$$

and the update

$$\overline{\lambda}^{\ell+1} = \overline{\lambda}^\ell + \rho(\overline{\mathbf{z}}^{\ell+1} - \overline{\mathbf{a}}^{\ell+1}).$$

The update (7a) of the variables $\mathbf{z}_i$ of the individual RESs given by the solution of the minimization problem

$$\operatorname*{argmin}_{\mathbf{z}_i \in D_i} f_i(\mathbf{z}_i) + \frac{\rho}{2}\left\|\mathbf{z}_i + \frac{\overline{\lambda}^\ell}{\rho} - \mathbf{a}_i^\ell\right\|^2$$

involves the variable $\mathbf{a}_i$ which differs for all $i \in \{1, \ldots, \mathcal{I}\}$ and hence has to be transmitted to every RES individually. To avoid the need to communicate individual $\mathbf{a}_i$'s, define

$$\Pi^\ell := \frac{\overline{\lambda}^\ell}{\rho} + \overline{\mathbf{z}}^\ell - \overline{\mathbf{a}}^\ell.$$

Then Equation (9) can be rewritten in the form

$$\frac{\lambda_i^\ell}{\rho} - \mathbf{a}_i^\ell = -\mathbf{z}_i^\ell + \frac{\overline{\lambda}^\ell}{\rho} + \overline{\mathbf{z}}^{\ell+1} - \overline{\mathbf{a}}^\ell = -\mathbf{z}_i^\ell + \Pi^\ell.$$

If the variable Π^ℓ is known by the individual systems then the update $\mathbf{z}_i^{\ell+1}$ can be computed by

$$\operatorname*{argmin}_{\mathbf{z}_i \in D_i} f_i(\mathbf{z}_i) + \frac{\rho}{2}\left\|\mathbf{z}_i - \mathbf{z}_i^\ell + \Pi^\ell\right\|^2 \tag{11}$$

without the knowledge of individual variables specific to RES i.

C. The hierarchical distributed optimization algorithm

Algorithm 1 summarizes the ideas of this section and splits the ADMM iterates in tasks which can be carried out by the individual RESs in parallel and tasks which have to be done by the central entity or the grid operator, respectively. Algorithm 1 provides several properties beneficial for our application including:

- Only the parameter Π is transmitted to the RESs and not the energy demand $\mathbf{z}_i$. Hence privacy of data between the individual RESs is maintained. Furthermore, the dimension of Π is independent of the number of RESs. Thus, the communication overhead scales well with the size of the microgrid.
- The number of unknowns in the optimization problem of the grid operator is independent of the number of RESs. Thus, the computational complexity of the central entity and the RESs are independent of $\mathcal{I}$.
- Safeguarded by Theorem 3.3, ADMM recovers an optimal solution of a centralized optimization algorithm.
- The RESs do not need to know the objective functions g and h defined by the central entity. This allows the

Algorithm 1 Hierarchical distributed optimization algorithm

Phase 1 (RES i, $i \in \mathbb{N}_{\mathcal{I}}$): Receive Π^{ℓ}.

- Solve the minimization problem

$$\mathbf{z}_i^{\ell+1} := \underset{\mathbf{z}_i \in D_i}{\operatorname{argmin}} \; f_i(\mathbf{z}_i) + \frac{\rho}{2} \left\| \mathbf{z}_i - \mathbf{z}_i^{\ell} + \Pi^{\ell} \right\|^2$$

and send $\mathbf{z}_i^{\ell+1}$ to the central entity.

Phase 2 (Central Entity): Receive $\mathbf{z}_i^{\ell+1}$, $i = 1, 2, \ldots, \mathcal{I}$.

- Compute the average $\overline{\mathbf{z}}^{\ell+1} = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \mathbf{z}_i^{\ell+1}$.
- Solve the minimization problem

$$(\overline{\mathbf{a}}^{\ell+1}, \mathbf{s}^{\ell+1}) = \underset{(\overline{\mathbf{a}}, \mathbf{s}) \in P}{\operatorname{argmin}} \quad \Big(g(\overline{\mathbf{a}}) + h(\mathbf{s}) + \tfrac{\rho \cdot \mathcal{I}}{2} \left\| \overline{\mathbf{z}}^{\ell+1} - \overline{\mathbf{a}} + \frac{\overline{\lambda}^{\ell}}{\rho} \right\|^2 \Big).$$

- Update the Lagrange multiplier

$$\overline{\lambda}^{\ell+1} = \overline{\lambda}^{\ell} + \rho \left(\overline{\mathbf{z}}^{\ell+1} - \overline{\mathbf{a}}^{\ell+1} \right).$$

- Compute and broadcast

$$\Pi^{\ell+1} = \overline{\mathbf{z}}^{\ell+1} - \overline{\mathbf{a}}^{\ell+1} + \frac{\overline{\lambda}^{\ell+1}}{\rho}.$$

Increment the iteration counter ℓ.

grid operator to modify the objective function without communicating this to the RESs.

IV. RECEDING HORIZON CONTROL

The final point in the preceding list is of particular interest when Algorithm 1 is embedded in a receding horizon scheme described in Algorithm 2, since it allows the grid operator to change the objective function at every time step k without changing the network or communication structure and without changing the optimization problem on the local level. Moreover, the grid operator does not need to react to changes in the local system dynamics (1) and (2) or to changes in the constraints (3).

Algorithm 2 Distributed model predictive control

1) **Initialization:**
   RES i, ($i \in \mathbb{N}_{\mathcal{I}}$):
   - Measure the initial state of charge of the battery $x_i(k) = x_i^0$ and predict the net consumption $\mathbf{w}_i$.

   Central Entity:
   - Define the objective functions g and h.
2) **Distributed optimization:** Apply Algorithm 1 to compute the solutions $\mathbf{u}_i^{+\star}$ and $\mathbf{u}_i^{-\star}$ for $i = 1, \ldots, \mathcal{I}$.
3) Apply $\mathbf{u}_i^{+\star}(k)$ and $\mathbf{u}_i^{-\star}(k)$ for $i = 1, \ldots, \mathcal{I}$ and increment the time index k

V. ISLANDED OPERATION OF A MICROGRID

In this section, we set up an optimal control problem which serves two purposes: when solved once at time k, its solution tells us the number of time steps $\bar{k} \in \mathbb{N}_0$ the grid can be operated in *islanded mode* after a given time instant $k + k^{\star}$, $k^{\star} \in \mathbb{N}_0$ (i.e., the microgrid can be disconnected from the main grid at time $k + k^{\star}$, visualized in Figure 1 using the switch S, without failing to meet the local energy demand). When solved iteratively within Algorithm 2, it yields the control strategy for keeping the microgrid in islanded mode from $k + k^{\star}$ to $k + k^{\star} + \bar{k}$.

We have the following two distinct applications in mind.

- $k^{\star} \geq 1$: A scheduled disconnection from the grid for an *a priori* specified time window.
- $k^{\star} = 0$: An unscheduled disconnection.

The main difference between the two cases is that in the first scenario the microgrid can specifically prepare itself in advance by charging the batteries until time $k + k^{\star}$, neglecting (possibly conflicting) other objectives. Despite these differences, both settings can be handled with the proposed methodology by adequately adapting the objective function and the constraints in Algorithm 1, which shows the flexibility of our approach.

The possibility of disconnecting the grid at time $k^{\star}$ is equivalent to the existence of $\mathbf{z}_i \in D_i$ (for $i \in \{1, \ldots, \mathcal{I}\}$) such that $\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \mathbf{z}_i(k^{\star}) \leq 0$ is satisfied, i.e., the overall power demand is less or equal to zero at time $k^{\star}$. To find the maximal consecutive number of time steps from $k^{\star}$ to $k^{\star} + q^{\star}$ ($q^{\star} \geq 0$) such that $\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \mathbf{z}_i(k^{\star} + q) \leq 0$ holds for all $q \in \{0, \ldots, q^{\star}\}$ we define the following minimization problem.

Definition 5.1: For a given time index $k^{\star} \in \{0, \ldots, N-1\}$, set $M = N - k^{\star}$ and define the grid disconnection problem as

$$(\overline{\mathbf{a}}^{\star}, \mathbf{s}^{\star}) \in \underset{(\overline{\mathbf{a}}, \mathbf{s}) \in P_s}{\operatorname{argmin}} \; h(\mathbf{s})$$

where

$$P_{\mathbf{s}} = \left\{ (\overline{\mathbf{a}}, \mathbf{s}) \in \mathbb{R}^{N+M} \;\middle|\; \begin{array}{l} \begin{pmatrix} 0 & I \end{pmatrix} \overline{\mathbf{a}} - \mathbf{s} \leq 0 \\ \overline{\mathbf{a}} = \frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \mathbf{z}_i \\ \mathbf{s} \in \mathcal{S} = \mathbb{R}^M_{\geq 0}, \;\; \mathbf{z}_i \in D_i \end{array} \right\}. \quad (12)$$

and the objective function $h : \mathbb{R}^M_{\geq 0} \to \mathbb{R}$ is defined as

$$h(\mathbf{s}) = \sum_{q=1}^{M} (M + 1 - q)^{\kappa} \cdot \mathbf{s}(q)$$

for a positive constant $\kappa > 0$.

We will show, that the number of leading zeros of a possibly non-unique optimal solution $\mathbf{s}^{\star}$ provides the maximal disconnection time if the weighting factor $\kappa > 0$ is chosen appropriately. Observe that the objective function h is linear and places a heavier penalty on the smaller indices of $\mathbf{s}$.

To give an illustrative motivation for choice of the objective function and the choice of κ, we assume that, for all $i \in \{1, \ldots, \mathcal{I}\}$, $\alpha_i = 1$ before we prove the general case in Theorem 5.2.

Since the weighting parameters in the objective function h are positive, the constraint $\mathbf{s} \in \mathbb{R}^M_{\geq 0}$ implies $\overline{\mathbf{a}}(k^{\star} - 1 + q) = \mathbf{s}(q)$ for all optimal $\mathbf{s}^{\star}$ with $\overline{\mathbf{a}}(k^{\star} - 1 + q) \geq 0$, $q \in \{1, \ldots, M\}$. For simplicity, consider an isolated (power) exchange between $\mathbf{s}(q_1)$ and $\mathbf{s}(q_2)$ ($q_1 < q_2$) of a feasible

solution $\mathbf{s}$. Due to the linear system dynamics, reducing $\mathbf{s}(q_1)$ by $\gamma_i\varepsilon$ leads to an increase of $\mathbf{s}(q_2)$ by $\varepsilon\beta_i^{-1}$ in the case that losses have maximal impact. This is for example the case if $\overline{\mathbf{a}}(k^\star-1+q_1) = \mathbf{s}(q_1)$ and $\overline{\mathbf{a}}(k^\star-1+q_2) = \mathbf{s}(q_2)$ and $\overline{\mathbf{a}}(k^\star-1+q_1)$ can only be decreased by using a smaller $u_i^-(k^\star-1+q_1)$ by at least one RES i (i.e., discharge ε more from the battery at time $k^\star-1+q_1$) and simultaneously increase $u_i^+(k^\star-1+q_2)$ (i.e., charge ε more at time $k^\star-1+q_2$). Charging more at time $k^\star-1+q_2$ could for example be necessary to prevent that the battery constraints (3a) are violated at time steps $k^\star-1+q$ for $q > q_2$. Moreover, these considerations show, that decreasing $\mathbf{s}(q_1)$ by $\gamma_i\varepsilon$ can always be compensated by maximally increasing $\mathbf{s}(q_2)$ by ε/β_i.

If this power exchange results in a new feasible $\tilde{\mathbf{s}}$ with

$$\tilde{\mathbf{s}}(q_1) = \mathbf{s}(q_1) - \gamma_i\varepsilon \quad \text{and} \quad \tilde{\mathbf{s}}(q_2) = \mathbf{s}(q_2) + \varepsilon/\beta_i$$

and $\tilde{\mathbf{s}}(q) = \mathbf{s}(q)$ for all $q \notin \{q_1, q_2\}$, and since $q_1 < q_2$ by assumption, κ has to be chosen such that the value of the objective function decreases, i.e., $h(\tilde{\mathbf{s}}) < h(\mathbf{s})$ holds. Hence we obtain the estimate

$$\begin{aligned} 0 &> h(\tilde{\mathbf{s}}) - h(\mathbf{s}) \\ &= -(M+1-q_1)^\kappa\gamma_i\varepsilon + (M+1-q_2)^\kappa\varepsilon/\beta_i \end{aligned} \tag{13}$$

or equivalently

$$\gamma_i\cdot\beta_i > \left(\frac{M+1-q_2}{M+1-q_1}\right)^\kappa.$$

Since this inequality has to hold for arbitrary $q_1 < q_2$, $q_1, q_2 \in \{1,\dots,M\}$ the estimate

$$\gamma_i\cdot\beta_i > \left(\frac{M-1}{M}\right)^\kappa > \left(\frac{M+1-q_2}{M+1-q_1}\right)^\kappa$$

has to be satisfied which leads to the condition

$$\kappa > \frac{\log(\gamma\cdot\beta)}{\log\left(\frac{M-1}{M}\right)}.$$

That this condition indeed suffices is rigorously shown in the following theorem.

Theorem 5.2: Consider the grid disconnection problem defined in Definition 5.1. For an arbitrary optimal solution $(\overline{\mathbf{a}}^\star, \mathbf{s}^\star) \in P_s$ and an arbitrary feasible solution $(\overline{\mathbf{a}}^\sharp, \mathbf{s}^\sharp) \in P_s$, we define $q^\star, q^\sharp \in \{1,\dots,M+1\}$ as the maximal indices such that, for all $q < q^\star$, $\mathbf{s}^\star(q) = 0$ and, for all $q < q^\sharp$, $\mathbf{s}^\sharp(q) = 0$. Let $\beta := \min_{i=1,\dots,\mathcal{I}}\{\beta_i\}$ and $\gamma := \min_{i=1,\dots,\mathcal{I}}\{\gamma_i\}$ define the maximal losses of the battery models.

If κ is chosen such that

$$\kappa > \log(\beta\cdot\gamma) \Big/ \log\left(\frac{M-1}{M}\right) \tag{14}$$

then $q^\star \geq q^\sharp$ holds, i.e., the grid can be disconnected for at most $q^\star-1$ time steps.

Proof: Assume that $\alpha_i = 1$ and let $(\overline{\mathbf{a}}^\star, \mathbf{s}^\star) \in P$ be an optimal solution of the minimization problem with κ chosen according to Equation (14). Let $q^\star$ denote the first entry of $\mathbf{s}^\star$ which is unequal to zero, i.e., we have $\mathbf{s}^\star(q) = 0$ for all $q < q^\star$ and $\mathbf{s}^\star(q^\star) > 0$. (In the case $\mathbf{s}^\star = 0$, the statement of the theorem is trivially satisfied, and hence we can assume that $q^\star \leq M$.)

Assume there exists a feasible solution $(\overline{\mathbf{a}}^\sharp, \mathbf{s}^\sharp) \in P_s$ such that $\mathbf{s}^\sharp(q) = 0$ holds for all $q < q^\sharp$ and $q^\star < q^\sharp$. We will show that the existence of $(\overline{\mathbf{a}}^\sharp, \mathbf{s}^\sharp)$ contradicts the optimality of $(\overline{\mathbf{a}}^\star, \mathbf{s}^\star)$. Additionally, we assume without loss of generality that $\overline{\mathbf{a}}^\star(k^\star-1+q) = \mathbf{s}^\star(q)$ and $\overline{\mathbf{a}}^\sharp(k^\star-1+q) = \mathbf{s}^\sharp(q)$ holds for all q with $\overline{\mathbf{a}}^\star(k^\star-1+q) \geq 0$ and $\overline{\mathbf{a}}^\sharp(k^\star-1+q) \geq 0$, respectively.

Since $\mathbf{s}^\sharp(q^\star) = 0$ and $\mathbf{s}^\star(q^\star) = \overline{\mathbf{a}}(k^\star-1+q^\star) > 0$ there is an index $i \in \{1,\dots,\mathcal{I}\}$ such that $u_i^{+\star}(k^\star-1+q^\star) > 0$ or $u_i^{-\star}(k^\star-1+q^\star) > \underline{u}_i$, i.e., one of the constraints (3b) and (3c) is not active and it is possible to decrease $u_i^{+\star}(k^\star-1+q^\star)$ and/or $u_i^{-\star}(k^\star-1+q^\star)$ to reduce $\mathbf{z}_i^\star(k^\star-1+q^\star)$ and consequently also $\overline{\mathbf{a}}(k^\star-1+q^\star)$ and $\mathbf{s}^\star(q^\star)$. If we reduce $\mathbf{u}_i^{-\star}(k^\star-1+q^\star)$ by $\varepsilon > 0$, i.e.,

$$\tilde{u}_i^{-\star}(k^\star-1+q^\star) := u_i^{-\star}(k^\star-1+q^\star) - \varepsilon \tag{15}$$

then the state x_i decreases to

$$\tilde{x}_i^\star(k^\star-1+q) := x_i^{-\star}(k^\star-1+q) - \varepsilon \tag{16}$$

for all $q > q^\star$. If $x_i^{-\star}(k^\star+q) > 0$ for all $q > q^\star$ then $u_i^{-\star}(k-1+q^\star)$ can be changed without violating the constraints (3) and the variable $\mathbf{s}^\star(q^\star)$ can be reduced by $\gamma_i\cdot\varepsilon/\mathcal{I}$ which contradicts the optimality. The same argument applies if it is possible to change $u_i^{+\star}(k^\star-1+q^\star)$.

Hence, we can assume, that it is only possible to change $u_i^{-\star}(k^\star-1+q^\star)$ (or $u_i^{+\star}(k^\star-1+q^\star)$, respectively) by simultaneously changing $u_i^{-\star}(q)$ (or $u_i^{+\star}(q)$) at a time $q < k^\star-1+q^\star$ or $q > k^\star-1+q^\star$. Note that one of these options needs to be possible due to the existence of the solution $(\overline{\mathbf{a}}^\sharp, \mathbf{s}^\sharp)$ and since the constraints of the systems are decoupled, one can concentrate on one index $i \in \{1,\dots,N\}$ only.

If it is possible to increase $x_i^\star(q)$ by increasing $u_i^{-\star}(q)$ or $u_i^{+\star}(q)$ at time steps $q < k^\star-1+q^\star$ without increasing $\mathbf{s}^\star(q)$ for all $q < k^\star-1+q^\star$ (i.e., there exist time steps $q < k^\star-1+q^\star$ such that $\overline{\mathbf{a}}^\star(q)$ can be increased without changing $s^\star$) then this strategy increases in particular $x_i^\star(k^\star-1+q^\star)$. Hence, it is possible to decrease $u_i^{+\star}(k^\star-1+q^\star)$ or $u_i^{-\star}(k^\star-1+q^\star)$ without violating the constraints $x_i^\star(q) \geq 0$ for $q > k^\star-1+q^\star$, i.e., $\mathbf{s}^\star(q^\star)$ can be reduced which violates the optimality of $\mathbf{s}^\star$.

If the strategy (15) leads to $x_i^{-\star}(k^\star-1+q) < 0$ for some $q > q^\star$, again a contradiction to optimality can be derived based on the estimate (13) and the choice of κ by decreasing $\mathbf{s}^\star(q^\star)$ and increasing $\mathbf{s}^\star(q)$ for $q > q^\star$. Hence, $(\overline{\mathbf{a}}^\sharp, \mathbf{s}^\sharp) \in P_s$ does not exist, which completes the proof for $\alpha_i = 1$.

Assume that $\alpha_i < 1$, for at least one $i \in \{1,\dots,\mathcal{I}\}$. The strategy of reducing $s(q^\star)$ by increasing $\overline{\mathbf{a}}^\star(q)$ for $q < q^\star$ is applicable in the same way as in the case $\alpha_i = 1$. Moreover, if $\mathbf{s}(q^\star)$ can be decreased using the idea of Equation (16) and simultaneously increasing $\mathbf{s}(q)$ for $q > q^\star$, then the amount of energy which is lost due to self discharge for $q > q^\star$ decreases (i.e., the corresponding $u_i^{+\star}(k^\star-1+q^\star)$ or $u_i^{-\star}(k^\star-1+q^\star)$ can be decreased

more before $x_i^{+\star}(k^\star - 1 + q) - \varepsilon = 0$ becomes active) which increases the amount $s(q^\star)$ can be reduced to. ■

It has been shown in Theorem 5.2 that an optimal pair $(\overline{\mathbf{a}}^\star, \overline{\mathbf{s}}^\star)$ provides the maximal disconnection time if κ is chosen such that Condition (14) holds. However, for very large M, large values of κ are required, which lead to a numerically unstable scaling of the cost function h. Nevertheless, if a maximal disconnection time can be estimated, the presented approach can be easily generalized such that κ remains reasonably sized, e.g. the maintenance work requires at most eight hours of the 24 hours within the prediction horizon N. Moreover, numerical experiments indicate that $\kappa = 1$ works well even if Condition (14) is violated.

Remark 5.3: If no losses are considered, i.e., $\beta = \gamma = 0$, then any value $\kappa > 0$ can be used in the objective function h. For the values $M = 48$ and $\beta = \gamma = 0.95$ we obtain $\kappa > 4.88$ from Condition (14).

Remark 5.4: If the maximal duration of the islanded mode is a priori specified, the constraints $\begin{pmatrix} 0 & I & 0 \end{pmatrix} \overline{\mathbf{a}} - \mathbf{s} \leq 0$ can be used instead of $\begin{pmatrix} 0 & I \end{pmatrix} \overline{\mathbf{a}} - \mathbf{s} \leq 0$ to obtain a smaller value M and, hence, a smaller κ.

Remark 5.5: Since the objective function is convex and defined on a convex and compact set (compactness of $\mathcal{S}$ can be easily enforced), Assumptions 3.1 hold and convergence of Algorithm 1 can be concluded from Theorem 3.3.

VI. OTHER PERFORMANCE MEASURES

In Section V, we concentrated on operation of a microgrid in islanded mode. However, the grid operator may also be interested in reducing consumption peaks. The introduced cost function allows to combine this objective with the previously presented operation in islanded mode. To this end, we use the degrees of freedom regarding the choice of the objective function and the constraints to optimize a so called *peak-to-peak* performance metric and, then, couple it with the *islanded mode*. Furthermore, the goals of the individual RESs can be taken into account by suitably choosing the *local* objective functions f_i and the corresponding constraints D_i, $i \in \{1, 2, \ldots, \mathcal{I}\}$.

A. Peak-to-peak performance

If the microgrid is operated in a 'normal' mode, i.e., no outage is expected and no maintenance is scheduled, as argued in [7] and [9] it is beneficial to minimize the fluctuations in the energy demand and to penalize the deviation from a given reference value, for example the average power demand $\overline{\zeta} = \frac{1}{\mathcal{I}N} \sum_{i=1}^{\mathcal{I}} \mathbb{1}^T \mathbf{z}_i$. Hence, a penalty from a given reference for the next $k^\star \in \{0, \ldots, N-1\}$ time steps can be realized using the cost function $g_{k^\star}(\overline{\mathbf{a}}) = \sum_{j=0}^{k^\star - 1} \left(\overline{\mathbf{a}}(j) - \overline{\zeta}\right)^2$.

B. Combination of cost functions

As already argued, the grid operator can change the objective function in every MPC iteration without notifying the RESs. Consider the minimization problem of the grid operator

$$\min_{(\overline{\mathbf{a}}, \mathbf{s}) \in P_s} \eta \cdot g_{k^\star}(\overline{\mathbf{a}}) + \nu \cdot h(\mathbf{s}) \qquad \text{s.t.} \quad \begin{pmatrix} 0 & I & 0 \end{pmatrix} \overline{\mathbf{a}} - \mathbf{s} \leq 0$$

with positive weights $\eta, \nu \in \mathbb{R}_{>0}$. In this setting the deviation from the average is minimized until the point of disconnection $k^\star$ is reached, and the function h makes sure the length of the disconnection time is optimized.

Remark 6.1: Note that the maximal disconnection time according to Theorem 5.2 can only be guaranteed for $\eta = 0$. Hence η has to be chosen appropriately to keep the focus on the islanded mode.

VII. NUMERICAL EXPERIMENTS

In this section we visualize the results obtained by Algorithm 1 and Algorithm 2. We consider a setting of 300 RESs using the constants $C_i = 4$[kWh], $-\underline{u}_i = \overline{u}_i = 0.9$, $(\alpha_i, \beta_i, \gamma_i) = (0.96, 0.94, 0.98)$ and $x_i^0 = 2$[kWh] for $i = 1, \ldots, 300$. Moreover we use a discretization of $T = 0.5$[h] and a prediction horizon $N = 48$. The parameter ρ in the ADMM formulation is set to $\rho = 10$. To indicate the disconnection time, we use the notation $h_{k^\star}$ instead of h. The sequences $(w_i(k))_{k \in \mathbb{N}}$ for $i = 1, \ldots, 300$ are taken from a dataset provided by the Australian electricity distribution company Ausgrid. For a detailed analysis of the dataset see [19].

A. Results of Algorithm 1

In Figure 3 the solutions of Algorithm 1 using the objective functions

$$F(\mathbf{z}, \mathbf{s}) = \eta_p \cdot g_{24}\left(\frac{1}{\mathcal{I}} \sum_{i=1}^{\mathcal{I}} \mathbf{z}_i\right) + \frac{\mathcal{I}}{20} \cdot h_{24}(\mathbf{s}) \tag{17}$$

with $\eta_1 = 0$ and $\eta_2 = 10^3$ are visualized. For the objective function h_{24} we use $\kappa = 2.5$ which satisfies the condition given in Theorem 5.2. Recall that the subscript 24 indicates a planned disconnection from $k^* = 24$ (i.e., after 12 hours).

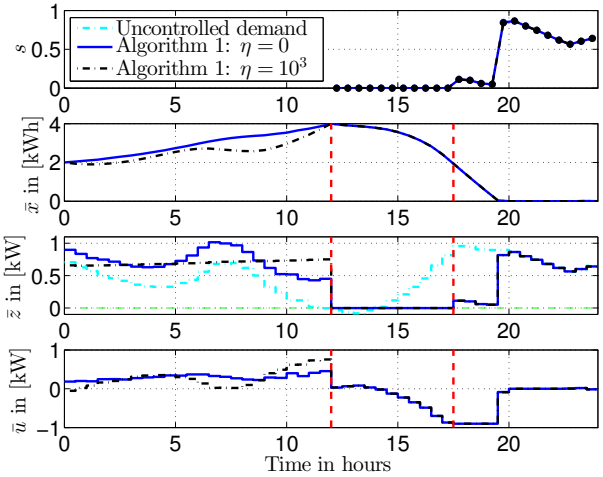

Fig. 3. Visualization of the variable $\mathbf{s}$ and the average values $\overline{x}$, $\overline{z}$, and $\overline{u}$ for a single minimization problem with different weights. The microgrid is disconnected after 12 hours and can stay islanded for 5 hours. Additionally, the uncontrolled power demand without storage devices is shown for comparison.

For the given initial state and parameters, the grid can be disconnected for 5 hours. In the case $\eta \neq 0$, additionally

the vertical grid load is minimized in the first 12 hours. Observe that at the time the microgrid must be reconnected, the average state of charge of the batteries is still at 50%. Hence, the requirement that the microgrid is reconnected is not due to a shortage of locally stored energy, but rather due to the maximal discharging rate being too small to satisfy the microgrid demand.

B. Closed-loop simulation of Algorithm 2

In Figure 4 the closed loop performance of the receding horizon Algorithm 2 is visualized. The grid operator wants to disconnect the grid after 48 time steps. Hence, the peak-to-peak variation is penalized and the disconnection time is maximized.

After 24 hours the grid is disconnected and stays disconnected for 14 time steps. After the RESs are connected again, the simulation is continued by minimizing the deviation from the average using the function g_{48}. As already pointed out, since only the cost function of the grid operator changes in this process, the RESs do not need to change anything on the local level.

For the closed loop simulation the weights $\eta = 10$ and $\nu = \mathcal{I}/20$ are used and κ is set to $\kappa = 1$. Even though $\kappa = 1$ does not satisfy condition (14) of Theorem 5.2, the maximal disconnection time is returned, which shows, that condition (14) is very conservative in our application.

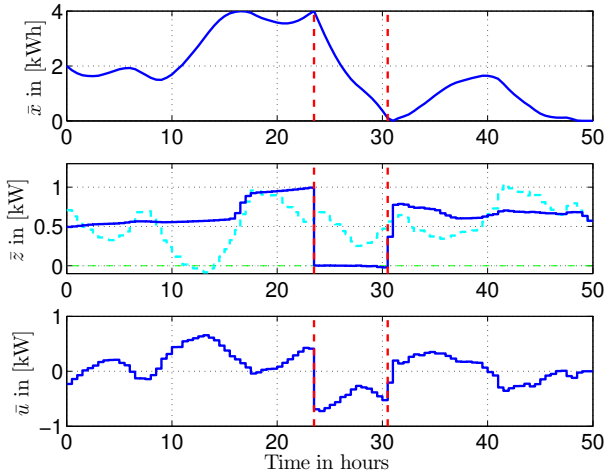

Fig. 4. Visualization of the average values $\overline{x}$, $\overline{z}$, and $\overline{u}$ for a simulation of 50 hours. During the first 24 hours, the deviation with respect to the average demand is penalized and the maximal disconnection time is computed. Afterwards, the microgrid is disconnected for 14 time steps (7 hours) before again the deviation from the average is penalized. The controlled power demand (blue) can be compared with the uncontrolled power demand (cyan) in the second graph.

VIII. CONCLUSIONS

In this paper we presented a hierarchical distributed optimization algorithm based on the alternating direction method of multipliers and designed for the control of a microgrid coordinated by a grid operator. We have shown how the flexibility in the objective function can be used to design an objective function to compute the maximal time interval that the microgrid can be operated in an islanded mode. The theoretical results are visualized by numerical simulations.

Future research will concentrate on the design of additional objective functions focusing on different objectives of the grid operator. Additionally we will investigate the speed of convergence of the distributed optimization algorithm especially in the context of model predictive control using warm-start techniques.

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