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Research Paper

The interaction factor method for energy pile groups

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ABSTRACT

Prior to this study, no simplified yet rational methods were available for estimating the vertical displacements of energy pile groups subjected to thermal loads. Observing such a challenge, the goal of this study has been threefold: (i) to extend the interaction factor concept from the framework of conventional pile groups to that of energy pile groups, (ii) to present charts for the analysis of the displacement interaction between two identical energy piles over a broad range of design conditions, and (iii) to propose, apply and validate the interaction factor method for the displacement analysis of energy pile groups.

1. Introduction

Over the last century, a substantial amount of research has been devoted to the analysis and design of conventional pile foundations because of their extensive application in the support of many structures and infrastructures. Classically, pile foundations have been applied to exploit adequate bearing capacities from soils of favourable strength and deformability characteristics, as well as to limit the use of surface area in densely built zones. In recent years, pile foundations have been increasingly used in an innovative form of energy piles to couple the aforementioned advantages associated with the structural support role of conventional deep foundations with the advantages associated with the role of the geothermal heat exchanger for satisfying the energy needs of building environments [1]. When addressing energy piles, a new challenge arises for civil engineers: the consideration of the mechanisms and phenomena induced by the application of thermal loads, in conjunction with those associated with the conventionally applied superstructure mechanical loads, on the mechanical behaviour of such foundations.

In the framework of the analysis and design of pile groups in which the piles are located sufficiently close to each other that their individual responses differ from that of an isolated pile because of the so-called "group effects" (i.e., closely spaced pile groups), two main aspects need to be considered (with reference to, e.g., the serviceability performance): (i) the vertical displacement – differential and average – of the piles in the group and (ii) the load redistribution among the piles in the group. The former aspect represents the subject matter of this paper.

To address the vertical displacement estimation of conventional pile groups subjected to mechanical loads, various numerical and analytical methods have been proposed. These methods include the finite element method [e.g., 2,3], the boundary element method [e.g., 4,5], the finite difference method [e.g., 6], the interaction factor method [e.g., 7,8–11], the equivalent pier and raft methods [e.g., 12–14], and the settlement ratio method [e.g., 15]. The finite element method, while providing the most rigorous and exhaustive representation of the pile group-related problem, is generally computationally expensive and considered mainly a research tool rather than a design tool. Conversely, the versatility of simplified (approximate) methods, such as the interaction factor approach that allows capturing the (e.g., vertical) displacements of any general pile group by the analysis of the displacement interaction between two identical piles and by the use of the elastic principle of superposition of effects, makes them attractive as design tools because they allow for the use of expedient parametric studies under various design conditions.

In contrast to the various approaches that have been used to estimate the vertical displacements of conventional pile groups subjected to mechanical loads, to date, only the finite element method [e.g., 16,17] has been applied for the same purpose for energy pile groups subjected to thermal loads and no detailed studies have been available on the analysis of the displacement behaviour of such foundations. This is because no simplified yet rational methods were available prior to this study for the vertical

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displacement estimation of energy pile groups subjected to thermal loads.

To address these challenges, the goal of this study has been threefold: (i) to extend the interaction factor concept from the framework of conventional pile groups to that of energy pile groups, (ii) to present charts for the analysis of the displacement interaction between two identical energy piles under a broad range of design conditions, and (iii) to propose and apply the interaction factor method for the analysis of the vertical displacement of energy pile groups subjected to thermal loads.

In addressing aspect (i), the key contributions concerning the displacement interaction between two identical semi-floating energy piles (i.e., the simplest system representing a pile group) subjected to a temperature change are described. In contrast to the description of the displacement interaction originally proposed by Poulos [7] for conventional piles subjected to mechanical loads, which was based on boundary element analyses, the description of the displacement interaction for energy piles subjected to thermal loads presented in this study is based on thermo-mechanical finite element analyses.

In addressing aspect (ii), the effects of many variables, including the pile spacing, the pile slenderness ratio, the pile-soil stiffness ratio, the Poisson's ratio of the soil, the depth of a finite layer, the non-uniform soil modulus and the soil-pile thermal expansion coefficient ratio, are investigated. According to the approach described by Poulos [18], although it is not possible to present theoretical solutions that cover all possible cases, those presented in this paper are considered to be sufficient to enable an approximation of the vertical displacement of energy pile groups to be made for most cases likely to be encountered in practice.

In addressing aspect (iii), the interaction factor concept defined for a group of two energy piles is first applied to the displacement analysis of symmetrical energy pile groups by exploiting the elastic principle of superposition of effects. This concept is next validated based on a comparison with results of 3-D thermo-mechanical finite element analyses. Then, a simplified (approximate) method for the displacement analysis of general energy pile groups with any configuration of piles in the group is formulated, although the solutions proposed in this paper refer to square groups of energy piles containing up to twenty-five piles. Finally, the interaction factor method is validated based on the comparison with results of 3-D thermo-mechanical finite element analyses on general energy pile groups subjected to thermal loads surrounded by soils with different thermal expansion coefficients.

2. The interaction factor concept

2.1. The problem: a group of two energy piles

The simplest system representing an energy pile group can be considered as consisting of two semi-floating energy piles in a deep soil layer. In the considered problem, the energy piles are (i) subjected to a thermal load, (ii) free of superstructure mechanical loads, and (iii) free to move vertically at their head.

The thermal load (i.e., aspect (i)) applied to the energy piles is a result of the geothermal operation of these elements. Cooling and/or thermal energy storage operations of energy piles can be associated to positive temperature changes applied to these elements. Heating operations of energy piles can be associated to negative temperature changes applied to these elements.

Reference to a situation in which no superstructure mechanical load is applied to the energy piles (i.e., aspect (ii)) allows focusing on the impact of the thermal load on the response of these elements.

The consideration of piles free to move at their head (i.e., aspect (iii)) has been generally accounted for in the analysis of conventional pile groups subjected to mechanical loads for estimates of the vertical displacement on the safety side. This approach appears to also be valuable for displacement analysis of energy pile groups and is considered in the following.

2.2. Idealisation

The previously described system is idealised considering the following assumptions. The energy piles are two identical isotropic, homogeneous and uniform cylindrical solids. The soil layer is a semi-infinite, isotropic, homogeneous and uniform mass. The same uniform temperature change is applied along the length of each of the energy piles. No mechanical load is applied to the energy piles. No head restraint is present (i.e., perfectly flexible slab). No slip or yielding occurs between each of the energy piles and the adjacent soil (perfect contact between the pile and soil is assumed). The energy piles are characterised by a linear thermo-elastic behaviour, whereas the soil is characterised by a linear elastic behaviour (i.e., the soil is an infinite heat reservoir that remains at a fixed constant temperature). Thus, reference is made to loading situations in which elastic (i.e., reversible) conditions prevail. Although not valid in situations where mechanical and thermal loads of significant magnitudes are applied to energy piles (especially if semi-floating) [19], these conditions have been demonstrated to characterise normal working situations based on the results of full-scale experimental tests [20–22] and numerical analyses [16,17].

The application of the temperature change to the energy piles involves a thermally induced deformation of these elements. An expansion of the energy piles is observed for cooling and/or thermal energy storage operations of these elements (positive temperature changes applied to the energy piles) whereas a contraction of the energy piles is observed for heating operations of these elements (negative temperature changes applied). In the former case, the upper portion of each energy pile displaces upwards, whereas the lower portion displaces downwards around a setting characterised by zero thermally induced displacements (defined as the null point referring to one-dimensional conditions [23]). In the latter case, the upper portion of each energy pile displaces downwards, whereas the lower portion displaces upwards[1]. The considered elastic assumption involves that the null point does not move depending on whether positive or negative temperature changes are applied to the energy piles. Hence, the displacement variation along the length of these elements for a unitary temperature change associated to their heating or cooling is the same in absolute value. The displacement field generated in each of the energy piles is transmitted in the adjacent soil. Interaction of the displacement fields generated by the thermally induced deformation of the energy piles thus occurs.

Assuming that the resulting deformation field of a group of two energy piles subjected to a temperature difference can be representatively decomposed through the elastic principle of superposition of effects, two (e.g., symmetrical) individual systems can be considered to describe the analysed problem. Fig. 1 provides an example of this decomposition for a situation in which a positive temperature change is applied to the energy piles. This decomposition approach has been widely proved to be suitable for

[1] It is worth noting that the phenomena characterising energy pile-related problems involve a remarkably different behaviour of the piles compared to that characterising most conventional pile-related problems in which a superstructure mechanical load (e.g., downward) is applied at the head of the piles inducing their overall settlement.

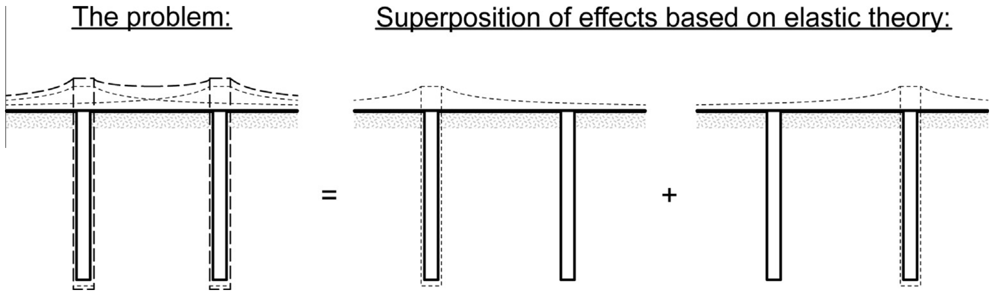

Fig. 1. The modelling approach.

describing the displacement interaction between conventional piles subjected to mechanical loads [12,24].

The elementary unit (cf., Fig. 2) composing the problem described above involves a source pile i subjected to a temperature change ΔT (i.e., thermally loaded) and a receiver pile j located at a certain spacing (i.e., centre-to-centre distance between the piles) s in the soil layer. As previously specified, the energy piles have the same length L and shaft diameter D.

2.3. Finite element analysis

2.3.1. Numerical models

Finite element modelling with the software COMSOL Multiphysics [25] is used in this paper as an analysis and validation tool.

3-D and axisymmetric finite element simulations are used as the analysis tool (i) to propose a description of the displacement field characterising the elementary unit described above and a single isolated energy pile, respectively; (ii) to present the interaction factor concept for energy pile groups; and (iii) to introduce design charts for the analysis of the displacement interaction between two identical energy piles.

3-D finite element simulations are used as a validation tool to compare the results obtained in the displacement analysis of symmetrical and general energy pile groups through the simplified method presented in this paper with a more rigorous approach.

In the analyses, unless otherwise specified, the idealised semi-infinite soil mass is approximated to a model of depth of $h = 25L$. This model has a width of $x = 500D$ in the axisymmetric simulations, whereas a width of $x = 1000D + (\tilde{n}_{EP} - 1)s$ and a breadth of $y = 1000D + (\tilde{n}_{EP} - 1)s$ are used in the 3-D simulations, where $\tilde{n}_{EP}$ is the number of energy piles along a row or a column of the group in plan view in the considered direction. Extremely fine tetrahedral meshes are used to describe the energy pile and soil domains.

2.3.2. Mathematical formulation

Two types of thermo-mechanical finite element analyses are performed: stationary and time-dependent analyses. The stationary analyses refer to idealised problems, such as those presented in Section 2.2, in which the energy pile and soil domains exhibit linear thermo-elastic and elastic behaviours, respectively (i.e., the soil is an infinite heat reservoir that remains at a fixed constant temperature). The time-dependent analyses refer to problems closer to reality in which both the energy pile and soil domains follow linear thermo-elastic behaviours (i.e., the soil is a mass that can be subjected to temperature changes and thermally induced volumetric variations).

In the following, compressive stresses, contractive strains and downward displacements (i.e., settlements) are considered to be positive.

The equilibrium equation can be written as

$$\nabla \cdot \sigma_{ij} + \rho g_i = 0 \tag{1}$$

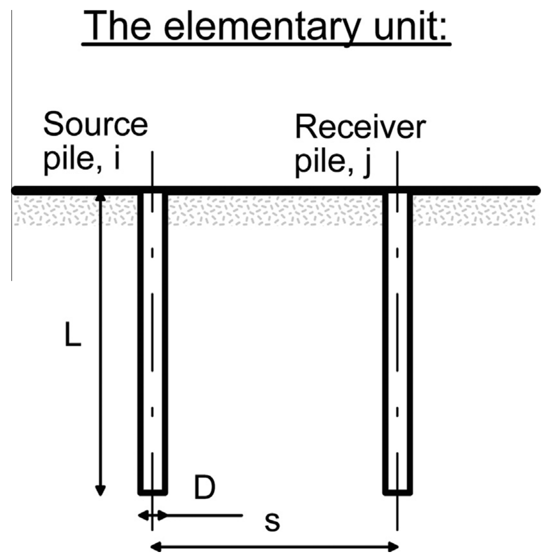

Fig. 2. The source and receiver pile constituting the elementary unit.

where $\nabla \cdot$ denotes the divergence, σ_{ij} denotes the total stress tensor, ρ denotes the bulk density of the material, and g_i is the gravity vector. The stress tensor can be expressed as

$$\sigma_{ij} = C_{ijkl}(\varepsilon_{kl} + \alpha I_{kl}\theta) \tag{2}$$

where C_{ijkl} is the stiffness tensor, which contains the material parameters (i.e., Young's modulus E and Poisson's ratio υ); ε_{kl} is the total strain tensor; α is the linear thermal expansion coefficient of the material; I_{kl} is the identity matrix; and $\theta = T - T_0 = \Delta T$ is the temperature variation.

Under time-dependent conditions, the energy conservation equation reads

$$\rho c_p \frac{\partial T}{\partial t} - \nabla \cdot (\lambda \nabla T) = 0 \tag{3}$$

where c_p is the specific heat, t is the time, λ is the thermal conductivity, and ∇ represents the gradient. It is worth noting that, under steady-state (stationary) conditions, the first term in Eq. (3) vanishes. It is also worth noting that the material properties of the energy pile/s and soil are considered to be insensitive to the temperature variations involved in the engineering application in this study.

2.3.3. Boundary and initial conditions

Restrictions are applied to both the vertical and horizontal displacements on the bases of the models (i.e., pinned boundary) and to the horizontal displacements on the sides (i.e., roller boundaries).

Table 1
Material properties used for the numerical analysis.

Reinforced concrete pile parameters	Value (thermo-elastic description)	Soil parameters	Value (elastic description)	Value (thermo-elastic description)
E_{EP}: [MPa]	30,000	G_{soil}: [MPa]	30[a]	
ν_{EP}: [–]	0.25	ν_{soil}: [–]	0.30[a]	
ρ_{EP}: [kg/m^3]	2450	ρ_{soil}: [kg/m^3]	1537	
α_{EP}: [1/°C]	$1 \cdot 10^{-5}$	α_{soil}: [1/°C]	–	$1 \cdot 10^{-5}$[a]
λ_{EP}: [W/(m °C)]	1.47	λ_{soil}: [W/(m °C)]	–	0.25
$c_{p,EP}$: [J/(kg °C)]	854	$c_{p,soil}$: [J/(kg °C)]	–	961

[a] Parameter varied throughout the simulations (cf., Table 3).

Table 2
Parameters of interest for analysis of energy pile groups.

Pile	Notation	Soil	Notation
Length	L	Depth of layer	h
Diameter	D	Shear modulus	G_{soil}
Spacing	s	Poisson's ratio	ν_{soil}
Young's modulus	E_{EP}	Linear thermal expansion coefficient	α_{soil}
Linear thermal expansion coefficient	α_{EP}		

Table 3
Dimensionless groups of parameters of interest for analysis of energy pile groups, typical values and values used in this study.

Dimensionless group	Notation	Practical range	Considered values
Pile spacing ratio	s/D	3–10	1.05, 1.25, 1.5, 2, 2.5, 3, 5, 10, 20
Pile breadth ratio	D/s	0.33–0.1	0.95, 0.8, 0.67, 0.5, 0.4, 0.33, 0.2, 0.1, 0.05
Pile slenderness ratio	L/D	10–50	10, 25, 50
Pile-soil stiffness ratio	$\Lambda = E_{EP}/G_{soil}$	100–10,000	10, 100, 500, 1000, 2000, 10,000
Poisson's ratio of soil	ν_{soil}	0.1–0.5	0.1, 0.15, 0.2, 0.3, 0.4, 0.5
Soil-pile thermal expansion coefficient ratio	$X = \alpha_{soil}/\alpha_{EP}$	0.25–4	0, 0.5, 1, 2
Depth of layer	h/L	–	1, 1.05, 1.1, 1.25, 2.5, $\rightarrow \infty$

The initial stress state due to gravity in the pile/s and the soil is considered to be geostatic and assumes a coefficient of Earth pressure at rest of $K_0 = 0.43$. No residual stresses from the installation of the piles are considered in these elements and in the adjacent region of soil. This hypothesis may not be completely representative of reality but can be applied successfully in methods of pile group deformation analysis by choosing appropriate values of the soil moduli [12].

In the steady-state analyses, the initial temperature of all the nodes of the energy pile/s is set to $T = 15$ °C. Throughout these simulations, a temperature change of $\Delta T = 10$ °C is applied to these nodes. The soil domain is treated as an infinite heat reservoir.

In the time-dependent analyses, the initial temperature of all the nodes of the energy pile/s and soil domains is set to $T = 15$ °C. Throughout these simulations, a temperature change of $\Delta T = 10$ °C is applied to all the nodes of the energy pile/s for a time of $t = 6$ months. The temperature of the external vertical and horizontal (bottom) boundaries of the model is fixed to $T = 15$ °C. The horizontal (top) boundary described by the soil surface is treated as adiabatic.

The impact of the elastic assumption on the location of the null point highlighted in Section 2.2 involves that, in all of the modelled pile/s-soil systems, a temperature change of $\Delta T = -10$ °C induces a symmetrical (equal in absolute value) response of the pile/s and soil to that observed for a temperature change of $\Delta T = 10$ °C.

2.3.4. Material properties and parameters of interest

The material properties considered in the analyses are reported in Table 1. The properties of the energy pile/s are typical of reinforced concrete. The soil properties have been successfully employed by Rotta Loria et al. [26] to model the behaviour of energy piles in dry Nevada sand with reference to physical observations.

Table 2 reports parameters that are considered of interest for the analysis of energy pile groups. Table 3 lists groups of dimensionless parameters that are considered useful for the same purpose, their typical ranges of variation and the values that are used for the purposes of this study.

In the present analyses, reference to a pile diameter of $D = 1$ m is generally made.

2.4. The interaction factor

The displacement fields characterising the source and receiver piles of the elementary unit (cf., Section 2.2) and a single isolated pile subjected to the same temperature change applied to the source pile are analysed in the following. The solutions have been obtained through stationary finite element analyses (cf., Sections 2.3.2 and 2.3.3).

Fig. 3 presents the evolution of the normalised vertical head displacement of the piles of the elementary unit with a normalised centre-to-centre distance between the piles. The normalised vertical head displacement of the single isolated pile and the evolution of the normalised vertical displacement of the adjacent soil at the ground surface as a function of horizontal distance are also plotted. The vertical displacement is normalised with respect to the head displacement of the single pile under free thermal expansion conditions, $w_{th,free} = -\alpha_{EP}\Delta TL/2$.

The essence of the displacement interaction between a source pile subjected to a temperature change and a receiver surrounding pile is shown.

2.4.1. Displacement of the source pile

As highlighted in Section 2.2, the application of the thermal load to the source pile induces a thermally induced deformation of this element that involves a modification of the displacement field along its length. This displacement is lower for smaller centre-to-centre distances to the receiver pile, whereas this displacement increases and tends to the displacement of a single isolated pile subjected to the same temperature change for centre-to-centre distances that approach infinity (cf., Fig. 3(a)). This result is caused by the effect of the stiffness of the receiver pile on the deformation of the source pile.

2.4.2. Displacement of the receiver pile

The thermally induced deformation of the source pile is transmitted to the surrounding soil and influences the displacement

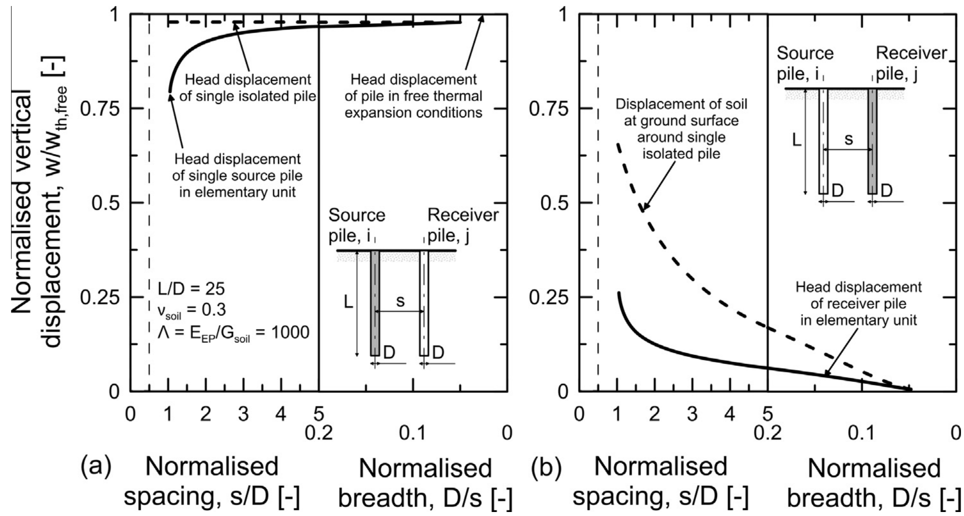

Fig. 3. Vertical head displacement characterising a source and receiver pile in the elementary unit, as well as a single isolated pile subjected to the same temperature change applied to the source pile.

field of the receiver pile (cf., Fig. 3(b)). This displacement is equal to that of the source pile for zero spacing between the two (i.e., one pile superimposed on the other, corresponding to the case of a single isolated pile subjected to a temperature change), whereas this displacement decreases and tends to zero for centre-to-centre distances that approach infinity. Yet, this displacement is always smaller than the displacement characterising the soil at the ground surface around a single isolated pile subjected to the same temperature change applied to the source pile in the elementary unit. This result is caused by the higher stiffness of the receiver pile compared to the stiffness of the soil.

The displacement interaction between piles implies that, when subjected to loading in a group, they present greater displacements compared to the case in which they are isolated and characterised by the same loading.

The additional displacement of a pile due to the loading (e.g., thermal) of an adjacent pile is expressed in this paper in terms of an interaction factor Ω, where

$$\Omega = \frac{\text{additional displacement due to adjacent pile}}{\text{displacement of single isolated pile}} = \frac{w_j}{w_i} \tag{4}$$

In defining the interaction factor, w_j is the vertical head displacement of a receiver pile in a pair, whereas w_i is the vertical head displacement of a single isolated pile subjected to the same load applied to the source pile in the elementary unit. This definition of the interaction factor relates the effect of loading a source pile on a receiver pile in a pair with the response of the source pile in an isolated case.

Fig. 4 presents the typical evolution of the interaction factor with a normalised centre-to-centre distance between two piles in the case of thermal and mechanical loading. The interaction decreases with increasing centre-to-centre distance between the piles. Mechanical loading causes a more pronounced displacement interaction between the piles compared to thermal loading.

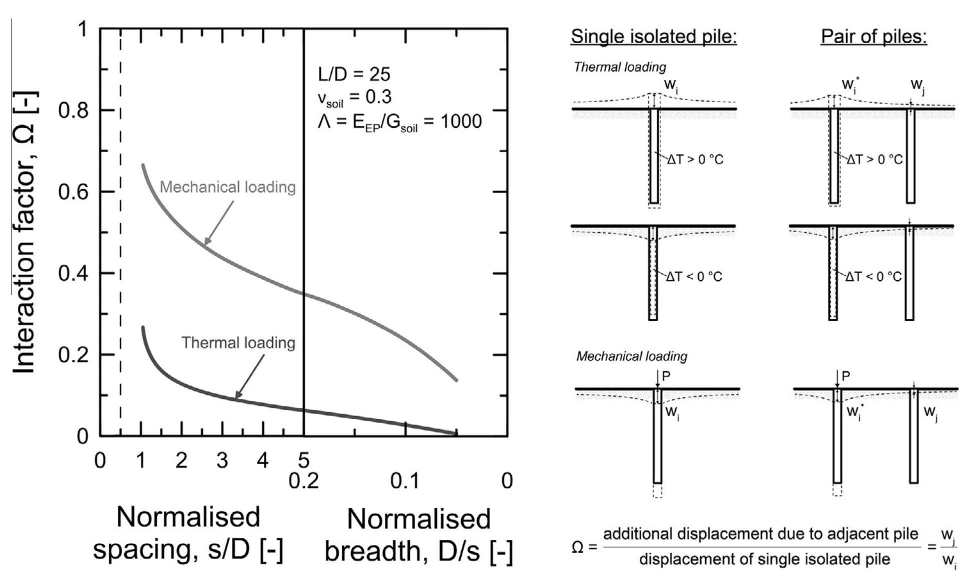

Fig. 4. Displacement interaction between two piles in a deep soil layer.

3. Design charts

The evolution of the interaction factor for varying design features characterising a group of two energy piles, including the pile spacing, the pile slenderness ratio, the pile-soil stiffness ratio, the Poisson's ratio of the soil, the depth of a finite layer, non-uniform soil moduli and the soil-pile thermal expansion coefficient ratio, is presented in the following. Unless otherwise specified, the solutions have been obtained through stationary finite element analyses (cf., Sections 2.3.2 and 2.3.3) and are valid for both positive and negative temperature changes applied to the energy piles (cf., Section 2.2).

3.1. Effect of pile spacing, pile slenderness ratio and pile-soil stiffness ratio

Figs. 5–7 present the evolution of the interaction factor as a function of the normalised centre-to-centre distance between the piles for various slenderness ratios L/D and pile-soil stiffness ratios $\Lambda = E_{EP}/G_{soil}$. The decreasing interaction with increasing centre-to-centre distance is shown according to the aforementioned comments. The interaction increases as L/D increases and Λ decreases, i.e., as the piles become slender or less stiff. The latter result indicates an opposite role of the stiffness compared to that found by Poulos [7] for conventional piles subjected to mechanical loads, i.e., increasing interaction as Λ increases and thus as the piles become stiffer.

3.2. Effect of Poisson's ratio of soil

Fig. 8 presents the effect of the Poisson's ratio of the soil ν_{soil}, where a correction factor N_ν is plotted for $L/D = 25$ and $\Lambda = 1000$. The interaction factor for any value of ν_{soil} is given by

$$\Omega = N_\nu \Omega_{\nu_{soil}=0.3} \tag{5}$$

where $\Omega_{\nu_{soil}=0.3}$ is the interaction factor for $\nu_{soil} = 0.3$. The interaction increases as the value of ν_{soil} decreases. This effect becomes more

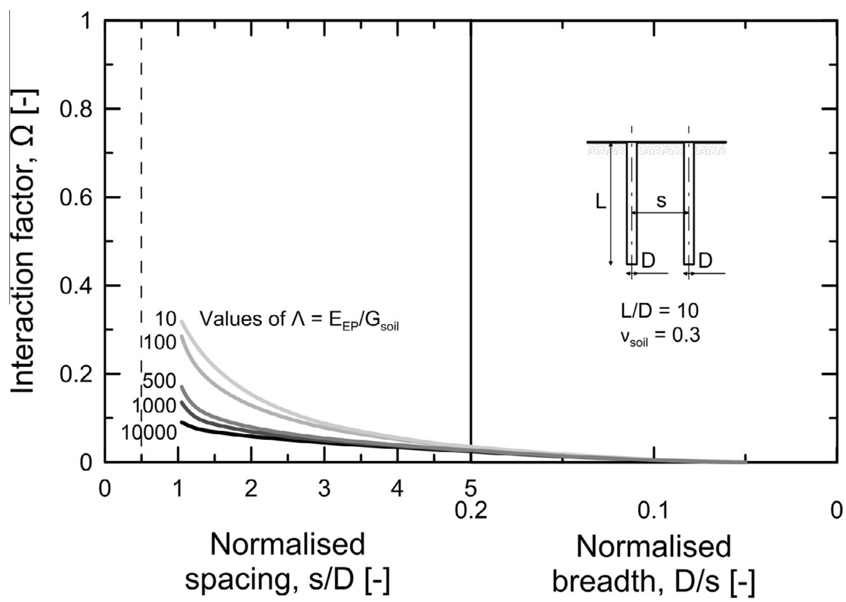

Fig. 5. Interaction factors for $L/D = 10$.

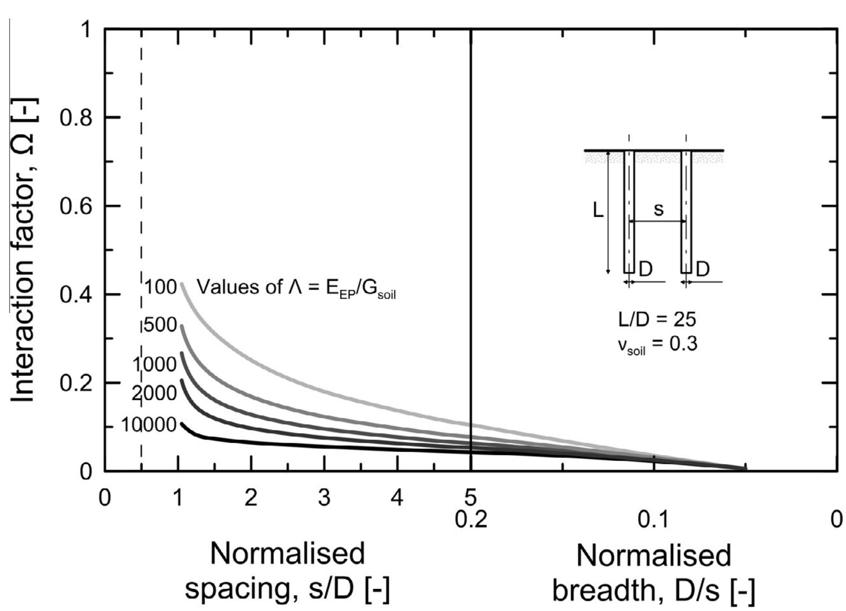

Fig. 6. Interaction factors for $L/D = 25$.

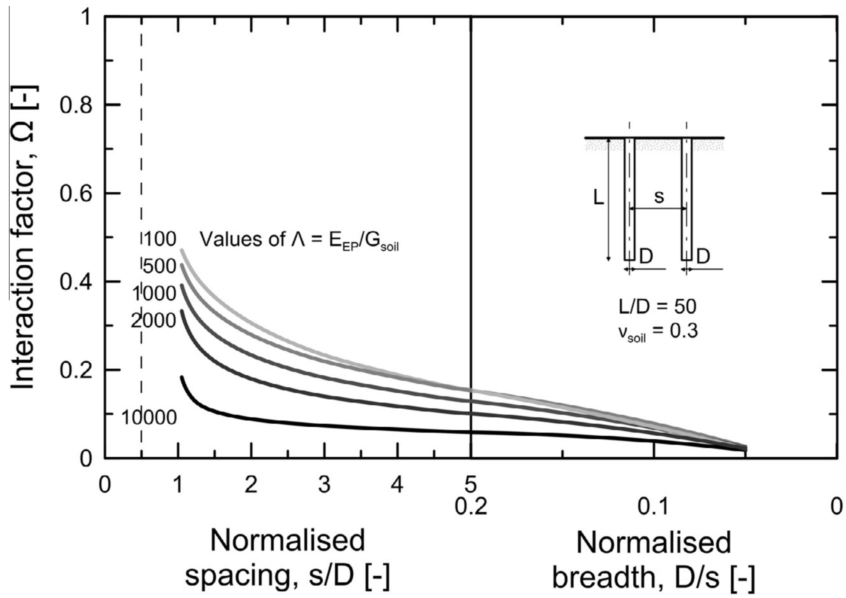

Fig. 7. Interaction factors for $L/D = 50$.

notable as the centre-to-centre distance between the piles increases.

3.3. Effect of finite layer depth

Fig. 9 presents the effect of the finite layer depth h/L, where a correction factor N_h is plotted for $L/D = 25$ and $\Lambda = 1000$. The interaction factor for any value of h/L is given by

$$\Omega = N_h \Omega_{h/L \to \infty} \quad (6)$$

where $\Omega_{h/L \to \infty}$ is the interaction factor for the deep soil layer ($h/L \to \infty$). The interaction increases as the value of h/L decreases. This effect becomes more notable as L/D increases and Λ decreases. Although presented for specific values of L/D and Λ, the values of the factor N_h presented in Fig. 9 can be approximately applied for other values of L/D and Λ. It is worth noting that, for $h/L = 1$ (e.g., end-bearing pile/s resting on a perfectly rigid stratum), the interaction factor is on average 43% higher than that characterising semi-floating piles. The latter result demonstrates an opposite role of the depth of the soil layer compared to that found by Poulos [7] for conventional piles subjected to mechanical loads, i.e., decreasing interaction as h/L decreases.

3.4. Effect of non-uniform soil modulus

The solutions presented above for the interaction factor all assume a uniform soil modulus along the pile shaft. In some cases, a closer approximation to reality is to consider the soil modulus as increasing linearly with depth (Gibson's soil) [7].

Fig. 10 shows the effect of a non-uniform soil modulus along the pile shaft on the interaction factor. A comparison between the interaction factor evolutions with normalised centre-to-centre distance between the piles for a constant and a linearly increasing soil modulus with depth (the latter being equal, on average, to the con-

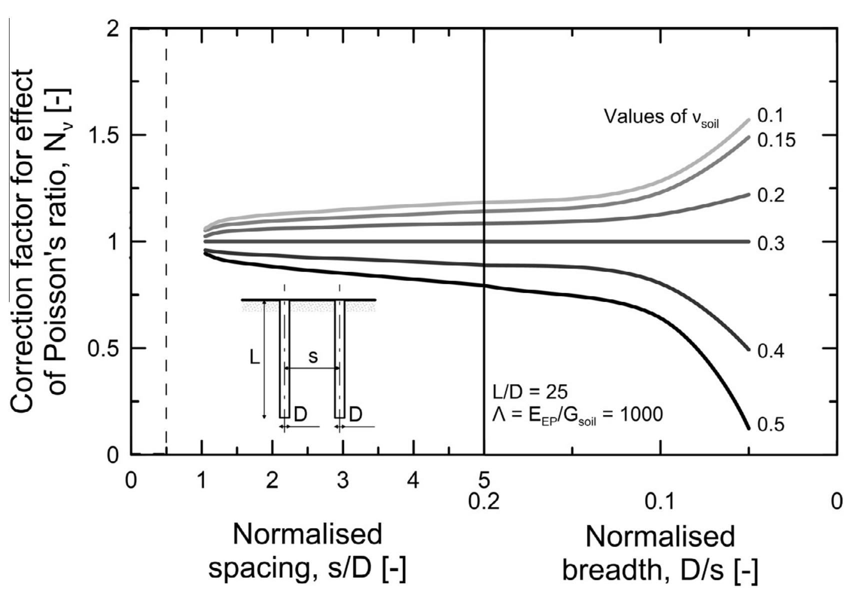

Fig. 8. Correction factor N_ν for effect of Poisson's ratio of soil.

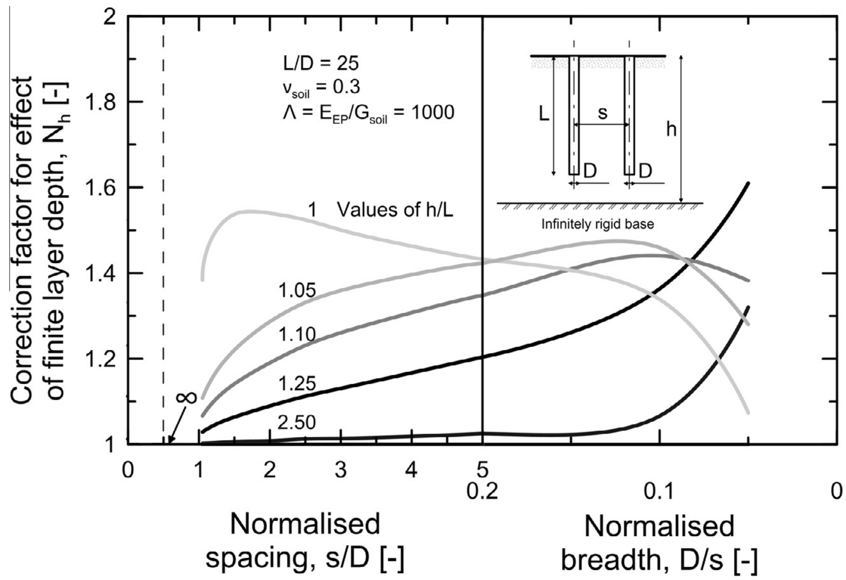

Fig. 9. Correction factor N_h for effect of finite layer depth.

stant distribution along the pile length) is presented. The evolution law for the soil modulus is as follows (the depth z from the top of the soil layer is considered to be negative)

$$E_{soil}(z) = 0.5E_{soil} - \frac{z}{L}E_{soil} \tag{7}$$

The value of Ω for the piles in the non-homogeneous soil is up to 2% smaller than for the homogeneous soil at any considered centre-to-centre distance. Thus, the use of interaction factors for the case of piles in a homogeneous soil gives conservative estimates of the interaction for cases in which the modulus increases with depth. The effect of non-uniform soil moduli on the displacement interaction between piles subjected to thermal loads is less marked compared to that characterising piles subjected to mechanical loads, for which the difference with the uniform case was from 20 to 25% [7].

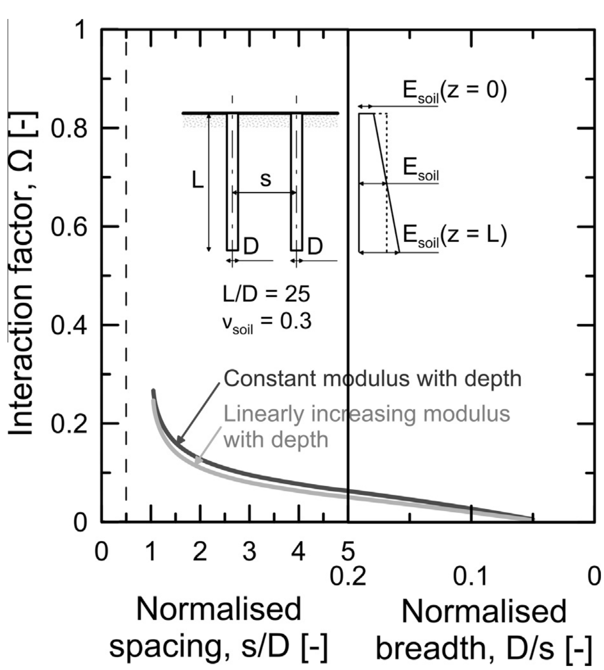

Fig. 10. Effect of non-uniform soil modulus on interaction factor.

3.5. Effect of soil-pile thermal expansion coefficient ratio

The solutions presented above for the interaction factor all assume an isothermal soil (infinite heat reservoir) that is characterised by an elastic behaviour. The effect of the ratio between the thermal expansion coefficient of soil and that of the pile (e.g., linear), namely, $X = \alpha_{soil}/\alpha_{EP}$, on the interaction factor is investigated herein. The solutions have been obtained through time-dependent finite element analyses (cf., Sections 2.3.2 and 2.3.3). In these analyses, the thermal expansion coefficient of the receiver pile is set to zero to highlight only the effect of thermally induced volumetric variations in the soil on the interaction previously defined with reference to the elastic soil.

Fig. 11 presents the effect of the soil-pile thermal expansion coefficient ratio on the interaction factor. The interaction increases with increasing thermal expansion coefficient of the soil. Values of $X = 0.5$ and 1 have a similar impact on the interaction factor compared to $\nu_{soil} = 0.15$, i.e., they induce a relative average increase of 12% compared to the increase of 15% from the reference value of Ω. Values of the thermal expansion coefficient of soil greater than that of the pile (e.g., $X = \alpha_{soil}/\alpha_{EP} = 2$) have a considerably stronger effect on the interaction, i.e., up to an average increase compared to the reference value of Ω of 200%. The time (and thus the spatial extent) of heat diffusion in the soil crucially characterises the effect of the soil-pile thermal expansion coefficient ratio on the interaction because it involves varying amounts of mobilised thermal expansion coefficient of soil.

4. Application and validation of the interaction factor concept

4.1. Analysis of symmetrical energy pile groups

In considering the displacement interaction between conventional piles subjected to mechanical loading, Poulos [7] remarked that the analysis of the interaction between two piles can be extended to any number of piles, provided that the arrangement of the piles in the group is such that they all behave identically. Such pile groups are defined as symmetrical pile groups. In a symmetrical pile group, all the piles are equally spaced around the circumference of a circle, and each pile displaces equally while carrying the same amount of load.

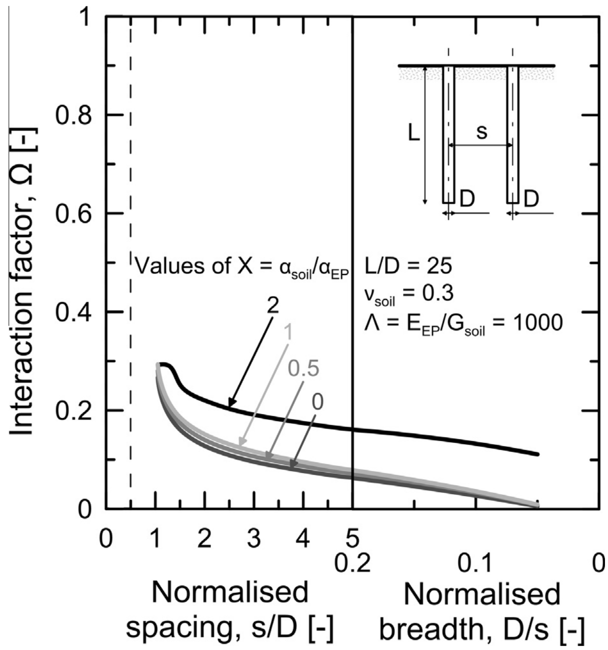

Fig. 11. Effect of soil thermal expansion coefficient on interaction factor.

The approach described above, which is based on the elastic principle of superposition of effects, is also adopted here for applying the interaction factor concept to the analysis of symmetrical energy pile groups. Groups of 2, 3 and 4 energy piles subjected to the same temperature change in an elastic soil are considered.

4.2. Application of the interaction factor concept

In analysing the displacement characterising energy pile groups, knowledge of a parameter defined in this paper as the displacement ratio R_d appears convenient. The definition of the displacement ratio has been extended from that of the settlement ratio R_s proposed by Poulos [7] for conventional piles subjected to mechanical loads. The displacement ratio is

$$R_d = \frac{\text{average displacement of group}}{\text{displacement of single pile subjected to same average load}} \tag{8}$$

Reference is made to the pile head vertical displacement. It is worth noting that the general definition of "displacement of single pile subjected to same average load" allows R_d to be determined in two ways.

The first way is to determine the displacement ratio with reference to the displacement of a single isolated energy pile subjected to a temperature change, that is, w_i, by determining analytically the increase in displacement of the group in which all the piles are subjected to the same temperature change through superposition with the use of the interaction factor Ω. In this case, the displacement ratio is

$R_d = \frac{w_i(1+\Omega_{s/D})}{w_i} = 1 + \Omega_{s/D}$ for a 2-pile group,
$R_d = \frac{w_i(1+2\Omega_{s/D})}{w_i} = 1 + 2\Omega_{s/D}$ for a 3-pile group, and
$R_d = \frac{w_i(1+2\Omega_{s/D}+\Omega_{s\sqrt{2}/D})}{w_i} = 1 + 2\Omega_{s/D} + \Omega_{s\sqrt{2}/D}$ for a 4-pile group,

where $\Omega_{s/D}$ is the interaction factor between two piles at any normalised centre-to-centre distance and $\Omega_{s\sqrt{2}/D}$ is the interaction factor between two piles in the 4-pile group along the diagonal of the square, whose side has a normalised length of s/D.

The second way is to determine the displacement ratio based on the results of a more rigorous approach, such as the finite element method, by calculating (through two different analyses) the displacement of a single source pile in the group subjected to a temperature change, that is, w^*, and the average displacement of the group in which all the piles are subjected to the same temperature change, that is, w_{ave} (for symmetrical pile groups, corresponding to the displacement of any single pile in the group). In this case, for any normalised centre-to-centre distance between the energy piles, the displacement ratio is

$R_d = \frac{2w_l/2}{w_l^*} = \frac{w_{ave}}{w_l^*}$ for a 2-pile group,
$R_d = \frac{3w_m/3}{w_m^*} = \frac{w_{ave}}{w_m^*}$ for a 3-pile group, and
$R_d = \frac{4w_n/4}{w_n^*} = \frac{w_{ave}}{w_n^*}$ for a 4-pile group,

where w_l, w_m and w_n are the displacements of the piles composing the 2-, 3- and 4-pile groups when all the piles are subjected to the same temperature change, whereas w_l^*, w_m^* and w_n^* are the displacements of the single source pile subjected to the same average temperature change in the 2-, 3- and 4-pile groups. It is worth noting that $w_n > w_m > w_l$ because of the more pronounced interactions in pile groups with higher numbers of piles. It is also worth noting that $w_n^* < w_m^* < w_l^*$ because of the greater effect of the stiffness of the receiver piles on the deformation of the source pile in pile groups with higher numbers of piles.

4.3. Validation of the interaction factor concept

Fig. 12 presents the evolution of the displacement ratio as a function of the normalised centre-to-centre distance between the piles constituting the 2-, 3- and 4-energy-pile groups subjected to the same temperature change. The figure shows the case for piles with a slenderness ratio of $L/D = 25$, pile-soil stiffness ratio of $\Lambda = E_{EP}/G_{soil} = 1000$ and Poisson's ratio of soil (considered as an infinite heat reservoir at a fixed constant temperature) of $\nu_{soil} = 0.3$. The displacement ratio has been calculated according to the two approaches proposed in Section 4.2, i.e., analytical application of the interaction factor concept and finite element analysis. The displacement ratio increases with increasing number of piles and decreases with increasing centre-to-centre distance between the piles because of the less pronounced interaction. Each of the symmetrical pile groups is characterised by a higher average displacement compared to that of a single pile subjected to the same average temperature change. For example, the displacement ratio of the 3-pile group for $s/D = 5$ is $R_d = 1.13$, i.e., the average vertical head displacement of the group is 13% higher than the vertical head displacement of a single pile subjected to the same average temperature change. For a limited number of piles in the group (e.g., up to 4 piles), the influence of the interactions between the piles (group effects) on the displacement behaviour of the group may be considered small and negligible for practical purposes. This consideration appears to be valid for groups of floating and semi-floating piles. It is not necessarily valid for groups of end-bearing piles because interactions have been proved to be on average 43% higher than those characteristics of the previous situations for a wide range of pile spacing.

The evolutions of the displacement ratio defined through the analytical and finite element approaches are well comparable in all cases, despite the accuracy of the analytical method decreasing with increasing number of piles in the group. The decrease in accuracy of the analytical method with increasing number of piles is caused by the variation in the stress field characterising the lengths of these elements compared to that of the 2-pile group. This variation in the stress field involves a different displacement field

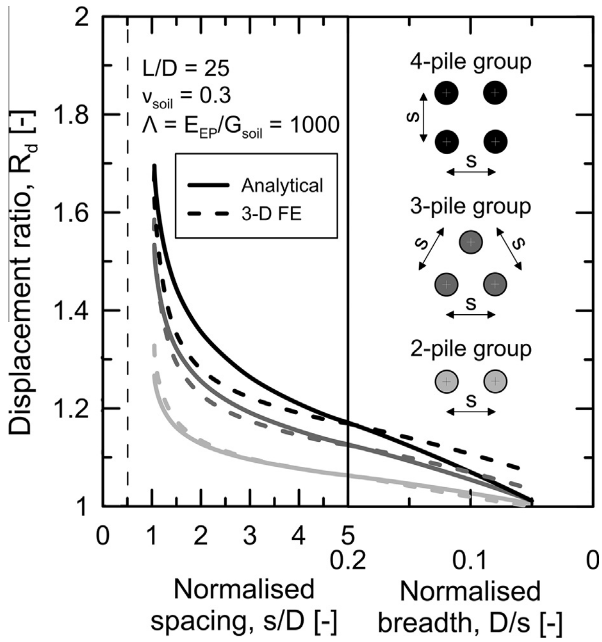

Fig. 12. Displacement ratio for symmetrical energy pile groups.

and associated location of the null point along the length of the piles in groups of more than 2 piles. Such an effect is not considered by the application of the interaction factor concept through the elastic principle of superposition of effects because reference is made to the displacement interaction between two piles.

A result of the different distribution of stress along the length of a single isolated energy pile compared to that characterising each of the energy piles in the 2-pile group as well as in the 3- and 4-pile groups for to the same positive temperature change applied to these elements is presented in Fig. 13. Fig. 13(a) presents the evolution of the normalised base resistance[2] with the normalised centre-to-centre distance between the energy piles in the group compared to the case of a single isolated energy pile. The interaction causes a decrease in the stress transmitted to the base of each energy pile in the group compared with the case of a single isolated energy pile, with such a mechanism becoming increasingly marked as the number of piles in the group increases. The aforementioned mechanism is opposite to that characterising conventional piles subjected to mechanical loads, in which interaction causes an increase in the stress transmitted to the base of each pile in the group compared with the case of a single isolated pile [7]. Fig. 13(b) depicts the distribution of the normalised shaft friction[3] along the normalised depth compared to the case of a single isolated energy pile for $s/D = 3$. As the number of piles in the group increases, a redistribution of shear stress occurs at the shaft to ensure equilibrium as a consequence of the different amounts of stress transmitted to the base. The distribution of shear stress becomes sharper along the major part of the length of the pile compared to that characterising a single isolated energy pile, and a more pronounced increase in shear stress occurs at the pile ends.

The applicability of the elastic principle of superposition of effects in approximately describing the displacement interaction in groups of 3 and 4 energy piles suggests that the head displacement of any symmetrical energy pile group may be estimated through analytical calculations by applying the interaction factor concept without having to resort to more rigorous albeit time-consuming approaches. Further, it appears reasonable to extend the use of the elastic principle of superposition of effects and the application of the interaction factor to the analysis of the displacement behaviour of general energy pile groups.

5. The interaction factor method

5.1. Analysis of general energy pile groups

Supposing that the elastic principle of superposition of effects holds for the analysis of a general group of total number of piles n_{EP}, the vertical head displacement of any pile k in the group can be estimated as

$$w_k = w_1 \sum_{i=1}^{i=n_{EP}} \Delta T_i \Omega_{ik} \quad (9)$$

where w_1 is the vertical head displacement of a single isolated pile per unit temperature change, ΔT_i is the applied temperature change to pile i, and Ω_{ik} is the interaction factor for two piles corresponding to the centre-to-centre distance between pile i and pile k. The displacement analysis of any general pile group in which some or all piles are subjected to a temperature change may therefore be performed based on the knowledge of the unitary head displacement of a single isolated pile w_1 and on the relationship between the interaction factor Ω and the centre-to-centre distance between the piles s/D for a group of two piles. This represents the essence of the interaction factor method.

The procedure used to apply the interaction factor method for the displacement analysis of general energy pile groups consists of three key steps (cf., Fig. 14):

1. The analysis of a single isolated pile subjected to a temperature change to define w_1. This analysis can be carried out by referring to Fig. 15, in which absolute values of w_1 for the design situations considered thus far are depicted. Otherwise, it may be performed with any analytical or numerical method available for such purpose.
2. The definition of Ω for a pair of two piles at any given centre-to-centre distance. This step can be accomplished by referring to the design charts proposed in this paper.
3. The analytical analysis of the displacement behaviour of the pile group. This analysis can be developed by applying Eq. (9).

The proposed approach for the displacement analysis of energy pile groups, as well as all present approaches based on the interaction factor concept for the estimation of the displacements of conventional pile groups subjected to mechanical loads [7–11], is indeed a simplified method because it involves approximations to obtain an answer even for the idealised situation. Because it is based on the analysis of the displacement interaction between two piles in a pair, in considering general pile groups, the method suffers from the drawbacks of not accounting for the following: (i) the redistribution of forces among the piles, which involves a variation in the displacement field compared to that characterising a group of two piles (cf., Section 4.3); (ii) the presence of piles between the pile at which the displacement is calculated and the

[2] The total vertical stress mobilised at the pile base $\sigma_{v,b}$ is normalised by the ultimate base resistance of the single isolated energy pile that is calculated based on a "rigid approach" as $q_b = \rho_{soil} g L$ to account for a limited contribution in base capacity provided by the considered soil. The considered value of base capacity is much smaller than the values that may be estimated through usual analytical formulae based on bearing capacity factors for deep foundations [see, e.g., 27]. This limited contribution in base capacity has been found to characterise piles of similar length than that considered in this study based on data available to the Authors. The tested pile may thus be considered as an almost fully floating pile.

[3] The shear stress mobilised at the pile shaft τ is normalised by the average shaft resistance of the single isolated energy pile that is calculated as $q_s = (\rho_{soil} g L/2) K_o \tan\delta + c$ (where δ is the pile-soil interface angle of shear strength and c is the soil cohesion, cf., Section 2.3.4).

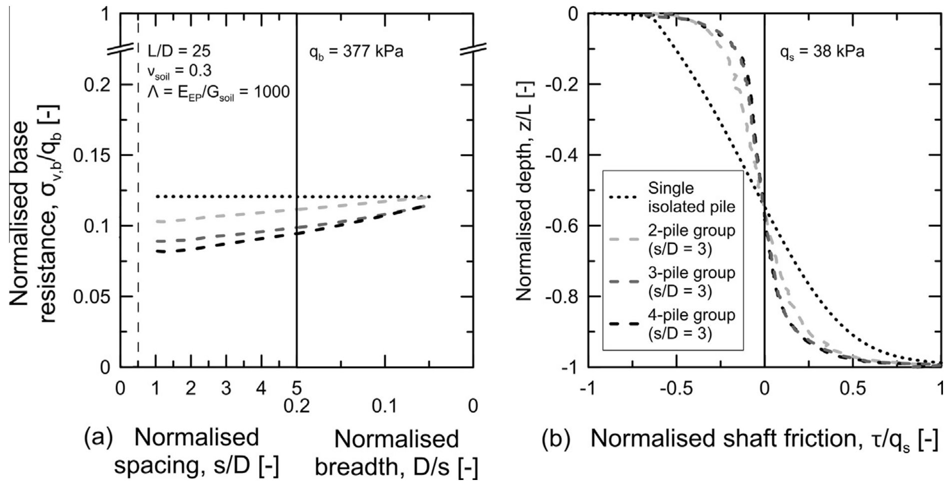

Fig. 13. (a) Effect of interaction on mobilised base resistance. (b) Effect of interaction on distribution of mobilised shaft friction.

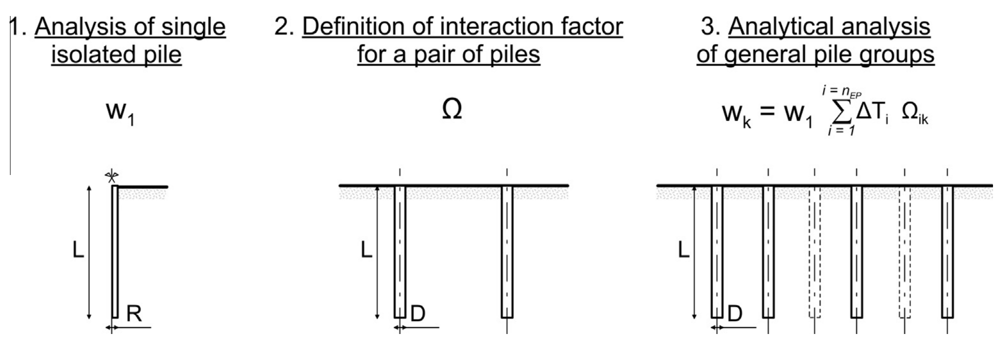

Fig. 14. Key steps for the application of the interaction factor method for the displacement analysis of energy pile groups.

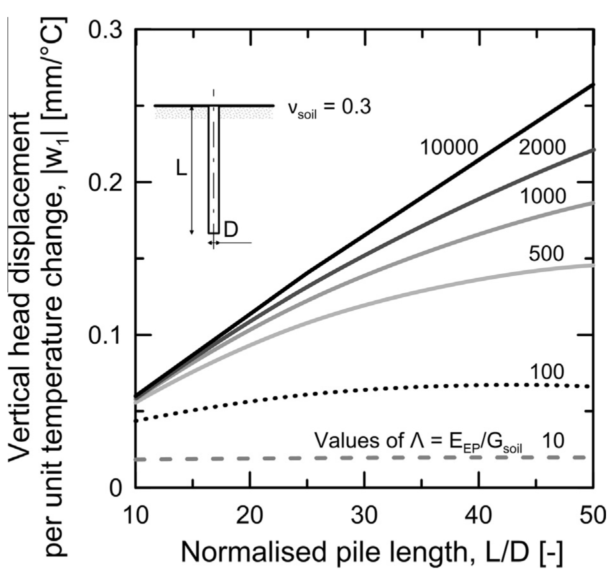

Fig. 15. Vertical head displacement per unit temperature change for different L/D.

pile whose influence is considered, which involves a reinforcing effect of the soil that tends to vary the influence of a pile on another pile compared to that characterising a group of two piles [7]; and (iii) the effect of the stiffness of the receiver piles on the deformation of the source pile/s in the group, which involves an approximation of the unitary displacement of these elements, especially for greater numbers of piles and shorter centre-to-centre distances between the piles.

Some inaccuracy may result from the approximations made in the formulation of the proposed method. According to Poulos [7], these approximations appear however to be justified because their consideration would result in an increase in complexity of the solution not commensurate with any increase in accuracy that might be obtained. The proposed approach thus represents a simplified yet rational method for the displacement analysis of energy pile groups. Further justification of the method of analysis is provided later based on a comparison between estimates of the vertical head displacement of general energy pile groups obtained through the considered approach and the more rigorous finite element method.

A notable feature of the proposed method is that it allows performing the displacement analysis of general energy pile groups also in situations in which some of the piles may be subjected to significant temperature changes that may induce non-linear phenomena (e.g., plastic strains) in the narrow region of soil adjacent to or in the vicinity of such elements, i.e., the pile-soil interface. The reason is because displacement interactions among piles are essentially elastic [11]. Reference to situations in which the soil surrounding the piles may be approximated to behave according to an elastic-perfectly plastic constitutive law allows considering that only the elastic component of deformation is transmitted from source piles (e.g., subjected to significant loads) to receiver piles (e.g., potentially subjected to less pronounced loads). Yet, in situations in which the soil region adjacent to receiver piles is in a (perfectly) plastic state[4] no deformation is transmitted from source piles to receiver piles. The reason for these phenomena is that when the shear strength of the pile-soil interface is fully mobilised, full slippage between the soil and the pile can be considered to occur. This fact includes the formation of a displacement discontinuity at the pile-soil interface. Therefore, while the displacement resulting from an elastic component of deformation that may be associated to a plastic state at the pile-soil interface of a source pile is transmitted in the surrounding bulk of the soil, the displacement induced by an elastic component of deformation from a source pile is no more transmitted to the receiver pile if its interface is in a plastic state. The elastic character of displacement interactions involves that a non-linear response of piles subjected to significant loads can be co-present to a linear response of piles subjected to less remarkable loads in the same group [28]. In these situations, the estimation of the displacement of the piles in the group may be performed by calculating through a suitable analysis the displacement of these elements with reference to a single isolated case and by applying the interaction factors only up to the limit corresponding to the elastic component of this displacement as well as only to the piles whose interface is not in a plastic state.

5.2. Application of the interaction factor method

Solutions for the displacement behaviour of general energy pile groups obtained through the analytical application of the proposed interaction factor method are presented in the following. Reference is made to a situation in which all the piles are subjected to the same temperature change and are surrounded by an elastic soil (i.e., infinite heat reservoir that remains at a fixed constant temperature). In obtaining these solutions, variables that crucially characterise the behaviour of pile groups are considered, including the number of piles in the group, the centre-to-centre distance between the piles in the group, the slenderness ratio of the piles in the group, the Poisson's ratio of the soil and the relative depth of the soil layer. Attention is devoted to square groups of 4, 9, 16 and 25 energy piles, which are referred to in the following as 2×2, 3×3, 4×4 and 5×5 pile groups, respectively.

5.2.1. Maximum average vertical head displacement

Fig. 16 presents the evolution of the displacement ratio with the normalised centre-to-centre distance between the piles. Groups of 3×3, 4×4 and 5×5 energy piles of slenderness ratio of $L/D = 25$ and pile-soil stiffness ratio of $\Lambda = E_{EP}/G_{soil} = 1000$ are considered. The displacement ratio increases with increasing number of piles in the group, with such a mechanism becoming less pronounced for increased centre-to-centre distances between the piles in the group because of the weaker interactions. The displacement ratio for the same group of energy piles in a soil mass with a greater Poisson's ratio decreases because of the weaker interactions among the piles. These results are in accordance with the analyses presented above.

5.2.2. Maximum vertical head displacement

Fig. 17 presents the evolution of the normalised vertical head displacement for the centre, side and corner energy piles in a 3×3 group of piles with normalised centre-to-centre distance between the piles. Energy piles characterised by a $\Lambda = 1000$ and a soil of $\nu_{soil} = 0.3$ are considered. For a general square group of energy piles in which all the piles are subjected to the same temperature change, the maximum vertical head displacement occurs at the centre pile/s, whereas the minimum displacement occurs at the corner piles. The vertical head displacement of the side piles is intermediate. This result is also found in groups of conventional piles subjected to mechanical loads because of the more pronounced interaction among the piles in the centre zone of the group. The vertical displacement of piles, whose arrangement in two corresponding groups is the same, increases with increasing slenderness ratio of the piles. This phenomenon is in accordance with the analyses presented above.

5.2.3. Maximum differential vertical head displacement

Fig. 18 shows the evolution of the maximum differential displacement normalised by the maximum displacement as a function of the normalised centre-to-centre distance between the piles. Groups of 3×3, 4×4 and 5×5 energy piles of $L/D = 25$ and $\Lambda = 1000$ are considered. The normalised maximum differential displacement increases as the number of piles in the group increases, although increasingly less markedly for greater numbers of piles in the group. The normalised differential displacement increases with decreasing depth of the soil layer. This result is in accordance to the results presented thus far.

5.3. Illustrative example

This section presents an introduction to the types of predictions possible through the application of the proposed interaction factor method.

The square group of four energy piles represented in Fig. 19 with the material properties specified in Table 1 is considered. The energy piles are all (i) subjected to the same temperature change of $\Delta T = 10\,°C$, (ii) free of superstructure mechanical load and (iii) without any head restraint.

The objective of the analysis is to estimate the maximum average vertical head displacement of the group that may reasonably occur in a corresponding real case. While the analysis of this problem may be considered of limited practical importance because of the weak displacement interactions expected among the piles, its purpose is to highlight the features of the proposed method in an effective way. In the analytical estimation of the displacement, reference is made to an idealised group of energy piles surrounded by an elastic soil that behaves as an infinite heat reservoir at a fixed constant temperature. A comparison between the obtained displacement value and those derived from more rigorous 3-D thermo-mechanical finite element analyses is also made. Stationary analyses are performed (cf., Sections 2.3.2 and 2.3.3) to consider the problem wherein the soil behaves as an infinite heat reservoir at a fixed constant temperature and is characterised by a zero linear thermal expansion coefficient. Time-dependent analyses are performed (cf., Sections 2.3.2 and 2.3.3) to consider the problem wherein the soil can be subjected to thermally induced volumetric variations according to linear thermal expansion coefficients of $\alpha_{soil} = 0.5$, 1 and $2 \cdot 10^{-5}$ 1/°C.

Following the procedure described in Section 5.1, the average displacement of the pile group can be determined as follows:

1. An axisymmetric (stationary, cf., Sections 2.3.2 and 2.3.3) finite element analysis of a single pile subjected to the considered temperature change in an elastic soil gives
   $w_1 = -0.122$ mm/°C
   i.e., a vertical head displacement of the energy pile of $w_i = -1.22$ mm.
2. The charts presented in this paper enable the definition of the interaction factors for the two characteristic centre-to-centre distances between the piles

$$\Omega_{s_1} = 0.063$$

$$\Omega_{s_2} = 0.045.$$

[4] This condition may potentially be induced by the application of the same significant load imposed to the source piles or by the effect of extremely pronounced displacement interactions.

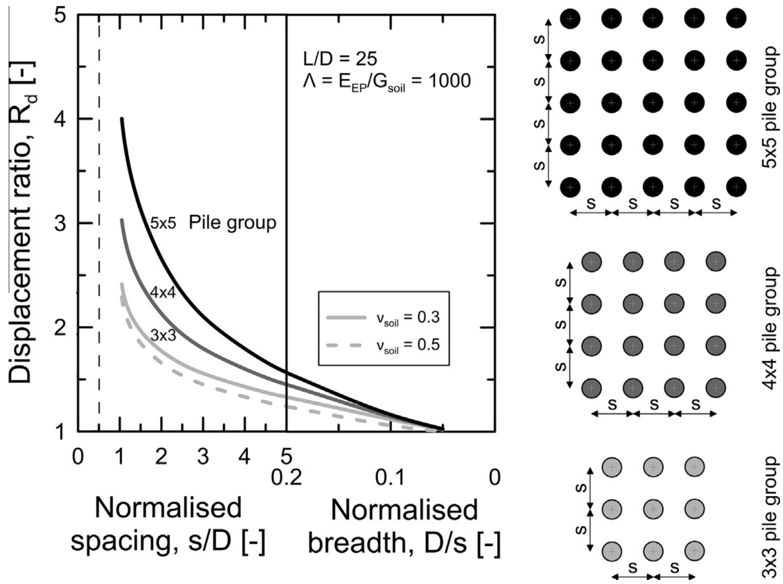

Fig. 16. Effect of Poisson's ratio of soil on the displacement ratio.

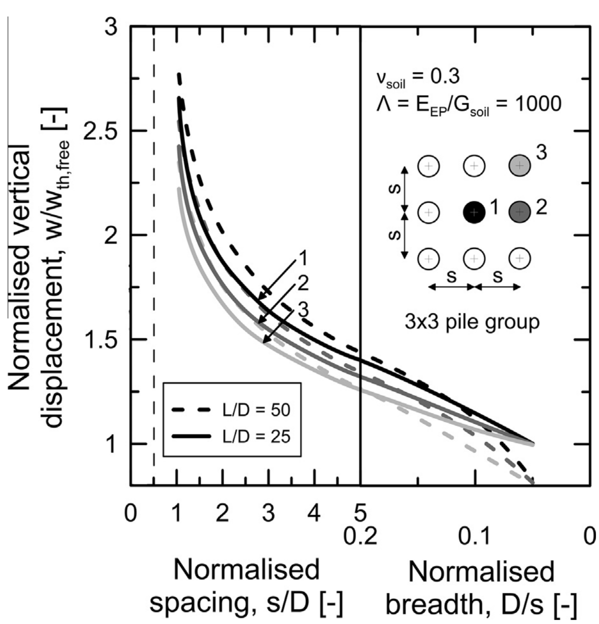

Fig. 17. Effect of position on the vertical displacement of the piles.

3. By applying the elastic principle of superposition of effects, the average vertical head displacement of the group is determined analytically as

$$w_k = w_{ave} = w_1 \sum_{i=1}^{i=n_{EP}} \Delta T_i \Omega_{ik} = w_i + 2(w_1 \Delta T \Omega_{s_1}) + w_1 \Delta T \Omega_{s_2} = -1.43 \text{ mm}$$

Table 4 presents a comparison between the average vertical head displacement of the group estimated through the interaction factor method and that determined through the more rigorous finite element method. The percentage error obtained when applying the proposed simplified method and the finite element approach is also presented. For the analysed pile group, the consideration of a displacement of unity, obtained through an axisymmetric finite element analysis in which the soil is characterised by an elastic behaviour (as in the idealisation of the pile group when applying the interaction factor concept), enables an estimate of the average vertical head displacement of the group that can be considered on the side of safety for most practical cases in which the soil thermal expansion coefficient is lower or equal than that of the piles. This does not appear to be the case for soil-pile thermal expansion coefficient ratios of greater that unity. This result is highlighted for example in the case of $X = \alpha_{soil}/\alpha_{EP} = 2$, for which an underestimation of the displacement of 11.40% made by the analytical prediction occurs. An approach that appears suitable for overcoming this issue is the consideration of a unitary displacement obtained through an axisymmetric finite element analysis (or any other available approach suitable for such purpose) in which the soil, as the energy pile, is characterised by a thermo-elastic behaviour (in contrast to the idealisation of the pile group when applying the interaction factor concept). In such a case, the resulting analysis of the pile-soil interaction for the single isolated pile better approaches reality. The use of this displacement value in applying the interaction factor concept allows one to obtain an estimate of the average vertical head displacement of the group that can be considered on the side of safety for most practical cases, including those in which the soil thermal expansion coefficient is higher than that of the piles. This result is corroborated by the data proposed in the last column of Table 4 and the results presented later.

5.4. Validation of the interaction factor method

Fig. 20 presents a comparison between the results obtained through the application of the proposed interaction factor method and more rigorous 3-D thermo-mechanical finite element analyses devoted to investigating the displacement behaviour of square groups of 4, 9, 16 and 25 energy piles subjected to the same positive temperature change considered thus far. The evolution of the normalised average vertical head displacement with practical values of the normalised centre-to-centre distance between the piles is presented. The finite element analyses consider the aforementioned pile groups in soil deposits with different thermal expansion coefficients.

The estimates of average vertical head displacement of the considered pile groups are always greater than the vertical head dis-

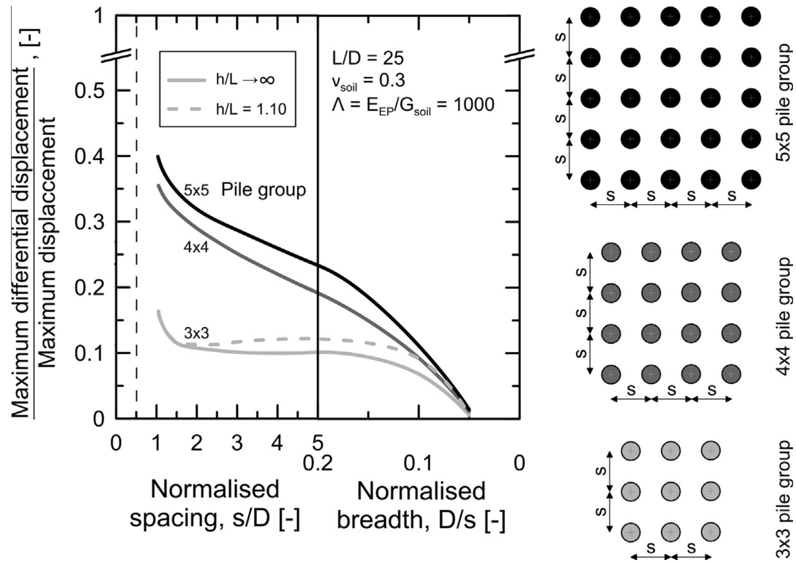

Fig. 18. Effect of finite layer depth on the differential displacement of the piles.

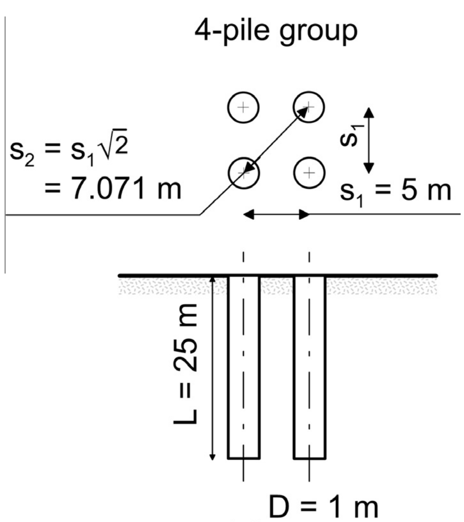

Fig. 19. Configuration of the practical example.

placement characterising a single isolated pile subjected to the same temperature change. In particular, variations of the average displacement of the pile groups comparable and even greater than those that characterise a single isolated pile can occur for increasing number of piles, such an effect becoming more pronounced for situations in which the soil-pile thermal expansion coefficient ratio exceeds unity. This phenomenon is in accordance with the considerations presented in Sections 3 and 4.

According to the remarks in Section 5.1 on the approximations of the proposed method, the inaccuracy between the displacement values estimated through the interaction factor method and the finite element method is greater at closer spacing between the energy piles, with such an effect becoming more pronounced as the number of piles in the group increases. A more accurate description of the problem is observed for centre-to-centre distances between the piles greater than or equal to $s/D = 5$, for which the application of the considered method is suggested. It is worth noting that normalised spacing comprised between $s/D = 3$ and 5 (in correspondence of which the more pronounced lack of accuracy of the proposed method is observed) are rarely considered for practical applications of geothermal heat exchangers such as energy piles. This design choice often allows limiting thermal interactions between the energy piles and ensuring their optimal energy performance. The lack of accuracy of the proposed method for centre-to-centre distances between the piles smaller than $s/D = 5$ appears thus to be acceptable for most of the practical analyses of energy piles. Although the suggested application of the method for normalised spacing between the piles greater than or equal to $s/D = 5$ is associated to a weaker increase in displacement of the energy piles compared to smaller spacing because of the weaker interaction effects among these elements, this application is considered important in the analysis and design of energy pile groups. The estimates of the increase in displacement that correspond to the application of the proposed method in the considered situations appear paramount for a thorough analysis of the dis-

Table 4
Comparison of predicted results for a 2 × 2 group of energy piles.

	Reference prediction analysis	Interaction factor method – Analytical (use of $w_{i,\alpha_{soil}/\alpha_{EP}=0}$)	Interaction factor method – Analytical (use of $w_{i,\alpha_{soil}/\alpha_{EP}=2}$)
	Estimated average head displacement [mm]	−1.43	−1.56
Type of alternative analysis	Calculated average head displacement [mm]	Prediction error [a] [%]	Prediction error [a] [%]
3-D FE – Elastic isothermal soil – $\alpha_{soil}/\alpha_{EP} = 0$	−1.37	4.38	12.29
3-D FE – Thermo-elastic soil – $\alpha_{soil}/\alpha_{EP} = 0.5$	−1.43	0.43	8.67
3-D FE – Thermo-elastic soil – $\alpha_{soil}/\alpha_{EP} = 1$	−1.48	−3.51	5.05
3-D FE – Thermo-elastic soil – $\alpha_{soil}/\alpha_{EP} = 2$	−1.60	−11.40	−2.18

[a] Positive sign indicates a prediction on the side of safety.

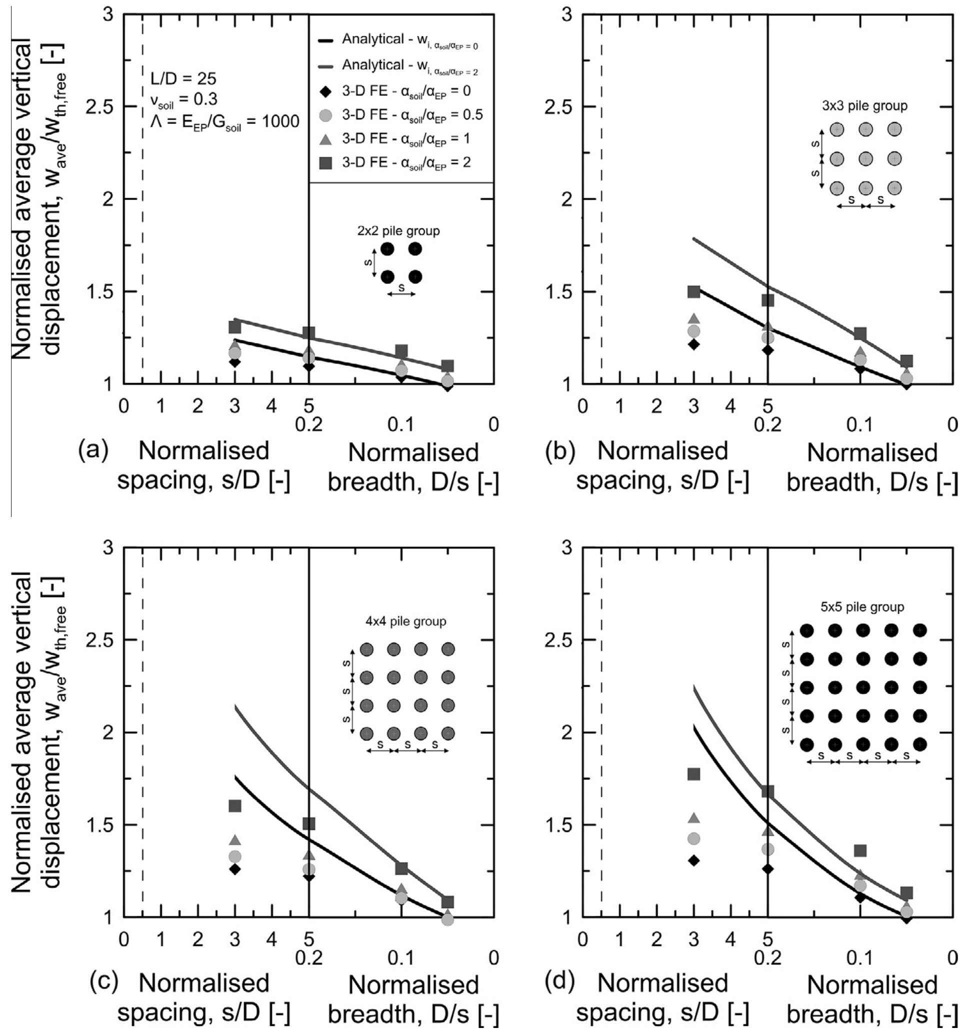

Fig. 20. Comparison between the results obtained through the proposed analytical approach and those obtained through more rigorous 3-D thermo-mechanical finite element analyses.

placement behaviour of energy pile groups. This statement seems to be particularly valid for energy pile groups comprising a notable number of piles and of further importance when such groups may rest on stiff soil strata or may be socketed in soil layers involving a soil-pile thermal expansion coefficient ratio greater than unity.

The use of the two approaches described in Section 5.3, i.e., reference to a unitary displacement of a single isolated energy pile in an elastic or thermo-elastic soil mass for estimating the displacement behaviour of the group, is considered to be validated based on the obtained results. The estimates of the average vertical head displacement appear to be on the side of safety in most of the considered cases. In those cases in which an underestimation is noted, the difference is small compared to the values obtained through the more rigorous finite element solutions. In particular, the estimates of the average vertical head displacement are considered sufficiently accurate for practical purposes.

From the foregoing considerations, it may generally be concluded that the theoretical approach described in this paper is capable of predicting the magnitude of vertical head displacements within the group with reasonable accuracy for most practical values of spacing between the energy piles.

According to the remarks made by Poulos [7] when describing the usefulness of the interaction factor method in addressing conventional pile groups, it must be borne in mind that the presented theory does not consider various aspects that may influence the behaviour of pile groups such as the order of the driving of the piles, the residual stresses at the pile shaft, the layering of the soil and potential construction imperfections. This theory also does not consider cyclic aspects related to the exploitation of the energy piles as geothermal heat exchangers and the pile-slab-soil interaction. At present, the consideration of the energy piles with no head restraint appears however to include any such action in the displacement analysis of these foundations through an approach on the side of safety that may be exploited for the preliminary design of such foundations.

6. Concluding remarks

The method of analysis presented in this paper allows the displacement behaviour of any general configuration of pile groups subjected to thermal loads to be estimated by considering only the effects of interactions between two piles and superimposing the individual effects of adjacent piles in the group. This method can be coupled to currently available classical interaction factor approaches to estimate the displacement behaviour of pile groups subjected to mechanical loads [e.g., 7–9] for performing a complete analysis of the impact that both thermal and mechanical loads have on the behaviour of such foundations. This appears a notable advance in the analysis and design of energy pile groups, for which no simplified yet rational methods for investigating the group displacement behaviour were available prior to this study.

Design charts have been proposed for the analysis of the displacement interaction between semi-floating energy piles in a broad range of conditions. These charts address the displacement interaction between energy piles subjected to positive and negative temperature changes that may be associated to cooling and/or thermal energy storage operations and heating operations of these elements, respectively. The impact of many variables on this interaction, including the pile spacing, the pile slenderness ratio, the pile-soil stiffness ratio, the Poisson's ratio of soil, the depth of a finite layer, non-uniform soil moduli and the soil-pile thermal expansion coefficient ratio, has been investigated.

The influence of the number of piles in the group, the spacing between the piles in the group, the slenderness ratio of the piles in the group, the Poisson's ratio of the soil and the relative depth of the soil layer on the behaviour of symmetrical and general energy pile groups has also been analysed.

Some of the main conclusions that can be drawn from this work are as follows:

- For a group of two piles subjected to a temperature change, the displacement interaction between the piles at a given centre-to-centre distance increases for (i) higher slenderness ratios of the piles L/D, (ii) lower pile-soil stiffness ratios $\Lambda = E_{EP}/G_{soil}$, (iii) lower values of the Poisson's ratio of the soil ν_{soil}, (iv) lower layer depths h/L, (v) higher uniformity of the soil modulus E_{soil}, and (vi) higher soil-pile thermal expansion coefficient ratios $X = \alpha_{soil}/\alpha_{EP}$. Interaction decreases with increasing centre-to-centre distance between the piles. Interaction is strongly characterised by the slenderness ratio of the piles.
- The effects of the aforementioned variables on the displacement interaction between two piles have a proportional influence on the displacement behaviour of any general pile group. Larger displacements are generally caused by stronger interactions among the piles in the group.
- For any given spacing between the piles, the displacement behaviour of a general pile group is markedly influenced by the number of piles. The displacement ratio R_d, which has been proposed in this paper as a parameter suitable for representing the increase in displacement in a pile group in which all the piles are subjected to the same temperature change compared to the displacement of a single pile subjected to the same average temperature change, increases with increasing number of piles. For a limited number of piles in the group (e.g., up to 4 piles), the influence of the interactions between the piles (group effects) on the displacement behaviour of the group may be considered small and negligible for practical purposes. This consideration appears to be valid for groups of floating and semi-floating piles. It is not necessarily valid for groups of end-bearing piles because interactions have been proved to be on average 43% higher than those characteristics of the previous situations for a wide range of pile spacing. The influence of the interactions between the piles on the displacement behaviour of the group is generally considered to be significant as the number of piles increases, especially for situations in which the soil-pile thermal expansion coefficient ratio exceeds unity. Comparable and even greater variations of the average displacement of any pile group subjected to a given average temperature change than the displacement of a single isolated pile of the same geometry and material properties of the piles of the group, and subjected to the same temperature change, can occur.
- For a general group of energy piles in which all the piles are subjected to the same temperature change, the maximum vertical displacement occurs at the centre pile/s, whereas the minimum displacement occurs at the corner piles. The vertical displacement of the side piles is intermediate. This phenomenon occurs because of the more pronounced interaction among the piles in the centre zone of the group. Such a result is a consequence of the greater number of surrounding piles (and stronger associated interaction) characterising a given pile in the centre zone of the group compared to an external pile.
- The normalised maximum differential displacement increases as the number of piles in the group increases, although increasing less markedly for greater numbers of piles in the group.
- The proposed interaction factor method for the displacement analysis of general energy pile groups consists of three key steps:
  1. The analysis of a single isolated pile subjected to a temperature change to define its displacement per unit temperature change, that is, w_1. This analysis can be carried out by referring to a chart proposed in this paper that depicts absolute values of w_1 for the considered design situations. Otherwise, it may be performed with any analytical or numerical method currently available for such purpose.
  2. The definition of the interaction factor Ω for a pair of two piles at any given centre-to-centre distance. This step can be accomplished by referring to the design charts proposed in this paper.
  3. The analytical analysis of the displacement behaviour of the pile group. This analysis can be performed by employing the elastic principle of superposition of effects with reference to the displacement interaction for a group of two piles and by applying the equation proposed in this work.
- Comparisons with results of more rigorous 3-D thermo-mechanical finite element analyses allow one to conclude that the theoretical method described in this paper is capable of predicting the magnitude of vertical head displacements within the group with reasonable accuracy for most of the spacing between energy piles encountered in practice.

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