Robust Smith Predictor Design for Time-Delayed Systems with H_∞ Performance

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Abstract A new method for robust fixed-order H_∞ controller design for uncertain time-delay systems is presented. It is shown that the H_∞ robust performance condition can be represented by a set of convex constraints with respect to the parameters of a linearly parameterized primary controller in the Smith predictor structure. Therefore, the parameters of the primary controller can be obtained by convex optimization. The proposed method can be applied to stable SISO and MIMO models with uncertain dead-time and with multimodel and frequency-dependent uncertainty. It is also shown that how the design method can be extended to unstable SISO models. The design of robust gain-scheduled dead-time compensators is also investigated. The performance of the method is illustrated for both SISO and MIMO systems by simulation examples.

1 Introduction

Most industrial processes present dead time in their dynamics. Generally, dead times are caused by the time needed to transport energy, mass or information, but they also can be caused by processing time or by accumulation of time lags in a sequence of simple dynamic systems interconnected in series ([16]).

The presence of dead times in the control loops has two main consequences: it greatly complicates the analysis and the design of feedback controllers and it

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makes satisfactory control performance more difficult to achieve ([19]). Dead-time compensators can be used to improve the closed-loop performance of classical controllers (PI or PID controllers) for processes with delay. The Smith Predictor (SP) (See Fig.1), proposed in the late 1950s by [24], was the first dead-time compensation structure used to improve the performance of the classical controllers and became the most known and used algorithm to compensate dead time in the industry.

Although the SP offers potential improvement of the closed-loop performance of process with large dead-time, it requires a good model since small modeling errors can lead to very poor performance. For this reason, research efforts have been focused on robustness issues of the SP. A tuning method for models with one uncertain parameter is proposed by [1]. Easy tuning rules for SP in the presence of dead-time uncertainty is addressed in [22] and a guideline for selection the closed-loop bandwidth based on the dead-time uncertainty bound is proposed. In [20], a robust tuning rule is developed which considers the modeling error in the dead time. Robust PID tuning for SP considering model uncertainty is proposed by [10]. In particular, first and second order plus dead-time systems which may contain uncertainty in multiple parameters of the model are considered. In [13], tuning guidelines are presented for setpoint tracking considering model mismatches in the dead-time.

Many researchers are interested in the optimal control of dead-time systems, especially H_∞ control, i.e., to find a controller to internally stabilize the system and to minimize the H_∞-norm of an associated transfer function. Many relevant results have been presented in this framework using modified versions of the SP. See, for instance, [14], [13] and [26]. Recently, the SISO SP has been extended and generalized for MIMO systems. In [21], a structured uncertainty approach was implemented for SP's with diagonal delay matrices. This method, however, does not consider general and distinct time delays for each element of the plant transfer matrix. A diagonal H_2 optimal controller for non-square plants is designed by factorization methods in [25]. In [15], a generalized predictive control (GPC) method is implemented on MIMO SP systems with multiple delays. Nonetheless, these control techniques are quite complex and their implementation can be involved.

This paper presents a new method to design fixed-order SP controllers that considers uncertainty simultaneously in the dead-time and in the rational part of the model. The performance specification, like the standard H_∞ control problem, is a constraint on the infinity norm of the weighted sensitivity function and is represented by a set of convex constraints in the Nyquist diagram. The extension to MIMO systems will be based on the idea presented in [5] for designing decoupling MIMO controllers. In [5], a convex optimization approach was implemented to design a linearly parameterized controller for a MIMO system. In this paper, this concept will be extended to MIMO SP's with process plants that possess uncertain time delays.

This paper is organized as follows: In Section 2 the class of models, controllers and the control objectives for SISO systems are defined. Section 3 will extend the class of controllers and control objectives to MIMO systems. Sections 2 and 3 will also discuss the control design methodology and stability conditions for the SISO and MIMO Smith predictor configurations, respectively. This methodology is based on the convex constraints in the Nyquist diagram. In Section 4 the results are extended to unstable time-delay SISO systems. Gain-scheduled SP is designed for time-delay systems in Section 5. Each section will end with an illustrative example. Finally the concluding remarks are given.

2 SISO Problem Formulation

2.1 *Class of models*

Consider the class of stable time-delay LTI-SISO systems with bounded infinity norm. It is assumed that the plant model can be represented by:

$$P(s) = G(s)e^{-\tau s} \tag{1}$$

where the time delay τ is unknown but belongs to a finite set $\{\tau_1, \tau_2, \dots, \tau_q\}$ and the dead-time free part of the model has unstructured multiplicative uncertainty described as:

$$G(s) = G_n(s)[1 + \Delta(s)W_2(s)] \tag{2}$$

where $W_2(s)$ is a known stable uncertainty filter, $G_n(s)$ the nominal dead-time free model and $\Delta(s)$ an unknown stable transfer function with $\|\Delta\|_\infty < 1$. Therefore, we can assume that $P(s)$ belongs to a set $\mathbb{P}$ of q models given by:

$$\mathbb{P} \triangleq \{P_i(s)[1 + \Delta(s)W_2(s)];\ i = 1, \dots, q\} \tag{3}$$

where $P_i(s) = G_n(s)e^{-\tau_i s}$.

2.2 *Class of controllers*

The SP control structure shown in Fig.1 is considered. The nominal model $P_0(s) = G_n(s)e^{-\tau_n s}$ with $\tau_n \in [\tau_1, \tau_q]$ is used for the implementation of the controller.

The primary controller $C(s)$ is linearly parametrized by

$$C(s) = \rho^T \phi(s) \tag{4}$$

where $\rho^T = [\rho_1, \rho_2, \dots, \rho_{n_c}]$ is an n_c dimensional vector of the controller parameters and $\phi^T(s) = [\phi_1(s), \phi_2(s), \dots, \phi_{n_c}(s)]$ is a vector of basis functions with $\phi_i(s)$ transfer functions with no RHP poles. For instance, a PID controller could be linearly parametrized by

$$\rho^T = [K_p, K_i, K_d] \quad , \quad \phi^T(s) = [1, \frac{1}{s}, \frac{s}{1 + T_f s}]$$

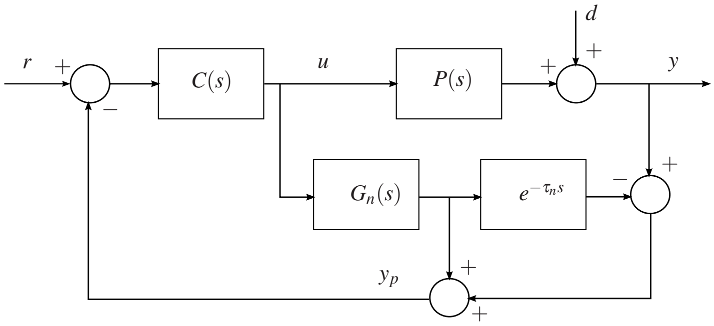

Fig. 1 Smith Predictor

2.3 Design specifications

As stated in [3], the sensitivity and complementary functions of a system are invoked to test the robust performance and robust stability conditions. From Fig. 1, the sensitivity functions for the nominal models $P_i(s)$ can be determined by obtaining the transfer function from the output disturbance d to the system output y:

$$S_i(s) = \frac{1 + C(s)H(s)}{1 + C(s)[H(s) + P_i(s)]} \tag{5}$$

where $H(s) = G_n(s) - P_0(s) = G_n(s)(1 - e^{-\tau_n s})$. The complementary sensitivity functions for the nominal models will be the transfer function from the reference input r to y (which is also equal to $1 - S_i(s)$):

$$T_i(s) = \frac{C(s)P_i(s)}{1 + C(s)[H(s) + P_i(s)]} \tag{6}$$

A standard robust control problem is to design a controller that satisfies $\|W_1 S_i\|_\infty < 1$ for a set of models where $W_1(s)$ is the performance weighting filter. If the model is described by unstructured multiplicative uncertainty, the necessary and sufficient condition for robust performance is given by [3]:

$$\| |W_1 S_i| + |W_2 T_i| \|_\infty < 1 \quad \text{for} \quad i = 1, \ldots, q \tag{7}$$

The goal of the proposed approach is to design the primary controller $C(s)$ in the SP structure to guarantee robust performance of the closed-loop system.

2.4 Proposed method

The robust performance condition (7) can be written as:

$$|W_1(j\omega)S_i(j\omega)| + |W_2(j\omega)T_i(j\omega)| < 1, \quad \forall\omega \tag{8}$$

for $i = 1,\dots,q$. Let $L_i(s,\rho)$ be defined as the open-loop transfer function of the SISO SP. From Fig. 1, the transfer function from the input of $C(s)$ to y_p will represent this open-loop transfer function:

$$L_i(s,\rho) = C(s,\rho)(H(s) + P_i(s)) \tag{9}$$

The dependency on frequency ω and s will be omitted for brevity but the dependency on the controller parameter ρ will be highlighted.

The main result of this section is given in the following theorem.

Theorem 1. *Consider the set of models $\mathbb{P}$ in (3) with multiplicative uncertainty filter $W_2(j\omega)$, then the linearly parametrized controller in (4) in the SP structure guarantees closed-loop stability and satisfy the following robust performance condition:*

$$\| |W_1 S_i| + |W_2 T_i| \|_\infty < 1 \quad \textit{for } i = 1,\dots,q \tag{10}$$

if

$$\begin{aligned} &\left[\left|W_1(j\omega)[1 + C(j\omega,\rho)H(j\omega)]\right| + \left|W_2(j\omega)C(j\omega,\rho)P_i(j\omega)\right| \right] |1 + L_d(j\omega)| \\ &\qquad - Re\{[1 + L_d^*(j\omega)][1 + L_i(j\omega,\rho)]\} < 0 \\ &\qquad\qquad \forall\omega \quad \textit{for } i = 1,\dots,q \end{aligned} \tag{11}$$

where $L_d(j\omega)$ is a strictly proper transfer function which does not encircle the critical point and $L_d^*(j\omega)$ is its complex conjugate.

Proof. Since the real part of a complex number is less than or equal to its magnitude, we have

$$Re\{[1 + L_d^*][1 + L_i(\rho)]\} \le |[1 + L_d^*][1 + L_i(\rho)]| \tag{12}$$

Then, using (11) and the fact that $|1 + L_d| = |1 + L_d^*|$, one obtains

$$\begin{aligned} &\left|W_1(1 + C(\rho)H)\right| + \left|W_2 C(\rho)P_i\right| - |1 + L_i(\rho)| < 0 \\ &\qquad \forall\omega \text{ for } i = 1,\dots,q \end{aligned} \tag{13}$$

Using $L_i(\rho) = C(\rho)(H + P_i)$ we have

$$\begin{aligned} &\frac{|W_1(1 + C(\rho)H)| + |W_2 C(\rho)P_i|}{|1 + C(\rho)(H + P_i)|} < 1 \\ &\qquad \forall\omega \text{ for } i = 1,\dots,q \end{aligned} \tag{14}$$

that leads directly to (10). To prove that all closed-loop transfer functions are stable, consider (11) which gives:

$$Re\{[1 + L_d^*(j\omega)][1 + L_i(j\omega,\rho)]\} > 0 \quad \forall\omega \tag{15}$$

or, alternatively,

$$wno\{[1+L_d^*(j\omega)][1+L_i(j\omega,\rho)]\}=0 \tag{16}$$

where wno stands for winding number around the origin. Since both $L_d^*(j\omega)$ and $L_i(j\omega,\rho)$ are constant or zero for the semi-circle with infinity radius of the Nyquist contour the wno depends only on the variation of s in the imaginary axis. Thus,

$$wno\{[1+L_d(j\omega)]\}=wno\{[1+L_i(j\omega,\rho)]\} \tag{17}$$

Since $L_d(j\omega)$ satisfies the Nyquist stability criterion $L_i(j\omega,\rho)$ will do so and all zeros of $1+L_i(j\omega,\rho)$ will be in the left-hand side of the complex plan. Since the zeros of $1+L_i(j\omega,\rho)$ are the closed-loop poles, the system will be internally stable. □

2.5 Primary controller design

The problem of minimizing the upper bound γ of the infinity norm of the weighted sensitivity function is considered. Therefore, the primary controller should be obtained from the following optimization problem:

$$\begin{aligned}&\min_{\rho}\gamma\\ &\text{Subject to:}\\ &\||W_1S_i|+|W_2T_i|\|_\infty<\gamma \quad \text{for } i=1,\dots,q\end{aligned} \tag{18}$$

This optimization can be convexified using Theorem 1 and solved by an iterative bisection algorithm. At each iteration j, γ_j is fixed and W_1 and W_2 are replaced by W_1/γ_j and W_2/γ_j. Then, a feasibility problem is solved under the convex constraints (11). If the problem is feasible, γ_{j+1} is chosen smaller than γ_j. Otherwise γ_{j+1} is increased.

Notice that the condition (11) is defined for every frequency ω leading to infinite number of constraints. In practice, a frequency grid can be used with a sufficiently large number of frequency points N (a finer grid can be used around the crossover frequency). The effect of gridding on the stability and performance of the closed loop system has been studied in [4].

Remark I: The constraint in (11) is an inner convex approximation of the non convex constraint in (10) or (8). The quality of this approximation depends on the choice of L_d. It can be shown that better approximation is achieved if L_d is chosen such that its frequency response is close to that of $L_i(\rho)$ ([8]).

Example 1

Consider the process described by (1) with multiplicative uncertainty as in (2) with

$$G_n(s)=\frac{1}{(5s+1)(10s+1)} \tag{19}$$

and

$$W_2(s) = \frac{-s^2 - 2s}{s^2 + 2s + 1} \tag{20}$$

The unknown time delay τ belongs to the set $\{4.5, 5, 5.5\}$. The nominal model used in the SP structure is chosen as $P_0(s) = G_n(s)e^{-5s}$. The performance specification is defined by the following filter:

$$W_1(s) = \frac{2}{(30s+1)^2} \tag{21}$$

A PID primary controller with $T_f = 0.01$ that minimizes $\||W_1 S_i| + |W_2 T_i|\|_\infty < \gamma$ for $i = 1, 2, 3$ should be computed.

Since the controller has an integrator, L_d is chosen as $L_d(s) = \omega_c/s$ where $\omega_c = 0.1$ rad/s which is 20% higher than open loop bandwidth. Then, the optimization problem (18) is solved considering $N = 100$ equally spaced frequency points between 10^{-3} and 10^3 rad/s. The resulting primary controller is:

$$C(s) = \frac{12.3s^2 + 3.28s + 0.2201}{0.01s^2 + s} \tag{22}$$

and leads to $\gamma = 0.313$. This controller is compared to that proposed in [9]. Kaya's controllers performs better than other controllers presented in the literature ([20], [6] and [7]). Fig. 2 depicts the performance of both controller on unitary step setpoint change considering the time-delay $\tau = 4.5s$, $\tau = 5.0s$ and $\tau = 5.5s$. As it can be seen, both controller performed well, however, the proposed controller achieves faster response.

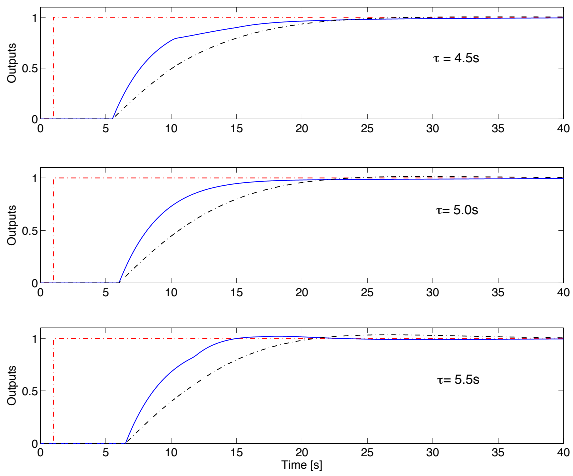

Fig. 2 Example 1: Blue solid line: proposed; black dot-dashed line: ref [9]

3 MIMO Problem Formulation

In this section, the SP for MIMO systems with generalized time delays will be investigated . An example of how to design a linearly parameterized MIMO controller for such a system will be presented at the end of this section. For notation purposes, bold face characters will represent transfer function matrices.

3.1 Class of models

Let n_o and n_i represent the number of outputs and the number of inputs of a system, respectively. The set of all LTI-MIMO strictly proper uncertain models with uncertain time delays can be defined as follows:

$$\mathscr{P} = \{\mathbf{P}_c(s)[\mathbf{I} + \Delta\mathbf{W}_2]; c = 1, \ldots, m\} \tag{23}$$

where each element in $\mathbf{P}_c(s)$ possesses a time delay that can vary over a range of specified values, and $\mathbf{W}_2$ is a matrix that represents the multiplicative input uncertainty of the system. For simplicity, one model from the set $\mathscr{P}$ will be investigated, and the subscript c will be omitted. The uncertain $n_o \times n_i$ time delayed plant has the following form:

$$\mathbf{P}(s) = \begin{bmatrix} G_{11}(s)e^{-\tau_{11}s} & \cdots & G_{1n_i}(s)e^{-\tau_{1n_i}s} \\ \vdots & \ddots & \vdots \\ G_{n_o1}(s)e^{-\tau_{n_o1}s} & \cdots & G_{n_on_i}(s)e^{-\tau_{n_on_i}s} \end{bmatrix} \tag{24}$$

where $G_{qp}(s)$ is a strictly proper delay-free transfer function, and τ_{qp} is the uncertain time-delay of the process for $p = 1, \ldots, n_i$ and $q = 1, \ldots, n_o$.

3.2 Class of controllers

As stated in [5], an $n_i \times n_o$ matrix can be formed to represent the controller $\mathbf{K}(s,\rho)$. The elements of $\mathbf{K}(s,\rho)$ will possess linearly parameterized elements $K_{pq}(s) = \rho_{pq}^T\phi_{pq}(s)$, where ρ_{pq}^T is a vector of parameters, and $\phi_{pq}(s)$ is a vector of stable transfer functions chosen from a set of orthogonal basis functions. The non-diagonal elements of $\mathbf{K}(s,\rho)$ strive to decouple the system, while the diagonal elements aim to control the single-loop subsystems. As with the SISO case, the main purpose of parameterizing the controller in this manner is due to the fact that the components of the open loop transfer function can be written as a linear function of the control parameters ρ,

$$\rho = [\rho_{11}, \ldots, \rho_{1n_i}, \ldots, \rho_{n_o1}, \ldots, \rho_{n_on_i}] \tag{25}$$

3.3 Design specifications

Fig. 3 displays the SP for the MIMO case, where $\mathbf{G}_n(s)$ is an $n_o \times n_i$ nominal delay-free transfer function matrix with elements $G_{qp}(s)$, and $\mathbf{P}_n(s)$ is an $n_o \times n_i$ nominal transfer function matrix which includes the nominal values of the time delays, which is comprised of elements $G_{qp}(s)e^{-\zeta_{qp}}$ (where ζ_{qp} represents the qp-th nominal time delay). Both $\mathbf{Y}(s)$ and $\mathbf{X}(s)$ are $n_o \times 1$ column vectors that possess elements $y_q(s)$ and $x_q(s)$, respectively. The transfer function from the inputs of $\mathbf{C}(s)$ to $\mathbf{Y}_p(s)$ will represent the open-loop transfer function,

$$\mathbf{L}(s) = [\mathbf{P}(s) + \mathbf{H}(s)]\mathbf{C}(s) \tag{26}$$

where $\mathbf{H}(s) = \mathbf{G}_n(s) - \mathbf{P}_n(s)$. Notice that if $\mathbf{P}(s) = \mathbf{P}_n(s)$, then $\mathbf{L}(s) = \mathbf{G}_n(s)\mathbf{C}(s)$. Since the class of controllers to be designed for this system are linearly parameterized, the elements of the controller $\mathbf{C}(s)$ will actually be a function of the controller parameters ρ. Therefore, $\mathbf{C}(s)$ will be represented as $\mathbf{C}(s,\rho)$.

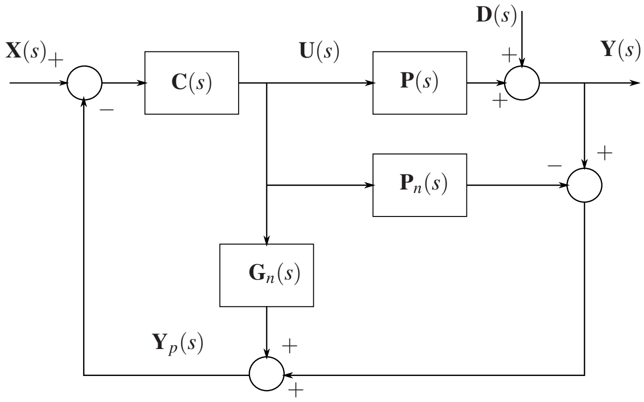

Fig. 3 MIMO representation of the Smith Predictor

The transfer function from the output disturbance $\mathbf{D}(s)$ to $\mathbf{Y}(s)$ is the output sensitivity function $\mathbf{S}(s,\rho)$, while the transfer function from $\mathbf{X}(s)$ to $\mathbf{Y}(s)$ is the complementary sensitivity function $\mathbf{T}(s,\rho)$:

$$\begin{aligned} \mathbf{S}(s,\rho) &= [\mathbf{I} + \mathbf{H}(s)\mathbf{C}(s,\rho)]\mathbf{Z}^{-1}(s,\rho) \\ \mathbf{T}(s,\rho) &= \mathbf{P}(s)\mathbf{C}(s,\rho)\mathbf{Z}^{-1}(s,\rho) \end{aligned} \tag{27}$$

where $\mathbf{Z}(s,\rho) = [\mathbf{I} + \mathbf{L}(s,\rho)]$. As with the SISO case, the goal here is to determine the controller $\mathbf{C}(s,\rho)$ that will guarantee the robust performance and robust stability of the closed-loop SP system.

3.4 Proposed method

Suppose that $\mathbf{S}(s,\rho)$ and $\mathbf{T}(s,\rho)$ are diagonal transfer matrices (the closed-loop system is fully decoupled). Then the MIMO sensitivity and complementary functions can essentially be treated as functions containing independent SISO subsystems. Let $\mathbf{W}_1(s)$ be a diagonal filter with diagonal elements W_{1_q} and a diagonal filter $\mathbf{W}_2(s)$ with diagonal elements W_{2_q} representing, respectively, the nominal performance and multiplicative uncertainty for the SISO subsystem. Therefore, the robust criterion that was proved for the SISO case in section (2.4) will be satisfied for each SISO subsystem of the decoupled MIMO system. Thus it is judicious to express the robust criterion for the decoupled system as follows:

$$\begin{aligned} &\| |W_{1_q} S_{qq}| + |W_{2_q} T_{qq}| \|_\infty < 1 \\ &\text{for } q = 1, \ldots, n_o \end{aligned} \tag{28}$$

where S_{qq} and T_{qq} are the q-th diagonal elements of $\mathbf{S}(s,\rho)$ and $\mathbf{T}(s,\rho)$, respectively.

The objective is to effectuate decoupling while simultaneously optimize the diagonal elements to achieve the desired single-loop performance. The proposed method will be to define a diagonal open-loop transfer function matrix $\mathbf{L}_D(s)$, where the diagonal elements satisfy the desired single open-loop response. Therefore, by minimizing the objective function $\|\mathbf{L}(s,\rho) - \mathbf{L}_D(s)\|_2^2$, a controller can be designed to simultaneously minimize the magnitudes of the off-diagonal elements of $\mathbf{L}(s,\rho)$ and drive the diagonal elements to be approximately equal to $L_{D_q}(s)$ (where $L_{D_q}(s)$ is the q-th diagonal element in $\mathbf{L}_D(s)$).

However, the resulting controller will stabilize the closed-loop system only if it is fully decoupled. In practice, with a finite order controller, it is not always possible to make the off-diagonal elements of $\mathbf{L}(j\omega,\rho)$ equal to zero. In this case, the generalized Nyquist stability criterion should be used to guarantee the stability of the MIMO system. According to this theorem, the eigenvalues of the open-loop transfer function (26) should not encircle the critical point. However, these eigenvalues are non-convex functions of the linear control parameters, which complicates the design process. A possible solution to this problem is to implement the Gershgorin band theorem in order to approximate the eigenvalues of $\mathbf{L}(j\omega,\rho)$. The Gershgorin bands represented by disks centered at the diagonal elements of a matrix that include the eigenvalues. For the open-loop transfer matrix $\mathbf{L}(j\omega,\rho)$, the radius of these disks are computed by:

$$r_q(\omega_k,\rho) = \sum_{p=1, p\neq q}^{n_o} |L_{qp}(j\omega_k,\rho)| \tag{29}$$

which is convex with respect to the control parameter ρ. Therefore, the closed-loop stability of the MIMO system is guaranteed if these disks do not encircle the critical point. This condition can be approximated with a convex constraint as it is shown in [5].

3.5 Primary controller design

In designing the controller $\mathbf{C}(s,\rho)$ for the MIMO SP, one must consider all of the possible combinations of the uncertain delay parameters τ_{qp}. Suppose that the cardinality of τ_{qp} is β_{qp}. Then the total number of possible combinations that must be considered in the design of the controller is,

$$m = \prod \beta_{qp}$$
$$\forall\ q = 1,\ldots,n_o;\ p = 1,\ldots,n_i \tag{30}$$

Therefore, one can define the following optimization problem for the multimodel system:

$$\min_{\rho} \sum_{c=1}^{m} \sum_{k=1}^{N} \|\mathbf{L}_c(j\omega_k,\rho) - \mathbf{L}_{D_c}(j\omega_k)\|_F$$

Subject to:

$$|r_{q_c}(j\omega_k,\rho)[1+L_{D_{q_c}}(j\omega_k)]| - R_e\{[1+L^*_{D_{q_c}}(j\omega_k)][1+L_{qq_c}(j\omega_k,\rho)]\} < 0$$
$$\{|W_{1_{q_c}}(j\omega_k)[1+M_{qq_c}(j\omega_k,\rho)]| + |W_{2_{q_c}}(j\omega_k)N_{qq_c}(j\omega_k,\rho)|\}|1+L_{D_{q_c}}(j\omega_k)|$$
$$-R_e\{[1+L^*_{D_{q_c}}(j\omega_k)][1+L_{qq_c}(j\omega_k,\rho)]\} < 0$$
$$\text{for } k = 1,\ldots,N;\ q = 1,\ldots,n_o;\ c = 1,\ldots,m \tag{31}$$

where

$$M_{qq_c}(j\omega_k,\rho) = \sum_{z=1}^{n_o} G_{qz_c}(j\omega_k)(1-e^{-j\omega_k\zeta_{qzc}})C_{zq_c}(j\omega_k,\rho)$$
$$N_{qq_c}(j\omega_k,\rho) = \sum_{z=1}^{n_o} P_{qz_c}(j\omega_k)C_{zq_c}(j\omega_k,\rho)$$
$$L_{qp_c}(j\omega_k,\rho) = \sum_{z=1}^{n_i} G_{qz_c}(j\omega_k)(1+e^{-j\omega_k\tau_{qzc}}-e^{-j\omega_k\zeta_{qzc}})C_{zp_c}(j\omega_k,\rho)$$

and $\|\cdot\|_F$ is the Frobenius norm of a matrix. Note that the first inequality shows that the Gershgorin bands do not encircle the critical point and so the MIMO system remains stable even if it is not fully decoupled. The second inequality guarantees the robust performance for the SISO subsystems of the decoupled MIMO system. Note also that $M_{qq_c}(j\omega_k,\rho)$ represents the diagonal elements of $\mathbf{H}_c(j\omega_k)\mathbf{C}_c(j\omega_k,\rho)$ and $N_{qq_c}(j\omega_k,\rho)$ represents the diagonal elements of $\mathbf{P}_c(j\omega_k)\mathbf{C}_c(j\omega_k,\rho)$. The objective function in (31) is convex with respect to the controller parameters ρ.

Example 2

The proposed optimization problem will now be applied to an uncertain time delayed MIMO system. Consider a 2×2 plant model (i.e., $x_q(s)$ and $y_q(s)$ for $q = 1,2$), similar to the system defined in [5], with uncertain time delays as:

$$\mathbf{P}(s) = \begin{bmatrix} \dfrac{10e^{-\tau_{11}s}}{8s+1} & \dfrac{5e^{-\tau_{12}s}}{30s+1} \\ \dfrac{-8e^{-\tau_{21}s}}{40s+1} & \dfrac{2e^{-\tau_{22}s}}{10s+1} \end{bmatrix} \tag{32}$$

where the time delays τ_{qp} possess values in the following sets:

$$\begin{aligned} \tau_{11} &= \{3,9\} & \tau_{12} &= \{7,13\} \\ \tau_{21} &= \{9,15\} & \tau_{22} &= \{5,11\} \end{aligned} \tag{33}$$

The nominal model with time delays is defined as:

$$\mathbf{P}_n(s) = \begin{bmatrix} G_{11}(s)e^{-6s} & G_{12}(s)e^{-10s} \\ G_{21}(s)e^{-12s} & G_{22}(s)e^{-8s} \end{bmatrix} = \begin{bmatrix} \dfrac{10e^{-6s}}{8s+1} & \dfrac{5e^{-10s}}{30s+1} \\ \dfrac{-8e^{-12s}}{40s+1} & \dfrac{2e^{-8s}}{10s+1} \end{bmatrix} \tag{34}$$

where the time scale is defined in minutes. The elements $G_{qp}(s)$ for $q = 1,2$ and $p = 1,2$ represent the strictly proper delay-free transfer functions in $\mathbf{G}_n(s)$. The relative gain array (RGA) analysis of this system shows that this process is not diagonally dominant.

The performance and uncertainty filters chosen for this example will be identical to those in [5],

$$W_{1_q} = 0.5 \qquad W_{2_q} = 0.5\left(\frac{2s+1}{s+1}\right) \qquad q = 1,2 \tag{35}$$

The desired diagonal open-loop transfer function $\mathbf{L}_D(s)$ will be chosen as simple integrators with time constants equal to 7 minutes (i.e, $\mathbf{L}_D(s) = diag\left(\frac{1}{7s}\right)$). For simplicity, a PI controller will be designed for this process. Since $\beta_{11} = \beta_{12} = \beta_{21} = \beta_{22} = 2$, there will be a total of $m = 16$ possible cases to consider in the design process. The optimization problem in (31) can now be solved by repeating the stability constraints for each combination of the uncertainties in (33). The frequency grid will be chosen to be between 10^{-2} and 10^{1} rad/min with $N = 150$ equally spaced points. The PI MIMO controller obtained from the optimization problem is as follows:

$$\mathbf{C}(s) = \begin{bmatrix} \dfrac{0.06234s + 0.001464}{s} & \dfrac{-0.04803s - 0.005408}{s} \\ \dfrac{0.1585s + 0.0168}{s} & \dfrac{0.3113s + 0.005995}{s} \end{bmatrix} \tag{36}$$

Fig. 4 displays the closed-loop MIMO response to a step input. Notice that with this controller, the MIMO process achieves robust performance while simultaneously decoupling the system. The Gershgorin bands are depicted in Fig. 5 for the system possessing the largest delay time uncertainty ($\tau_{11} = 9$, $\tau_{12} = 13$, $\tau_{21} = 15$, $\tau_{22} = 11$). The red and blue bands possess a radius of $|r_q(j\omega_k)|$ for $q = 1,2$ and $k = 1,\dots,N$.

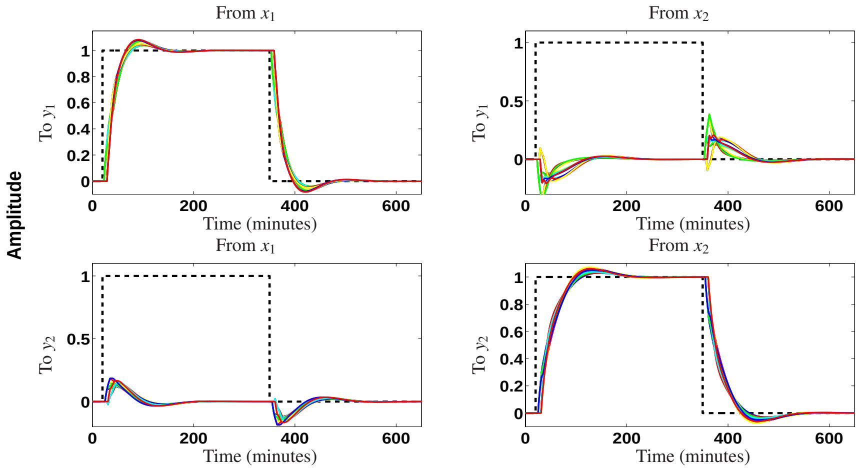

Fig. 4 MIMO response to a unit step input: reference signal (black,dash), the remaining $\Omega = 16$ closed-loop responses are for all possible combinations of the time delay parameters in (33).

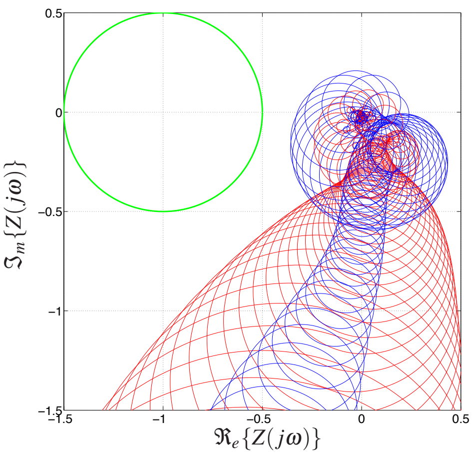

Fig. 5 Gershgorin bands for L_{qq} with the largest time delay combination in (33): performance filter with $|W_{1_q}| = 0.5$ (green circle), Gershgorin bands corresponding to $q = 1$ (blue circles), Gershgorin bands corresponding to $q = 2$ (red circles). Note that $Z(j\omega)$ is simply the complex number representation of each circle in the plot.

Notice how the Gershgorin bands never intersect with the performance circle centered at $(-1+j0)$. This proves that the MIMO system is stable, robust, and satisfies the optimization criterion in (31).

4 Extension to Unstable SISO Systems

The SP in the scheme shown in Fig. 1 cannot be used for unstable plants since the controller will contain zeros in right-hand side of the s-plan which cancel the unstable poles in the plant and leads to instability. To avoid this unstable zero-pole cancellation, the control structure shown in Fig.1 should be changed. Several alternatives are available in literature to cope with unstable processes with dead-time (see, for example, [2, 12, 11, 16, 17]).

Consider, for instance, the SP with modified dead-time free model depicted in Fig. 6 which is discussed in [16]. In this case, the dead-time free model is defined

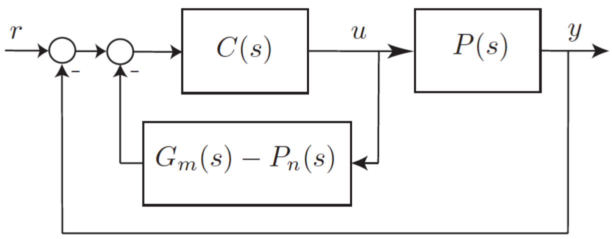

Fig. 6 Smith Predictor with modified dead-time free model

as $G_m = \frac{N_m}{D_n}$ and $H = G_m - P_0 = (N_m - N_n e^{-\tau s})\frac{1}{D_n}$. Therefore, N_m must be tuned such that the zeros of $N_m - N_n e^{-\tau s}$ cancel the unstable poles in D_n. Once N_m has been properly designed, the primary controller can be obtained by solving the optimization problem in (18) redefining $H = G_m - P_0$ and $L_i(\rho) = (G_m - P_0 + P_i)C(\rho)$. Here, care should be taken in the choice of L_d. As it has been shown, the *wno* of $1 + L_i$ equals the *wno* of $1 + L_d$. Therefore, L_d should be chosen such that the number of encirclement of the critical point $(-1+0j)$ by its Nyquist plot is equal to the number of unstable poles in P_i.

Example 3

Consider the model studied in [13] given by:

$$P(s) = \frac{k}{s-a}[1 + \Delta(s)W_2(s)]e^{-\tau s} \tag{37}$$

where $k = 1$, $a = 1$, $\tau = \tau_n \pm 0.02$ and $\tau_n = 0.2$. The interval of variation of τ is gridded using $q = 3$ equally spaced points. A finer grid just increases the number of constraints and for this example does not change significantly the final controller. The performance and uncertainty filters are respectively chosen as:

$$W_1(s) = 2\left(\frac{s+1}{10s+1}\right) \quad \text{and} \quad W_2(s) = 0.2\left(\frac{s+1.1}{s+1}\right) \tag{38}$$

Here, we use the SP with modified dead-time free model (Fig. 6) due to its simplicity. The dead-time free model $G_m(s)$ is chosen as

$$G_m(s) = \frac{T_m s + 1}{s-1}. \tag{39}$$

T_m is computed in order to obtain $H(s) = G_m(s) - P_0(s)$ without a pole in $s = 1$. Since

$$H(s) = \frac{1}{s-1}[T_m s + 1 - e^{-0.2s}], \tag{40}$$

if $T_m = e^{-0.2} - 1$, then $s = 1$ is a zero of $H(s)$.

A PI as the primary controller is designed. The first step is to choose the transfer function $L_d(s)$, which must encircle the critical point in the Nyquist diagram once and must contain one integrator. Therefore, it is chosen as

$$L_d(s) = 10\frac{s+1}{s(s-1)}. \tag{41}$$

Optimization problem (18) is solved considering $N = 100$ equally spaced frequency points between $\omega = 10^{-3}$rad/s and $\omega = 10^3$rad/s and the following controller is obtained:

$$C_0(s) = (3.582s + 0.5838)/s \tag{42}$$

which yields $\gamma = 0.6854$. This result can be further improved by using a new $L_d(s)$ based on $C_0(s)$ in the optimization problem. With this new $L_d(s) = G_m(s)C_0(s)$ the optimal primary controller is:

$$C(s) = (2.994s + 0.4612)/s \tag{43}$$

and $\gamma = 0.6074$. Figure 7 depicts the function

$$\Gamma_i(j\omega) = |W_1(j\omega)S_i(j\omega)| + |W_2(j\omega)T_i(j\omega)|$$

where S_i and T_i are respectively given by (5) and (6) with $H = G_m - P_0$ and P_i is obtained by gridding of τ. Note that the maximum value of the function is 0.6072, which occurs when $\tau = \tau_n + 0.02 = 0.22$, is close to the bound γ. It is worth to point out that, although the conditions given in Theorem 1 are only sufficient to guarantee $\|\Gamma_i\|_\infty < \gamma$, with a proper choice of L_d it is possible to obtain a solution with very low conservatism. Furthermore, the resulting controller is a standard PI which can be implemented in a straightforward manner and has great practical significance.

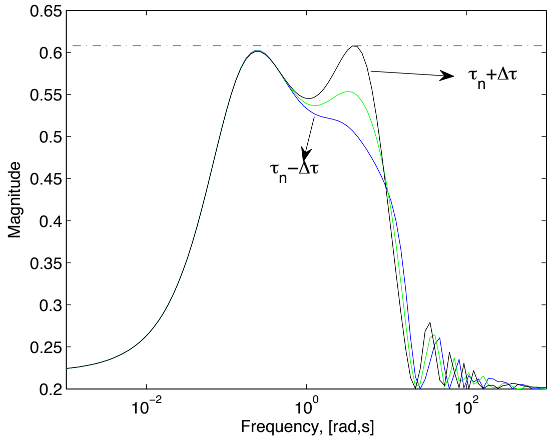

Fig. 7 Example 2: Blue solid line: Γ for $\tau = 0.18$; green solid line: Γ for $\tau = 0.2$; black solid line: Blue solid line: Γ for $\tau = 0.22$; red dot-dashed line: γ.

For the same example, controller designed in [13] leads to the optimal $\gamma = 0.9407$ which is 55% higher than the value obtained with the proposed method. It should be mentioned that the true robust performance criterion in (18) is not minimized in [13]. Instead, the maximum singular value of $[W_1(j\omega)S(j\omega) \quad W_2(j\omega)T(j\omega)]$ for all ω is minimized by the H_∞ control theory.

5 Gain-scheduled Controller Design

Consider an uncertain plant $P(s,\theta)$ belonging to the set:

$$\mathsf{P}_\theta = \{G(s,\theta)e^{-\tau_i(\theta)s},\ i = 1,\ldots,q\} \tag{44}$$

where the dead-time free part of the model has unstructured multiplicative uncertainty and is described as:

$$G(s,\theta) = G_n(s,\theta)[1 + \Delta(s)W_2(s)] \tag{45}$$

and θ is a vector of scheduling parameters that belongs to a finite set $\Theta = \{\theta_1, \theta_2, \ldots, \theta_m\}$ (corresponding e.g. to the different operating point parameters). It is assumed that the operating point does not frequently change (the stability and performance are achieved for the frozen scheduling parameter). The dead-time is also a function of the scheduling parameter and uncertain, so for a given value of θ it belongs to the set $\{\tau_1(\theta), \tau_2(\theta), \ldots, \tau_q(\theta)\}$.

We will consider the SP shown in Fig. 8 where both, the nominal model $P_0(s,\theta) = G_n(s,\theta)e^{-\tau_n(\theta)s}$ and the primary controller $C(s,\theta)$ are functions of the scheduling parameter vector θ. The goal is to compute a primary gain-scheduled controller for this scheme that meets the H_∞ robust performance specification.

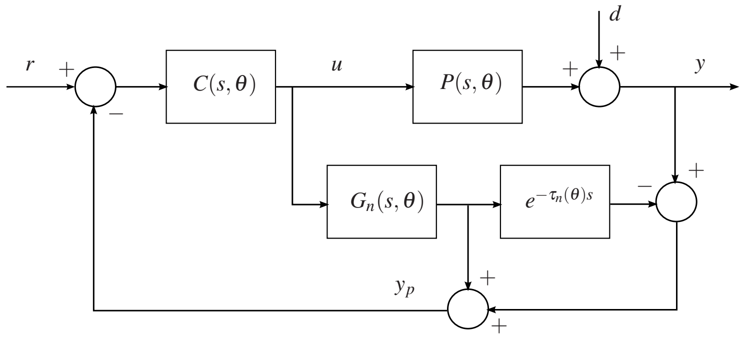

Fig. 8 Gain-Scheduled Smith Predictor

The primary controller $C(s,\theta)$ is linearly parametrized by: $C(s,\theta) = \rho^T(\theta)\phi(s)$, where the basis function vector $\phi(s)$ is defined as in section (2.2) and $\rho^T(\theta)$ is given by

$$\rho^T(\theta) = [\rho_1(\theta), \rho_2(\theta), \dots, \rho_n(\theta)] \tag{46}$$

Every gain is a polynomial function of order δ of the scheduling parameters and is defined as

$$\rho_i(\theta) = (\nu_{i,\delta})^T \theta^\delta + \ldots + (\nu_{i,1})^T \theta + \nu_{i,0} \tag{47}$$

and θ^k denotes element-by-element power of k of vector θ. In Fig. (8), the transfer function from the output disturbance d to y is the sensitivity function $S_i(s,\theta)$, while the transfer function from r to y is the complementary function $T_i(s,\theta)$:

$$\begin{aligned} S_i(s,\theta) &= \frac{1 + C(s,\theta)H(s,\theta)}{1 + C(s,\theta)(H(s,\theta) + P_i(s,\theta))} \\ T_i(s,\theta) &= \frac{C(s,\theta)P_i(s,\theta)}{1 + C(s,\theta)(H(s,\theta) + P_i(s,\theta))}, \ \forall\theta \in \Theta \end{aligned} \tag{48}$$

where $H(s,\theta) = G_n(s,\theta) - P_0(s,\theta)$. The primary controller is obtained from the following optimization problem:

$$\begin{aligned} &\min_{\rho} \gamma \\ &\text{Subject to:} \\ &\| |W_1 S_i(s,\theta)| + |W_2 T_i(s,\theta)| \|_\infty < \gamma \\ &\text{for } i = 1, \dots, q, \quad \forall\theta \in \Theta \end{aligned} \tag{49}$$

Optimization problem (49) is again solved using an iterative bisection algorithm as previously presented. At each iteration, a feasibility problem is solved with the following convex constraints:

$$\Big[|W_1(j\omega_k)[1+C(j\omega_k,\theta_l)H(j\omega_k,\theta_l)|+|W_2(j\omega_k)C(j\omega_k,\theta_l)P(j\omega_k,\theta_l)|\Big]$$
$$\times|1+L_d(j\omega_k)|-Re\{[1+L_d^*(j\omega_k)][1+L_i(j\omega_k,\theta_l)]\}<0$$
$$\text{for } k=1,\dots,N, \quad i=1,\dots,q, \quad l=1,\dots,m \quad (50)$$

Example 4

The design method is applied on a simulated system having a resonance whose frequency changes as a function of a scheduling parameter θ. Consider the following plant model

$$P(s,\theta)=G(s,\theta)e^{-\tau s} \quad (51)$$

where $G(s,\theta)=G_n(s,\theta)[1+\Delta(s)W_2(s)]$ and

$$G_n(s,\theta)=\frac{(2+0.2\theta)^2}{s^2+0.2(2+0.2\theta)s+(2+0.2\theta)^2} \quad (52)$$

$$W_2(s)=0.8\frac{1.1337s^2+6.8857s+9}{(s+1)(s+10)} \quad (53)$$

and $\theta\in[-1,-0.5,0,0.5,1]$. Consider also that the dead-time is within the interval $\tau\in[2.7,3.0,3.3]$ but its exact value is unknown in runtime. The objective is to design a primary gain-scheduling PID controller for the Smith Predictor structure considering the performance filter $W_1(s)=\frac{2}{(20s+1)^2}$. The parameters ρ of the primary controller will be affine functions of the scheduling parameter θ. The filter of the derivative action is chosen to have a time constant of $T_f=0.01$s.

Finally, optimization problem (49) is solved considering $L_d=1/s$ and $N=100$ equally spaced frequency points between 10^{-2} and 10^2 rad/s. The resulting gain-scheduled controller is given by: $K_p(\theta)=-0.0168\theta+0.2152$, $K_i(\theta)=0.0144\theta+2.4736$, $K_d(\theta)=-0.1224\theta+0.6424$.

This controller leads to:

$$\begin{aligned}&\||W_1S_i(s,\theta_l)|+|W_2T_i(s,\theta_l)|\|_\infty<\gamma=0.8928\\&l=1,\dots,5, \quad i=1,2,3\end{aligned} \quad (54)$$

The gain-scheduled controller is evaluated considering $\theta=-1,0,1$ and $\tau=3.3$s. The performance is compared to a fixed-gain PID designed for the nominal case ($\theta=0$ and $\tau=3$s). Figure 9 shows the step response of the gain-scheduled controller in all conditions (blue, red and green solid lines) compared with the fixed PID controller (black dashed line, highly oscillating).

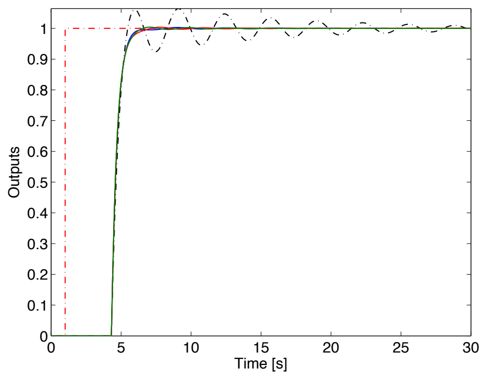

Fig. 9 Example 3: Blue, red and green solid line: gain-scheduled PID Smith Predictor and G_2 using $\theta_1 = -1$, $\theta_3 = 0$ and $\theta_5 = 1$ respectively; black dashed line: fixed PID Smith Predictor using $\theta = -1$.

6 Conclusions

This paper presents a new method to design a robust Smith Predictor for uncertain SISO and MIMO time-delay systems using convex optimization techniques. The proposed approached allows one to design PI/PID as well as higher order primary controllers in the Smith Predictor structure which provide robust H_∞ performance for systems with uncertain dead-time and multiplicative or multimodel uncertainty in the dead-time free model of the system. The method is based on a convex approximation of the H_∞ robust performance criterion in the Nyquist diagram. This approximation relies on the choice of a desired open-loop transfer function L_d for the dead-time free model of the plant. For the SISO case, a bisection algorithm was implemented to solve the convex constrained problem. For the MIMO case, a controller was designed such that the system became decoupled and simultaneously optimized the single-loop performances of the SISO subsystems.

References

1. C. B. Brosilow. The structure and design of Smith predictors from the viewpoint of inferential control. In Proceedings of Joint American Control Conference, Denver, Colorado, 1979.
2. A. M. De Paor. A modified Smith predictor and controlled for unstable processes with time delay. International Journal of Control, 41(4):1025–1036, 1985.
3. C. J. Doyle, B. A. Francis, and A. R. Tannenbaum. Feedback Control Theory. Mc Millan, New York, 1992.
4. G. Galdos, A. Karimi, and R. Longchamp. Robust controller design by convex optimization based on finite frequency samples of spectral models. In 49th IEEE Conference on Decision and Control, Atlanta, USA, 2010a.

5. G. Galdos, A. Karimi, and R. Longchamp. H_∞ controller design for spectral MIMO models by convex optimization. Journal of Process Control, 20(10):1175–1182, 2010b.
6. T. Hagglund. A predictive PI controller for processes with long dead times. IEEE Control Systems Magazine, 12 (1):57–60, 1992.
7. C. C. Hang, Q. Wang, and L. S. Cao. Self-tuning Smith predictors for processes with long dead time. International journal of adaptive control and signal processing, 9(3):255–270, 1995.
8. A. Karimi and G. Galdos. Fixed-order H_∞ controller design for nonparametric models by convex optimization. Automatica, 46(8):1388–1394, 2010.
9. I. Kaya. Tuning Smith predictors using simple formulas derived from optimal responses. Industrial and engineering chemistry research, 40(12):2654–2659, 2001.
10. D. Lee, M. Lee, S. Sung, and I. Lee. Robust PID tuning for Smith predictor in the presence of model uncertainty. Journal of Process Control, 9(1):79–85, 1999.
11. T. Liu, Y. Z. Cai, D. Y. Gu, and W. D. Zhang. New modified Smith predictor scheme for integrating and unstable processes with time delay. IEE Proceedings- Control Theory and Applications, 152(2):238–246, 2005.
12. S. Majhi and D. P. Atherton. A new Smith predictor and controller for unstable and integrating processes with time delay. In Proceedings of the 37th IEEE Conference on Decision and Control, pages 1341–1345, 1998.
13. G. Meinsma and H. Zwart. On H_∞ control for dead-time systems. IEEE Transactions on Automatic Control, 45 (2):272–285, 2000.
14. L. Mirkin. On the extraction of dead-time controllers and estimators from delay-free parametrizations. IEEE Transactions on Automatic Control, 48(4):543–553, 2003.
15. J. E. Normey-Rico and E. F. Camacho. Multivariable generalised predictive controller based on the Smith predictor. IEEE Proceedings in Control Theory Applications, Vol. 147, No. 5, 2000.
16. J. E. Normey-Rico and E. F. Camacho. Control of dead-time processes. Springer Verlag, 2007.
17. J. E. Normey-Rico and E. F. Camacho. Unified approach for robust dead-time compensator design. Journal of Process Control, 19(1):38–47, 2009.
18. Z. J. Palmor. Stability properties of Smith dead-time compensator controllers. International Journal of Control, 32:937–949, 1980.
19. Z. J. Palmor. Time-delay compensation Smith predictor and its modifications. In W. Levine, editor, The Control Handbook. CRC Press, Boca Raton, FL, 1996.
20. Z. J. Palmor and M. Blau. An auto-tuner for Smith dead time compensator. International Journal of Control, 60 (1):117–135, 1994.
21. R. S. Sanchez-Pena, Y. Bolea and V. Puig. MIMO Smith predictor: Global and structured robust performance analysis. Journal of Process Control 19:163-177, 2009.
22. C. Santacesaria and R. Scattolini. Easy tuning of Smith predictor in presence of delay uncertainty. Automatica, 29(6):1595–1597, 1993.
23. S. Skogestad and I. Postlethwaite. Multivariable Feedback Control; Analysis and Design. John Wiley and Sons, England, 1996.
24. O. J. M. Smith. Closer control of loops with dead time. Chemical Engineering Progress, 53(5):217–219, 1957.
25. W. Zhang and C. Lin. Multivariable Smith Predictors Design for Nonsquare Plants. IEEE Transactions on Control Systems Technology, Vol. 14, No. 6, 2006.
26. Q. C. Zhong. On standard H_∞ control of processes with a single delay. IEEE Transactions on Automatic Control, 48(6):1097–1103, 2003.
27. K. Zhou and J. Doyle. Essentials of Robust Control. Prentice-Hall, Upper Saddle River, New Jersey, USA, 1998.