

In brief

- First fully turbulent SOL simulations self-consistently coupled to a kinetic neutral model implemented in GBS.
- Neutral kinetic equation with Krook operators for ionization, recombination and charge-exchange.
- Two fluid drift-reduced Braginskii equations are solved for the plasma.
- Development of a more refined two-point model in very good agreement with GBS
- Initial study of gas puff imaging with fluctuating neutrals.
- The details of the model in [C. Wersal and P. Ricci, 2015 *Nucl. Fusion* **55** 123014].

A model for neutral atoms in the SOL

Kinetic equation with Krook operators

$$\frac{\partial f_n}{\partial t} + \mathbf{v} \cdot \frac{\partial f_n}{\partial \mathbf{x}} = -\nu_{iz} f_n - \nu_{cx} n_n \left(\frac{f_n}{n_n} - \frac{f_i}{n_i} \right) + \nu_{rec} f_i \quad (1)$$

$$\nu_{iz} = n_e \nu_{iz} = n_e \langle \nu_{eiz}(\mathbf{v}_e) \rangle, \quad \nu_{cx} = n_i \nu_{cx} = n_i \langle \nu_{i\text{rel}cx}(\mathbf{v}_{\text{rel}}) \rangle, \quad \nu_{rec} = n_e \nu_{rec} = n_e \langle \nu_{e\text{rec}}(\mathbf{v}_e) \rangle$$

Boundary conditions: particle conservation, i.e.

$$f_n(\mathbf{x}_b, \mathbf{v}) = (1 - \alpha_{\text{refl}}) \Gamma_{\text{out}}(\mathbf{x}_b) \chi_{\text{in}}(\mathbf{x}_b, \mathbf{v}) + \alpha_{\text{refl}} [f_n(\mathbf{x}_b, \mathbf{v} - 2\mathbf{v}_p) + f_i(\mathbf{x}_b, \mathbf{v} - 2\mathbf{v}_p)] \quad (2)$$

with Γ_{out} the ion and neutral particle outflow, α_{refl} the reflection coefficient, \mathbf{v}_p the velocity perpendicular to the wall. The distribution function of absorbed and re-emitted particles is

$$\chi_{\text{in}}(\mathbf{x}_b, \mathbf{v}) = \frac{3}{4\pi} \frac{m^2}{T_b^2} \cos(\theta) \exp\left(-\frac{m\mathbf{v}^2}{2T_b}\right) \quad (3)$$

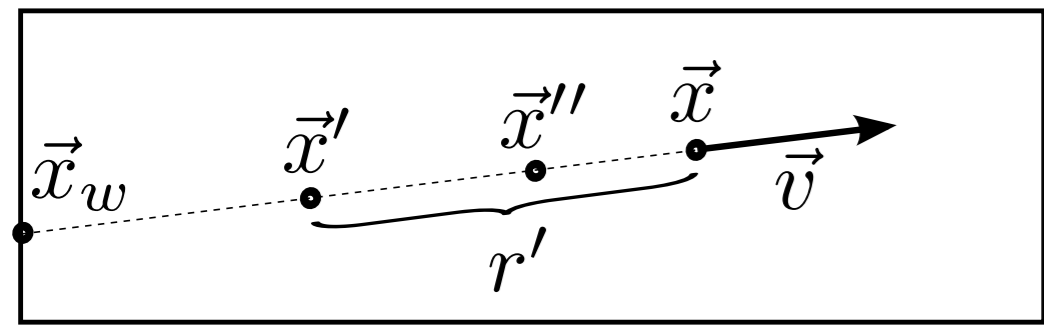
with θ the angle between \mathbf{v} and vector normal to the surface, and wall temperature T_b .

Two assumptions: $\tau_{\text{neutral losses}} < \tau_{\text{turbulence}}$ and $\lambda_{\text{mfip, neutrals}} \ll L_{\parallel, \text{plasma}}$

The method of characteristics

The formal solution of Eq. (1) is

$$f_n(\mathbf{x}_{\perp}, \mathbf{v}) = \int_0^{r'_{\perp b}} \left[\frac{S(\mathbf{x}'_{\perp}, \mathbf{v})}{v_{\perp}} + \delta(r'_{\perp} - r_{\perp b}) f_n(\mathbf{x}'_{\perp b}, \mathbf{v}) \right] \exp\left[-\frac{1}{v_{\perp}} \int_0^{r'_{\perp}} \nu_{\text{eff}}(\mathbf{x}'_{\perp}) dr'_{\perp}\right] dr'_{\perp} \quad (4)$$



$$S(\mathbf{x}, \mathbf{v}) = \nu_{cx}(\mathbf{x}) n_n(\mathbf{x}) \Phi_i(\mathbf{x}, \mathbf{v}) + \nu_{rec}(\mathbf{x}) f_i(\mathbf{x}, \mathbf{v})$$

$$\nu_{\text{eff}}(\mathbf{x}) = \nu_{iz}(\mathbf{x}) + \nu_{cx}(\mathbf{x})$$

$$r' = |\mathbf{x} - \mathbf{x}'|$$

An **integral equation** for neutral density is obtained by integrating Eq. (4) over \mathbf{v} .

$$n_n(\mathbf{x}_{\perp}) = \int d\mathbf{v} f_n(\mathbf{x}_{\perp}, \mathbf{v}) = \int_D n_n(\mathbf{x}'_{\perp}) \nu_{cx}(\mathbf{x}'_{\perp}) K_{p \rightarrow p}(\mathbf{x}_{\perp}, \mathbf{x}'_{\perp}) dA' + n_{n, \text{rec}}(\mathbf{x}_{\perp}) + n_{n, \text{walls}}(\mathbf{x}_{\perp}) \quad (5)$$

$$K_{p \rightarrow p}(\mathbf{x}_{\perp}, \mathbf{x}'_{\perp}) = \int_0^{\infty} \frac{1}{r'_{\perp}} \Phi_{\perp i}(\mathbf{x}'_{\perp}, \mathbf{v}_{\perp}) \exp\left[-\frac{1}{v_{\perp}} \int_0^{r'_{\perp}} \nu_{\text{eff}}(\mathbf{x}'_{\perp}) dr'_{\perp}\right] dv_{\perp} \quad (6)$$

$K_{p \rightarrow p}$ only depends on plasma quantities. Equation (5) and boundary conditions are spatially discretized, leading to a linear system of equations

$$\begin{bmatrix} n_n \\ \Gamma_{\text{out}} \end{bmatrix} = \begin{bmatrix} K_{p \rightarrow p} & K_{b \rightarrow p} \\ K_{p \rightarrow b} & K_{b \rightarrow b} \end{bmatrix} \cdot \begin{bmatrix} n_n \\ \Gamma_{\text{out}} \end{bmatrix} + \begin{bmatrix} n_{n, \text{rec}} \\ \Gamma_{\text{out, rec}} + \Gamma_{\text{out, i}} \end{bmatrix} \quad (7)$$

which is solved with standard methods. n_n is used to compute f_n and its moments using Eq. (4).

The GBS code

GBS is a **3D, flux-driven, global** turbulence code in limited geometry.

GBS solves the **two fluid drift-reduced Braginskii equations** [Ricci *et al.*, PPCF 2012], $k_{\perp}^2 \gg k_{\parallel}^2$, $d/dt \ll \omega_{ci}$

$$\frac{\partial n}{\partial t} = -\frac{1}{B} [\phi, n] + \frac{2}{eB} [C(\rho_e) - enC(\phi)] - \nabla_{\parallel} (n\nu_{ie}) + D_n(n) + S_n + n_n \nu_{iz} - n \nu_{rec} \quad (8)$$

$$\frac{\partial \tilde{\omega}}{\partial t} = -\frac{1}{B} [\phi, \tilde{\omega}] - \nu_{ij} \nabla_{\parallel} \tilde{\omega} + \frac{B^2}{m_i n} \nabla_{\parallel} j_{\parallel} + \frac{2B}{m_i n} C(\rho) + D_{\tilde{\omega}}(\tilde{\omega}) - \frac{n}{n} \nu_{cx} \tilde{\omega} \quad (9)$$

$$\frac{\partial \nu_{ie}}{\partial t} = -\frac{1}{B} [\phi, \nu_{ie}] - \nu_{ie} \nabla_{\parallel} \nu_{ie} + \frac{e}{\sigma_{\parallel} m_e} j_{\parallel} + \frac{e}{m_e} \nabla_{\parallel} \phi - \frac{T_e}{m_e n} \nabla_{\parallel} n - \frac{1.71}{m_e n} \nabla_{\parallel} T_e + D_{\nu_{ie}}(\nu_{ie}) + \frac{n}{n} (\nu_{en} + 2\nu_{iz})(\nu_{in} - \nu_{ie}) \quad (10)$$

$$\frac{\partial \nu_{ij}}{\partial t} = -\frac{1}{B} [\phi, \nu_{ij}] - \nu_{ij} \nabla_{\parallel} \nu_{ij} - \frac{1}{m_i n} \nabla_{\parallel} p + D_{\nu_{ij}}(\nu_{ij}) + \frac{n}{n} (\nu_{iz} + \nu_{cx})(\nu_{in} - \nu_{ij}) \quad (11)$$

$$\frac{\partial T_e}{\partial t} = -\frac{1}{B} [\phi, T_e] - \nu_{ie} \nabla_{\parallel} T_e + \frac{4T_e}{3eB} \left[\frac{T_e}{n} C(n) + \frac{7}{2} C(T_e) - eC(\phi) \right] + \frac{2T_e}{3n} \left[\frac{0.71}{e} \nabla_{\parallel} j_{\parallel} - n \nabla_{\parallel} \nu_{ie} \right] + D_{T_e}(T_e) + D_{\nu_{ie}}(T_e) + S_{T_e} + \frac{n}{n} \nu_{iz} \left[-\frac{2}{3} E_{iz} - T_e + m_e \nu_{ie} \left(\nu_{ie} - \frac{4}{3} \nu_{in} \right) \right] - \frac{n}{n} \nu_{en} m_e \frac{2}{3} \nu_{ie} (\nu_{in} - \nu_{ie}) \quad (12)$$

$$\frac{\partial T_i}{\partial t} = -\frac{1}{B} [\phi, T_i] - \nu_{ij} \nabla_{\parallel} T_i + \frac{4T_i}{3eB} \left[C(T_e) + \frac{T_e}{n} C(n) - \frac{5}{3} C(T_i) - eC(\phi) \right] + \frac{2T_i}{3n} \left[\frac{1}{e} \nabla_{\parallel} j_{\parallel} - n \nabla_{\parallel} \nu_{ij} \right] + D_{T_i}(T_i) + D_{\nu_{ij}}(T_i) + S_{T_i} + \frac{n}{n} (\nu_{iz} + \nu_{cx}) \left[T_n - T_i + \frac{1}{3} (\nu_{in} - \nu_{ij})^2 \right] \quad (13)$$

$$\nabla_{\perp}^2 \phi = \omega, \quad \rho_* = \rho_s / R, \quad \nabla_{\parallel} f = \mathbf{b}_0 \cdot \nabla f, \quad \tilde{\omega} = \omega + \tau \nabla_{\perp}^2 T_i, \quad p = n(T_e + \tau T_i)$$

► A set of fluid boundary conditions applicable at the magnetic pre-sheath entrance where the magnetic field lines intersect the limiter is used [Loizu *et al.*, PoP 2012]

Some achievements of GBS (see also http://spc.epfl.ch/research_theory_plasma_edge):

- SOL width scaling as a function of dimensionless/engineering plasma parameters
- Origin and nature of intrinsic toroidal plasma rotation
- Non-linear turbulent regimes in the SOL
- Mechanism regulating the equilibrium electrostatic potential

A refined two-point model

Two-point models describe the relation between target (HFS limiter) and upstream (LFS mid-plane) T_e . They are derived from 1-D models along B and widely used experimentally.

Simplest two-point model for T_e in the limited SOL is

$$\nabla_{\parallel} \left(\frac{5}{2} \Gamma T_e \right) - \chi_{e0} \nabla_{\parallel} \left(T_e^{5/2} \nabla_{\parallel} T_e \right) = S_Q$$

$$\nabla_{\parallel} \Gamma = \nabla_{\parallel} (n\nu_{ij}) = S_n$$

with $\nabla_{\parallel} T_{e,u} = 0$, $Q_t = \gamma_e \Gamma T_{e,t}$, $\gamma_e \approx 5$, $\chi_{e0} = 3/2 \bar{n} \kappa_{e\parallel}$, and constant S_Q and S_n .

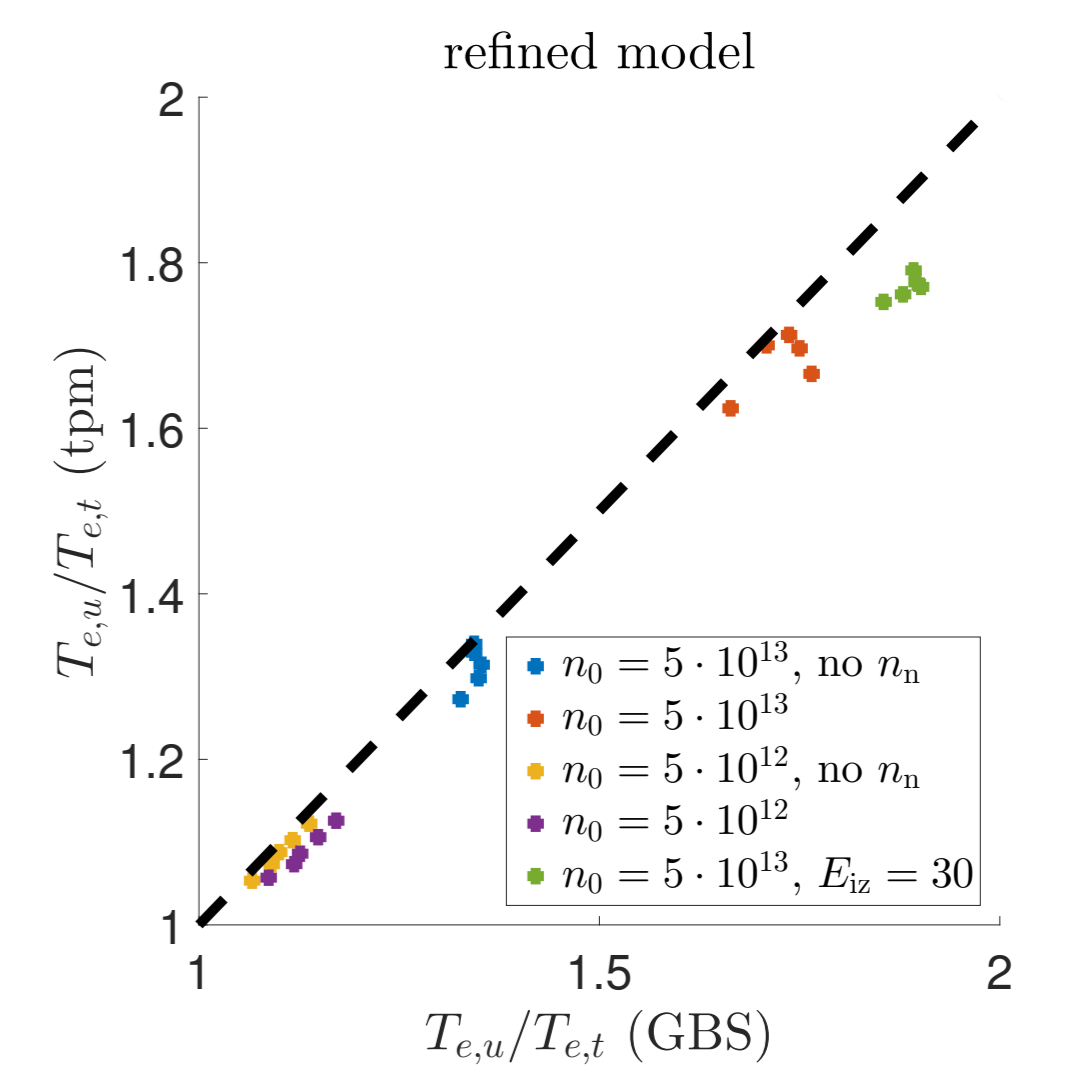
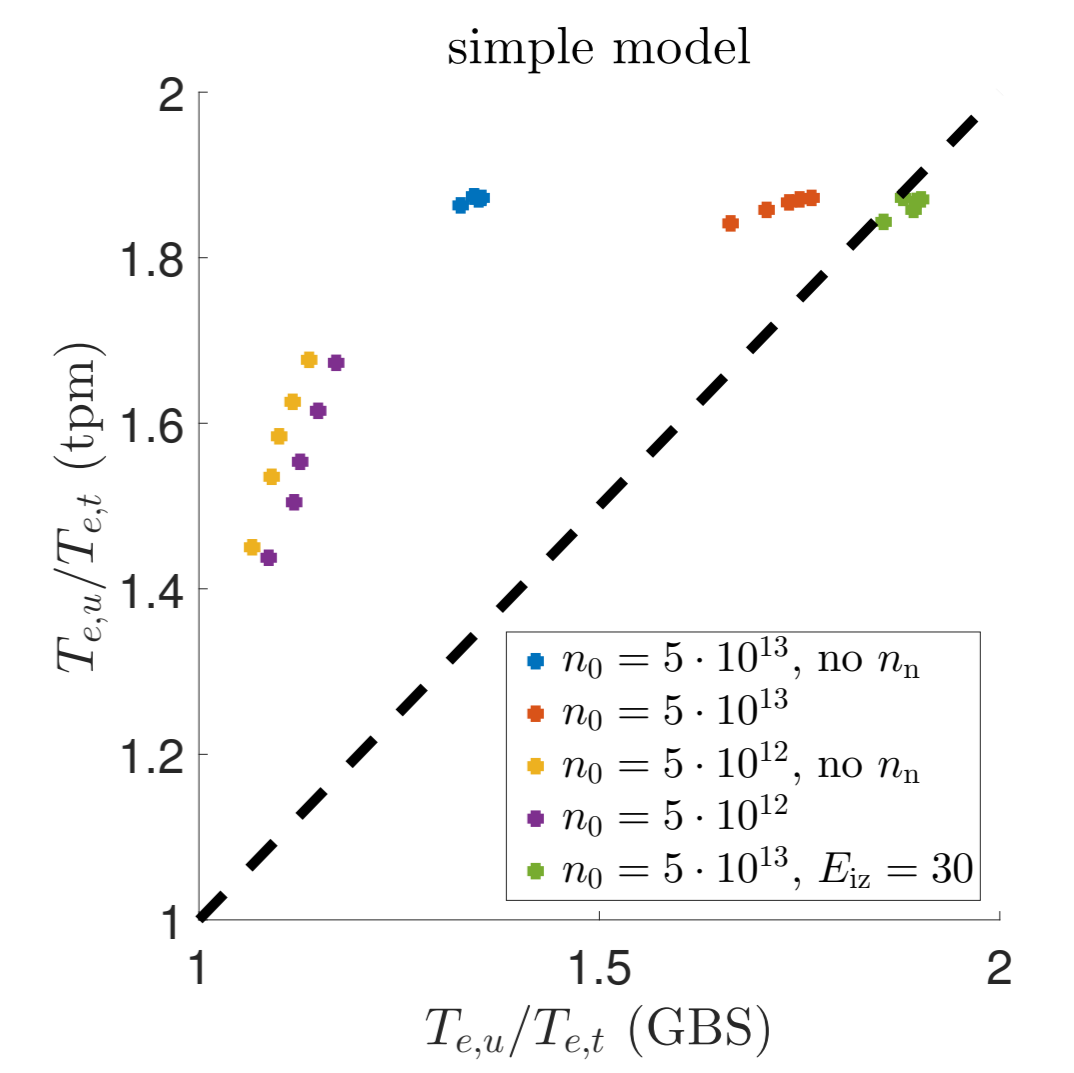
A **refined two-point model** is derived from the drift-reduced Braginskii equations

$$\nabla_{\parallel} \left(\frac{5}{2} \Gamma T_e \right) - \chi_{e0} \nabla_{\parallel} \left(T_e^{5/2} \nabla_{\parallel} T_e \right) - v_{\parallel} \nabla_{\parallel} (n T_e) = S_Q - n_n \nu_{iz} (T_e) E_{iz}$$

$$\nabla_{\parallel} \Gamma = \nabla_{\parallel} (n\nu_{ij}) = S_n + n_n \nu_{iz} (T_e)$$

with $\nabla_{\parallel} T_{e,u} = 0$ and the assumptions

- v_{\parallel} is linear from $-c_s$ to c_s
- $c_s = \sqrt{T_{e,t} + T_{i,t}} \approx \sqrt{2T_{e,t}}$
- Cosine-shaped S_Q and S_n
- n_n is decaying exponentially from limiter with λ_{mfip}
- Third input parameter, S_{iz} , the total ionization source [C. Wersal, P. Ricci, and J. Loizu, 2016 submitted to *PPCF*]



Neutral fluctuations and gas puff imaging

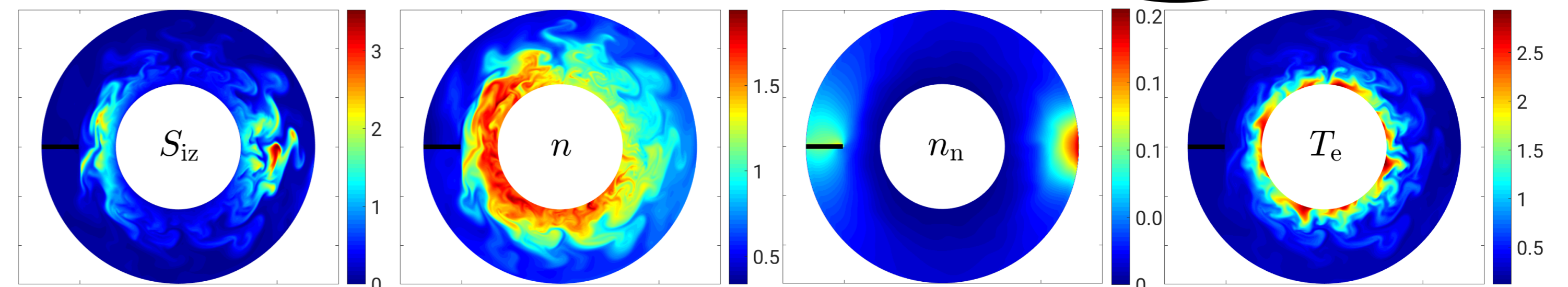
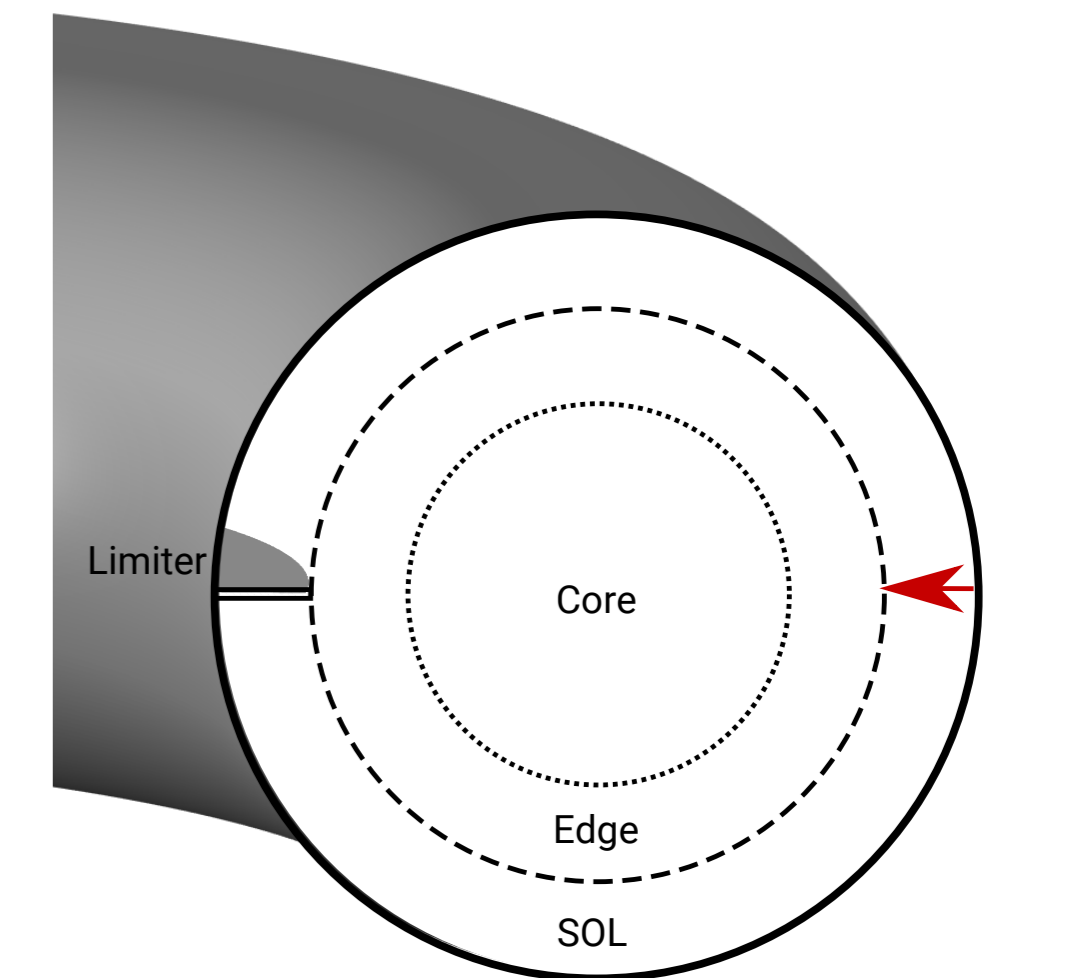
► Simulation with **SOL and edge**

► **Gas puff from LFS**

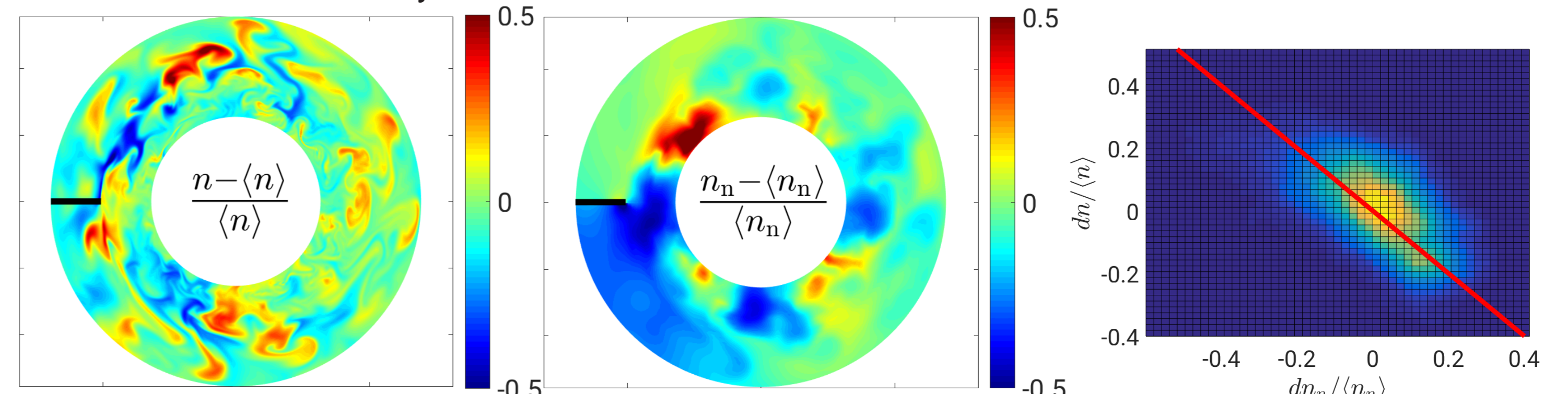
► Small constant main wall recycling

► $n_0 = 2 \cdot 10^{13} \text{cm}^{-3}$, $T_0 = 20 \text{eV}$, $q_0 = 3.87$, $\rho_*^{-1} = 500$, $a_0 = 200 \rho_s$, $\rho_s \approx 1 \text{mm}$, $R/c_s \approx 10 \mu\text{s}$

► $S_{iz} = n_n n r_{iz}(T_e)$ is approximately proportional to light emission



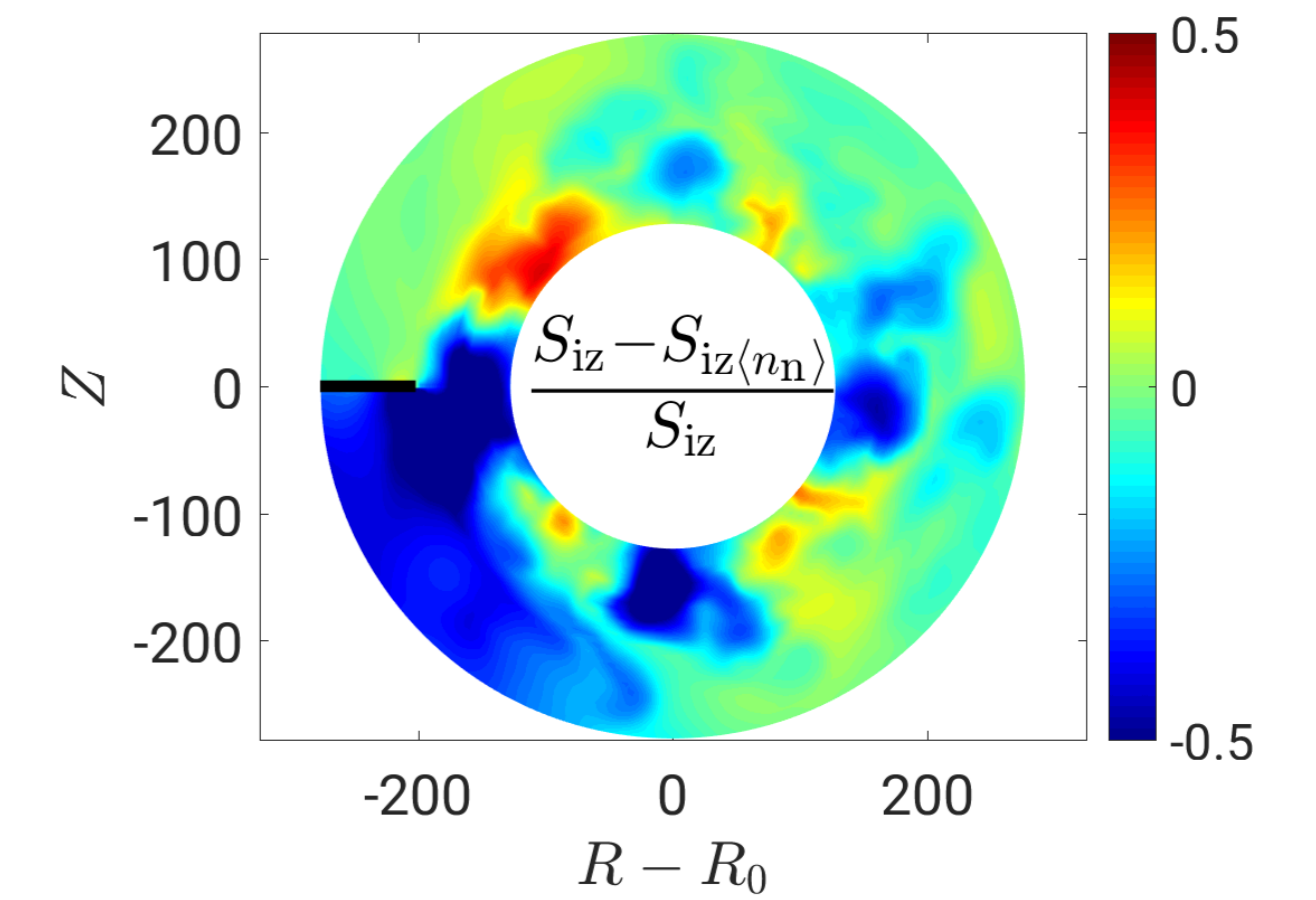
Plasma and neutral density show **anti-correlation**



Evaluating S_{iz} without neutral density fluctuations leads to relative errors of up to 50% in the emission rate.

$$S_{iz} = n_n n r_{iz}(T_e)$$

$$S_{iz, \langle n_n \rangle} = \langle n_n \rangle n r_{iz}(T_e)$$



Towards a simpler neutral model

Repeat a HFS gas puff simulation without neutral fluctuations

► (left) Average n_n , ν_n , and T_n

($S_{iz} = \langle n_n \rangle n r_{iz}$)

→ no significant differences

► (right) Average $S_{iz} = \langle n_n n r_{iz} \rangle$ and neglect other neutral-plasma terms

→ large differences

