A flexible numerical scheme for simulating plasma turbulence in the tokamak scrape off layer

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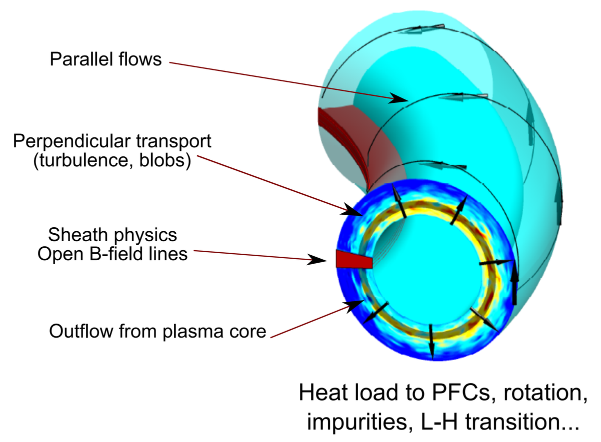
GBS, a tool to study SOL turbulent plasma

- In the tokamak scrape-off layer (SOL) magnetic field lines are open, channeling heat onto device wall
- The plasma behavior in this region governs the overall confinement properties of the device



- Global Bragiskii Solver, GBS:
  - Solves fluid equations for the plasma and kinetic equations for the neutrals atoms
  - Reproduces SOL turbulent dynamics
  - Treats magnetic equilibria with elongation and triangularity
  - Simulates medium size tokamaks: TCV, RFX, Alcator C-mod
- So far, tokamak SOL simulations performed in limited geometry
- GBS capabilities are now extended to **diverted X-point configurations**

Drift-reduced fluid equations for plasma turbulence

- GBS evolves the drift-reduced Braginskii equations, with ordering $k_\perp \gg k_\parallel$, $d/dt \ll \omega_{ci}$ [Ricci *et al.*, PPCF 2012]
- Plasma and heat outflowing from the core is mimicked by localised plasma and heat sources
- Boundary conditions described in [Loizu *et al.*, Phys. Plasmas 2012]

$$\frac{\partial n}{\partial t} = -\rho_\star^{-1}[\phi, n] + \frac{2}{B}[C(p_e) - nC(\phi)] - \nabla_\parallel(n v_{\parallel e}) + S_n$$

$$\frac{\partial \nabla_\perp^2\phi}{\partial t} = -\rho_\star^{-1}[\phi, \nabla_\perp^2\phi] - v_{\parallel i}\nabla_\parallel\nabla_\perp^2\phi + \frac{B^2}{n}\nabla_\parallel j_\parallel + \frac{2B}{n}C(p)$$

$$\frac{\partial v_{\parallel e}}{\partial t} = -\rho_\star^{-1}[\phi, v_{\parallel e}] - v_{\parallel e}\nabla_\parallel v_{\parallel e} + \frac{m_i}{m_e}\left(\nu\frac{j_\parallel}{n} + \nabla_\parallel\phi - \frac{1}{n}\nabla_\parallel p_e - 0.71\nabla_\parallel T_e\right) + \frac{4}{3}\eta_{0,e} n \nabla_\parallel^2 v_{\parallel e}$$

$$\frac{\partial v_{\parallel i}}{\partial t} = -\rho_\star^{-1}[\phi, v_{\parallel i}] - v_{\parallel i}\nabla_\parallel v_{\parallel i} - \frac{1}{n}\nabla_\parallel p + \frac{4}{3}\eta_{0,i} n \nabla_\parallel^2 v_{\parallel i}$$

$$\frac{\partial T_e}{\partial t} = -\rho_\star^{-1}[\phi, T_e] - v_{\parallel e}\nabla_\parallel T_e + \frac{4}{3}\frac{T_e}{B}\left[\frac{1}{n}C(p_e) + \frac{5}{2}C(T_e) - C(\phi)\right] + \frac{2}{3}T_e\left[0.71\nabla_\parallel j_\parallel - \nabla_\parallel v_{\parallel e}\right] + S_{T_e}$$

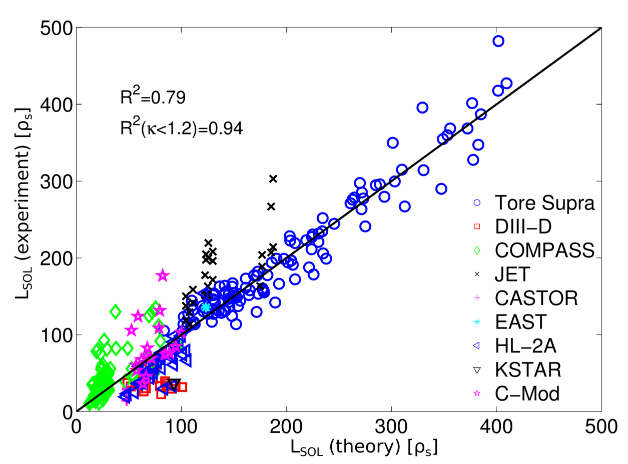
$$\frac{\partial T_i}{\partial t} = -\rho_\star^{-1}[\phi, T_i] - v_{\parallel i}\nabla_\parallel T_i + \frac{4}{3}\frac{T_i}{B}\left[C(T_e) + \frac{T_e}{n}C(n) - C(\phi)\right] + \frac{2}{3}T_i(v_{\parallel i} - v_{\parallel e})\frac{\nabla_\parallel n}{n} - \frac{2}{3}T_i\nabla_\parallel v_{\parallel e} - \frac{10}{3}\frac{T_i}{B}C(T_i)$$

$$[\phi, f] = \mathbf{b}\cdot(\nabla\phi\times\nabla f), \quad C(f) = B/2(\nabla\times\mathbf{b}/B)\cdot\nabla f, \quad \rho_\star = \rho_s/R$$

- Normalized units used throughout: $L_\perp \to \rho_s$, $L_\parallel \to R$, $t \to R/c_s$, $\nu = ne^2c_s/(m_i\sigma_\parallel R)$

Achievements of GBS

- Characterization of **non-linear turbulent regimes** in the SOL
- **SOL width scaling** as a function of dimensionless / engineering plasma parameters
- Origin and nature of **intrinsic toroidal plasma rotation** in the SOL
- Mechanisms regulating the SOL **equilibrium electrostatic potential**



[Halpern *et al.*, PPCF 2016]

Analytical and numerical development of *flexible* GBS for diverted scenarios

Challenges behind X-point simulations

- For axisymmetric magnetic fields:

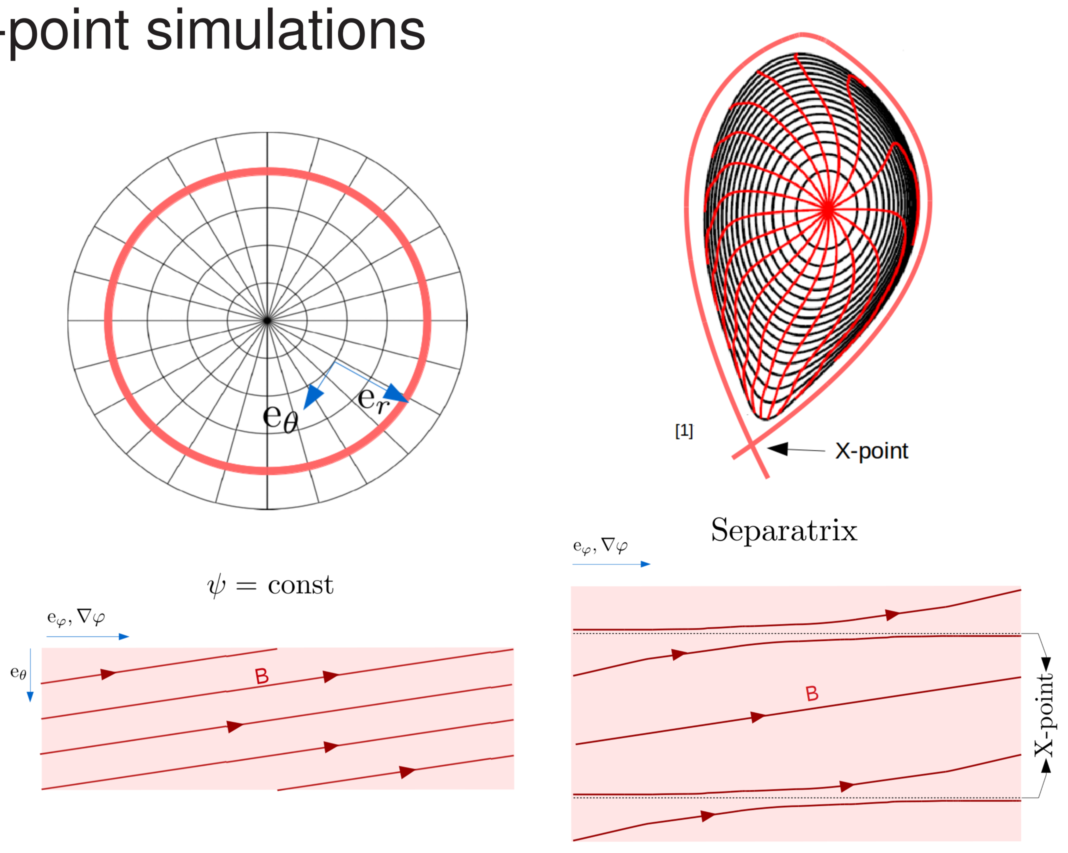
$$\mathbf{B} = F(\psi)\nabla\varphi + \nabla\psi\times\nabla\varphi$$

where φ is the toroidal angle and ψ is the poloidal magnetic flux.

- At the X-point the magnetic field is only toroidal

$$\mathbf{B}_{Xpt} = B^\varphi\mathbf{e}_\varphi = F(\psi)\nabla\varphi$$

- Therefore, the jacobian is not defined at the X-point as $\nabla\psi_{Xpt} = 0$ and $J_{Xpt} = (\nabla\psi\cdot\nabla u^2\times\nabla u^3)^{-1} = \infty$
- **It is not possible to use flux label coordinates** (ψ, u_2, u_3) at the X-point



[1] Image credit: E. Strumberger 2012

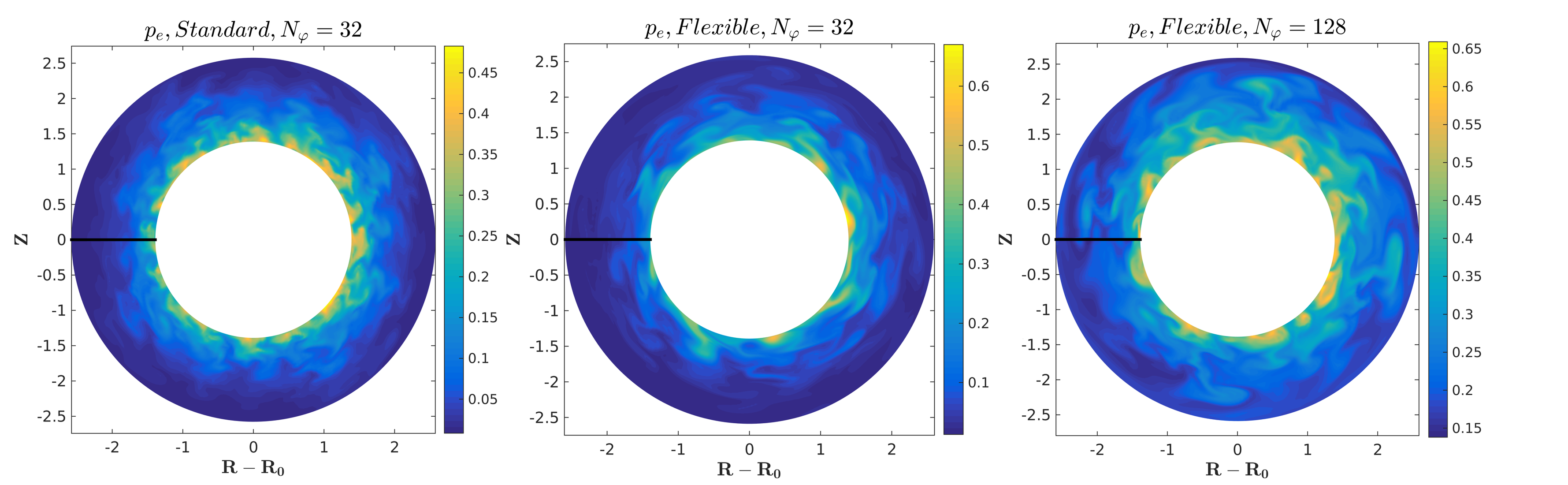
In order to extend GBS capabilities to treat diverted configurations, we moved from a field aligned to a non-aligned coordinate system.

	STANDARD GBS	NEW FLEXIBLE GBS
Coordinates	field aligned $(\psi, \theta_*, \varphi_*)$	geometrical $(\hat{r}, \hat{\theta}, \varphi) = (\frac{a}{\rho_s}\bar{r}, \frac{a}{\rho_s}\theta, \varphi)$, $r = \bar{r}a$
Magnetic equilibria allowed	With shaping (δ, κ)	(Any) $B = B_0R_0\nabla\varphi + \nabla\varphi\times\nabla\psi$
Parallel derivative $\nabla_\parallel$	$b^{\varphi*}\partial_{\varphi_*}$	$\frac{R_0}{R}\partial_\varphi + \frac{a}{\rho_s\bar{r}}\partial_{\bar{r}}\bar{\psi}\partial_{\hat{\theta}} - \frac{a}{\rho_s\bar{r}}\partial_\theta\bar{\psi}\,\partial_{\hat{r}}$
Curvature operator C	$C^\psi\partial_\psi + C^{\theta^*}\partial_{\theta*}$	$\sin\theta\partial_{\hat{r}} + \frac{\cos\theta}{\bar{r}}\partial_{\hat{\theta}}$
Parallel laplacian $\nabla_\parallel^2$	$b^{\varphi*2}\partial^2_{\varphi*\varphi*} + b^{\varphi^*}\partial^*_\varphi b^{\varphi^*}\partial_{\varphi*}$	$c_1\partial^2_{\varphi\varphi} + c_2\partial^2_{\hat{\theta}\theta} + c_3\partial^2_{\hat{r}\hat{r}} + c_4\partial^2_{\varphi\hat{\theta}} + c_5\partial^2_{\varphi\hat{r}} + c_6\partial^2_{\hat{r}\hat{\theta}} + c_7\partial_{\hat{r}} + c_8\partial_{\hat{\theta}}$
Numerical Grid	Staggered in φ_*	Staggered in both $\hat{\theta}$ and φ
Discretization Method	Finite differences $O(2)$	Finite differences $O(4)$

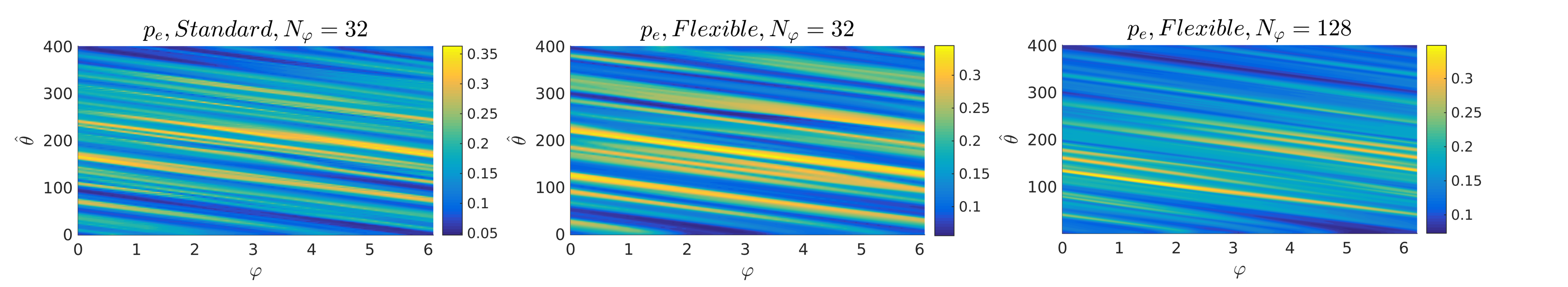
- In the non field aligned flexible code, the magnetic field topology information is contained in ψ and its derivatives which appear in the operators

Verification of the *flexible* GBS in limited configuration

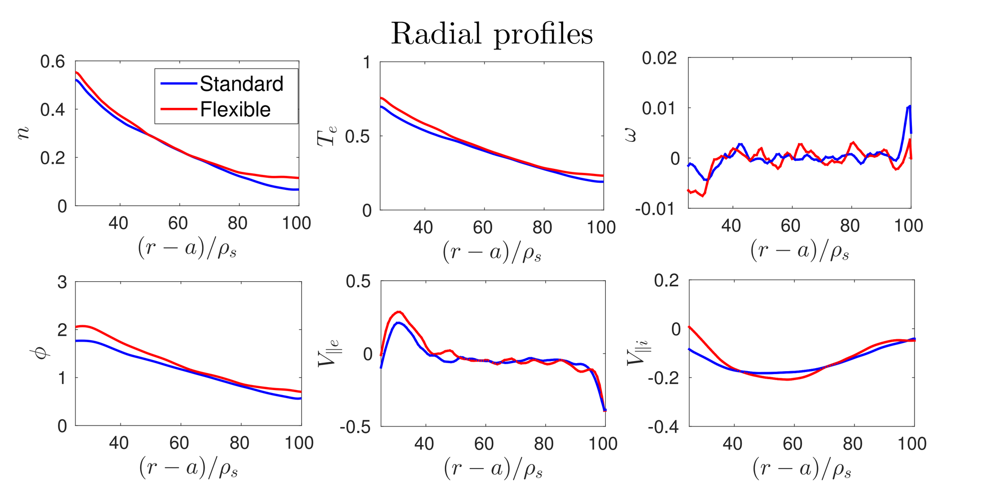
- *Flexible* version with its new non-field aligned operators is tested in limited domain with circular flux surfaces
- The magnetic equilibrium is defined through the poloidal flux $\bar{\psi}(\bar{r}, \theta) = \bar{\psi}(\bar{r}) = -\frac{\bar{r}^2}{2q}$
- When using the same grid resolution, the *standard* field aligned code shows better resolution. By increasing toroidal resolution similar turbulence structures are recovered.

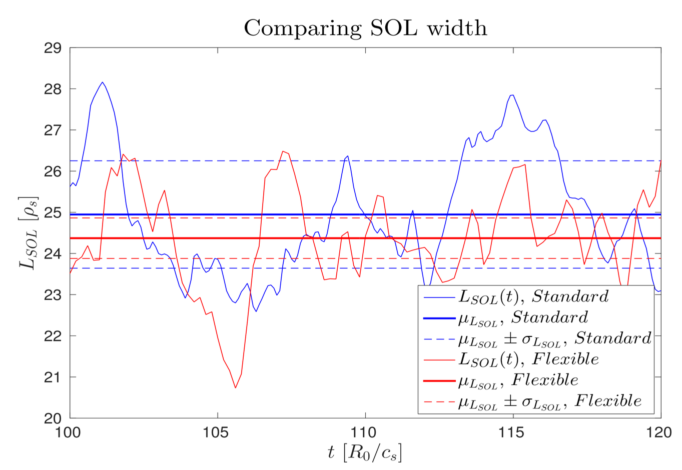


- The turbulence structures are still field aligned even when using non field aligned operators.



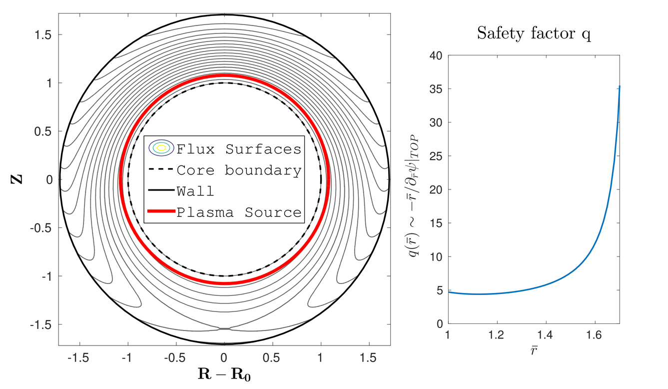
- For radial time averaged profiles, there is good agreement between simulation, even with same grid

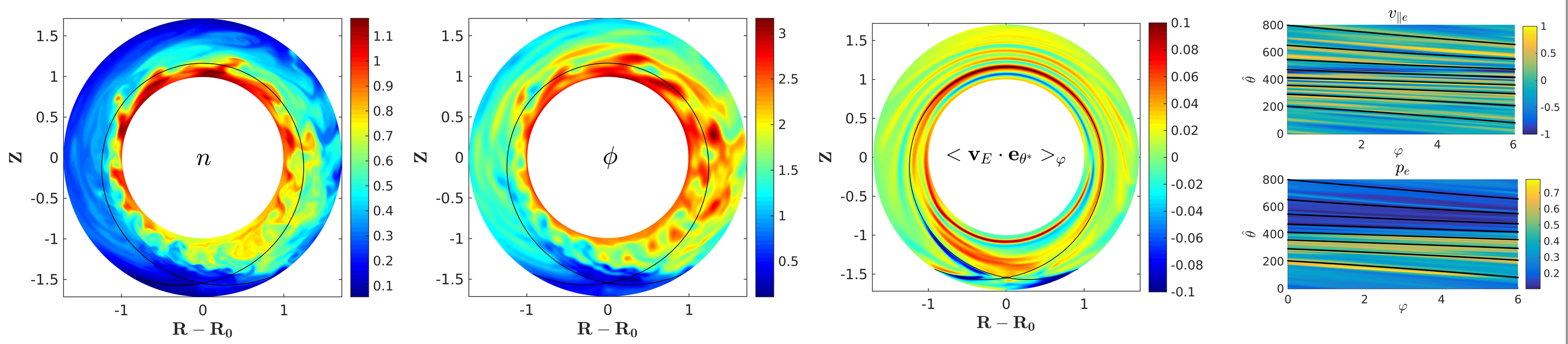




Flux surfaces with X-point

- First X-point equilibrium implemented in GBS
- ψ depends on θ: $\bar{\psi}(\bar{r}, \theta) = k(2t^3 - 2t^2 - (3/2 + \cos\theta)t + 1)$, $t = (\bar{r} - 1)/(r_{max} - r_{min})$, $k = 0.06$
- Physical boundary conditions applied at the wall
- The turbulence structures still appear to be field aligned





Conclusions

- GBS code capabilities succesfully extended to simulate diverted equilibria, applying physical boundary conditions at the wall
- The *flexible* version removes the constraint of a field aligned numerical grid, allowing also for non-local and diverted magnetic equilibria
- Even with a fixed (r, θ, φ) grid, the turbulence structures still appear to be field aligned
- Higher computational costs and finer toroidal grid are required

Outlook and GBS development plans

- Perform convergence studies and apply the method of manufactured solution to verify the code
- Implement shaping of the wall to allow for wider variety of equilibria
- Investigate spectral methods for the toroidal direction to lower computational cost
- Study the physics introduced by the X-point, possible simulations with neutral atoms [see talk Wersal on Thursday]