LABYRINTH WEIRS: DEVELOPMENTS UNTIL 1985

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ABSTRACT

The weir is a fundamental structure in hydraulic engineering, serving to retain a water body, to control a water level, facilitate flow diversion, or to measure discharge. Under particular site conditions, the cross-sectional width at the weir location is limited so that either higher overflow depths or a compressed weir expansion are set. A form of the latter arrangement is the so-called labyrinth weir, which is composed of rectangular, trapezoidal or triangular plan shaped weirs, so that the geometrical crest length is increased. Along with the recently developed Piano Key Weir, labyrinth weirs represent economically and hydraulically sound alternative for increasing spillway discharge capacity. The present paper describes their historical development, reviews the main advances until the 1980s, summarizes current design guidelines, and presents the main individuals having participating in their development.

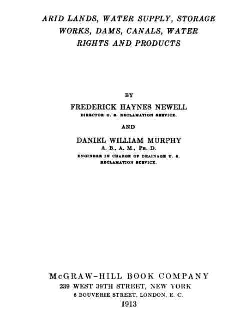
Keywords: Biography, Discharge, History, Hydraulics, Weir.

1. INTRODUCTION

A weir is defined as a barrier across a water conveyance, inhibiting flow into the tailwater for approach flow depths below the weir crest. The discharge of rectangular weirs is over-proportional, so that weirs are particularly apt for discharge retention up to the weir crest, generating high discharge once it is surpassed, as typically during floods. The weir is a fundamental hydraulic structure, by which discharge can be measured, rivers are made navigable, or flooding is prevented, particularly if gates are added. Given that the overflow zone is often laterally restricted, the overflow depths can become excessive. As a result, non-linear weirs e.g., Labyrinth (LW) and Piano Key Weirs (PKW) have been developed to increase weir length and discharge capacity within a fixed overflow width. The labyrinth weir and the PKW have an increased weir length through the use of a polygonal weir plan. In what follows, the historical development of the Labyrinth Weir shall be reviewed.

Murphy (1909) appears to be the first who highlighted the advantages of the LW. Figure 1 shows an example of the weir shape adopted. It is stated that the effective crest length was more than three times that of a straight weir, discharging 17 m^3/s into the tailwater. Given the narrow space at the overflow section, as shown in Fig. 2, seven bays or cycles were arranged in the plan. These were 3 m long and 1.5 m wide, resulting in a total crest length of 60 m in the 18 m wide channel. The inlet bays were designed with reducing cross-sectional areas along their length to generate nearly uniform overflow. Tests indicated that the discharge was practically identical with that of the standard straight-crested weir of equivalent length.

Daniel William Murphy was born on 18.01. 1865 at Defiance OH, and passed away on 15.05. 1931 at Los Angeles CA. He obtained the AB degree from Indiana University, the AM degree from the Leland Stanford University in 1892, and the PhD degree in 1895 from *Physikalische Technische Reichsanstalt*, the University of Berlin, Germany. He was from 1896 to 1900 assistant professor of physics at Stanford University, until 1904 then had his private engineering practice at Los Angeles, was further until 1908 with the US Reclamation Service during which period he was responsible for the Klamath Project in Oregon State. From 1909 to 1911 Murphy was in charge of engineering works at Washington DC, from when he was engaged as drainage engineer until 1918, thereby planning and supervising the construction of drainage works for 15 irrigation projects involving thousands of acres, specializing since 1918 in irrigation, drainage, and underground water developments. He was from then consulting engineer on the drainage of the Salt River Valley in Arizona, and the Imperial Valley in California. Murphy authored with Frederick H. Newell (1862-1932) the book Principles of irrigation engineering (1913), and his book Drainage engineering (1920). Murphy was member of the American Society of Civil Engineers ASCE.


PRINCIPLES
OF
IRRIGATION ENGINEERING

ARID LANDS, WATER SUPPLY, STORAGE WORKS, DAMS, CANALS, WATER RIGHTS AND PRODUCTS

BY
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1913

The first hydraulic study on the LW was conducted by the Italian Bruno Gentilini (1907-1998) at the Politecnico di Milano (Polytechnic School, Milan) under Giulio De Marchi (Gentilini 1941). First, the plan of the weir shape is explained: Let *L* be the straight structural length, and α the intended angle. Figure 3 shows a number of weir arrangements tested, all of which feature a sharp standard weir crest shape. Gentilini (1941) found that the head-discharge relationship per cycle was independent of the number of cycles, e.g., weir types 1, 2, and 3, or types 5 and 6 in Fig. 3. It was further observed that the data of weir types 1 to 4 and 5 to 8 both with α=90°, and 9, 10 with slightly different plan geometry, respectively, had identical discharge-head relations. Note that the notation in this paper is based on that of the various papers discussed, so that an identical abbreviation may apply to various parameters.

As to the approach flow direction perpendicular to the weir, the discharge coefficient μ_n in the free weir flow equation generally varies with the relative head on the weir [$h/(h+p)$] with h as the overflow depth and p as the weir height, and the Reynolds and Weber numbers accounting for viscosity and surface tension effects. If clean water is employed, and absolute overflow depths are larger than h=0.05 m, say, then the latter two effects remain insignificant. For oblique approach flow, the corresponding discharge coefficient μ depends in addition on the relative overflow depth h/L and α, with L as the projected weir length perpendicular to the approach flow direction. Gentilini (1941) shows a plot μ/μ_n versus h/L for α=30°, 45°, and 60° (Fig. 4), indicating that μ/μ_n=1 for h/L=0, from where all curves drop exponentially versus a final plateau value. The drop rate changes inversely with α; no drop at all occurs for α=180°. Note also from Fig. 4 that for e.g. h/L=1, $\mu/\mu_n(\alpha=60°)$=0.90, $\mu/\mu_n(\alpha=45°)$=0.79, and $\mu/\mu_n(\alpha=30°)$=0.65, indicating a massive loss of discharge capacity as α reduces on the one hand, which is made up on the other hand by the increase in crest length, resulting in total in an absolute increase of discharge capacity.

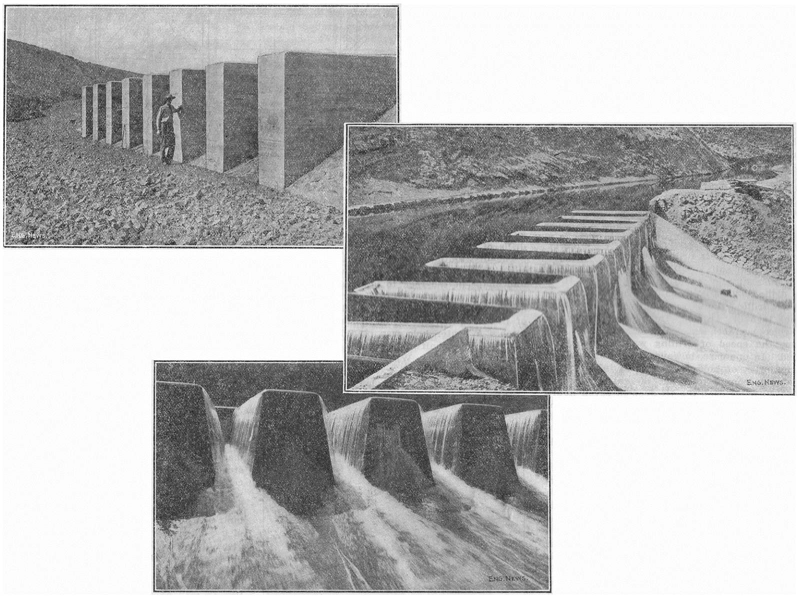


Figure 1. Photographs of reinforced-concrete spillway of Keno Canal, near Klamath Falls OR (Murphy 1909)

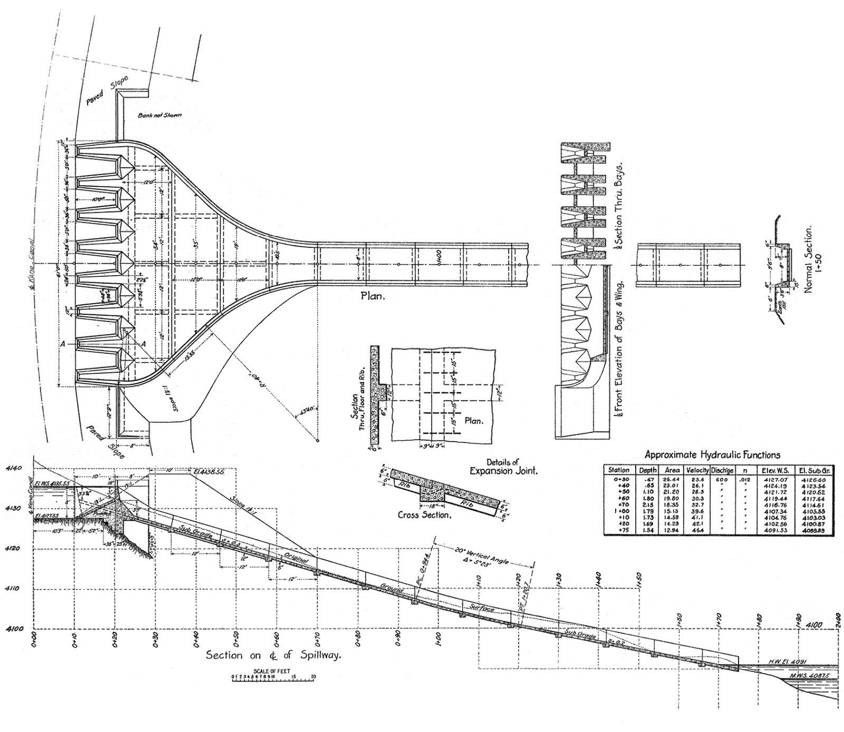


Figure 2. Keno Canal Spillways with concentrated crest length (Murphy 1909)

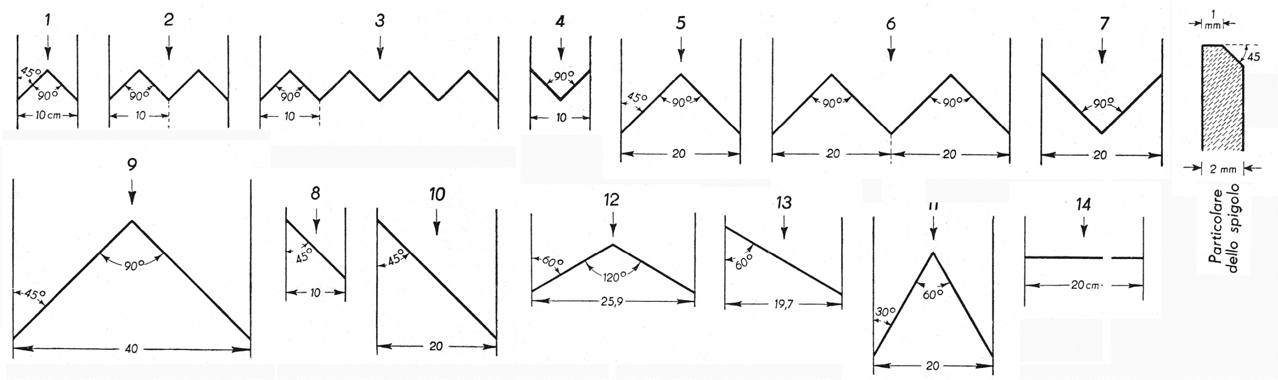


Figure 3. Plan geometries of LWs tested by Gentilini (1941)

The effect of LW discharge modification is finally addressed by Gentilini (1941). Figure 5 shows the ratio of discharges of the LW and the normal weir in terms of the variable *h*/*L*, with *L* as the length of the LW across the channel for various numbers *n* of singular oblique weirs and α=45°. As in Fig. 4, all curves start at $Q/Q_n(h/L=0)=1.41$ for all *n*, reducing exponentially as *h*/*L* increases to asymptotically attain $Q/Q_n(h/L\rightarrow\infty)=1$. The reduction of the discharge ratio remains small for *n*=1, but increasing with *n*. A slightly modified result applies as the angle α is changed. The work of Gentilini (1941) results in a definite increase of discharge capacity of the LW, particularly for relative small overflow depths *h*/*L*.

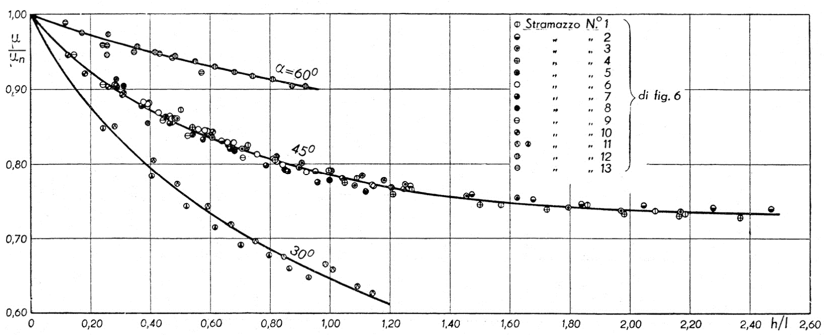


Figure 4. Effects of relative overflow depth *h*/*L*, and α on discharge coefficient ratio μ/μ_n (Gentilini 1941)

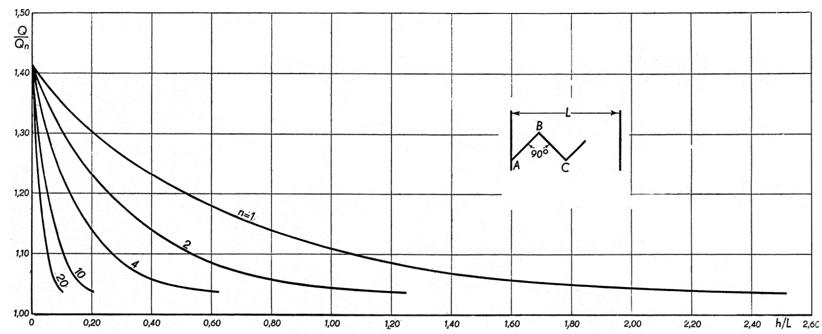


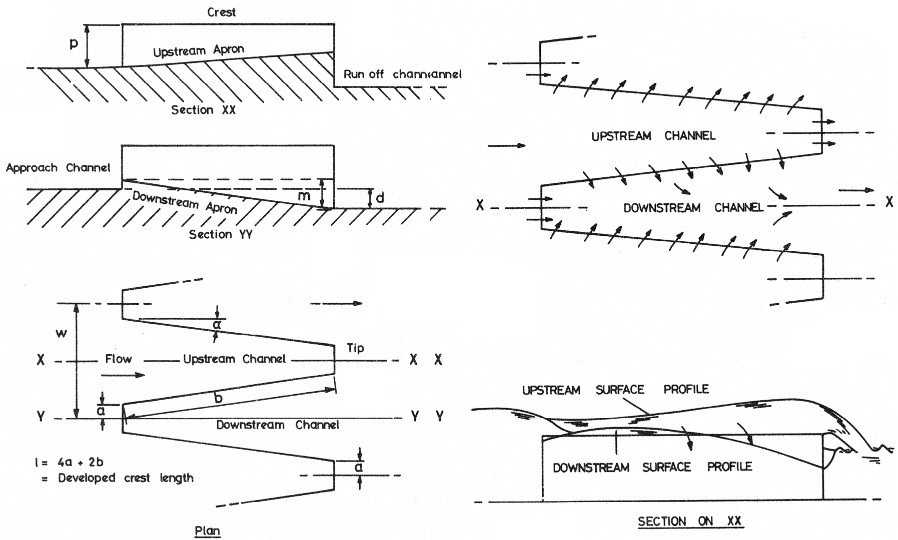
Figure 5. Effects of relative overflow depth *h*/*L*, and obliquity angle α on discharge ratio Q/Q_n (Gentilini 1941)

Bruno Gentilini was born on 20.05. 1907 at Luzzara, in the Reggio Emilia Region, Italy, and passed away on 01.10. 1998 at Milan, Italy. He was educated as a civil engineer at Politecnico di Milano, Milan. From 1930 to 1947 he was an assistant to Giulio De Marchi (1890-1972) and obtained his PhD degree. He then left for the University of Cagliari, Sardinia, as an associate professor until 1949 and was later appointed professor of hydraulic engineering at Torino University, Italy. He returned in 1965 as professor of hydraulic structures to the Politecnico di Milano, from where he was retired in 1982.

Gentilini was an active Italian hydraulician from 1935 to 1950. As a scholar of De Marchi, he not only concentrated on national works but always had an international outlook. His research topics included free and submerged weir flows, uniform channel flow, scour at check dams, sideweir flow to check the approach of De Marchi, gate flow, labyrinth weirs, all of which were dealt with both theoretically and experimentally, and a historical paper on Gian Battista Venturi (1746-1822). Once having left Milan in 1947, the scientific activities of Gentilini stopped, even after having returned to his Alma Mater.

2. DEVELOPMENTS FROM THE 1970s

This period started with the paper of Hay and Taylor (1970). It is stated that the only relevant hydraulic criterion for the weir performance is the ratio between the discharges of the labyrinth and the normal weir configurations Q_L/Q_N in a channel of common width. Figure 6 shows a scheme of the LW and the surface profiles generated. Note the flow contraction shown in Fig. 6b due to water flowing from the up- to the downstream channel. In contrast, along the upstream channel up to the crest region, the surface profile increases, and only then falls into the downstream channel. From this perspective, the application of the usual weir discharge formula was questioned. Figure 7 shows further details considered, namely the plan geometry of the models tested, varying from the trapezoidal, to the rectangular, and the triangular shapes, and the parameter ranges tested in terms of the relative weir cycle width w/p, the length magnification l/w with l here as the length of one weir cycle, the number of weir cycles n, and the weir plan shapes.



(a) (b)

Figure 6. Labyrinth Weir (a) definition of geometry, (b) flow features (Hay and Taylor 1970)

Hay and Taylor (1970) concluded that the weir discharge performance Q_L/Q_N depends exclusively on the parameters h/p, w/p, l/w, α, and n. Note that all weirs tested were made of Perspex sheets provided with a standard crest shape. It was not mentioned that this relation applies only for water overflow with a minimum overflow depth of 50 mm, so that effects of viscosity and surface tension are excluded. The normal discharge Q_N was expressed again with the equation provided by Kindsvater and Carter, whereas submerged flow was normalized with the proposal of Villemonte (1947). The effects of these parameters were systematically discussed based on model experimentation.

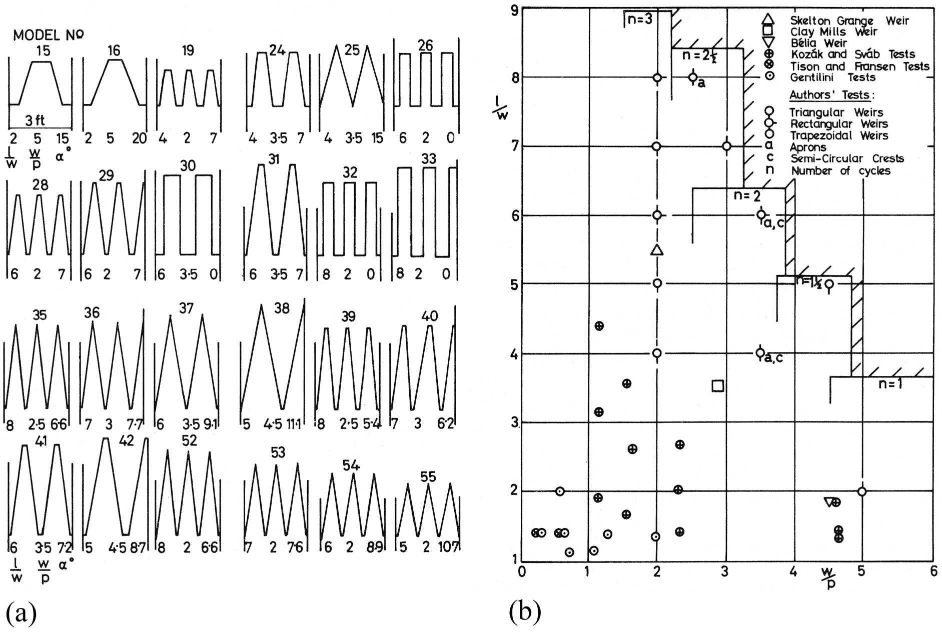


Figure 7. Labyrinth Weir (a) plan geometry of models, (b) parameter ranges (Hay and Taylor 1970)

As to the head-to-crest height ratio $h/p \rightarrow 0$ for all weir types, $Q_L/Q_N \rightarrow l/w$ i.e., the relative increase in labyrinth weir discharge, relative to the linear weir becomes equal to the weir length magnification factor at small upstream heads. As the relative overflow depth h/p increases, then $Q_L/Q_N \rightarrow 1$ (in theory) because of the interference of the various overflow jets, head-losses, and possibly tailwater effects. As to the sidewall angle α, varying from 0 for the rectangular to the maximum for the triangular plan shapes, the data indicate that the discharge performance increases with increasing α. Accordingly, the triangular plan shape renders the highest weir discharge performance. Note the limits of this statement, because effects of flow interference increase as the angle α decreases. No effect of the vertical aspect ratio w/p was noted in Hay and Taylor's data. The effect of nappe interference is less pronounced for trapezoidal plan shape. As to the number of weir cycles n, no effect was observed. The effect of downstream interference due to problems with nappe aeration was noted, so that an adequate air supply should be considered. As to the submergence effect, the ratio Q_L/Q_N reasonably follows Villemonte's equation. Hay and Taylor (1970) state that the LW should not be submerged with a tailwater elevation higher than the weir crest elevation, in analogy to normal weir flow conditions. As to crest shapes other than sharp-crested, no effect was found if the normal discharge is determined with the appropriate overflow relation, including e.g. broad-crested or round-crested weirs.

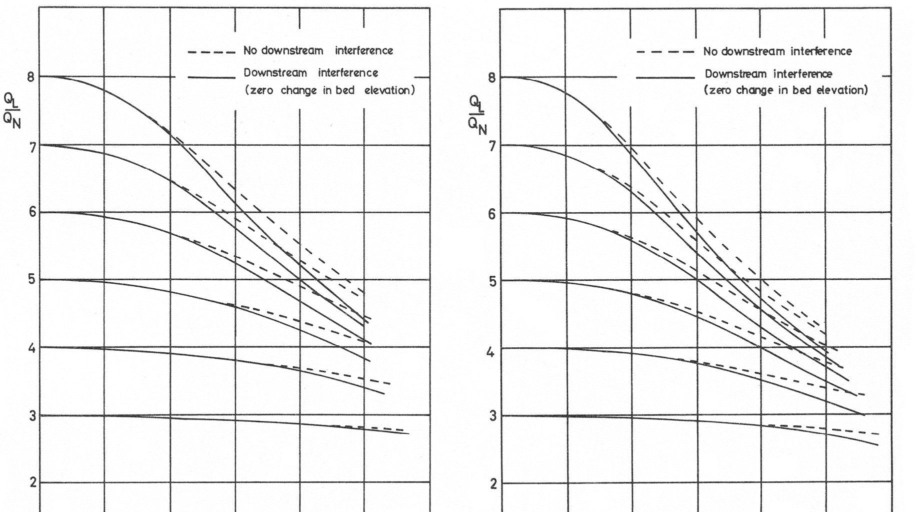
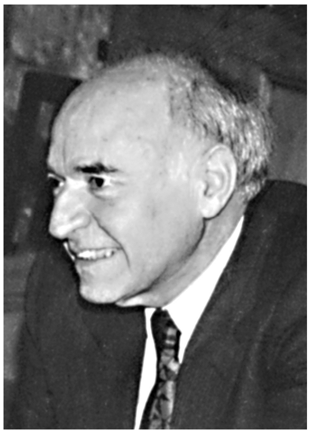


Figure 8. Labyrinth Weir (a) triangular plan form with $w/p \geq 2.5$ and $\alpha = 0.75\alpha_{max}$, (b) trapezoidal plan form with $w/p \geq 2.0$ and $\alpha = 0.75\alpha_{max}$ (Hay and Taylor 1970)

As to the LW design guidelines, Hay and Taylor provide discharge performance diagrams $Q_L/Q_N[h/p,\ l/w]$ for the triangular and the trapezoidal weir plan shapes for a sidewall angle of α=0.75α_{max} with and without downstream interferences. Note from Fig. 8 that the weir discharge performance remains excellent if h/p<0.1, whereas it is too small for h/p>0.5. The design guidelines further include comments relating to the crest length magnification, the vertical aspect ratio, sidewall angle, the apron elevation difference, crest geometry, submergence, and other limitations.



Nessim Israel Hay was born on 13.01. 1927 at Baghdad, Iraq, passing away on 06.10. 2001 at Nottingham UK. He obtained the civil engineering degree BSc in 1951, and the PhD degree in 1955 from University of Nottingham. He was from 1956 to 1957 manager of the Chevrolet Co., Baghdad, then until 1960 mechanical engineer of the United Contracting Co., Iraq, from when he joined as R&D engineer Hawker Siddeley Aircrafts, London UK. He moved then as Lecturer to the Dept. of Mechanical Engineering, the University of Nottingham, Nottingham UK, becoming later Reader until his retirement in 1992. He continued his consulting work for eight years at Montreal, Canada. Sadly he was diagnosed with prostate cancer in 2000 and passed away in 2001.

The research interests of Hay included hydraulics and hydraulic engineering during his early career at Nottingham, and later heat transfer, combustion, and internal combustion engines. He is known for two papers, namely the 1958 research conducted with Eric Markland (1923-) on the geometry of weir flow trajectories by applying the method of the electrolytic tank, and that on labyrinth weirs. He took over from 1993 to 1999 as Director and a member of the Board of management in two companies at Montreal, Canada.

A notable research on the LW, referred to by the Authors as the polygonal weir, was conducted by Indlekofer and Rouvé (1975). Figure 9 shows weir plan shape arrangements including curved and straight weir crests, of which only the latter were considered under fully-aerated nappe. A so-called Corner Weir consists of two or a multiple of straight vertical weirs with intended angle α varying between 0<α≤180°. Model tests for α=47°, 62°, 90°, and 123° and a standard crest shape were made. The resulting discharge was related to the Rehbock (1929) equation. Based on laboratory observations, various overflow zones affect the discharge. A computational approach was presented including the overflow length, and the related disturbance coefficients, resulting in a complex generalized Rehbock discharge equation.

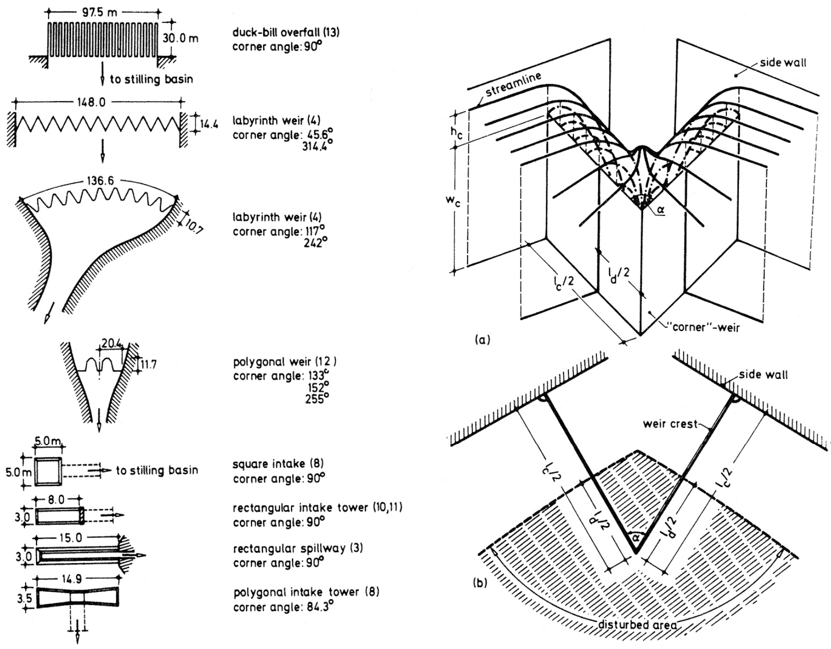


Figure 9. (a) Polygonal Weir crests, (b) Corner Weir from upstream and plan (Indlekofer and Rouvé 1975)

Horst Indlekofer was born in 1941 at Wertheim, Germany. He obtained in 1968 the civil engineering degree from the Technical University of Stuttgart. From 1969 he was an assistant at the Institute of Hydraulic Engineering and Water Resources, Rheinländisch-Westfälische Technical University (RWTH), Aachen, Germany, under Gerhard Rouvé (1927-2008). He was promoted in 1970 to Head of the Hydraulic Laboratory, obtaining the PhD degree on the work discussed above in 1972. From 1976 to 1977, Indlekofer was a post-doc engineer at Colorado State University, Fort Collins CO, where he was involved in computational techniques of shallow-water waves and sediment transport. From 1983 to 2008, he was professor of hydraulic engineering and water resources at Fachhochschule Aachen, concerned with projects in environmental engineering, hydrology, and the management of river valleys.

The research interests of Indlekofer include hydraulics, hydraulic engineering, water and vegetation, roughness in hydraulics, numerical stability in water wave modeling, and bed-load transport.

Cassidy et al. (1985) describe a spillway providing protection against the possible maximum flood of an earth dam and reservoir in north central Oregon. The zoned-filled earth dam is 31 m high, across Six Mile Canyon. To counter a dam failure under maximum flood conditions, a labyrinth type spillway was integrated into the dam crest. After a description of the spillway requirements for two stages of reservoir development, various spillway types are discussed. Among the straight crest, the controlled crest, and the labyrinth crest types, the latter was selected for the final design for reasons stated previously. Figure 10 shows a plan of the overflow reach, including pressure tap locations. The straight crest length was 110 m, so that two symmetrical triangular plan-shape labyrinth portions of 39° intended angle were selected along the 36.6 m wide overflow section. To increase the discharge coefficient, the crest shape adopted was semi-circular of diameter 0.46 m. The weir height varied from 2.8 m upstream to 4.3 m downstream of the structure, with an average of p=3.5 m. Based on Hay and Taylor (1970), the discharge magnification ratio varied from 2.7 to 3, which followed also from model tests. The free surface profiles were determined along the weir crest to estimate the freeboard along the weir section.

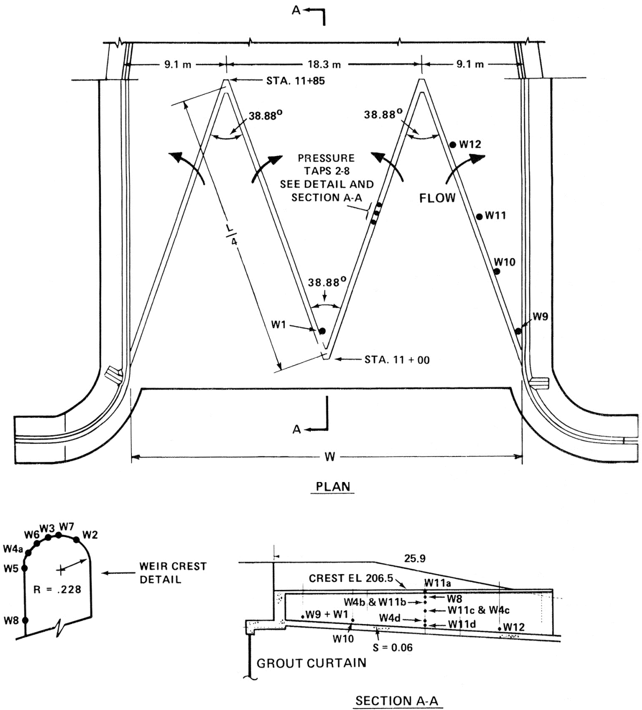
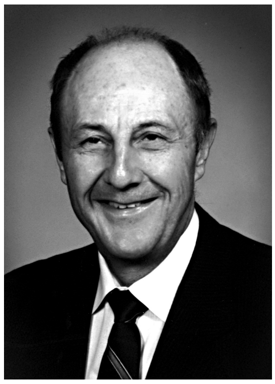


Figure 10. Boardman labyrinth-crest spillway (Cassidy et al. 1985)



John (Jack) Joseph Cassidy obtained the BS degree from the Montana State College, Bozeman MT, the MS in 1960, and the PhD in 1964 from the University of Iowa, Iowa City IA. He was from 1963 to 1972 from assistant professor to professor of civil engineering at the University of Missouri, Columbia MO, from when he was with the Bechtel Inc., San Francisco CA, as assistant chief hydraulic engineer, and from 1975 as chief hydrologic engineer. Cassidy is ASCE Member, was in 1978 chairman of its Hydraulic Division, and chairman of its National Water Policy Committee in 1984. He served as chairman of the Intl. Commission of Large Dams ICOLD, and on its Committee on hydraulics for large dams from 1987. He was the 1962 Ford Foundation Fellow, and Bechtel Fellow in 1986.

His research interests include flood hydrology, reservoir operation, water resources, and design of hydraulic structures. The 1965 paper deals with spillway flow using the potential flow theory to study free surface profiles, discharge coefficients and pressure distribution. Information on minimum boundary pressures and design guidelines are also given.

3. CONCLUSIONS

The labyrinth spillway was 'invented' a century ago, allowing for a reduced overflow width normally at the spillway crest region. The so-called discharge magnification factor as the ratio between discharges of the labyrinth to the normal weir arrangements is the true measure of the hydraulic performance. This research details the developments of this particular hydraulic structure by accounting for the outstanding research additions up to 1985. It demonstrates that labyrinth weirs apply not for all hydraulic conditions, particularly under high submergence, high relative overflow depth, and poor plan geometry. This structure is a predecessor of the Piano Key Weir, which currently is popular because of even increased discharge characteristics as compared with the labyrinth weir. The future development of both weir types may be improved with this historical outlook for further progress both in hydraulic and economical features.

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