

# EFFECT OF COMPOSITE BEAM ACTION ON THE HYSTERETIC BEHAVIOR OF FULLY-RESTRAINED BEAM-TO-COLUMN CONNECTIONS UNDER CYCLIC LOADING

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## ABSTRACT

This paper assesses the composite beam effects on the hysteretic behavior of fully-restrained beam-to-column connections as part of steel moment-resisting frames (MRFs) designed in highly seismic regions. A practical approach is developed based on available experimental data to simulate the hysteretic behavior of composite beams including the effects of asymmetric deterioration of the beam flexural strength and stiffness. A system-level analytical study is then performed that evaluates the collapse resistance of steel frame buildings designed with steel MRFs including the composite beam effects. It is demonstrated that when steel MRFs are designed with a SCWB ratio larger than 1.5 a tolerable probability of collapse is achieved over the life cycle of the steel frame building. It is also shown that controlled panel zone yielding can be achieved while reducing the required number of welded doubler plates in beam-to-column panel zone joints.

## 1. INTRODUCTION

Past experimental studies on fully restrained composite beam-to-column connections reveal that: (a) the flexural strength of a steel beam typically increases especially when the slab is in compression; (b) the strong-axis moment of inertia of a composite steel beam is typically larger than that of the bare steel beam; and (c) the cyclic deterioration in flexural strength and stiffness of a composite beam becomes asymmetric (e.g., Jones et al. 2002; Tremblay et al. 1997; Zhang and Ricles 2006). Experimental studies on steel frame systems (Cordova et al. 2004; Nakashima et al. 2007; Ohsaki et al. 2008) indicate that the neutral axis of a composite steel beam shifts into its upper half due to the presence of the slab. This shift as well as the role of the composite action in delaying local buckling in the top flange is more pronounced when there is beam and slab continuity, which is not evident in typical cruciform subassemblies that their beam ends are free to translate. For the seismic design of steel special moment frames (SMFs) in North America (AISC 2010a), the *moment ratio* rule [also referred to as the strong-column/weak-beam (SCWB) criterion] is employed in order to avoid column flexural yielding. In this check a set of adjustments for strain hardening and the material variability is considered depending on the beam-to-column connection type. However, the contribution of the composite beam effects is typically ignored. There is a perception that this assumption is typically conservative; however,

the considerable flexural strength increase of a beam due to the presence of the slab may shift flexural yielding to occur in the steel column rather than the beam even if the SCWB ratio was employed. The same issue may occur because of the redistribution of forces within a steel SMF due to strength deterioration of the steel beams because of geometric instabilities (i.e., local buckling). The aforementioned issues affect the collapse resistance of steel SMFs when subjected to earthquakes with low probability of occurrence.

This paper quantifies the effects of the composite action on the hysteretic behavior of steel beams as part of fully restrained beam-to-column connections. A rational approach is then developed that captures such effects within a numerical model. Finally, the effects of the composite action on the seismic performance of typical SMFs are investigated through rigorous nonlinear response history analyses based on a set of archetype steel frame buildings designed in the West Coast of the US.

## 2. NONLINEAR MODELING OF COMPOSITE BEAM-TO-COLUMN CONNECTIONS

This section proposes an approach to capture the effect of the composite action on the hysteretic behavior of beam-to-column connections. For this reason, a set of 22 experiments on interior joint beam-to-column connections was gathered from a searchable *W*-shape database of steel beams (Lignos and Krawinkler 2011; Lignos et al. 2010). Due to brevity, a detailed description of the test data can be found in (Elkady and Lignos 2014, 2015).

### 2.1 Modeling of Composite Steel Beams

The modified Ibarra-Medina-Krawinkler (IMK) model (Ibarra et al. 2005; Lignos and Krawinkler 2011) has been calibrated extensively to simulate the hysteretic behavior of bare steel beams (Lignos and Krawinkler 2011, 2013). An example of such calibration is shown in Figure 1a for a bare steel beam with a reduced beam section (RBS). The same numerical model was modified to simulate the asymmetric hysteretic behaviour of a composite steel beam, its residual strength due to local buckling stabilization as well as the ductile tearing due to low cycle fatigue (Lignos et al. 2011). The modified IMK deterioration model is bounded by a backbone curve as shown in Figure 1. This backbone curve is defined based on: (1) the elastic flexural stiffness  $K_e$  of the steel beam; (2) the effective yield moment  $M_y$ ; (3) the capping-to-effective yield moment ratio  $M_c/M_y$ ; and (4) the residual-to-effective yield moment ratio  $M_r/M_y$ . From Figure 1, three deformation parameters fully define the backbone curve of the IMK model: (1) the pre-capping plastic rotation  $\theta_p$ ; (2) the post-capping plastic rotation  $\theta_{pc}$ ; and (3) the ultimate rotation  $\theta_u$ . The cyclic deterioration of the flexural strength and stiffness of a steel beam is controlled through the reference energy dissipation capacity ( $\Delta$ ) and the rates of cyclic deterioration in the positive and negative loading directions ( $D^+$  and  $D^-$ ).

From Figure 1, the differences between the hysteretic behavior of a bare and a composite steel beam can be qualitatively assessed. In brief, due to the presence of the slab, composite steel beams have an asymmetric hysteretic behavior (see Figure 1b). In particular, a higher flexural strength and plastic deformation capacities are observed in the positive loading direction (i.e., slab in compression) than the corresponding parameters in the negative loading direction (i.e., slab in tension). The former is attributed to the restraint that the slab provides to the top flange of a steel beam. The latter is attributed to the beam neutral axis shifting towards its upper flange. In this case, the bottom flange is susceptible to lateral torsional buckling.

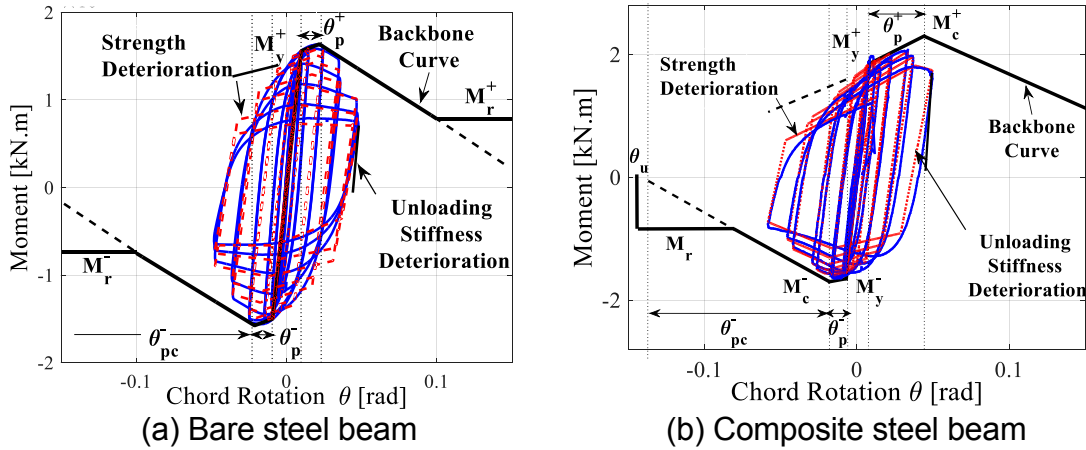


Figure 1: Modified IMK component model [data from Uang and Fan( 2001) and Zhang and Ricles 2006)

To quantify these values and develop modeling recommendations for composite beams, the hysteretic response of the modified IMK deterioration model is calibrated with respect to the set of experiments summarized in Elkady and Lignos (2014). The calibrated parameters of the modified IMK deterioration model for both the positive and negative loading directions are summarized in Table 1. From this table, the deformation parameters are normalized with respect to those of the respective bare steel beam. The effective yield flexural strength in the positive and negative loading directions are normalized with respect to the effective flexural strength of the bare steel beam  $M_y$ , calculated as  $M_y = 1.1Z_xF_{ye}$ , in which  $Z_x$  is the plastic modulus about the beam's strong-axis; and  $F_{ye}$  is the expected yield stress of the steel material. The 1.1 factor is considered to represent approximately the effects of cyclic hardening on the steel beam flexural strength (Lignos and Krawinkler 2011). Table 1 summarizes the counted mean and coefficient of variation (COV) of the normalized values of each input parameter of the modified IMK model for composite beams. These values can be used to adjust the backbone curve of a bare steel beam to account for the composite action in a practical manner.

Table 1: Normalized deterioration parameters for composite steel beams

Param.	$I_c$	$\frac{M_y^+}{M_y}$	$\frac{M_y^-}{M_y}$	$\frac{M_c^+}{M_y^+}$	$\frac{M_c^-}{M_y^-}$	$\frac{M_r^+}{M_y^+}$	$\frac{M_r^-}{M_y^-}$	$\frac{\theta_p^+}{\theta_p}$	$\frac{\theta_p^-}{\theta_p}$	$\frac{\theta_{pc}^+}{\theta_{pc}}$	$\frac{\theta_{pc}^-}{\theta_{pc}}$	$\Lambda$	$D^+$	$D^-$
	$I_s$	$\frac{M_y^+}{M_y}$	$\frac{M_y^-}{M_y}$	$\frac{M_c^+}{M_y^+}$	$\frac{M_c^-}{M_y^-}$	$\frac{M_r^+}{M_y^+}$	$\frac{M_r^-}{M_y^-}$	$\frac{\theta_p^+}{\theta_p}$	$\frac{\theta_p^-}{\theta_p}$	$\frac{\theta_{pc}^+}{\theta_{pc}}$	$\frac{\theta_{pc}^-}{\theta_{pc}}$			
Mean	1.4	1.35	1.25	1.30	1.05	0.30	0.20	1.80	0.95	1.35	0.95	1.0	1.15	1.0
COV	0.12	0.13	0.13	0.08	0.02	0.31	0.23	0.32	0.48	0.21	0.13	0.31	0.19	0.0

In brief, when the slab is in compression, the effective yield flexural strength,  $M_y^+$  and capping flexural strength,  $M_c^+$  is larger by 30 to 35% compared to that of the bare steel beam. After the crushing of the concrete slab and the formation of beam flange local buckling, the flexural strength of a composite beam reaches to a residual strength of about 20% to 30% of the respective effective yield strength in the loading direction of interest. If a designer knows the geometric and material properties of the steel deck and concrete slab, the composite beam flexural strength can be directly calculated based on the approach discussed in AISC (2010b). The following assumptions should be considered: (a) full composite action between the concrete slab and the steel beam; (b) the geometry of the reduced beam section for beams with RBS; (c) the effective width of the composite beam as calculated based on AISC

(2010b) (see Section I 3.1a); and (d) the effective stress in the concrete is taken as 0.85 of the specified concrete stress  $f_c'$ .

## 2.2 Modeling of Composite Panel Zones

The “parallelogram model” proposed by Gupta and Krawinkler (2000) is utilized to represent the beam-to-column panel zone joint within a steel moment-resisting frame (MRF). This model simulates the nonlinear relation between the shear force,  $V$  and the shear distortion angle,  $\gamma$  within a panel zone. The trilinear backbone curve proposed by Krawinkler (1978) is employed to simulate the backbone curve of a panel zone as shown in Figure 2a. The concrete slab affects the hysteretic behavior of the panel zone. Therefore, the backbone curve of the panel zone should be adjusted for this reason. In particular, the effective depth of the panel zone depends on the loading direction (see Figure 2b). When the slab is in tension the effective depth is similar to that of a bare steel beam-to-column panel zone joint; hence the negative yield flexural strength of the panel zone should be calculated as discussed in Gupta and Krawinkler (2000). When the slab is in compression the effective depth becomes larger than that of the bare steel panel zone joint. The positive yield flexural strength of the panel zone joint can be calculated based on Equation 1,

$$M_y^- = V_y d_{eff}^- = V_y (d_b - t_f) \quad (1)$$

$$M_y^+ = V_y d_{eff}^+ = V_y (d_b - d_{rib} + 0.5t_s - 0.5t_f) \quad (2)$$

in which,  $d_{rib}$  is the depth of the ribbed section of the steel deck and  $t_s$  is the thickness of the slab. This increase in the effective depth reflects the higher stiffness and yield flexural strength of the panel zone due to the composite action (Kim and Engelhardt 2002). For composite panel zones as part of interior beam-to-column connection joints, their backbone curve remains symmetric because in both loading directions the slab is effective (Elkady and Lignos 2014). However, composite panel zones as part of exterior beam-to-column connection joints have an asymmetric backbone curve.

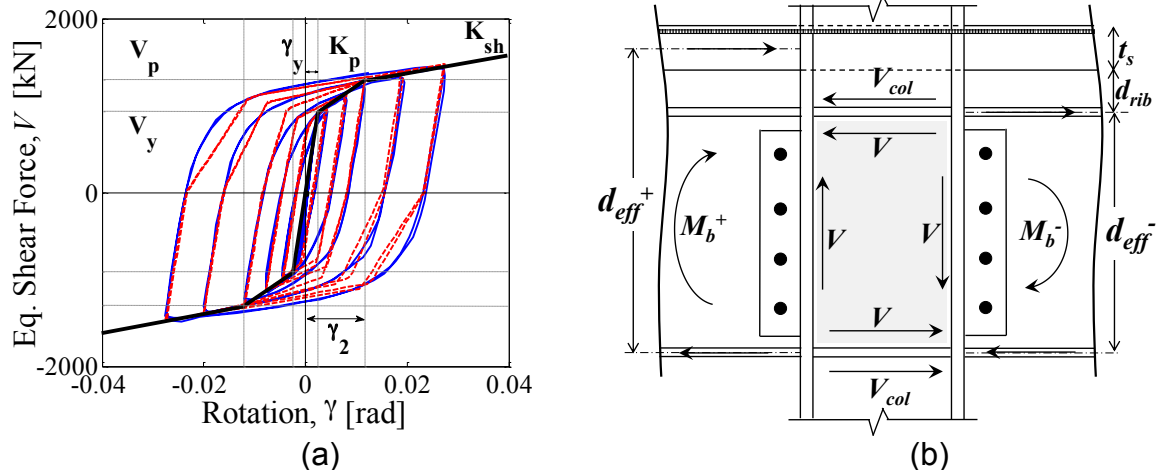


Figure 2: (a) Panel zone hysteretic material model [Data from Jones et al. (2002)]; (b) boundary forces acting on interior composite beam-to-column panel zone joints

The current seismic provisions for steel MRFs in the US (AISC 2010a; b) only consider the bare steel properties when sizing the panel zone thickness (i.e., doubler plate thickness). This implies that beam-to-column panel zone joints as part of steel

MRFs experience larger shear force demands and hence larger plastic deformations than expected. This is discussed in detail in the subsequent sections.

### 3. EFFECT OF COMPOSITE ACTION ON THE SEISMIC PERFORMANCE OF STEEL FRAME BUILDINGS WITH SPECIAL MOMENT FRAMES

This section discusses an analytical investigation of the seismic performance of archetype steel frame buildings with perimeter SMFs through collapse. The main goal is to assess the composite beam effects on the collapse resistance of the archetypes. The modeling recommendations discussed in Section 2 are utilized for this purpose.

#### 3.1 Description of Archetype Steel Frame Buildings

A set of 2- to 20-story archetype steel frame office buildings that utilize perimeter SMFs is designed according to AISC (2010a; b) and ASCE 7-10 (ASCE 2010). The archetypes are assumed to be located in urban California. The SMFs are designed with typical beams with RBS. Figure 3 shows a typical plan view of the archetypes including an elevation view of one of the SMFs (see Figure 3b). Two dimensional (2-D) analytical model representations of the archetype buildings are developed in the OpenSees simulation platform (Mckenna 1997). The steel beams and columns of the steel SMFs are modelled with elastic beam-column elements. Each element utilizes the modified IMK deterioration model at its ends. For the bare SMFs (noted as B-models) the input parameters of the deterioration model are defined based on earlier work by Lignos and Krawinkler (2011). For the composite SMFs (noted as C-models) the input parameters of the steel beams are defined based on the recommendations presented in Section 2. P-Delta effects are considered with a fictitious leaning column. The Rayleigh model is employed with 2% damping ratio to simulate viscous damping.

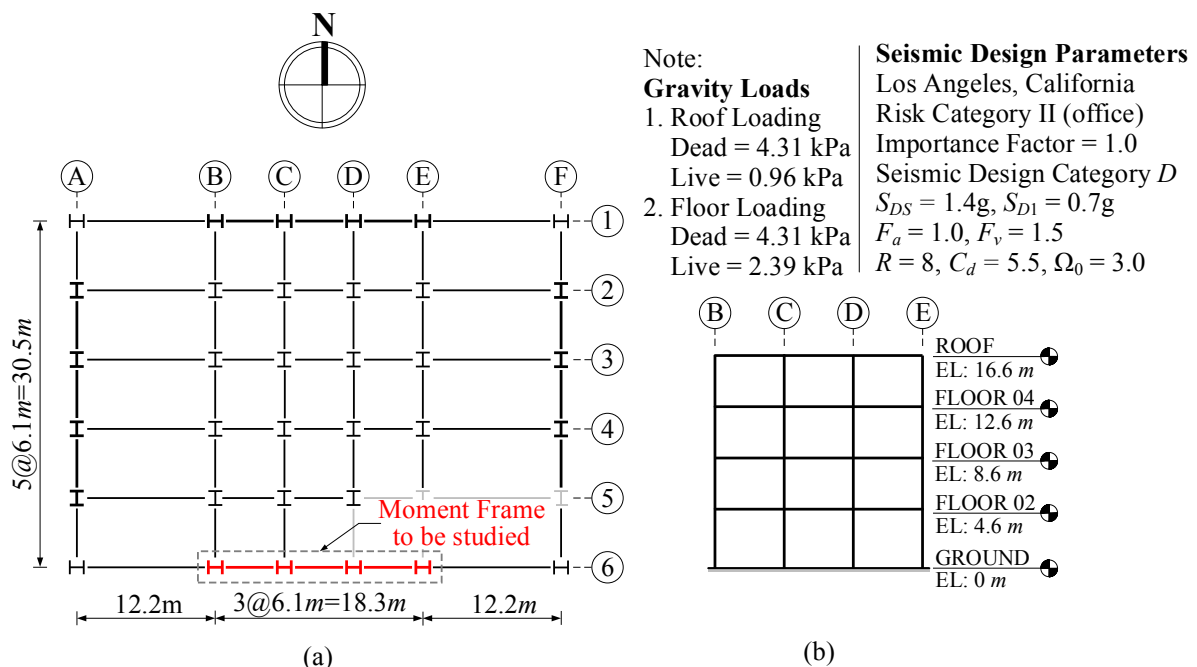


Figure 3: Typical archetype steel frame buildings: (a) plan view; (b) elevation of a four-story SMF

#### 3.2 Collapse Assessment of Archetype Steel Frame Buildings

The dynamic behavior of the archetype buildings is assessed by performing incremental

dynamic analysis (Vamvatsikos and Cornell 2002). Earthquake-induced collapse due to sidesway instability is simulated explicitly. The far field set of 44 ground motions from FEMA P695 (FEMA 2009) is utilized for this purpose. Figure 4a shows the incremental dynamic analysis curves for the 8-story steel SMF (B-model) in the East-West (EW) loading direction in terms of the spectral acceleration of the first mode period  $S_a(T_{1,5\%})$  of the SMF versus its maximum story drift ratio (SDR). Based on this figure, a collapse fragility curve is constructed for the B-model representation of the 8-story SMF (see Figure 4b). If we repeat the same process with the C-model of the same archetype its collapse fragility curve shifts to the right as shown in Figure 4b. The collapse resistance of the same SMF slightly increases when the composite action is considered.

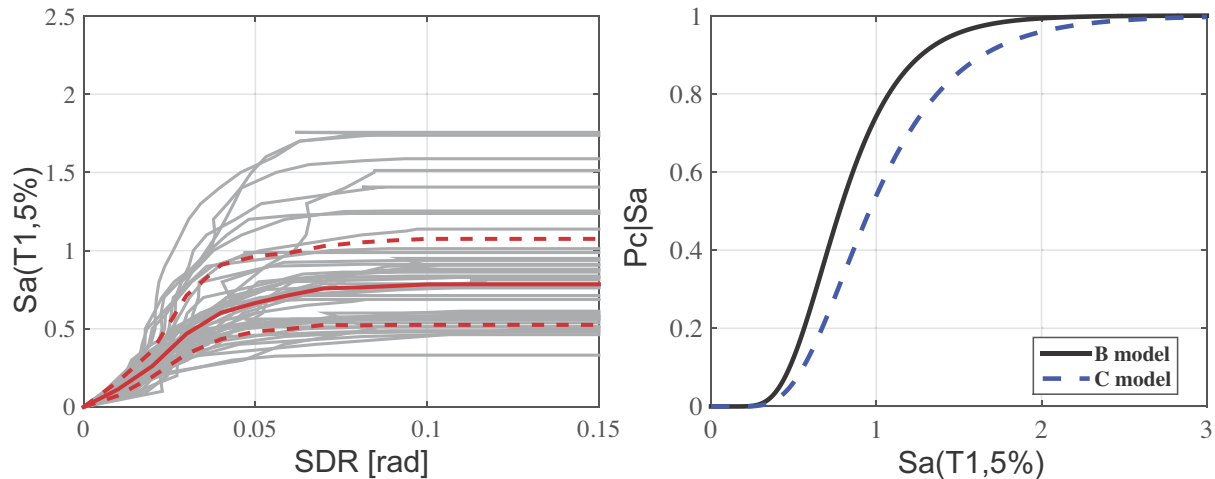


Figure 4: IDA and collapse fragility curves for the 8-story steel SMF

The added benefit of the composite beam action on the overall steel SMF seismic performance can be assessed through the mean annual frequency (MAF) of collapse,  $\lambda_c$ . This metric defines a collapse risk that is estimated by combining the probability of collapse of a steel SMF, given a seismic intensity, with a seismic hazard curve. The mean annual frequency of collapse can be translated to a probability of collapse over the life building expectancy. This value can be compared with the acceptable probability of collapse limit given in ASCE (2010). Figure 5 shows the  $\lambda_c$  values for the five archetypes that were considered in this paper. These values are computed based on the B- and C-models. From this figure, note that when a B-model is employed then mid-rise steel SMFs do not meet the requirements for a probability of collapse less than 1% over 50 years. It is evident that when the composite action is considered the estimated probability of earthquake-induced collapse is somewhat decreased compared to that predicted based on the B-models. This is mainly attributed to three reasons. The first relates to the added lateral stiffness that the composite action provides and therefore P-Delta effects do not easily shift the first order story shear of a steel SMF to zero. The second reason is due to the delay of flexural strength deterioration of a steel beam in the positive loading direction because of the slab restraint. The last reason is related to the higher inelastic demands that the panel zones experience due to the presence of the slab. Note that panel zone shear yielding is a stable yielding mechanism. This indicates the added benefit of a balanced panel zone design (Zhang and Ricles 2006).

The aforementioned issue can be further evaluated in Figure 6 that illustrates the moment-rotation relations of a column, the panel zone and the beam of the second story interior joint of the 8-story SMF for one of the employed ground motions scaled at the collapse intensity. In particular, from Figures 6a to 6c when a B-model is employed (i.e.,

composite action is neglected) most of the plastic deformation concentrates in the steel beam (see Figure 6c). The panel zone and the column essentially remain elastic. However, from Figures 6d to 6f when the nonlinear response history analysis is repeated with a C-model (i.e., composite action is considered) the panel zone yields extensively due to the increased flexural strength of the composite steel beam (see Figure 6e). In addition, flexural yielding occurs in the column (see Figure 6d). From this figure, excessive panel zone yielding at interior joints of steel SMFs should be treated with caution. The reason is that fracture may occur between the bottom flange of the steel beam and the column face. A reasonable approach to control the amount of panel zone yielding as well as to avoid flexural yielding in the steel columns of a steel SMF would be to consider a higher SCWB ratio as part of the design of the steel SMF compared to what is traditionally used in seismic design. This is examined in the following section.

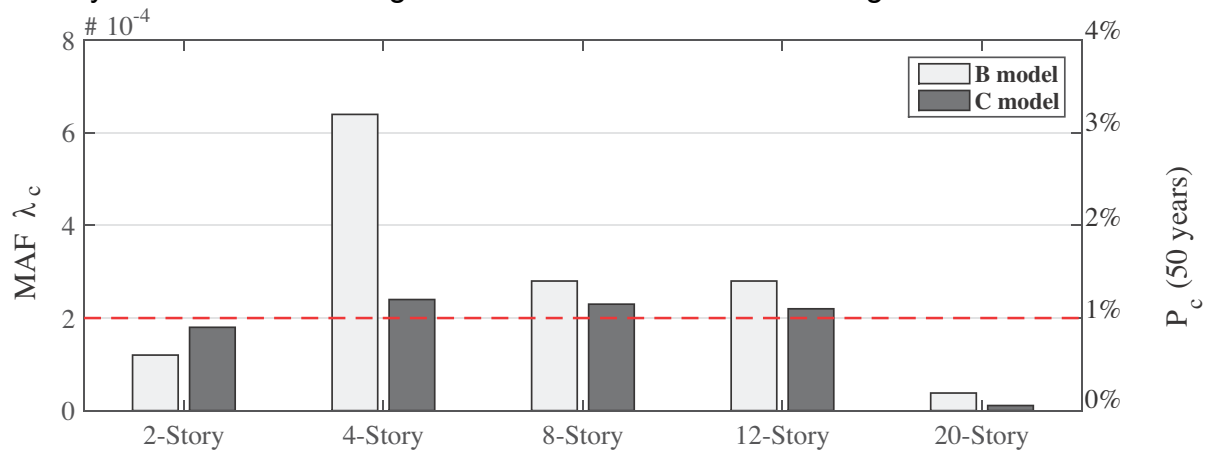


Figure 5: Mean annual frequency of collapse,  $\lambda_c$  for steel SMFs based on B- and C-models

#### 4. EVALUATION OF COMPOSITE SPECIAL MOMENT FRAMES DESIGNED WITH HIGHER STRONG-COLUMN/WEAK-BEAM RATIOS

The archetype steel frame buildings discussed in Section 3.1 are redesigned with a SCWB ratio  $>1.5$  and  $> 2.0$ . Because the steel columns of the redesigned SMFs have thicker webs, their panel zones experienced lower levels of shear distortion compared to the original designs. Therefore, the likelihood of bottom flange fracture due to excessive panel zone shear distortion is reduced. Furthermore, the thicker column webs reduce the dependency of the panel zone design on welded doubler plates. Therefore, fabrication costs may be reduced with an average column weight increase of not more than 149kg/m (100lbs/ft). Figure 7 illustrates the dependence of  $\lambda_c$  on the SCWB ratio and the corresponding probability of collapse in 50 years based on the C-models of the archetypes. From this figure, when a SCWB  $> 1.5$  is employed then a nearly uniform probability of collapse is achieved for all the archetypes. The probability of earthquake-induced collapse,  $P_c(50 \text{ years})$  becomes less than 1%, which is the acceptable limit per ASCE/SEI 7-10 (ASCE 2010) given that earthquakes follow a Poisson distribution in time. Because the 3-dimensional effects and the flexibility of the column bases is neglected in the present study it is recommended that the employed SCWB ratio for collapse prevention be at least larger than 2.0.

A higher SCWB ratio would typically lead to the use of heavier column sections compared to the current SCWB criteria. However, a considerable reduction in the required welded doubler plates to satisfy the panel zone strength requirements per AISC (2010a) is achieved. This reduces the fabrication cost and the likelihood of weld-related failures (Ibrahim et al. 2013).

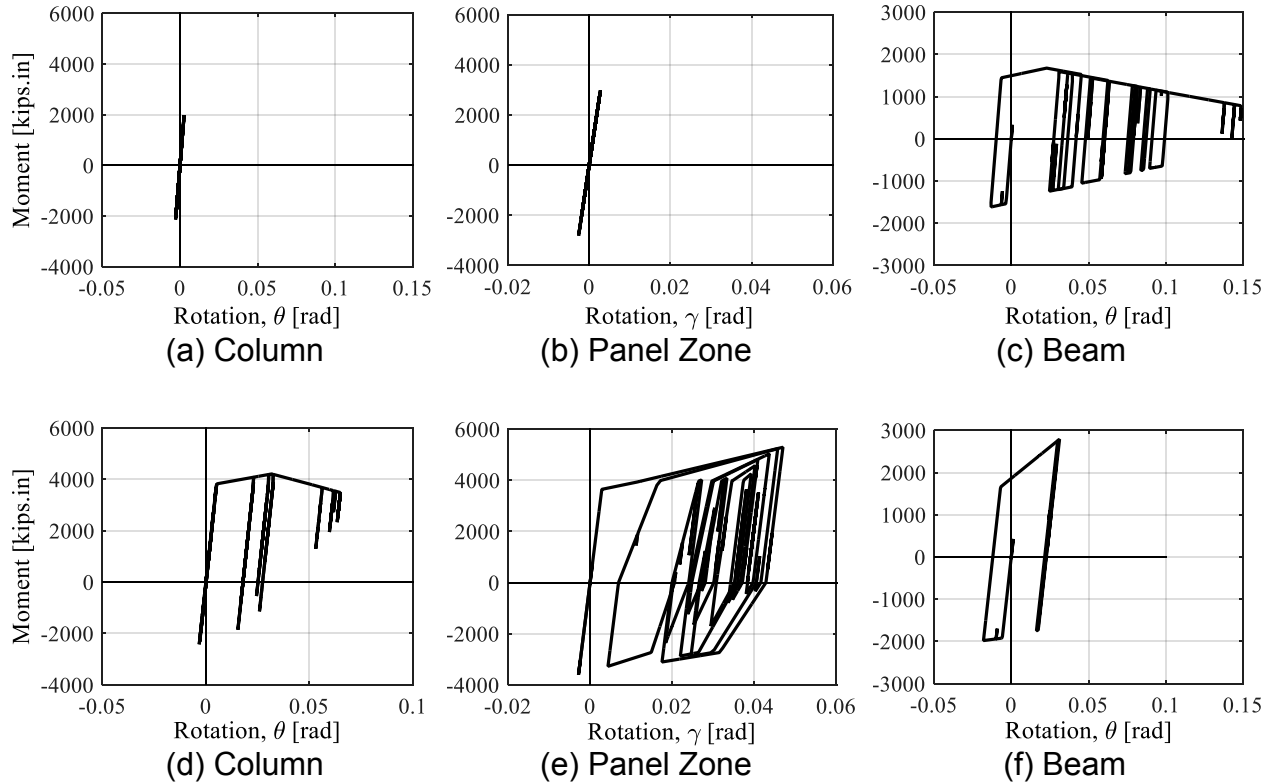


Figure 6: Moment-rotation relation of a second story interior joint at collapse intensity for the 8-story bare model (top) and composite model (bottom)

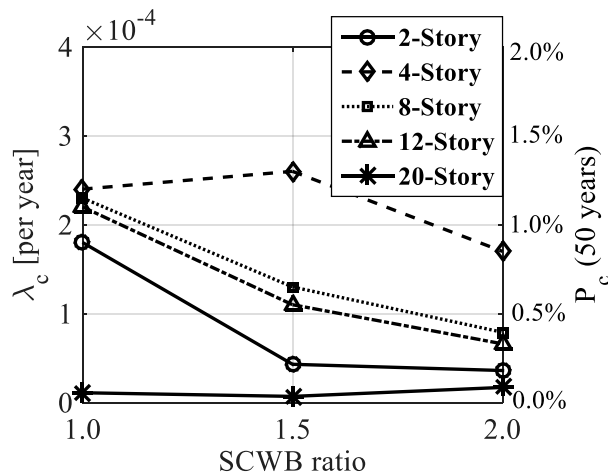


Figure 7: Mean annual frequency of collapse versus SCWB ratio for all composite models representations of the steel archetype buildings

## 5. CONCLUSIONS

This paper evaluates the effect of the composite action on the hysteretic behavior of fully restrained beam-to-column connections. The assessment is based on available test data from full-scale experiments of such connections. A state-of-the-art deterioration model was calibrated with the available experimental data and a rational approach was developed to simulate the asymmetric hysteretic behavior of composite steel beams and panel zones. This approach was then used to develop analytical model representations of archetype steel frame buildings with special moment



frames (SMFs) designed in highly seismic regions in North America. The main findings are summarized as follows:

- The flexural strength of a composite steel beam can be reasonably computed based on the ANSI/AISC 360-10 (AISC 2010b) provisions if the slab's geometry and material properties are available. Therefore, this value can be directly used in the strong-column/weak-beam (SCWB) ratio computation.
- The pre- and post-capping plastic rotation of a composite steel beam when the slab is in compression increase by 80% and 35%, respectively, compared to those of a bare steel beam. This is due to the lateral restraint that the slab provides to the top flange of the beam.
- When the composite beam effects are considered, excessive panel zone shear distortion was observed in steel SMFs designed with a SCWB  $> 1.0$  (i.e., current code requirement). This is due to the increased flexural strength of composite beams compared to bare ones. Controlled panel zone yielding is achieved only if a SCWB ratio  $> 1.5$  or  $2.0$  was employed as part of the seismic design process of the SMFs.
- Steel SMFs designed with SCWB ratios  $> 1.5$  or  $2.0$  achieved less than 1% probability of collapse over 50 years per ASCE/SEI 7-10 (ASCE 2010). The same designs typically utilized columns with thicker webs. This results into lower fabrication costs due to the lesser number of required welded doubler plates in the beam-to-column panel zone joints of the SMF.

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