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PRESSURE DROP AND HEAT TRANSFER IN A CYLINDRICAL HEAT EXCHANGER WITH ICE SLURRY FLOW

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In an earlier contribution it was shown that the Continuous-Properties Model (CPM) is an ideal theoretical model to calculate melting of ice slurries. Now with a shock-theoretical approach, it is proven that the CPM - in the limit toward a discontinuous melting - just yields the Stefan problem. This limit corresponds to the case when the additive content in an ice slurry tends toward zero.
In heat exchangers the ice fraction of an ice slurry is a decreasing function of the downstream space coordinate. The specific pressure drop R=-dp/dx can differ from the inlet to the outlet by more than a factor ten, because the viscosity and the critical shear stress decrease with increasing temperature. A simple analytical model to calculate the overall pressure drop of a cylindrical heat exchanger (with different boundary conditions) is presented.

1. INTRODUCTION

It now has become clear that Phase Change Slurries (PCS) - which change phase at a well defined temperature, respectively in some temperature domain - have an enormous potential as future thermal energy transport fluids. Their attractiveness is the high energy density given by the latent heat and the stabilization of temperature at the temperature of phase change, respectively in the phase change temperature interval, where the transition occurs. The PCS must be pumpable also at the lowest temperature of operation of the system. Therefore, at low temperatures it is allowed to freeze only partially. Suspensions with a large domain of melting yield a first solution. A Japanese steel company has developed a PCS on clathrate basis, which melts between 7 °C and 12 °C depending on the fraction of additive [1]. This temperature domain is ideal for air-conditioning applications. The slurry can be simply produced by cooling the fluid in a plate heat exchanger without any additional mechanical scraping. Moreover, it shows negligible buoyancy effects and so in storage tanks no mixing elements are required. A second method, which can guarantee fluidity to very low temperatures, is the technique of microencapsulation with characteristic capsule sizes of 10 to 300 μm. Such PCS were tested by the NASA [2]. A problem is the durability of the capsules in a technical flow system. By choosing carrier fluids, which show very low freezing points, energy transport fluids for outdoor applications can be designed, e.g. for solar engineering applications. Ice slurries yield a subclass of the overall domain of PCS.

2. MELTING OF ICE SLURRIES AND SHOCK THEORY

In 1994 the Continous-Properties Model (CPM) was published (see Ref. [3]). This article takes account of numerous theoretical subtleties. In a succeeding article, presented to the IIR ice slurry working party [4], it was shown that a melting ice slurry - from a macroscopic point of view - can be described as a substance showing continous physical properties (see Figure 1). Most approaches to determine the physical properties of ice slurries are based on this macroscopic view. A probe is assumed to have a very high number of ice particles (respectively a high ice particle density) and the ice particles are assumed to be very small in comparison to a characteristic overall flow length scale.

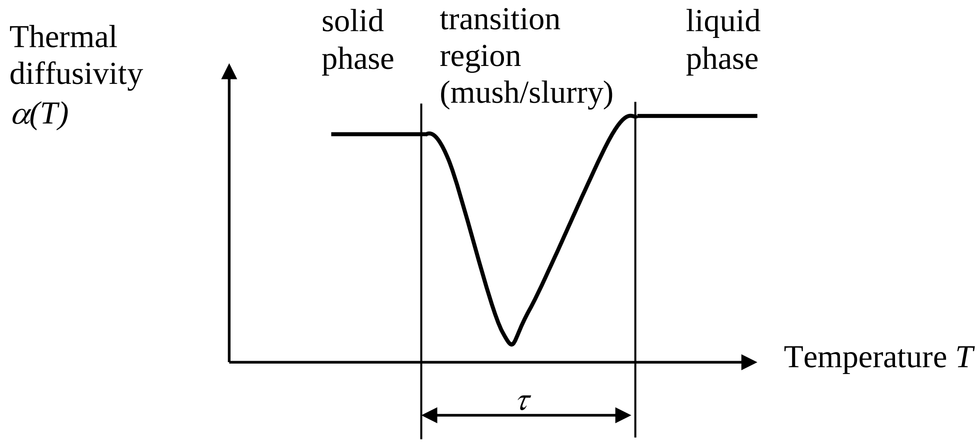

Figure 1: The thermal diffusivity of an ice slurry shows continuous behaviour. In Ref. [3] an example is shown how a measure of the width of the melting domain τ can be introduced.

The objective is to prove that the CPM contains the Stefan problem as a special case. For this purpose, just as in the derivation of the CPM, the energy conservation law is introduced

$$\rho \frac{\partial h}{\partial t} + \frac{\partial \dot{q}}{\partial x} = 0 . \tag{1}$$

The quantity ρ denotes the density, h the enthalpy density, T the temperature, t the time, and x the space co-ordinate. The heat flux density is given by Fick's law, which introduces the thermal conductivity k

$$\dot{q} = -k \frac{\partial T}{\partial x} . \tag{2}$$

It is clear that (1) and (2) are identical to

$$\rho \frac{\partial h}{\partial t} - \frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x} \right) = 0 . \tag{3}$$

Assuming constant pressure, with the definition

$$c_p = \frac{dh}{dT}, \tag{4}$$

and a product differentiation of (3), the nonlinear differential equation of the CPM is obtained

$$\frac{\partial T}{\partial t} = a(T)\frac{\partial^2 T}{\partial x^2} + b(T)\left(\frac{\partial T}{\partial x}\right)^2, \tag{5}$$

with

$$a(T) = \alpha(T) \qquad \text{and} \qquad b(T) = \alpha(T)\frac{1}{k}\frac{dk}{dT}. \tag{6a,b}$$

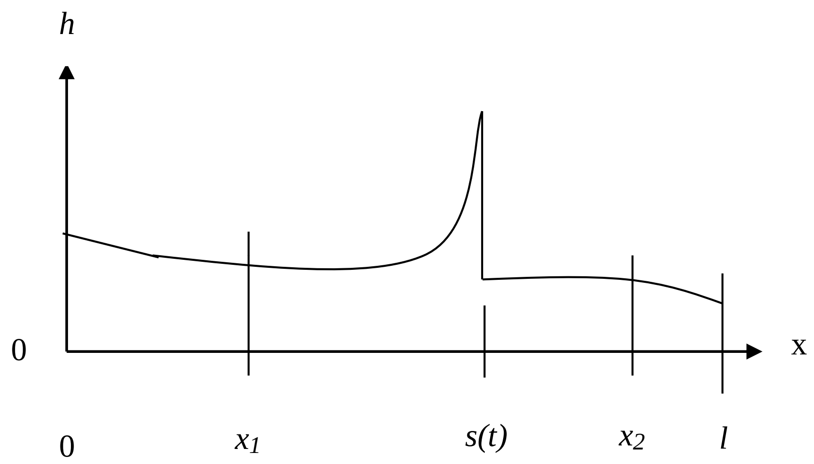

Figure 2: A moving discontinuity is described by its position *s(t)*, where *t* denotes the time. An arbitrary position with finite distance from *s* in front of the discontinuity is denoted by x_2 and analogous an arbitrary position behind *s* by x_1.

Figure 1 shows continuous behaviour, e.g. of an ice slurry, in parameter space. In Figure 2 a discontinuous disturbance is presented in real space. Notify that a continuous signal in real space has a width, which is dependant on the dynamics.

In this first approach the density is assumed to be constant. Then it follows that

$$\dot{q}(x_1,t) - \dot{q}(x_2,t) = \rho\left[\frac{d}{dt}\int_{x_1}^{s(t)} h(x,t)dx + \frac{d}{dt}\int_{s(t)}^{x_2} h(x,t)dx\right], \tag{7}$$

respectively

$$\dot{q}(x_1,t) - \dot{q}(x_2,t) = \rho\left[h(s^-,t)\frac{ds(t)}{dt} - h(s^+,t)\frac{ds(t)}{dt}\right] + \rho\int_{x_1}^{s(t)}\frac{\partial h(x,t)}{\partial t}\,dx + \rho\int_{s(t)}^{x_2}\frac{\partial h(x,t)}{\partial t}\,dx \tag{8}$$

The limit on the left-hand side of s is denoted with s^- and the limit on the right-hand side of s with s^+. Now the limiting cases $x_1 \to s(t)$ and $s(t) \leftarrow x_2$ are considered

$$\lim_{x_1 \to s^-} \lim_{x_2 \to s^+} \left[\dot{q}(x_1,t) - \dot{q}(x_2,t)\right] = \rho\left[h(s^-,t)\frac{ds(t)}{dt} - h(s^+,t)\frac{ds(t)}{dt}\right]. \tag{9}$$

In the two limits it is clear that the two integrals in equation (8) vanish. Therefore, the following equation remains

$$\dot{q}(s^-,t) - \dot{q}(s^+,t) = \rho\left[h(s^-,t) - h(s^+,t)\right]\frac{ds(t)}{dt}. \tag{10}$$

The velocity of the discontinuity is defined by

$$U(t) = \frac{ds(t)}{dt}. \tag{11}$$

Now it follows that

$$\dot{q}_1 - \dot{q}_2 = \rho U\left[h_1 - h_2\right] \qquad \Rightarrow \qquad \dot{q}_2 - \dot{q}_1 = \rho U\left[h_2 - h_1\right]. \tag{12a,b}$$

The differences in physical quantities are now described by parentheses. From (12b) it follows that

$$-\rho U[h] + [\dot{q}] = 0, \qquad [\chi] := \chi_2 - \chi_1, \qquad \chi \in \{\dot{q}, h, ...\} \tag{13a,b}$$

Studying the limiting procedure two laws have occurred. This becomes clear if equation (1) and (13) are compared

$$\frac{\partial}{\partial t} \to -U[...] \quad \text{and} \quad \frac{\partial}{\partial x} \to [...]. \tag{14a,b}$$

With these rules “arbitrary” partial differential equations can be transformed to algebraic equations, which describe discontinuities (see also Ref. [5]).

In a discontinuous case the physical properties of the two phases “solid” and “liquid” may be different. But in the single phases they are assumed to be constant. Therefore, in equation (6a,b) it follows that

$$a(T) = a_\chi = \text{constant} \qquad \chi \in \{1,2\}, \qquad b = 0. \tag{15a-c}$$

From equation (5) in the limit $\tau \to 0$ two equations are derived

$$\begin{aligned} \frac{\partial T}{\partial t} &= a_1 \frac{\partial^2 T}{\partial x^2}, \qquad 0 \le x < s(t) \\ \frac{\partial T}{\partial t} &= a_2 \frac{\partial^2 T}{\partial x^2}, \qquad s(t) < x \le l, \end{aligned} \tag{16a,b}$$

which describe the usual linear heat diffusion equations for the liquid and solid phase. The phases are now separated by a boundary with zero width. This discontinuity must be treated with the derived recipies. Applying the rules (14a,b) to equation (3) leads to

$$-\rho U[h] + \left[-k\frac{\partial T}{\partial x}\right] = 0\,, \qquad (17a,b)$$

respectively

$$\rho U(h_2 - h_1) + \left[k_2 \left.\frac{\partial T}{\partial x}\right|_2 - k_1 \left.\frac{\partial T}{\partial x}\right|_1\right] = 0\,. \qquad (18)$$

Equations (16a,b) and (18), together with the initial condition and the boundary conditions on the solid surfaces, define the Stefan problem for discontinuous melting and freezing.

3. THEORY OF PRESSURE DROP IN HEAT EXCHANGER TUBES

In Ref. [6] the calculation of pressure drops of laminar flowing isothermal Bingham fluids in tubes is presented in detail. Here only the most important aspects are briefly resumed. At the beginning of a pressure drop calculation the Reynolds and the Hedström number are calculated by applying the following equations

$$\mathrm{Re} = \frac{\bar{u}d}{\nu}\,, \qquad He = \frac{\tau_0 d^2 \rho}{\eta^2}\,, \qquad (19\ a,b)$$

where $\bar{u}$ denotes the mean velocity downstream, d the diameter of the tube, ν the kinematic viscosity, τ_0 the critical shear stress, ρ the density and η the dynamic viscosity. If one does not want to solve a transcendental equation, the friction factor λ can be taken from a graphical presentation, e.g. in Ref. [7]. Then the specific pressure drop follows directly by applying the two following equations

$$R = -\frac{dp}{dx} = \lambda \frac{L}{d} p_{dyn}\,, \qquad p_{dyn} = \frac{\rho}{2}\bar{u}^2\,. \qquad (20\ a\text{-}c)$$

In these equations p denotes the pressure, L the length of the tube and p_{dyn} the dynamic pressure.

The pressure drop of a cylindrical heat exchanger cannot be calculated by simply applying this theory for one condition. Because of the heat flux into the tube ice melts downstream and the viscosity and the critical shear stress decrease toward the end of the heat exchanger.

Therefore, at the inlet a much higher specific pressure drop is expected than at the outlet. As a consequence the value R(x)=-dp/dx is a continuous function of the spatial location downstream. The refrigeration expert is interested in the mean specific pressure drop $\bar{R}$. With knowledge of this value the pressure loss of the heat exchanger can be determined by simply multiplying $\bar{R}$ with the length of the heat exchanger tube.

In Ref. [4], applying a perturbation analysis, it is shown that small heat fluxes into a tube with flowing ice slurry - in first order - do not disturb the velocity profile. With this technique it is possible to develop linear corrections to the specific pressure drop in the downstream direction. But such corrections will lead to erraneous results if the heat exchangers are very long.

Because the velocity profile is very stable to thermodynamic perturbations, the isothermal calculation method of the pressure drop can be also applied to non-isothermal flows in heat exchangers with small energy flux densities. - One could develop the method to calculate the specific pressure drop at the inlet and at the outlet of the heat exchanger and then to determine the arithmetic mean value. The calculation of different cases shows that the specific pressure drop always decreases approximately exponential. This investigation leads to a calculation method for practical purposes, which is not based on first physical principles. The following Ansatz is made

$$R(x) = \alpha \exp(-\beta \frac{x}{L}) . \qquad (21)$$

Considering the boundary conditions

$$x = 0 : \quad R(0) = R_{In} , \qquad x = L : \quad R(L) = R_{Out} \qquad (22)$$

and setting

$$\xi = \frac{x}{L} , \qquad (23)$$

the following simple equation is obtained

$$R(\xi) = R_{In} \left(\frac{R_{Out}}{R_{In}} \right)^{\xi} . \qquad (24)$$

The mean value follows from an integration

$$\overline{R} = \int_0^1 R(\xi) d\xi = R_{In} \int_0^1 \left(\frac{R_{Out}}{R_{In}} \right)^{\xi} d\xi , \qquad (25)$$

which when performed yields the following final result

$$\overline{R} = \frac{R_{In}}{\log_e \frac{R_{In}}{R_{Out}}} \left(1 - \frac{R_{Out}}{R_{In}} \right) . \qquad (26)$$

To demonstrate the usefulness of the model, a numerical example is calculated and presented in detail. The heat exchanger produces a constant heat flux density boundary condition. Other cases (boundary conditions) are not more difficult to solve. The main data are the following:

- Ice slurry with 10 % water/talin solution
- Length of the tube $L = 10\ m$
- Inner diameter of the tube $d = 10\ mm$
- Total power $\dot{Q} = 10\ kW$

- Mass flow $\dot{m} = 0.111\ kg/s$
- Temperature at the inlet: $\vartheta_{In} = -6.0\ °C$
- Temperature at the outlet: $\vartheta_{out} = -4.0\ °C$.

The example was chosen in a manner that an approximately correct heat transfer coefficient occurs. Further quantities are listed in the two following tables:

Location (m)	**Temperature (°C)**	**Enthalpy (kJ/kg)**	**Ice fraction (Mass-%)**	**Density (kg/m^3)**	**Velocity (m/s)**	**Viscosity (mPas)**
0	- 6.0	-110	27	966	0.65	14
2.5	- 5.3	-87.5	17	971	0.65	8
5	- 4.7	-65	10	978	0.64	6
7.5	- 4.3	-42.5	6	983	0.64	5
10	- 4.0	-20	2	986	0.64	4

Location (m)	**Crit. Sh. St. (Pa)**	**Reynolds n. (-)**	**Hedström n. (-)**	**Friction fact. (-)**	**Dyn. Press. (Pa)**	**R (Pa/m)**
0	8	672	8870	0.297	205	4061
2.5	2.3	1183	7851	0.113	205	1544
5	1	1565	6113	0.067	200	893
7.5	0.5	1887	4424	0.047	201	630
10	0	2370	0	0.027	201	364

Table 1: An example of the specific pressure drop in a cylindrical heat exchanger. All the physical properties were taken from Ref. [4], [8] and [9].

The results of the specific pressure drop are shown in Figure 3. They decrease slightly more than exponential. Nevertheless, the exponential function is a useful approximation. The mean value of the specific pressure drop is calculated by applying equation (26)

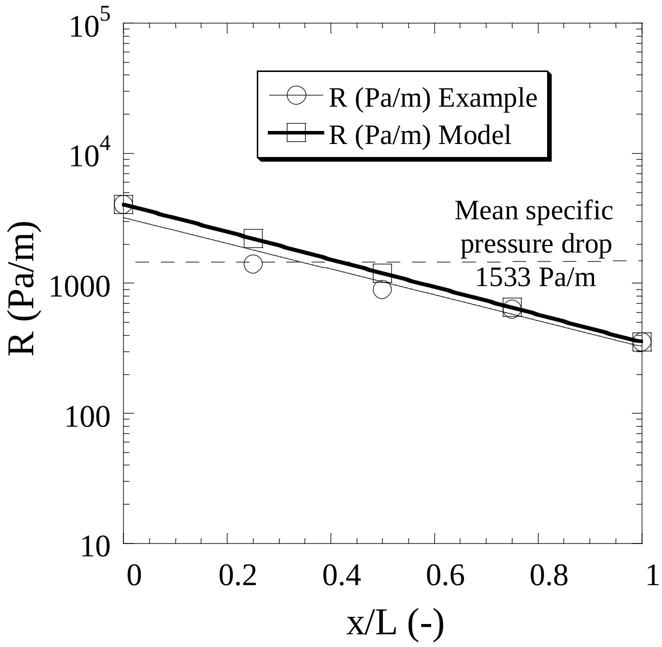

Figure 3: The specific pressure drop *R* in a heat exchanger is approximately exponentially decreasing. In this example *R* alters from the inlet to the outlet by more than a factor ten! “Example” denotes a step by step calculation downstream in the heat exchanger (e.g. Table 1). “Model” denotes results obtained by applying equations (24) and (27).

$$\overline{R} = \frac{4061}{\log_e \dfrac{4061}{364}}\left(1-\frac{364}{4061}\right) = 1533 \;\; Pa/m\,. \qquad (27)$$

If one would calculate $\overline{R}$ by applying the formula for the arithmetic mean between the specific pressure drop at the inlet and the outlet, the result would be 680 Pa/m too high!

4. OUTLOOK

The investigations shall be extended to high heat flux densities, which produce high non-linearities. At present at the University of Applied Sciences of Western Switzerland - for one of our industrial partners (BMS) - plate heat exchangers with ice slurry flows are investigated.

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