Gravity insensitive flexure pivots for watch oscillators

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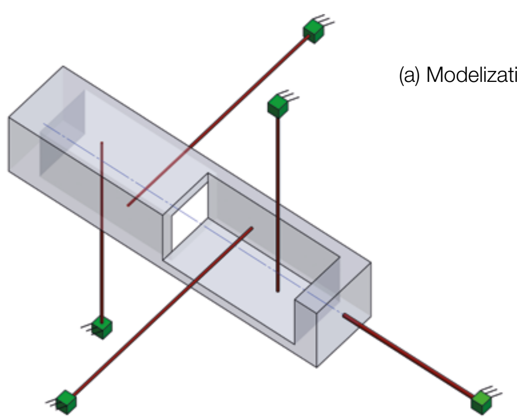
Abstract

Classical pivots have frictional losses leading to the limited quality factor of oscillators used as time bases in mechanical watches. Flexure pivots address these issues by greatly reducing friction. However, they have drawbacks such as gravity sensitivity and limited angular stroke. This paper analyses these problems for the cross-spring flexure pivot and presents an improved version addressing these issues. We first show that the cross spring pivot cannot be both insensitive to gravity and have a long stroke. A 10 ppm sensitivity to gravity acceptable for watchmaking applications occurs only when the leaf springs cross at about 87.3 % of their length, but the stroke is only 30.88 % of the stroke of the symmetrical cross-spring pivot. For the symmetrical pivot, gravity sensitivity is of the order of 10^4 ppm. Our solution is to introduce the *co-differential* concept which we show to be gravity insensitive. We then use the co-differential to build a gravity insensitive flexure pivot with long stroke. The design consists of a main rigid body, two co-differentials and a torsional beam. We show that our pivot is gravity insensitive and achieves 100 % of the stroke of symmetrical pivots.

Résumé

Les frottements dans les paliers lisses des oscillateurs utilisés comme bases de temps des montres mécaniques mènent à un facteur de qualité limité. Les guidages flexibles permettent de réduire ces pertes en éliminant le frottement. En revanche, ils ont des défauts en raison de leur sensibilité à la gravité et leur course limitée. Cet article analyse le pivot flexible à lames croisées et en présente une version améliorée. Dans un premier temps, nous démontrons que le pivot à lames croisées ne peut pas être à la fois insensible à la gravité et avoir une longue course. Une insensibilité de 10 ppm acceptable pour les montres se produit lorsque les lames-ressort se croisent à 87.3 % de leurs longueurs, mais la course n'est alors que 30.88 % de la course du pivot croisé symétrique. Pour le pivot symétrique, la sensibilité à la gravité est de l'ordre de 10^4 ppm. Notre solution est l'introduction du concept du codifférentiel que l'on démontre insensible à la gravité. On utilise le codifférentiel pour réaliser un pivot flexible à grande course. Le mécanisme est composé d'un corps rigide principal, de deux codifférentiels et d'une tige de torsion. Nous démontrons que notre pivot est insensible à la gravité et qu'il atteint 100 % de la course des pivots symétriques.



(a) Modelization

(b) Demonstrator.

Fig. 1 : New gravity insensitive flexure pivot (GIFP).

[1] At ICube – INSA, Strasbourg, France, since September 2015

1. Introduction and statement of results

1.1 Mechanical watch oscillators

The time base used in all mechanical watches is a harmonic oscillator consisting of a spiral spring attached to a balance wheel having a rigid pivot rotating on jewelled bearings, see figure 2(a). The pivoting motion on bearings causes significant friction and decreases watch autonomy as well as oscillator quality factor to the order of several hundred, this quantity believed to be the most significant indicator of chronometric performance [4].

It is well-known that flexure pivots drastically reduce friction, see [5, 6], so flexure-pivot based oscillators could improve mechanical watch time bases. In 2014, a flexure pivot was first used as mechanical watch time base, see figure 2(b), thereby increasing quality factor to several thousand and watch autonomy by an order of magnitude to approximately 30 days [3]. This flexure pivot has a special geometry designed to minimize the effect of gravity on stiffness.

In this paper, we describe a new flexure pivot minimizing the effect of gravity while retaining a long angular stroke, making it desirable as a time base for mechanical watches.

1.2 Oscillator flexure pivot specifications

Mechanical harmonic oscillators must obey Hooke's Law so spring stiffness should be constant. Chronometric performance means constant frequency and since this depends on spring stiffness, portable timekeepers such as watches must have oscillators whose spring stiffness is insensitive to outside influences such as temperature and the orientation of the force of gravity. Since mechanical watches are precise to within a few seconds per day, we will consider an effect to be negligible if it is of the order of 10 ppm (parts per million), in watchmaking terms, about a 1 s/d error.

In addition to being rotational bearings, flexure pivots provide an elastic restoring force so they can be used as springs for harmonic oscillators. However, their application to time bases can be limited by the following factors:

- **Limitation 1**. Spring stiffness can be a non-linear function of rotation angle.
- **Limitation 2**. By construction, the kinematics of flexure pivots closely approximate rotational motion around a fixed axis but small translation can occur as angular rotation increases, a so-called a *parasitic shift*.
- **Limitation 3**. Spring stiffness can be affected by the orientation of gravity load.
- **Limitation 4**. Limited stroke makes it difficult to maintain and count oscillation using classical watch escapements.

Non-linearity of beam stiffness under bending has been studied extensively [2] including the case of flexure pivots [8], and parasitic shift has also been investigated in the literature [9], so we do not consider these issues here.

We will focus on gravity sensitivity as this issue has not received much attention. We will also consider the stroke for a given aspect ratio and admissible stress level of the beams making up the flexure pivots.

Definition. We apply the term *gravity insensitive* to an oscillator if the relative change in its period caused by the effect of gravity on its stiffness is of order 10 ppm. Otherwise, we will say that it is *gravity sensitive*.

Definition. We define *stroke* to be the rotation angle of the pivot which leads to a maximum stress level in the beams equal to the admissible value σ_{adm}, [9, p. 29]. The value of σ_{adm} is taken to be the same for all pivots considered in this paper. Stroke is essentially the maximum amplitude of the oscillator.

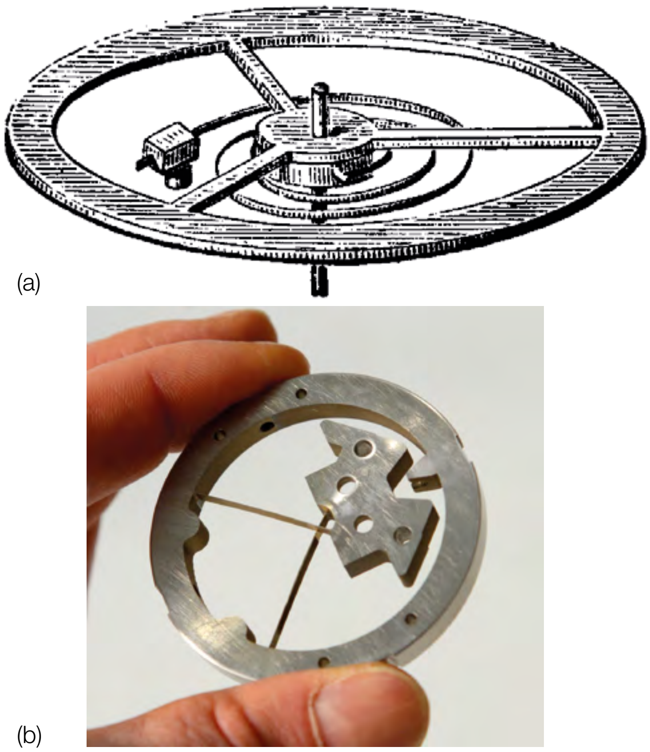
We can now define design goals for our flexure pivots.

Goal 1. Gravity insensitivity.

Goal 2. Maximum angular stroke for a given beam aspect ratio and given admissible stress and Young's modulus (material properties).

Since the main form of linear force affecting the chronometric performance of a portable timekeeper is gravity, we will use the term *gravity* to refer to any linear acceleration. The results of this paper apply to all linear accelerations.

1.3 The cross spring flexure pivot (CSFP)



(a)

(b)

Fig. 2: (a) Classical pivot watch time base [12], (b) Flexure pivot watch time base demonstrator.

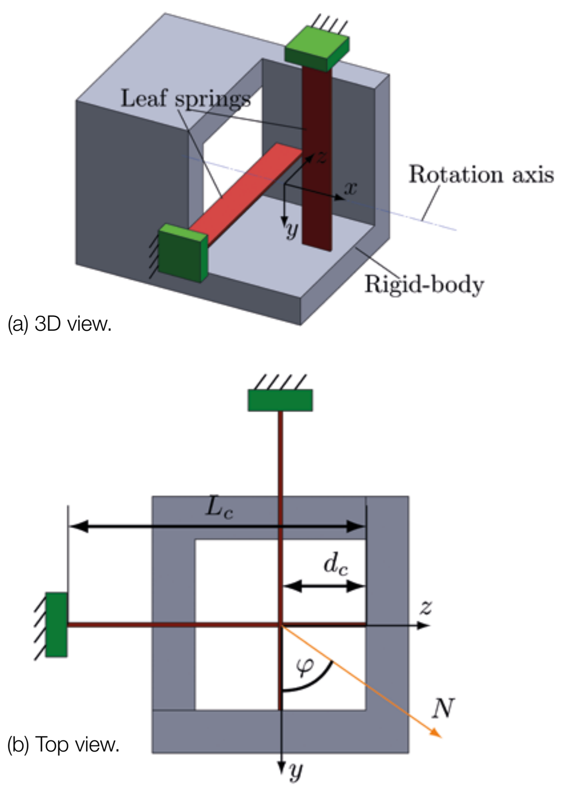
Our analysis begins with the study of the well-known *cross-spring flexure pivot* (CSFP) described by Wittrick [14] and applied to watchmaking in [3,10]. The CSFP consists of a rigid-body attached to the ground by two perpendicular leaf-spring beams, as illustrated in figure 3. The beam length is L_c and d_c denotes the distance between their crossing and the mobile end, as shown in figure 3(b).

The beam geometry is such that the CSFP has one DOF (degree of freedom), a rotation with axis lying on the intersection of the beam mid-planes [9, p. 97], the x-axis in figure 3(a).

Tab. 1 : Properties of the CSFP for special values of $\delta_c = d_c / L_c$.

δ_c / Specification	0	0.127	0.5	0.873	1
Stiffness error due to gravity [ppm]	10^3	10	10^4	10	10^3
Normalized angular stroke [%]	25	30.88	100	30.88	25

The performance of the CSFP is evaluated by varying the parameter $\delta_c = d_c / L_c$, with d_c and L_c as in figure 3(b). Table 1 shows that choosing δ_c = 0.127 or δ_c = 0.873 minimizes gravity sensitivity to the order of 10 ppm, satisfying goal 1 of section 1.2, where stiffness error due to gravity is defined in section 2.5. On the other hand, the angular stroke is only 30.88 % of the maximum angular stroke occurring at δ_c = 0.5, so goal 2 of section 1.2 is not satisfied. On the other hand δ_c = 0.5 satisfies goal 2 but not goal 1, since gravity sensitivity is of order 10^4 ppm.



(a) 3D view.

(b) Top view.

Fig. 3 : Cross-spring flexure pivot (CSFP).

1.4 New gravity insensitive flexure pivot (GIFP)

In order to have a wider range of gravity insensitivity, we design a new flexure pivot which we name *Gravity Insensitive Flexure Pivot* (GIFP). Figure 1 illustrates the GIFP as compared to the CSFP. The design is based on the co-differential concept and will be described and analysed completely in section 2.

1.5 Performance of GIFP

Tab. 2 : Properties of the GIFP for special values of $\delta = d / L$.

δ / Specification	0	0.127	0.5	0.873	1
Stiffness error due to gravity [ppm]	10	10	10	10	10
Normalized angular stroke [%]	25	30.88	100	30.88	25

The performance of the GIFP is again evaluated by varying $\delta = d / L$, with d and L as shown in figure 4, where stiffness error due to gravity is defined in section 2.5. Table 2 summarizes the performance of the GIFP for special values of δ. In particular, δ = 0.5 gives a long stroke gravity insensitive flexure pivot meeting all the goals of section 1.2.

1.6 Comparison between GIPF and CSFP

Tab. 3 : Comparison of CSFP with δ_c = 0.127 and GIFP with δ = 0.5.

Pivot type / Specification	CSFP, δ_c = 0.127	GIFP, δ = 0.5
Stiffness error due to gravity [ppm]	10	10
Normalized angular stroke [%]	30.88	100

Table 3 compares GIFP with δ = 0.5 and CSFP with δ_c = 0.127 and shows that GIFP is superior to CSFP in term of angular stroke while having the same order of gravity insensitivity.

2. Design and analysis of GIFP

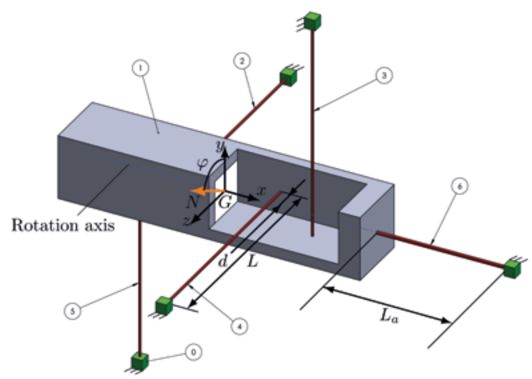
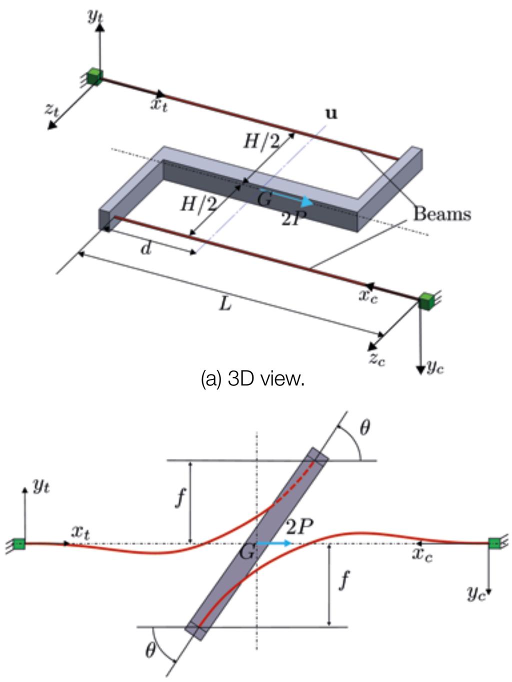


Fig. 4 : The gravity insensitive flexure pivot (GIFP).

The GIFP is illustrated in figure 4 and will be completely described in this section. The design is based on a new flexure element we have called the co-differential.

2.1 The co-differential



(a) 3D view.

(b) Side view under rotation θ about the u axis.

Fig. 5 : The co-differential.

We define co-differential to mean the flexure element consisting of two identical beams subjected to the same kinematic boundary conditions where the loading (due to gravity) is always such that their axial loads are opposite (have the same magnitude but opposite signs). In other words, when one beam is subjected to tensile axial load, the other one is subjected to compressive axial load with equal magnitude.

Figure 5(a) shows a co-differential consisting of two flexible beams of length L symmetrically positioned at a 180° rotation with respect to the point G (center of gravity) and in the x–z plane. Both beams have the same geometric properties and are made of the same material. The beam arrangement is such that there is one DOF, rotation about the axis u, see figure 5(a).

The total axial load applied to the co-differential at point G is denoted by $2P$, as shown in figure 5(b). Since both beams are at distance $H/2$ from the point G, the axial load on each beam is P. One beam is under tension and its coordinates at its base are x_t, y_t, z_t and the other under compression the coordinates at its base are x_c, y_c, z_c.

Both cantilever beams cross the rotation axis u at distance d and have the same kinematic boundary conditions at their mobile ends : deflection f and slope θ, as shown in figure 5(b).

2.2 Co-differential gravity insensitivity

We quantify gravity insensitivity by considering the behaviour of the stiffness matrix under axial load. In particular, the stiffness matrix K of a cantilever beam subject to an axial load P can be modeled by linear Euler-Bernoulli beam theory, see [1, 11], to give

$$\mathbf{K} = \mathbf{K}_0 + \frac{\lambda}{30}\mathbf{K}_1 - \frac{\lambda^2}{12600}\mathbf{K}_2 + \mathcal{O}\left(\lambda^3\right) \quad (1)$$

where $\lambda = PL^2/EI$ denotes the normalized axial load,

$$\mathbf{K}_0 = \begin{pmatrix} 12 & -6 \\ -6 & 4 \end{pmatrix}, \quad \mathbf{K}_1 = \begin{pmatrix} 36 & -3 \\ -3 & 4 \end{pmatrix}, \quad \mathbf{K}_2 = \begin{pmatrix} 18 & -9 \\ -9 & 22 \end{pmatrix},$$

and $\mathcal{O}$ represents the Landau notation: $A(\lambda) = \mathcal{O}(B(\lambda))$ if there is a constant C such that $\|A(\lambda)\| < C\|B(\lambda)\|$ for all small λ, see [7].

When the load P is taken to be gravity, the constant term K_0 represents the nominal stiffness, and the terms in K_1, K_2, represent the sensitivity to gravity. Therefore, gravity insensitivity is achieved by minimizing the non-constant terms in the stiffness power series expansion.

The geometry of the co-differential is such that when one beam is under tension, the other is under compression. We therefore compute the stiffness matrix of the co-differential K_{cd} by adding the stiffness matrix K_+ of the beam under tension and the stiffness matrix K_- of the beam under compression. Going from tension to compression changes the sign of P and therefore the sign of λ, so that $K_-(\lambda) = K_+(-\lambda)$. Substituting these values in equation (1) and adding gives

$$\mathbf{K}_{cd} = \mathbf{K}_+ + \mathbf{K}_- = 2\,\mathbf{K}_0 - \frac{\lambda^2}{6300}\mathbf{K}_2 + \mathcal{O}\left(\lambda^3\right) \quad (2)$$

A cancellation of the λ term occurs in equation (2) showing that the co-differential improves gravity insensitivity since the deviation from the constant term has gone from first order to second order.

2.3 GIFP design

We use the co-differential concept to design our gravity insensitive pivot. Since a pivot has one DOF and a rigid body has 6 DOF, the simplest design consists of a rigid main body and five beams blocking all but one DOF. The five beams consist of two co-differentials having two beams each and a fifth torsional beam insensitive to axial load. Note that the two co-differentials share the same rigid mass.

The configuration of GIFP is depicted in figure 4. The design comprises a rigid-body (1) attached to the ground (0) by five beams : a co-differential in the y direction with beams (3) and (5), a co-differential in the z direction with beams (2) and (4), and a single torsional beam (6) in the x direction. The single degree of freedom is rotation around the x axis.

The mechanism is statically determinate (isostatic) as there is only one DOF and there are no redundant constraints. The co-differential beams have length L and cross the axis of rotation at distance d from their mobile ends.

2.4 GIFP vs CSFP rotational stiffness

Since axial load has no effect on stiffness of the torsional beam (6) of figure 4, the x-component of the gravity load has no effect on the rotational stiffness of the pivot. We therefore assume that the gravity load N is applied in the y–z plane at angle φ with respect to y–axis.

We will compute the rotational stiffness k of the GIFP using standard linear Euler-Bernoulli beam theory. In order to do this, we write E for Young's modulus and I for the moment of inertia of each beam, and denote by $\bar{N} = NL^2/EI$ the normalized gravity load, noting that $\bar{N} < 1$ holds.

Writing k_a for the normalized torsional stiffness of the axial torsional beam, then it is known that $k_a = G_a J_a / L_a$, see [13], where L_a is the length of the torsional beam, G_a its shear modulus and J_a its polar area moment of inertia.

Applying linear Euler-Beam theory and neglecting the effect of parasitic shift then gives

$$k = k_a + \frac{16EI}{L}\left(3\delta^2 - 3\delta + 1\right) - \frac{EI}{12600L}\left(9\delta^2 - 9\delta + 11\right)\bar{N}^2 + \mathcal{O}(\theta^2 + \theta^2\bar{N}^2 + \bar{N}^4)\,. \quad (3)$$

With the same assumptions, we estimate the stiffness k_c of the CSFP. Once again, linear Euler-Bernoulli theory and neglecting parasitic shift gives

$$\begin{aligned} k_c &= \frac{8EI}{L}\left(3\delta_c^2 - 3\delta_c + 1\right) \\ &\quad + \frac{2EI}{15L}\left(9\delta_c^2 - 9\delta_c + 1\right)(\sin\varphi + \cos\varphi)\,\bar{N} \\ &\quad - \frac{EI}{6300L}\left(9\delta_c^2 - 9\delta_c + 11\right)\bar{N}^2 \quad + \mathcal{O}(\theta^2 + \theta^2\bar{N} + \bar{N}^3), \quad (4) \end{aligned}$$

where we have written δ_c for the ratio d/L of the CSFP. Equations (3) and (4) show that GIFP has improved gravity insensitivity as compared to CSFP since its stiffness k does not have a $\bar{N}$ term for all values of δ. For CSFP, the $\bar{N}$ term disappears only when $\delta_c = (3 \pm \sqrt{5})/6$, that is, $\delta_c = 0.127$ and $\delta_c = 0.873$.

Even when $\delta = 0.127$ or $\delta = 0.873$, GIFP is, in principle, gravity insensitive for a wider range of rotational angles θ, since the $\mathcal{O}(\bar{N}\theta^2)$ of equation (4) is replaced by $\mathcal{O}(\bar{N}^2\theta^2)$ in equation (3), which has smaller order.

2.5 GIFP vs CSFP gravity insensitivity

In order to quantify the effect of the gravity force on the rotational stiffness of GIFP, we define a nominal stiffness

$$k_0 = k_a + \frac{16EI}{L}\left(3\delta^2 - 3\delta + 1\right)$$

corresponding to the first two terms in equation (3). We then define the relative stiffness error ε due to gravity by

$$\varepsilon = \left|\frac{k - k_0}{k_0}\right| \times 10^6,$$

expressed in ppm's. We similarly define a nominal stiffness

$$k_{c,0} = \frac{8EI}{L}\left(3\delta_c^2 - 3\delta_c + 1\right)$$

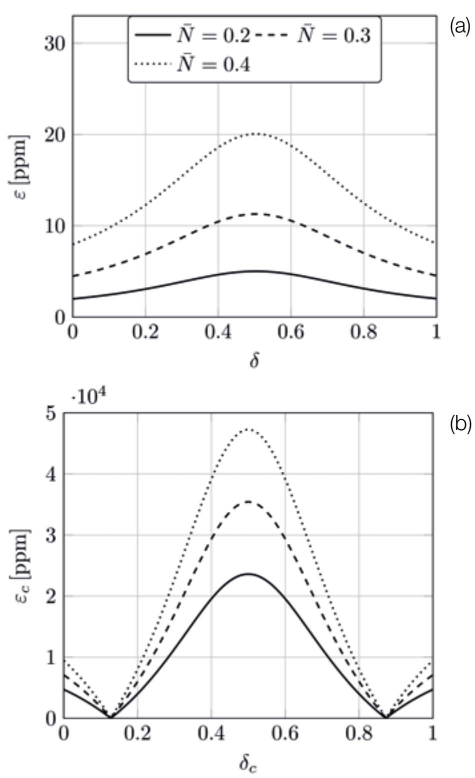


Fig. 6 : Stiffness error due to gravity versus δ for normalized gravity loads $\bar{N}$: (a) GIFP, (b) CSFP.

for the CSFP and the relative stiffness error ε_c due to gravity by

$$\varepsilon_c = \left|\frac{k_c - k_{c,0}}{k_{c,0}}\right| \times 10^6.$$

The error for GIFP is much less sensitive to δ and is in the order of 10 ppm for all values of the design parameter. Figure 6(a) shows that the relative stiffness error ε is of the order of 10 ppm, so satisfies specification 2 of section 1.2 and is relatively insensitive to the choice of δ.

On the other hand, figure 6(b) shows that relative stiffness error ε_c of CSFP is not only three orders of magnitude larger, 10^4 ppm versus 10 ppm, but is also more sensitive to the choice of δ_c.

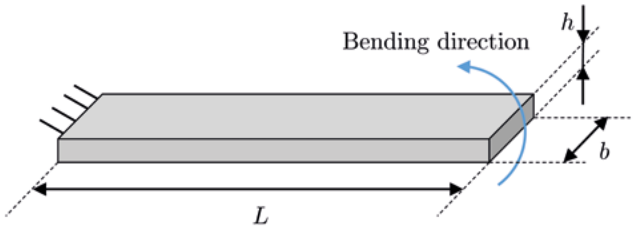


Fig. 7 : Geometry of a cantilever beam.

2.6 GIFP vs CSFP angular stroke

Based on [14] and [9, formula (3.3), page 29], we derive the following formula for the angular stroke θ_s for both GIFP and CSFP

$$\theta_s = \frac{1}{1+3\,|1-2\delta|}\,\frac{2\sigma_{\mathrm{adm}}}{E\alpha}, \tag{5}$$

where σ_{adm} is the maximum admissible stress of the beam [9, p. 29] and α is the aspect ratio of the beam. For a beam of rectangular cross-section with width b and thickness h as shown in figure 7, the aspect ratio is $\alpha = h/L$, and for a beam of circular cross-section with diameter D, the aspect ratio is $\alpha = D/L$. As given in section 1.2, we are seeking the maximum

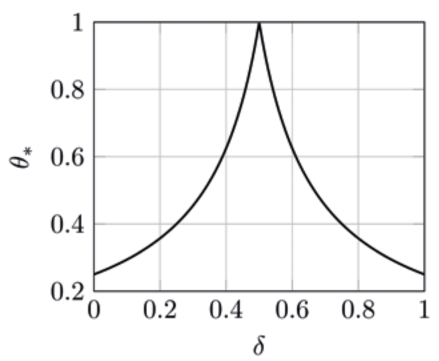


Fig. 8 : Normalized stroke for both GIFP and CSFP versus δ.

stroke assuming σ_{adm}, α and E have the same value for all the pivots under consideration. Formula (5) shows that for both GIFP and CSFP, the maximum stroke is achieved when $\delta = 0.5$ while $\delta = 0$ and $\delta = 1$ have minimum stroke.

Based on equation 5, we normalize the stroke by the dimensionless quantity

$$\theta_* = \frac{E\alpha\theta_s}{2\sigma_{\mathrm{adm}}}.$$

We note that $\theta_*(0.5) = 1$ and $\theta_*(0.127) = \theta_*(0.873) = 0.309$, so θ_* at $\delta = 0.5$, is 3.24 times the value of θ_* at $\delta = 0.127$ and $\delta = 0.873$.

3. Conclusion

Classical mechanical watch oscillators have pivots on jewelled bearings and their frictional losses lead to low quality factor, reducing autonomy and chronometric performance. A solution is to appeal to flexure pivots which greatly reduce friction. However, they have drawbacks such as gravity sensitivity and limited stroke. For watch oscillator applications, flexure pivots must limit gravity sensitivity and maximize stroke according to the specifications given in section 1.2.

Our study of suitable flexure pivots begins with an analysis of the cross spring flexure pivot already used as a watch oscillator. In section 1.3, we show that its performance is dependent on the geometric parameter $\delta_c = d_c/L_c$, but that there is no value of δ_c which ensures both gravity insensitivity and long stroke.

We therefore designed a new flexure pivot which we named the gravity insensitive flexure pivot (GIFP). Its design is based on the concept of the co-differential, as described in section 2.1, which we show to be insensitive to gravity in section 2.2. The GIFP consists of a rigid mass, two co-differentials and a polar beam, as described in section 2.3. Since a rigid mass has 6 DOF and there are 5 beams, there is only 1 DOF, a rotation, and the design is statically determinate (isostatic), a desirable feature.

In section 2.4 we show that the GIFP improves the gravity insensitivity of the CSFP by eliminating the first order effect of gravity on rotational stiffness. The GIFP also reduces the effect of gravity on stiffness from first order to second order for all rotational angles so, in principle, the GIFP should be gravity insensitive for a wider range of rotational angles.

In section 2.5, we quantify the gravity insensitivity by showing that the effect of gravity on GIFP stiffness is on the order of 10 ppm for all values of the geometric parameter $\delta = d/L$. So, GIFP achieves the goal 1 of the paper given in section 1.2 for all values of δ. By comparison, we show that the effect of gravity on CSFP stiffness changes from the order of 10 ppm to the order of 10^4 ppm, depending on the value of δ_c. The CSFP achieves the goal 1 of the paper only for $\delta_c = 0.127$ and $\delta_c = 0.873$.

Finally, section 2.6 provides an explicit formula for the stroke of the GIFP and CSFP and shows that the special value $\delta = \delta_c = 0.5$ leads to maximum stroke which achieves the goal 2 of the paper given in section 1.2. At this value, the effect of gravity on CSFP stiffness is in the order of 10^4 ppm showing that CSFP can not achieve both goals of the paper. On the other hand, GIFP achieves both goals of the paper since at $\delta = 0.5$, it has gravity insensitivity and maximum stroke at the same time. Section 2.6 shows that GIFP with $\delta = 0.5$ has stroke 3.24 times that of CSFP with $\delta_c = 0.127$ and $\delta_c = 0.873$, the only values of δ_c providing gravity insensitivity for CSFP.

Our analysis shows that, with respect to gravity sensitivity and stroke, GIFP is theoretically superior to CSFP.

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