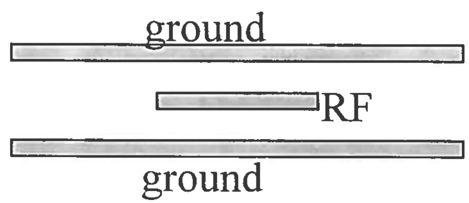
INT 206/02 September 2002

Power transfer efficiency to a RF plasma reactor via a transmission line (balanced, shielded stripline) of length comparable to the RF wavelength

A.A. Howling

Power transfer efficiency to a RF plasma reactor

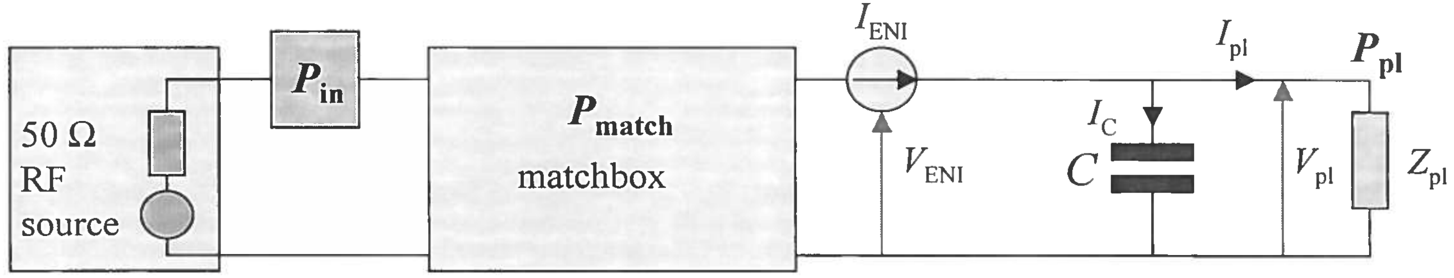
via a transmission line
(balanced, shielded stripline)



of length comparable
to the RF wavelength

Subtractive Method: Small reactor approximation

For a small reactor, the RF connection is short, so the line & reactor impedance is dominated by the capacitance of a short length of unterminated cable and a guard screen. This is the case for PADEX. The stripline and reactor are replaced by a simple capacitance C and the equivalent circuit becomes :



V_{pl},

Since this capacitance is much bigger than the plasma capacitance, for small area plasma sheaths, we have:
$I_C >> I_{pl}$ and so $I_{ENI} \approx I_C$ and also $I_{ENI} \approx V_{pl}\omega C$. The plasma is almost "short - circuited" by the vacuum circuit. Therefore, if we adjust the input power, in vacuum, to obtain the same value of V_{pl} or I_{ENI} as with plasma, then we can say that the currents in all the vacuum circuit are approximately the same as with plasma because the terminating impedance has not changed much.

By subtraction, the plasma power is therefore $P_{pl} = \left[P_{in}^{pl} - P_{in}^{vac}\right]_{\text{same } V_{pl} \text{ or same } I_{ENI}}$ (with no re-tuning!)

The above is the approach described also by Lieberman p383. See our PISA conference and JVST 1991 papers.

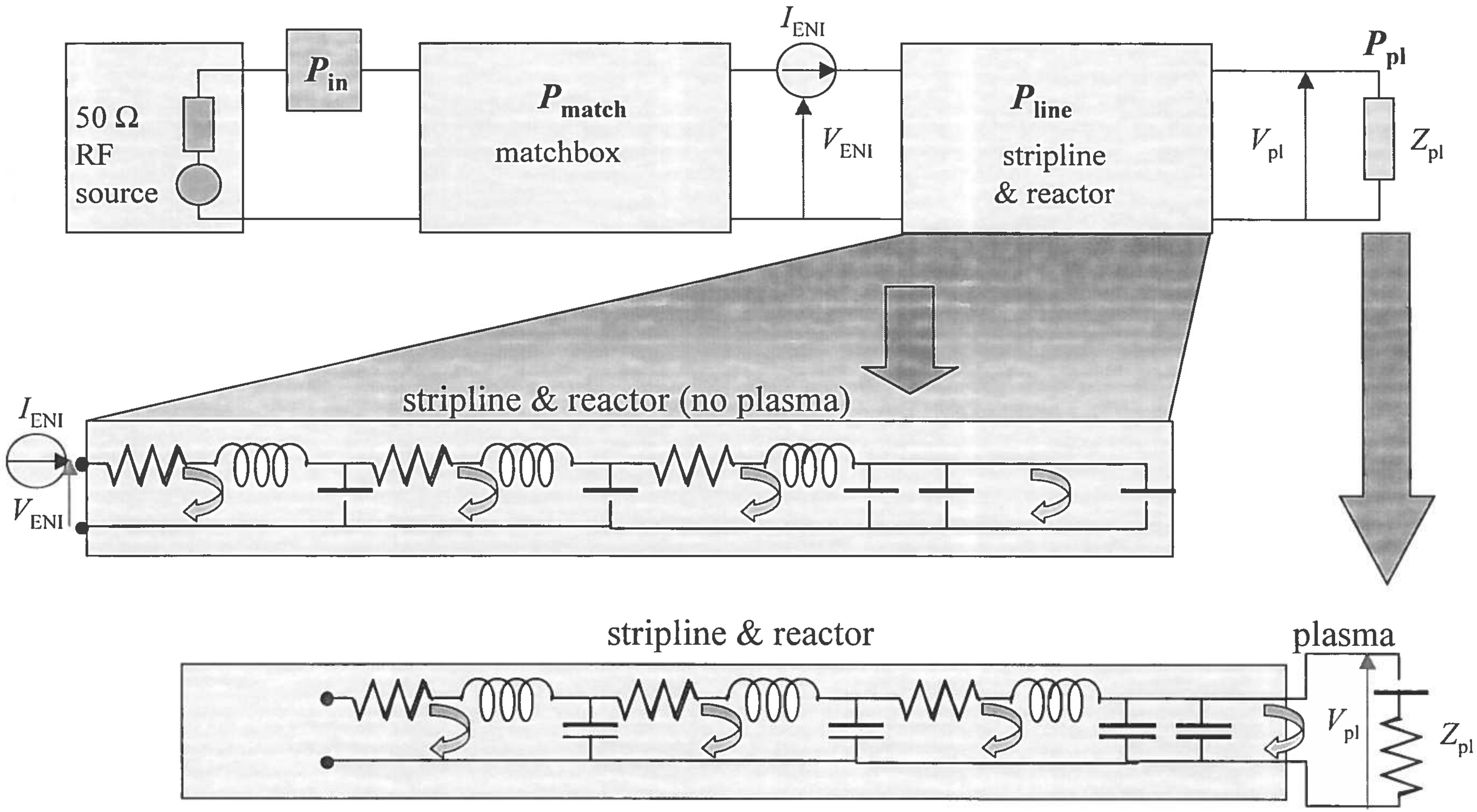
Subtractive Method Problems for Large Reactors:

Two complications arise:

i) The stripline-and-reactor circuit becomes a complicated 4-pole passive network with several current paths;

ii) The large-area plasma impedance is no longer large compared to the vacuum impedance.

Also, the ENI probe is physically and electrically separated from the plasma and voltage probe.



NOW, the plasma strongly changes the terminating impedance and alters the current distribution and power losses. The same electrode voltage does **NOT** reproduce the power loss in the vacuum circuit when the plasma is off.

The difference between networks and transmission line approaches:
(Link between T-section approach and line theory.)

Gardiol "Lossy Transmission Lines p157:
"For discrete circuit elements, the basic components exhibit frequency dependences such as ω^{-1}, ω^{0}, and ω^{+1}. The assembly of compnonents can be described by ratios of polynomials.
In transmission lines, however, the frequency dependence is always of a transcendental nature."

See Hannes' CTI: the transmission line is 90 cm long, and a free space quarter wavelength at 100 MHz is only 75 cm (52 cm in PTFE)

=> a transmission line treatment of transmitted power is better than a T-section equivalent circuit model.

Basic Revision of e/m Power Transfer.

Start by taking dot products of Maxwell's equations:

$$\vec{H} \bullet \left\{ \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \right\} \text{ ...subtract... } \vec{E} \bullet \left\{ \nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t} \right\} \text{ gives:}$$

$$\nabla \bullet \left(\vec{E} \times \vec{H} \right) = -\vec{H} \bullet \frac{\partial \vec{B}}{\partial t} - \vec{E} \bullet \frac{\partial \vec{D}}{\partial t} - \vec{E} \bullet \vec{j},$$ where $\vec{E} \times \vec{H}$ is Poynting's vector.

$$\nabla \bullet \left(\vec{E} \times \vec{H} \right) = -\frac{\partial}{\partial t}\left(\frac{\mu H^2}{2} \right) - \frac{\partial}{\partial t}\left(\frac{\varepsilon E^2}{2} \right) - \vec{E} \bullet \vec{j}.$$ Using the divergence theorem for surface area A enclosing volume V:

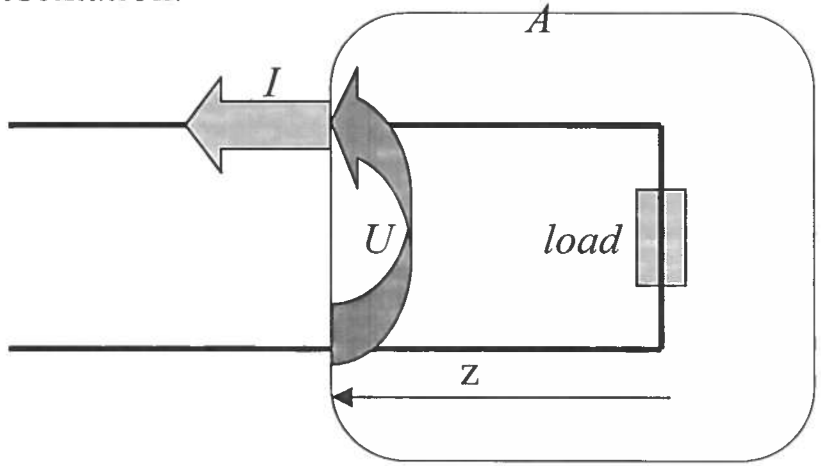
$$\oint_A \left(\vec{E} \times \vec{H} \right) \bullet d\vec{A} = -\frac{\partial}{\partial t} \int_V \left(W_m + W_e \right) dV - \int_V \left(\vec{E} \bullet \vec{j} \right) dV,$$ where $W_m + W_e$ are the magnetic and electric energy densities.

i.e. Outward flux of e/m power = (rate of decrease of magnetic + electric energy) - ohmic dissipation in the volume

Furthermore, it can be proven that the power transferred (delivered) along a transmission line is:

$$\boxed{Power(z,t)[\mathrm{W}] = \int_A \left(\vec{E} \times \vec{H} \right) \bullet d\vec{A} = U(z,t) I(z,t)}$$ (see Fred E. Gardiol "Lossy Transmission Lines" p59 - 61)

Schematic representation:



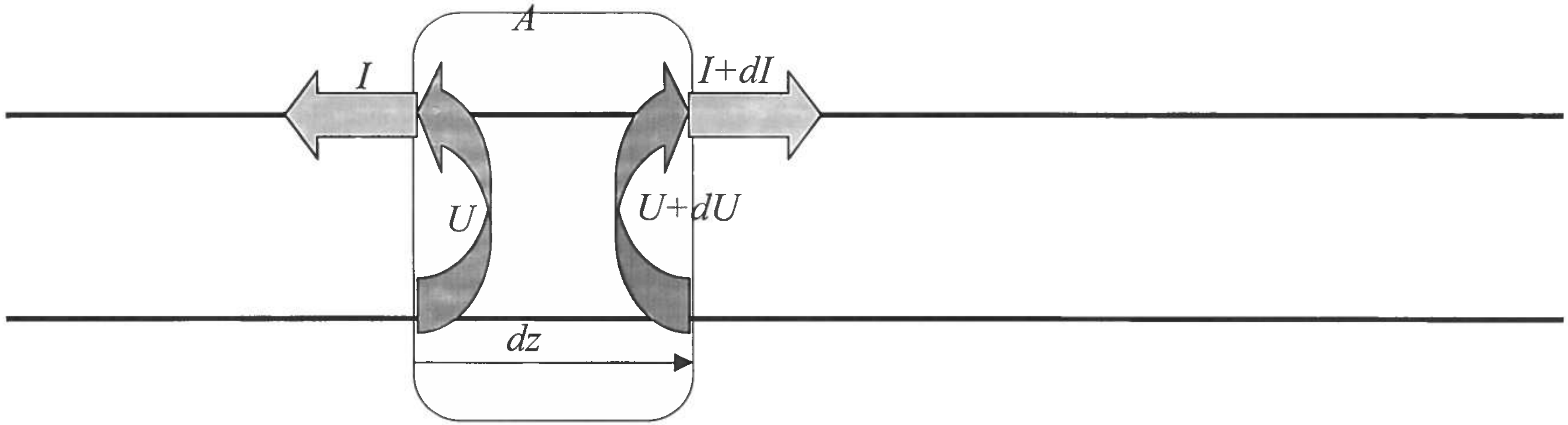
The net power transmitted from the surface A is

$$Power(z,t) = \underbrace{\underbrace{-\frac{\partial}{\partial t}\int_V (W_m + W_e)dV}_{\text{rate of change of electric and magnetic energy}}}_{\text{REACTIVE POWER}} - \underbrace{\underbrace{\int_V (\vec{j} \bullet \vec{E})dV}_{\text{rate of change of ohmic power}}}_{\text{REAL POWER}} = U(z,t)I(z,t).$$

The reactive power is purely oscillatory (no dc level), <u>provided no transients in amplitudes!</u>

$\therefore$ The cycle-averaged power $\overline{Power}(z) = \overline{U(z,t)I(z,t)}$ [W] = the time average of the ohmic dissipation.
i.e. the net power is purely ohmic dissipation within surface A (in the line and the load).

Consider an infinitesimal section of line:



The variation of power per unit length on the line is $\frac{\partial Power(z,t)}{\partial z} = \frac{\partial (UI)}{\partial z} = U\frac{\partial I}{\partial z} + I\frac{\partial U}{\partial z}$.

For a <u>lossless</u> line, using the infinitesimal equivalent circuit, substitute : $\frac{\partial U}{\partial z} = -L'\frac{\partial I}{\partial t}$ and $\frac{\partial I}{\partial z} = -C'\frac{\partial U}{\partial t}$

where L' and C' are respectively the inductance and capacitance per unit length.

$\therefore \frac{\partial Power(z,t)}{\partial z} = -\frac{1}{2}\frac{\partial}{\partial t}\left[L'I^2 + C'U^2\right]$ = temporal rate of change of magnetic and electric energy density per unit length;

- this is a purely oscillating power with double the RF frequency.

Average over a cycle: $\frac{\partial \overline{Power}}{\partial z} = 0$ and no net, time - averaged delivered power is dissipated on the lossless line,

provided we are in steady - state $\frac{\partial}{\partial t}$(amplitudes I_0 and U_0) $= 0$ i.e. no varying amplitudes such as transients.

Now consider the general phasor description.

$U(z,t) = U_0(z)\cos[\omega t + \varphi_u(z)] = \mathrm{Re}[\underline{U}(z)\exp(j\omega t)]$, where the phasor $\underline{U}(z) = U_0(z)\exp(j\varphi_u(z))$.
We assume that the amplitudes $U_0(z)$ and $I_0(z)$ are constant in time (steady state).

Firstly, using trigonometry, the transmitted power $P(z,t) = U(z,t)I(z,t)$

$$= U_0(z)\cos[\omega t + \varphi_u(z)]\, I_0(z)\cos[\omega t + \varphi_i(z)] = \frac{1}{2}U_0(z)I_0(z)\{\cos[2\omega t + \varphi_u(z) + \varphi_i(z)] + \cos[\varphi_u(z) - \varphi_i(z)]\},$$

which is an oscillating component (net time - averaged power = 0) and a net constant power transfer.

Averaging over a RF period, $\overline{Power}(z) = \overline{P}(z) = \frac{1}{2}U_0(z)I_0(z)\cos[\varphi_u(z) - \varphi_i(z)] = U_{\mathrm{rms}}(z)I_{\mathrm{rms}}(z)\cos[\varphi_u(z) - \varphi_i(z)]$.

Secondly, using phasors, the transmitted power $Power(z,t) = U(z,t)I(z,t)$

$$= \mathrm{Re}[\underline{U}(z)\exp(j\omega t)]\ \mathrm{Re}[\underline{I}(z)\exp(j\omega t)]$$

$$= \frac{1}{2}[\underline{U}(z)\exp(j\omega t) + \underline{U}^*(z)\exp(-j\omega t)]\frac{1}{2}[\underline{I}(z)\exp(j\omega t) + \underline{I}^*(z)\exp(-j\omega t)], \quad \text{using } \mathrm{Re}[z] = \frac{1}{2}[z + z^*]$$

$$= \frac{1}{4}[\underline{U}\underline{I}\exp(2j\omega t) + \underline{U}^*\underline{I}^*\exp(-2j\omega t) + \underline{U}\underline{I}^* + \underline{U}^*\underline{I}]$$

$$= \frac{1}{2}[\mathrm{Re}[\underline{U}\underline{I}\exp(2j\omega t)] + \mathrm{Re}[\underline{U}\underline{I}^*]],$$ which gives exactly the same result as with trigonometry above.

Averaging over a RF period, we get $\overline{Power}(z) = \overline{P}(z) = \frac{1}{2}\mathrm{Re}[\underline{U}\underline{I}^*]$.

...This result comes purely from time - averaging (see Born & Wolfe *Optics* p33).

Some authors use the concept of « Complex Power » (Gardiol p161). What is it?

Some authors define a complex power as $\underline{S} = \underline{U}\underline{I}^*/2$, where "complex power is the power calculated based on the impedance of a component" (Radmanesh 'RF and Microwave Electronics' pp 96 and 800). It doesn't seem to be useful.

In fact, $\underline{U}\underline{I}^*/2$ is the phasor of the *oscillating* real and imaginary component of the power :

$$\underline{U}\underline{I}^*/2 = U_0(z)\exp(j\varphi_u)I_0(z)\exp(-j\varphi_i)/2 = U_0 I_0 \exp(j(\varphi_u - \varphi_i))/2$$

$$= \frac{1}{2}U_0(z)I_0(z)\{\cos[\varphi_u(z) - \varphi_i(z)] + j\sin[\varphi_u(z) - \varphi_i(z)]\}$$

Take a general case with complex impedance $\underline{Z} = |Z|\exp(j\varphi_z) = R + j\omega L$.

$\underline{U} = \underline{I}\underline{Z} = I_0(z)\exp(j\varphi_i)|Z|\exp(j\varphi_z)$. (If φ_z is negative, use $\underline{I} = \underline{U}\underline{Y}$ in stead).

Then $\underline{U}\underline{I}^*/2 = I_0^2|Z|\exp(j\varphi_z)/2 = \frac{I_0^2|Z|}{2}\cos(\varphi_z) + j\frac{I_0^2|Z|}{2}\sin(\varphi_z) = I_0^2 R/2 + j\omega L\, I_0^2/2 = \overline{P} + j\overline{Q}$

where $\overline{P}$ is the amplitude of the oscillating resistive (real) power,

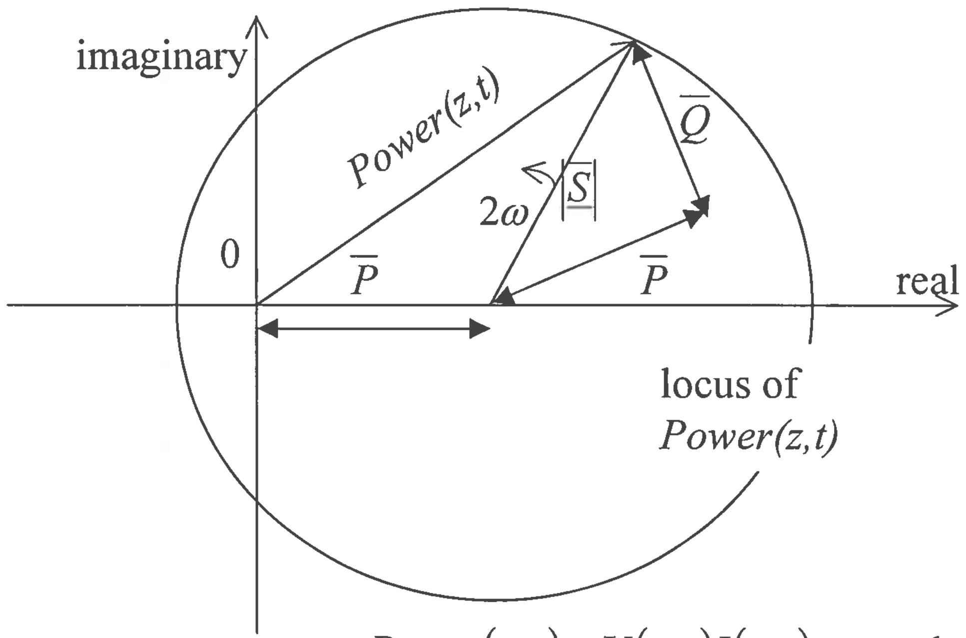
and $\overline{Q}$ is the amplitude of the oscillating reactive (imaginary) power.

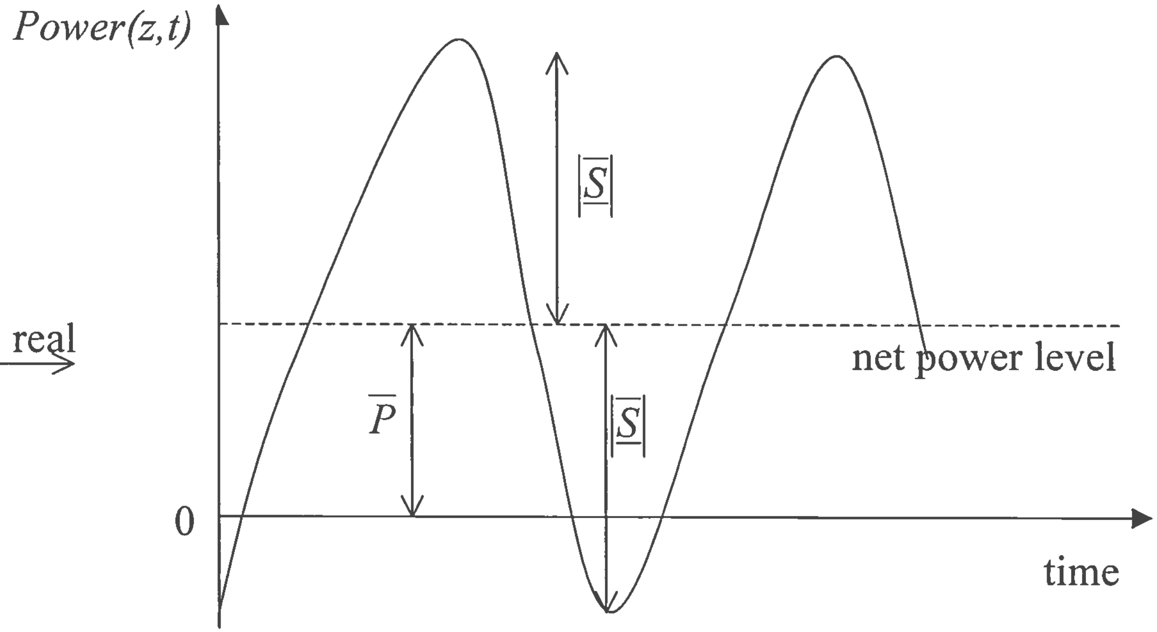
Note that the net delivered power is $\overline{Power(z)} = \mathrm{Re}(\underline{U}\underline{I}^*/2) = \overline{P}$ as previously,

and for the oscillating component, the amplitude is $U_0 I_0 = |\underline{S}| = |\underline{U}\underline{I}^*/2| = \sqrt{\overline{P}^2 + \overline{Q}^2}$

for which the resistive power oscilln. amp. is $\overline{P}$ which is in quadrature with the reactive power oscilln. whose amp. is $\overline{Q}$.

Schematic representation of Power(z,t)





$Power(z,t) = U(z,t)I(z,t)$, angular frequency $= 2\omega$

$\overline{P} = \cos(\varphi_z)U_0 I_0/2$ net power level, and amplitude of resistive power oscillations

$\overline{Q} = \sin(\varphi_z)U_0 I_0/2$ amplitude of reactive power oscillations

Complex power $\underline{\overline{S}} = \overline{P} + j\overline{Q}$; $|\underline{\overline{S}}|^2 = \overline{P}^2 + \overline{Q}^2$

$|\underline{\overline{S}}| = U_0 I_0/2$ amplitude of total power oscillations

How are U, I, Phase and Power Measured?

2 basic techniques:

1) Measure V (by capacitive pickup), I (by induction in current loop) and phase (e.g. by digital sampling). Then real power, reactive power, forward and reverse waves and power etc. are deduced quantities.
   These can be used anywhere in an RF system.
   For examples, see Scientific Systems PIM probe and ENI probe.

OR

2) Measure forward and reverse power directly (e.g. using toroidal current transformer, capacitive pickup, and diodes). Voltage and current could be deduced if required.
   These are generally used in a 50 ohm line before the matching network, because otherwise the forward and reverse powers are almost equally large and the estimation of delivered power is inaccurate.
   For examples, see Daiwa and Vectronics in-line power meters. The Bird Model 43 is related.

So what does the U, I, phase probe measure? (These expressions agree with the ENI formulae)

The PIM and ENI probes measure U_0, I_0, and phase difference φ_z.

The net delivered power $\overline{P}$ is $\cos(\varphi_z)U_0 I_0/2\,[\mathrm{W}]$ (this is also the amplitude of the oscillation in resistive power);

The total amplitude of the power oscillation is $U_0 I_0/2\,[\mathrm{VA}]$;

The amplitude of the oscillation in resistive power is also $\overline{P} = \cos(\varphi_z)U_0 I_0/2\,[\mathrm{W}]$.

The amplitude of the oscillation in reactive power $\overline{Q}$ is $\sin(\varphi_z)U_0 I_0/2\,[\mathrm{VA}]$.

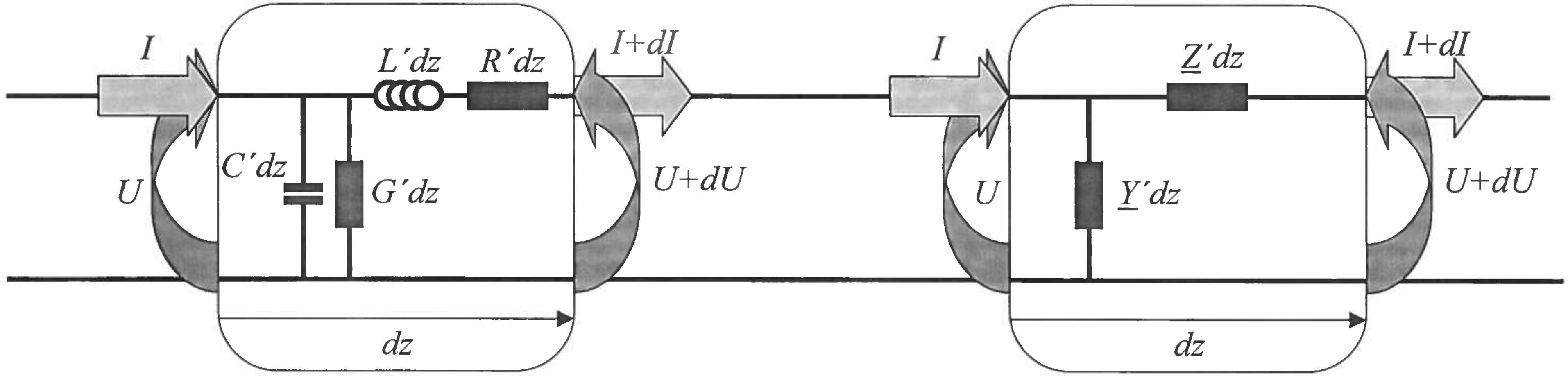
The so-called complex power $\underline{S} = \overline{P} + j\overline{Q}\,[\mathrm{VA}]$.

<u>Assuming $\underline{Z}_c$ is $50\,\Omega$ (true for the probe itself, but generally not true for the transmission line where it is implanted!):</u>

The forward wave voltage is $\underline{U}_+ = (\underline{U}_0 + \underline{Z}_c \underline{I}_0)/2$, the reverse wave voltage is $\underline{U}_- = (\underline{U}_0 - \underline{Z}_c \underline{I}_0)/2$.

The forward power $= \frac{1}{2} Y_c \left(|\underline{U}_+|^2\right)$, and the reverse power $= \frac{1}{2} Y_c \left(|\underline{U}_-|^2\right)$

Generalised Lossy Transmission Line



All primed symbols are values per unit length along the line.

Dielectric losses are respresented by a shunt conductance G' in parallel with the line capacitance.

Conduction losses are represented by a series resistance R'.

The general line consists of a series impedance $\underline{Z}'$ and a shunt admittance $\underline{Y}'$ where

$$\underline{Z}' = R' + j\omega L' \ [\Omega/\text{m}], \quad \underline{Y}' = G' + j\omega C' \ [\text{S/m}].$$

"Ohm's law" for the infinitesimal section is $\frac{\partial \underline{U}(z)}{\partial z} = -\underline{Z}'\underline{I}(z)$, and $\frac{\partial \underline{I}(z)}{\partial z} = -\underline{Y}'\underline{U}(z)$.

Combining gives $\boxed{\frac{\partial^2 \underline{U}(z)}{\partial^2 z} = \underline{Z}'\underline{Y}'\underline{U}(z)}$ or equivalently, $\frac{\partial^2 \underline{I}(z)}{\partial^2 z} = \underline{Z}'\underline{Y}'\underline{I}(z)$

General Solution for Lossy Transmission Line

The general solution of the wave equation $\frac{\partial^2 \underline{U}(z)}{\partial^2 z} = \underline{Z}'\underline{Y}'\underline{U}(z)$ is :

$\underline{U}(z) = \underline{U}_+ \exp(-\underline{\gamma} z) + \underline{U}_- \exp(+\underline{\gamma} z)$ which is a forward and backward wave

with $\underline{U}_+ = U_+ \exp(j\varphi_+)$ and $\underline{U}_- = U_- \exp(j\varphi_-)$ the 2 constants of integration.

The propagation factor $\underline{\gamma} = \sqrt{\underline{Z}'\underline{Y}'} = \alpha + jk$, α the attenuation/length and k the wavenumber, so that

$U(z,t) = U_+ \exp(-\alpha z)\cos(\omega t - kz + \varphi_+) + U_- \exp(\alpha z)\cos(\omega t + kz + \varphi_-)$.

Using $\frac{\partial \underline{U}(z)}{\partial z} = -\underline{Z}'\underline{I}(z)$ and $\frac{\partial \underline{I}(z)}{\partial z} = -\underline{Y}'\underline{U}(z)$ from above, and by comparison with

$\underline{U}(z) = \underline{Z}_c \underline{I}_+ \exp(-\underline{\gamma} z) + \underline{Z}_c \underline{I}_- \exp(+\underline{\gamma} z)$, and $\underline{I}(z) = \underline{Y}_c \underline{U}_+ \exp(-\underline{\gamma} z) - \underline{Y}_c \underline{U}_- \exp(+\underline{\gamma} z)$, we find

$$\underline{Y}_c = \frac{1}{\underline{Z}_c} = \frac{\underline{\gamma}}{\underline{Z}'} = \frac{\underline{Y}'}{\underline{\gamma}} = \sqrt{\frac{\underline{Y}'}{\underline{Z}'}}$$

for the characteristic admittance and impedance.

Losses on the line have two effects :

i) the propagation factor has a real component, causing attenuation;

ii) the characteristic impedance has an imaginary component (Z_c is complex), therefore U and I in each of the forward and reverse waves do not stay in phase along the line.

Characteristic Impedance, Wavenumber and Attenuation for a Lossy Transmission Line
Gardiol p171

The characteristic admittance $\underline{Y}_c = \sqrt{\dfrac{\underline{Y}'}{\underline{Z}'}} = \sqrt{\dfrac{G' + j\omega C'}{R' + j\omega L'}}$.

For small losses, i.e. $R' << \omega L'$ and $G' << \omega C'$, we can expand :

$$\underline{Y}_c \cong \sqrt{\frac{C'}{L'}}\sqrt{\frac{(1 - jG'/\omega C')}{(1 - jR'/\omega L')}} \cong \sqrt{\frac{C'}{L'}}(1 - jG'/2\omega C')(1 + jR'/2\omega L') \cong \sqrt{\frac{C'}{L'}}\{1 + jR'/2\omega L' - jG'/2\omega C'\}.$$

Since the characteristic admittance of a lossless line is $Y_c(lossless) = \sqrt{\dfrac{C'}{L'}}$, we have :

$$\boxed{\underline{Y}_c(small\ losses) = Y_c(lossless) \times \left\{1 + j\left(\frac{R'}{2\omega L'} - \frac{G'}{2\omega C'}\right)\right\}.}$$

Note the *difference* of dielectric and conductor losses

The propagation factor $\underline{\gamma} = \sqrt{\underline{Z}'\underline{Y}'} = \alpha + jk$, α the attenuation/length and k the wavenumber, where

$$\underline{\gamma} = \sqrt{(R' + j\omega L')(G' + j\omega C')} \cong j\omega\sqrt{L'C'} + \frac{\sqrt{L'C'}}{2}\{R'/L' + G'/C'\} = j\omega\sqrt{L'C'} + \left\{\frac{R'}{2}\sqrt{\frac{C'}{L'}} + \frac{G'}{2}\sqrt{\frac{L'}{C'}}\right\}.$$

$$\boxed{\underline{\gamma}(small\ losses) = k\left\{j + \left(\frac{R'}{2\omega L'} + \frac{G'}{2\omega C'}\right)\right\}} = k\left\{j + \frac{\alpha}{k}\right\}.$$

Note the *sum* of dielectric and conductor losses

Therefore, the attenuation per unit length, $\alpha = \{R'Y_c(lossless) + G'Z_c(lossless)\}/2$,

and the wavenumber $k = \omega\sqrt{L'C'}$ (note the phase velocity $v = \omega/k = 1/\sqrt{L'C'}$ as for a lossless line),

and so the forward wave $U_+(z,t) = \exp(j\omega t)\underline{U}_+ \exp(-\underline{\gamma} z) = \underline{U}_+ \exp(-\alpha z)\exp(j(\omega t - kz))$ as usual.

How to estimate G' for a Lossy Transmission Line?

Gardiol p115, 120.

Dielectric losses in imperfect dielectrics are respresented by a complex dielectric constant :

$\underline{D} = \underline{\varepsilon}\underline{E}$, where $\underline{\varepsilon} = \varepsilon_0 \underline{\varepsilon_r}$, define $\underline{\varepsilon_r} = \varepsilon_r(1 - j\tan\delta)$, where the definition of loss tangent is $\tan\delta = -\frac{\mathrm{Im}(\varepsilon_r)}{\mathrm{Re}(\varepsilon_r)}$.

To show that this leads to losses, consider Maxwell's equation for a lossy dielectric :

$$\nabla \times \underline{H} = \frac{\partial \underline{D}}{\partial t} = j\omega\underline{\varepsilon}\underline{E} = (\omega\varepsilon_0\varepsilon_r \tan\delta)\underline{E} + j\omega\varepsilon_0\varepsilon_r\underline{E}.$$

$(\omega\varepsilon_0\varepsilon_r \tan\delta)$ acts as an effective conductivity causing power dissipation.

The loss tangent will be taken to include all loss terms in the imperfect dielectric of the propagation medium, including any residual conductivity.

By analogy with a lossless dielectric,

the admittance per unit length can be written $= j\omega\underline{C}' = (j\omega\underline{\varepsilon})(\text{geometric form factor})$

$= (j\omega\varepsilon_0\varepsilon_r(1 - j\tan\delta))(\text{form factor}) = (j\omega\varepsilon_0\varepsilon_r)(\text{form factor}) + (\omega\varepsilon_0\varepsilon_r\tan\delta)(\text{form factor})$

$= j\omega C' + G'$, where $C' = (\varepsilon_0\varepsilon_r)(\text{form factor})$ and $G' = (\omega\varepsilon_0\varepsilon_r\tan\delta)(\text{form factor})$.

(See Gardiol p120).

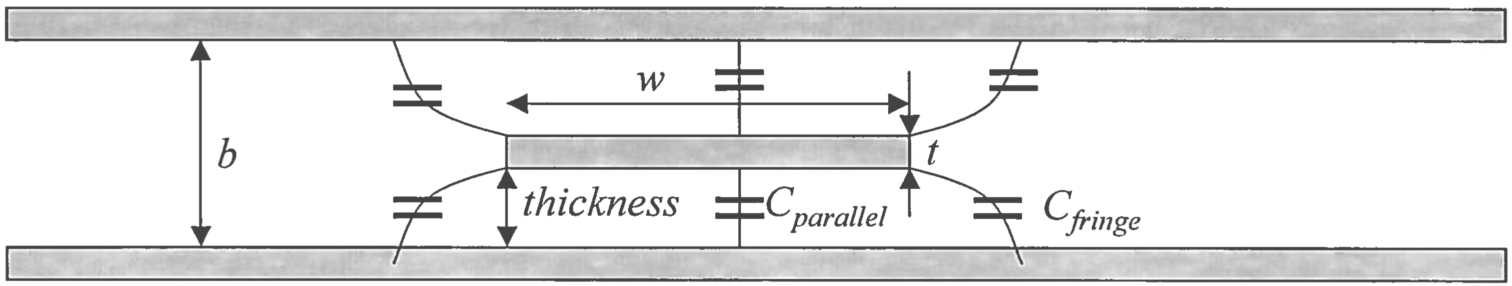
Therefore, if the dielectric loss tangent and the line capacitance per unit length are known,

the line conductivity is $G' = \omega C'\tan\delta$. Therefore, $\alpha_{\text{dielectric}} = \frac{kG'}{2\omega C'} = \frac{\pi\tan\delta}{\lambda}$, where λ is the wavelength in the dielectric.

For teflon at 24 °C and 100 MHz : $\tan\delta < 2.10^{-4}$, therefore the criterion for weak losses, $G' << \omega C'$, is satisfied.

Characteristic Impedance of a Finite-Thickness Balanced Stripline

(For the Feshbach p1248 profile, the current density IS uniform, so R' is the perimeter x the skin depth resistance.)



(see Balanis "Advanced Engineering Electromagnetics" p446 and Cohn PTO).

For a balanced stripline, the total capacitance $C'_t = 2C'_{parallel} + 4C'_{fringe}$; where $C'_{parallel} = \varepsilon_0\varepsilon_r \dfrac{2w}{b-t} = 2\varepsilon_0\varepsilon_r \dfrac{w/b}{1-t/b}$

and $C'_{fringe} = \dfrac{\varepsilon_0\varepsilon_r}{\pi}\left\{\dfrac{2}{1-t/b}\ln\left(\dfrac{1}{1-t/b}+1\right)-\left(\dfrac{1}{1-t/b}-1\right)\ln\left(\dfrac{1}{(1-t/b)^2}-1\right)\right\}$.

Using $Z_c = \dfrac{\sqrt{\mu_0\varepsilon_0\varepsilon_r}}{C'_t}$ we obtain $Z_c\sqrt{\varepsilon_r} = \dfrac{Z_0/4}{\dfrac{w/b}{1-t/b}+\dfrac{C'_{fringe}}{\varepsilon_0\varepsilon_r}} \approx \left(\dfrac{Z_0}{4}\right)\left(\dfrac{b-t}{w}\right)$ for simple parallel plates if $\dfrac{w}{b} >> \dfrac{2\ln 2}{\pi}$.

where $Z_0 = \sqrt{\dfrac{\mu_0}{\varepsilon_0}}$ and for the KAI stripline, $w \cong 0.08$ m, $b \cong 0.007$ m, $\varepsilon_r = 2.1$ for teflon with *thickness* = 0.003 m

and $t = 0.001$ m is the metal thickness. We find $Z_c \cong 4.66\,\Omega$, and so $Y_c = 1/Z_c = 0.21\,\Omega^{-1}$.

Now that we have a value for Z_c, we also have L' and C' from: $Z_c = \sqrt{\dfrac{L'}{C'}}$ and $v = \dfrac{1}{\sqrt{L'C'}} = \dfrac{c}{\sqrt{\varepsilon_r}}$:

$$C' = \frac{\sqrt{\varepsilon_r}}{Z_c c} = 1040 \text{ pF/m} \quad \text{and} \quad L' = \frac{Z_c\sqrt{\varepsilon_r}}{c} = 22.5 \text{ nH/m}.$$

How to estimate R' for a Lossy Transmission Line?
Gardiol p131-5.
R' can be deduced from L' See Wheeler and Cohn (PTO)
If so, C' gives access to all 4 values of C', L', G', R'.

R' represents the resistance per unit length to current flowing in the skin depth of the line conductors. It depends on the line geometry because the current density depends on the $\underline{E}$ and $\underline{H}$ distribution, for example, in an infinitely - thin stripline, the current density tends to infinity at the corners. (R. A. Pucel et al IEEE Trans. MTT - 16 no.6 p342 (June 1968) and Bleaney p236):

The resistance per unit length is $R' = 1/(\sigma\delta p)$, where σ is the conductivity (assumed $38 \cdot 10^6\ \Omega^{-1}\text{m}^{-1}$ but alloy is less), $\delta = \sqrt{2/(\sigma\omega\mu)}$ is the skin depth $\left(= \sqrt{2/\left(38 \cdot 10^6 2\pi 100 \cdot 10^6 4\pi \cdot 10^{-7}\right)}\,\text{m} \cong 8\ \mu\text{m}\right.$ for aluminium at 100 MHz), and p is a sort of perimeter around the line's transverse cross - section. For no fringing fields, here we have $p = w$

Therefore, $R' \cong 1/\sigma\delta w = \left[38 \cdot 10^6 \cdot 8 \cdot 10^{-6} \cdot 0.08\right]^{-1} = 4.11 \cdot 10^{-2}\ \Omega\text{m}^{-1}$

and the attentuation per unit length due to R' alone $\boxed{\alpha_{\substack{\text{conductor}\\ \text{no fringing}}} = \frac{R'}{2Z_c}} \cong 4.41 \cdot 10^{-3}\ \text{m}^{-1}$ (using $Z_c \cong 4.66\Omega$).

To compare the relative magnitude of losses in the characteristic admittance and propagation factor, we require

$$\frac{R'}{\omega L'} = \frac{4.11 \cdot 10^{-2}}{2\pi \cdot 100 \cdot 10^6 \cdot 22.5 \cdot 10^{-9}} = 2.92 \cdot 10^{-3}$$, which is $<< 1$, as required for small loss treatment.

$$\boxed{\text{Summary}: \frac{R'}{\omega L'} = 2.92 \cdot 10^{-3},\quad \frac{G'}{\omega C'} = 2 \cdot 10^{-4},\ \text{both are} << 1.}$$

How thin must the line be to strongly increase the line resistance due to current density concentration in the stripline corners?
To calculate R' correctly, see H. A. Wheeler Proc. IRE **30** p412-424 Eq. 18 (1942); and
S. B. Cohn IEEE Transactions on Microwave Theory & Techniques MTT-3 No.2 p119-126 (1955).

Above, we estimated the resistance per unit length $R' = 1/(\sigma\delta p)$, assuming that there is a uniform current density in a perimeter of length p. The problem is how to estimate the effective perimeter in $R' = 1/(\sigma\delta p) = R_s/p$, where R_s is the surface resistivity in ohms per square, $R_s = 1/(\sigma\delta) = \sqrt{\pi f \mu_0/\sigma}$.

Wheeler shows how to calculate R' from the line inductance L', via $R' = \dfrac{R_s}{\mu_0}\dfrac{\partial L'}{\partial n}$ where n is normal to all conductors.

Since $L' = Z_c\sqrt{\mu_0\varepsilon_0\varepsilon_r}$ and $\alpha_{\text{conductor}} = R'/2Z_c$, we have $\alpha = \dfrac{R_s\sqrt{\varepsilon_r}}{2Z_0 Z_c}\dfrac{\partial Z_c}{\partial n}\ \text{m}^{-1}$.

For a balanced stripline, geometry dictates that $\dfrac{\partial Z_c}{\partial n} = 2\left(\dfrac{\partial Z_c}{\partial b} - \dfrac{\partial Z_c}{\partial w} - \dfrac{\partial Z_c}{\partial t}\right)$. Diff'ation of Z_c expression above gives:

$$\therefore \alpha_{\text{conductor}} = \frac{4R_s\sqrt{\varepsilon_r}Z_c\sqrt{\varepsilon_r}}{Z_0^2 b}\left\{x + \frac{2wx^2}{b} + \frac{x^2(1+t/b)}{\pi}\ln\left(\frac{x+1}{x-1}\right)\right\}\text{m}^{-1},\ \text{where}\ x = \frac{1}{1-t/b}$$

Note in the limit of $t/b \to 0$, the resistance and attenuation coefft. both $\to \infty$ because all the current is at the strip edges!

How thin must the line be to strongly increase the line resistance due to current density concentration in the stripline corners?... continued

Now we have the relevant expressions both with and without edge effects for Z_c and $\alpha_{\text{conductor}}$:

The characteristic impedance including edge effects is :

$$Z_c\sqrt{\varepsilon_r} = \frac{Z_0/4}{\dfrac{w/b}{1-t/b}+\dfrac{C'_{fringe}}{\varepsilon_0\varepsilon_r}} \quad \text{where } C'_{fringe} = \frac{\varepsilon_0\varepsilon_r}{\pi}\left\{\frac{2}{1-t/b}\ln\left(\frac{1}{1-t/b}+1\right)-\left(\frac{1}{1-t/b}-1\right)\ln\left(\frac{1}{(1-t/b)^2}-1\right)\right\}.$$

The characteristic impedance for simple parallel plates is : $Z_c\sqrt{\varepsilon_r} = \dfrac{Z_0(b-t)}{4w}$, for $w/b >> \dfrac{2\ln 2}{\pi}$ and any t.

The attenuation coefficient including edge effects is :

$$\therefore \alpha_{\text{conductor}} = \frac{4R_s\sqrt{\varepsilon_r}Z_c\sqrt{\varepsilon_r}}{Z_0^2 b}\left\{x+\frac{2wx^2}{b}+\frac{x^2(1+t/b)}{\pi}\ln\left(\frac{x+1}{x-1}\right)\right\}\text{m}^{-1}\left(\text{where } x=\frac{1}{1-t/b}\right), \rightarrow\infty \text{ as } t\rightarrow 0.$$

The attn. coefft. excluding edge effects is : $\alpha_{\text{no fringe}} = \dfrac{R_s}{2wZ_c} \cong \dfrac{2R_s\sqrt{\varepsilon_r}}{Z_0(b-t)} \rightarrow \dfrac{2R_s\sqrt{\varepsilon_r}}{Z_0 b}$ as $t\rightarrow 0$ (depends only on b!)

The resistance per unit length including edge effects is : $R' = 2Z_c\alpha_{\text{conductor}}$

The resistance per unit length excluding edge effects is : $R' = \dfrac{R_s}{w}$ (depends only on w!)

CONCLUSION :

The effect of small thickness on attenuation and resistance per unit length is negligible (see *Zc_alpha.m*)

The strip has to be incredibly thin before the attenuation and resistance begin to increase noticeably.

Expressions for Y, α and line resistance in terms of experimental parameters

- The real value of the line admittance,

$$Y = \frac{cC'}{\sqrt{\varepsilon_r}} \cong \frac{2width \times \sqrt{\varepsilon_r}}{thickness \times Z_0} = \frac{2width \times \sqrt{\varepsilon_0 \varepsilon_r}}{thickness \times \sqrt{\mu_0}}.$$

- The attenuation per unit length,

$$\alpha = \frac{k}{2}\left(\frac{R'}{\omega L'} + \frac{G'}{\omega C'}\right) = \left(\frac{R'Y}{2} + \frac{k}{2}\frac{G'}{\omega C'}\right)$$

$$= \left(\frac{Y}{4\sigma\delta \cdot width} + \frac{\pi f\sqrt{\varepsilon_r}}{c}\tan\delta\right)$$

$$= \sqrt{\varepsilon_r}\left(\frac{\sqrt{\varepsilon_0}}{2\sqrt{\mu_0}\sigma\delta \cdot thickness} + \frac{\pi f}{c}\tan\delta\right)$$

$$\alpha = \sqrt{\varepsilon_0\varepsilon_r}\left(\frac{1}{2thickness}\sqrt{\frac{\pi f}{\sigma}} + \pi f\sqrt{\mu_0}\tan\delta\right).$$

- The resistance of the line $= R'l$

$$= \frac{length}{2width \cdot \sigma\delta} = \frac{length}{2width}\sqrt{\frac{\pi f \mu_o}{\sigma}}$$

Forward and Reverse Power on a Transmission Line

Starting from $\underline{U}(z) = \underline{U}_+ \exp(-\underline{\gamma} z) + \underline{U}_- \exp(+\underline{\gamma} z)$, choose arbitrarily $z = 0$ at the power meter :

$\underline{U}_0 = \underline{U}_+ + \underline{U}_- \quad \otimes$, where $\underline{U}_+$ and $\underline{U}_-$ each have their phases φ_+ and φ_- as defined above.

and also $\underline{I}_0 = \underline{I}_+ - \underline{I}_- \therefore \underline{Z}_c \underline{I}_0 = \underline{Z}_c \underline{I}_+ - \underline{Z}_c \underline{I}_- = \underline{U}_+ - \underline{U}_-$ i.e. $\underline{Z}_c \underline{I}_0 = \underline{U}_+ - \underline{U}_- \quad \oplus$.

From the simultaneous equations $\otimes$ and $\oplus$:

$\underline{U}_+ = (\underline{U}_0 + \underline{Z}_c \underline{I}_0)/2$ and $\underline{U}_- = (\underline{U}_0 - \underline{Z}_c \underline{I}_0)/2$ (Note if $\underline{U}_0 = \underline{Z}_c \underline{I}_0$, backward wave $= 0$).

i.e. we can always define a forward and reverse wave from the measured $\underline{U}_0$ and $\underline{I}_0$, along with an (arbitrary?) $\underline{Z}_c$.

Now we can revisit the expression for the transmitted power :

The cycle - averaged power $\overline{P} = \frac{1}{2}\mathrm{Re}[\underline{U}_0 \underline{I}_0 *] = \frac{1}{2} U_0 I_0 \cos(\varphi_u - \varphi_i)$ as before, but now in terms of $\underline{U}_+$ and $\underline{U}_-$ from $\otimes, \oplus$

$$\overline{P} = \frac{1}{2}\mathrm{Re}\left[(\underline{U}_+ + \underline{U}_-)\frac{(\underline{U}_+ - \underline{U}_-)*}{\underline{Z}_c *}\right] = \frac{1}{2}\mathrm{Re}[\underline{Y}_c * (\underline{U}_+ + \underline{U}_-)(\underline{U}_+ - \underline{U}_-)*] = \frac{1}{2}\mathrm{Re}\left[\underline{Y}_c * \left(|\underline{U}_+|^2 - |\underline{U}_-|^2 + 2j\,\mathrm{Im}(\underline{U}_+ * \underline{U}_-)\right)\right]$$

$\overline{P} = \frac{1}{2}\mathrm{Re}[\underline{Y}_c]\left(|\underline{U}_+|^2 - |\underline{U}_-|^2\right) + \mathrm{Im}[\underline{Y}_c](\mathrm{Im}(\underline{U}_+ * \underline{U}_-))$. We can identify the power contributions as :

$\overline{P}$ = forward power - reverse power + a cross - product term.

The cross - product term is zero if the characteristic admittance is real.

Note that "high reflected power" just means high voltage $(\underline{U}_0 = \underline{U}_+ + \underline{U}_-)$ for small current $(\underline{I}_0 = \underline{I}_+ - \underline{I}_-)$.

...this agrees with Gardiol Eq.(5.61) for z=0 (note Gardiol includes sqrt(2) in his phasors).

Forward and Reverse Power on a Transmission Line deduced from U, I, phase measurements

Summary for a purely real characteristic impedance:

(note that all of these terms refer to the real, net, time-averaged power $\overline{P}$)

The forward power $= \frac{1}{2} Y_c \left(|\underline{U}_+|^2 \right)$ and the reverse power $= \frac{1}{2} Y_c \left(|\underline{U}_-|^2 \right)$

where $\underline{U}_+ = (\underline{U}_0 + \underline{Z}_c \underline{I}_0)/2$ and $\underline{U}_- = (\underline{U}_0 - \underline{Z}_c \underline{I}_0)/2$. (Note if $\underline{U}_0 = \underline{Z}_c \underline{I}_0$, then line is matched, reverse power = 0).

i.e. we can always define a forward and reverse power from the measured $\underline{U}_0$ and $\underline{I}_0$, along with an (arbitrary?) $\underline{Z}_c$.

Note that the ENI pickup head is specified as having a characteristic impedance $\underline{Z}_c = 50.2 \pm 0.5\ \Omega$,

although it is a very short section inserted into a transmission line of a very different impedance.

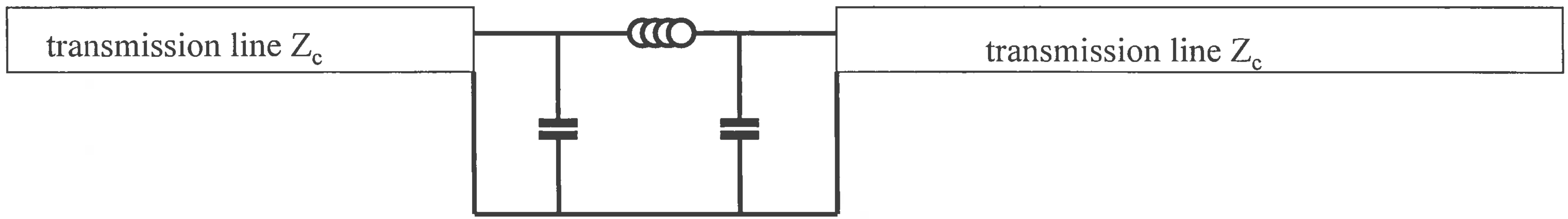
Because the probe head acts as the insertion of small parasitic impedances, there is almost no change to the circuit (i.e. negligible insertion loss).

Therefore the given values of forward and reverse power are in fact meaningless for the transmission line itself unless the true complex characteristic impedance of the line is used.

NB the delivered power, $\overline{P} = \frac{1}{2} Y_c \left(|\underline{U}_+|^2 - |\underline{U}_-|^2 \right) = \frac{1}{2} \mathrm{Re}[\underline{U}\,\underline{I}^*]$ and is independent of characteristic impedance

(provided that Y_c is real!).

probe head Z=50 ohm



Power Dissipated on a Lossy Transmission Line   Gardiol p161.

The variation in delivered power along the line at position z is :

$$\frac{\partial \overline{P}}{\partial z} = \mathrm{Re}\frac{\partial \underline{U}\,\underline{I}^*}{\partial z} = \mathrm{Re}\left[\underline{U}\frac{\partial \underline{I}^*}{\partial z} + \underline{I}^*\frac{\partial \underline{U}}{\partial z}\right] = \mathrm{Re}\left[-\underline{Y}'^*|\underline{U}|^2 - \underline{Z}'|\underline{I}|^2\right]$$

$$\frac{\partial \overline{P}}{\partial z} = -G'|\underline{U}|^2 - R'|\underline{I}|^2 = \text{power loss in dielectric} + \text{power loss in conductors},$$

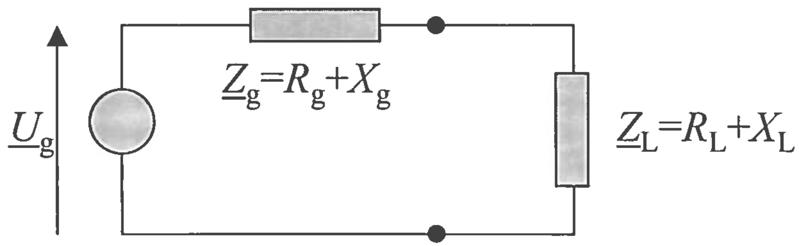
which is obvious from the elemental circuit.

The power dissipated in a length l of the line is $\overline{P}_{in} - \overline{P}_{out} = G'\int_0^l |\underline{U}|^2\,dz + R'\int_0^l |\underline{I}|^2\,dz.$

Unfortunately, to know the delivered power from the generator to the load, we need to know $\underline{U}$ and $\underline{I}$.

These are determined by the boundary conditions at the source and load.

Definitions of Matching   Gardiol p205-6.   There are TWO types of matching:



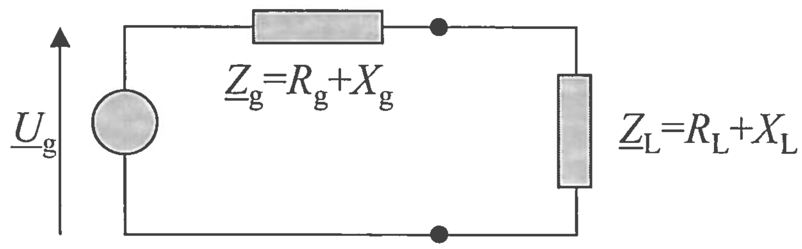
1) For high **quality**, distortionless signal transfer, a **reflectionless load** is required. Therefore choose a load (and generator) impedance the same as the line characteristic impedance, $Z_{load} = Z_{generator} = Z_c$, so that the forward wave is entirely absorbed by the load.

2) For high **power**, maximum power efficiency, a **conjugate load** is required so that the generator can deliver its maximum power into the load. $Z_{load} = Z^*_{generator}$.

For a 50 ohm generator impedance, relectionless and conjugate match are the same.

But next we will consider conjugate matching via networks which transform the 50 ohm generator impedance to a strongly reactive generator impedance in order to make a conjugate match with reactive loads.

Derivation of conjugate matching condition



For maximum power transfer, we want to make the best choice of, say, the load impedance $\underline{Z}_L$ (arbitrary choice).

Time - averaged power to load, $\overline{P}_L = \frac{1}{2}\mathrm{Re}[\underline{U}_L \underline{I}_L{}^*] = \frac{1}{2}\mathrm{Re}\left[\underline{Z}_L |\underline{I}_L|^2\right]$

$$= \frac{R_L}{2} \frac{|\underline{U}_g|^2}{|\underline{Z}_g + \underline{Z}_L|^2} = \frac{R_L}{2} \frac{|\underline{U}_g|^2}{\left|(R_g + R_L)^2 + (X_g + X_L)^2\right|}.$$

Clearly, the power is greater when the reactive component cancels: $X_L = -X_g$ for which:

$\overline{P}_L = \frac{R_L}{2} \frac{|\underline{U}_g|^2}{(R_g + R_L)^2}$. To determine R_L we put $\frac{\partial \overline{P}_L}{\partial R_L} = 0$ and find a maximum at $R_L = R_g$.

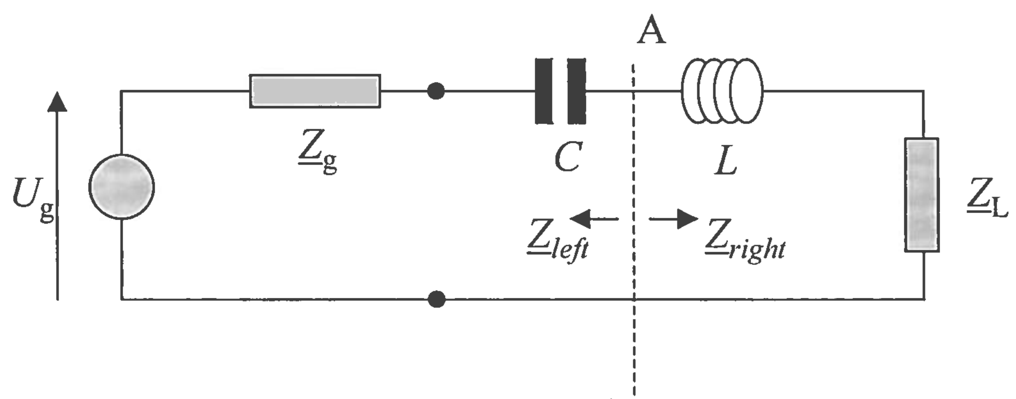
$$\therefore \max(\overline{P}_L) = P_M = \frac{|\underline{U}_g|^2}{8R_g} \text{ for } \boxed{\underline{Z}_L = \underline{Z}_g{}^*.}$$

Note: $\overline{P}_L$ is symmetric in $G \leftrightarrow L$, so we could equivalently adjust the *generator impedance* to the given *load*.

Maximum power when the load impedance is the conjugate of the generator impedance, hence "<u>conjugate matching</u>"

Extension of Conjugate Matching

Consider the same circuit, but with a series C, L circuit with resonance at the operating frequency (the circuit is therefore identical to the above circuit at this frequency).



At the operating frequency, we have chosen $\omega L = 1/\omega C$ and the original circuit is unchanged

and so $\underline{Z}_L = (1/j\omega C + j\omega L + \underline{Z}_g)^* = \underline{Z}_g{}^*$

for the conjugate matching as before, as seen from the generator output terminals.

But now if we now consider the impedances at point A,

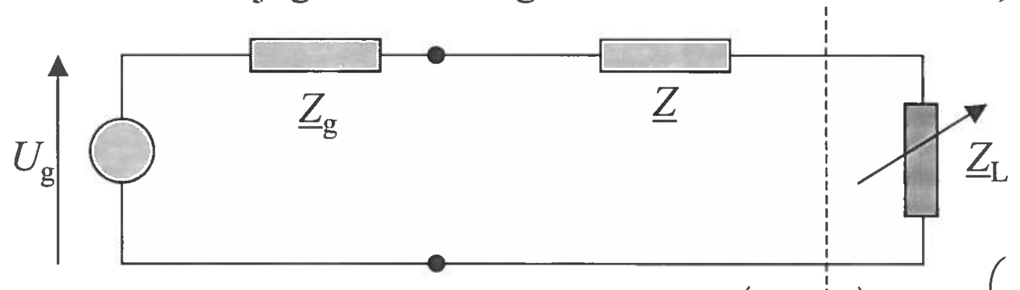
towards the generator we have $\underline{Z}_{left} = \underline{Z}_g + 1/j\omega C$; towards the load we have $\underline{Z}_{right} = \underline{Z}_L + j\omega L$;

However, note that the conjugate matching condition $\underline{Z}_{right} = \underline{Z}_{left}{}^*$ remains valid because of the resonance

It appears that the conjugate matching condition applies at every point in the circuit *for these lossless components*, not only at the generator output terminals.

Note : the series L - Z arm can be converted into an equivalent parallel arm, giving a matching configuration, therefore we could expect that the conjugate match condition pertains also when match netweorks are interposed.

General extension of Conjugate Matching. Consider the same circuit, but with a series impedance



Power dissipated in the load is $\overline{P_L} = \frac{1}{2}\mathrm{Re}\left(\underline{U}\cdot\underline{I}^*\right) = \frac{1}{2}\mathrm{Re}\left(|\underline{I}|^2 \underline{Z}_L\right) = \frac{1}{2}\mathrm{Re}\left(\frac{|\underline{U}|^2 \underline{Z}_L}{|\underline{Z}_g + \underline{Z} + \underline{Z}_L|^2}\right) = \frac{|\underline{U}|^2 R_L}{2|\underline{Z}_g + \underline{Z} + \underline{Z}_L|^2}$.

As before, the maximum generator power $P_M = \frac{|\underline{U}|^2}{8R_g}$

$\therefore$ Power fraction dissipated in the load is $\frac{P_L}{P_M} = \frac{4R_g R_L}{|\underline{Z}_g + \underline{Z} + \underline{Z}_L|^2}$. NB symmetric in $g \leftrightarrow L$

<u>Case 1 : Maximisation of the power fraction by varying the LOAD impedance $\underline{Z}_L$:</u>

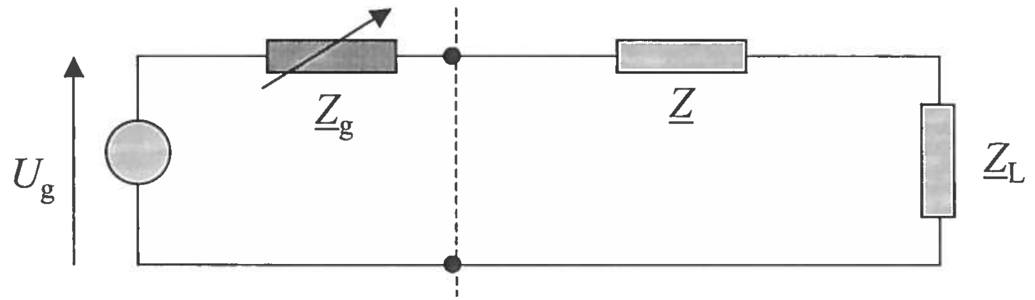
Clearly, first choose $X_L = -\left(X + X_g\right) \therefore \frac{P_L}{P_M} = \frac{4R_g R_L}{\left(R_g + R + R_L\right)^2}$.

Now partial differentiation wrt R_L :

$$\left.\frac{\partial}{\partial R_L}\left(\frac{P_L}{P_M}\right)\right|_{R_g,R} = 0 \text{ yields } R_L = R + R_g$$

Conclusion : for max power in load, when the load is varied, choose <u>conjugate match at load</u> :

$$\underline{Z}_L = \left(\underline{Z} + \underline{Z}_g\right)^*, \text{ for which } \underbrace{\max}_{\text{wrt load}}\left(\frac{P_L}{P_M}\right) = 1 - \frac{R}{R_L}.$$



Case 2 : Maximisation of the power fraction by varying the GENERATOR impedance $\underline{Z}_g$:

Clearly, first choose $X_g = -(X + X_L) \therefore \dfrac{P_L}{P_M} = \dfrac{4R_g R_L}{(R_g + R + R_L)^2}$.

Now partial differentiation wrt R_g :

$$\left.\frac{\partial}{\partial R_g}\left(\frac{P_L}{P_M}\right)\right|_{R_L,R} = 0 \text{ yields } R_g = R + R_L$$

Conclusion : for max power in load, when the generator is varied, choose conjugate match at generator :

$\underline{Z}_g = (\underline{Z} + \underline{Z}_L)^*$, for which $\underbrace{\max}_{\text{wrt Zg}}\left(\dfrac{P_L}{P_M}\right) = 1 - \dfrac{R}{R_g}$ (symmetric with above) $= \dfrac{1}{1 + R/R_L}$, a *different* value at the load.

Note that there are two different approaches for maximising the power in the load :

1) adjust the generator to the fixed line and load impedances; and

2) adjust the load to the fixed line and load impedances.

Each approach gives a different value for $\max\left(\dfrac{P_L}{P_M}\right)$.

If *both* generator *and* load impedance can be simultaneously varied, then we solve the simultaneous equations:

$$\frac{\partial}{\partial R_g}\left(\frac{P_L}{P_M}\right)\Bigg|_{R_L,R} = 0 \quad \text{and} \quad \frac{\partial}{\partial R_L}\left(\frac{P_L}{P_M}\right)\Bigg|_{R_g,R} = 0$$

(here, we have $R_L = R + R_g$ and $R_g = R + R_L$ with solution $R_g = R_L >> R \Rightarrow$ lossless cpt., $\underbrace{\max}_{\text{Zg \& load}}\left(\frac{P_L}{P_M}\right) \to 1$),

OR solve $\frac{\partial}{\partial R_g}\left(\frac{P_L}{P_M}\right)\Big|_{R_L,R} = 0$ for R_g and substitute to obtain $\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right)$, then solve $\frac{\partial}{\partial R_L}\left[\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right)\right] = 0$

to obtain $\underbrace{\max}_{\text{Zg \& load}}\left(\frac{P_L}{P_M}\right)$, optimised for R_g and R_L.

(for this example, we find $\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) = \frac{1}{1 + R/R_L}$ which is max. for $R_g = R_L >> R \Rightarrow$ lossless line, $\underbrace{\max}_{\text{Zg \& load}}\left(\frac{P_L}{P_M}\right) \to 1$).

SUMMARY

The maximum power transfer, as a component is varied, is obtained for a conjugate match at that component.

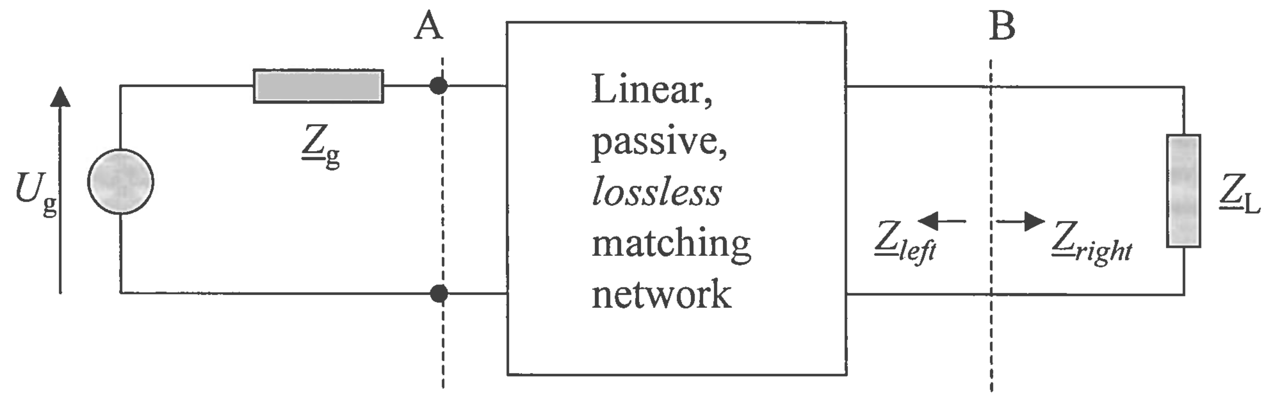
But:

- a conjugate match at one junction does not imply a simultaneous conjugate match at other junctions, unless the intermediate components are lossless,

and

- the power transfer fraction obtained at the load is different according to which component is conjugately matched. unless the intermediate components are lossless.

Extension of Conjugate Matching Through Matching Networks



When the load is matched for maximum power transfer,

i) Referred to point A :

the left and right impedances are conjugately matched, and $\underline{Z}_{right} = \underline{Z}_{left}{}^*$ (= 50Ω for most amplifiers),

and the reverse power is zero (provided it is a 50 Ω line).

ii) Referred to point B : If the matching network is *lossless*,

the left and right impedances are also conjugately matched, and $\underline{Z}_L = \underline{Z}_{(mn),g}{}^*$,

where $\underline{Z}_{(mn),g}$ is the generator impedance transformed through the matching network.

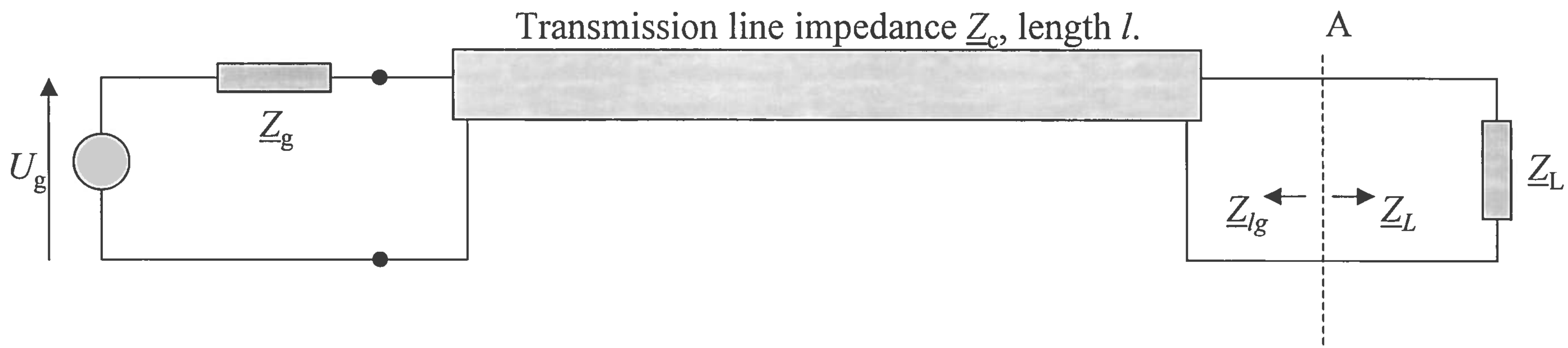
This has a practical application, because, after matching to a plasma, the line can be opened after the match box and the measured impedance (with a 50 Ω load replacing the generator) is the complex conjugate of the (line + reactor + plasma) load.

i.e. the conjugate matching condition can be extended through a *lossless* matching network.

(see Logan JVST 6120 (1969))

Extension of Conjugate Matching to Transmission Lines

The conjugate matching condition also applies at the end of transmission line:



The conjugate matching condition for maximum power transfer, by adjusting the load, is :

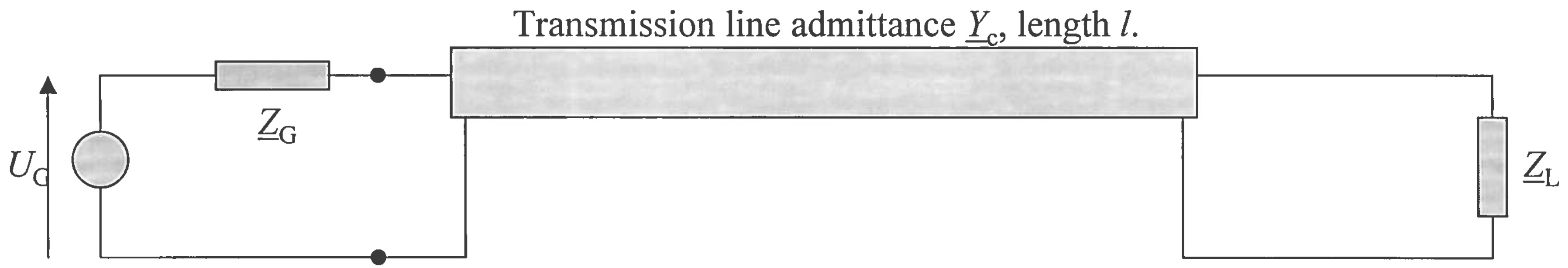
$$\underline{Z}_L = \underline{Z}_{lg}^*$$

where $\underline{Z}_{lg}$ is the generator impedance transformed by the transmission line

i.e. the conjugate matching condition can be extended through (lossy) transmission lines (Gardiol p208).

Optimisation of Stripline...

Gardiol p200 gives the power transfer for a load connected to a generator via a lossy transmission line:



The power fraction reaching load $\underline{Z}_L$ across a line $(\underline{Y}_c, \underline{\gamma})$ from a generator with internal impedance $\underline{Z}_G$ is :

$$\frac{P_L}{P_M} = \frac{4R_G R_L |\underline{Y}_c|^2}{\left|\left(\underline{Z}_G \underline{Z}_L \underline{Y}_c^{\,2} + 1\right)\sinh \underline{\gamma l} + (\underline{Z}_G + \underline{Z}_L)\underline{Y}_c \cosh \underline{\gamma l}\right|^2}$$ (Gardiol p200),

symmetric in $\underline{Z}_G$ and $\underline{Z}_L$, so we could match the load to the (line + generator), or the generator to the (load + line).

where $P_M = \dfrac{|\underline{U}_G|^2}{8R_G}$ is the largest power that the generator could supply (directly into a load $\underline{Z}_L = R_G$),

Group the terms containing the generator impedance $\underline{Z}_G$:

$$\frac{P_L}{P_M} = \frac{4R_G R_L |\underline{Y}_c|^2}{\left|\underline{Z}_G\left(\underline{Z}_L \underline{Y}_c^{\,2} \sinh \underline{\gamma l} + \underline{Y}_c \cosh \underline{\gamma l}\right) + \sinh \underline{\gamma l} + \underline{Z}_L \underline{Y}_c \cosh \underline{\gamma l}\right|^2}$$

Optimisation of Stripline continued...

$\frac{P_L}{P_M}$ can be written as : $\frac{P_L}{P_M} = \frac{4R_G R_{eq}}{\left|\underline{Z}_G + \underline{Z}_{lL}\right|^2}$ which resembles $\frac{4R_g R_L}{\left|\underline{Z}_g + \underline{Z}_L\right|^2}$ from the simplest circuit earlier,

where $R_{eq} = \frac{R_L \left|\underline{Y}_c\right|^2}{\left|\underline{Z}_L \underline{Y}_c^{\,2} \sinh \underline{\gamma l} + \underline{Y}_c \cosh \underline{\gamma l}\right|^2}$

and $\underline{Z}_{lL} = \frac{\sinh \underline{\gamma l} + \underline{Z}_L \underline{Y}_c \cosh \underline{\gamma l}}{\underline{Z}_L \underline{Y}_c^{\,2} \sinh \underline{\gamma l} + \underline{Y}_c \cosh \underline{\gamma l}} = \underline{Z}_c \frac{\underline{Z}_L \underline{Y}_c + \tanh \underline{\gamma l}}{\underline{Z}_L \underline{Y}_c \tanh \underline{\gamma l} + 1}$ = the load impedance seen through the transmission line.

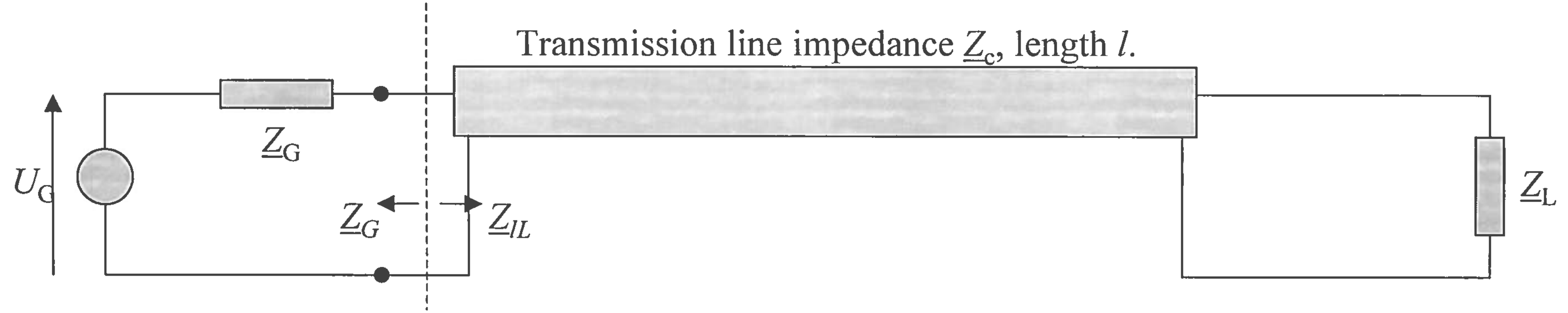
For the maximum power in the load, we put $X_G = -X_{lL}$ (as before), giving $\frac{P_L}{P_M} = \frac{4R_G R_{eq}}{(R_G + R_{lL})^2}$

and furthermore, by allowing the generator impedance to vary to find the matching (admittedly at its input...), we put :

$\frac{\partial(P_L/P_M)}{\partial R_G} = 0$ which gives $R_G = R_{lL}$ for maximum power with respect to variation of the *generator* impedance.

The condition for maximum power (with respect to the generator adjustment) is $\underline{Z}_G{}^* = \underline{Z}_{lL}$ which is conjugate matching of the generator to the (load + line).

NB the generator impedance includes the generator **and** matchbox here.



Consider conjugate matching at both ends of the line.

I have used the condition for maximum power $\underline{Z}_G{}^* = \underline{Z}_{lL}$ for when the *generator* impedance is varied which is conjugate matching of the generator to the line - transformed load.

The condition for maximum power in Gardiol p208 is $\underline{Z}_L = \underline{Z}_{Gl}{}^*$ for when the *load* impedance is varied; this gives conjugate matching of the load to the line - transformed generator.

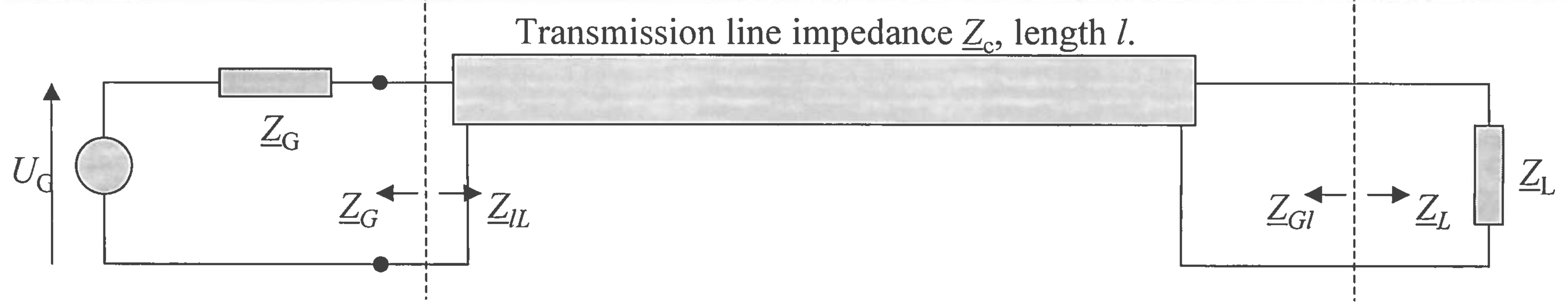
But does $\underline{Z}_L{}^* = \underline{Z}_{Gl}$ necessarily imply $\underline{Z}_G = \underline{Z}_{lL}{}^*$? Derive the latter starting from $\underline{Z}_L{}^* = \underline{Z}_{Gl}$:

$$\underline{Z}_L{}^* = \underline{Z}_{Gl} = \underline{Z}_c \frac{\underline{Z}_G \underline{Y}_c + \tanh \underline{\gamma} l}{\underline{Z}_G \underline{Y}_c \tanh \underline{\gamma} l + 1}.$$ Solve for $\underline{Z}_G$ to obtain :

$$\underline{Z}_G = \underline{Z}_c \frac{-\underline{Z}_L^* \underline{Y}_c + \tanh \underline{\gamma} l}{\underline{Z}_L^* \underline{Y}_c \tanh \underline{\gamma} l - 1} = \underline{Z}_c \frac{\underline{Z}_L^* \underline{Y}_c + \tanh^* \underline{\gamma} l}{\underline{Z}_L^* \underline{Y}_c \tanh^* \underline{\gamma} l + 1}$$ (using $\tanh^* \phi = -\tanh \phi$).

$$\therefore \underline{Z}_G = \underline{Z}_{lL}^* = \underline{Z}_c^* \frac{\underline{Z}_L^* \underline{Y}_c^* + \tanh^* \underline{\gamma} l}{\underline{Z}_L^* \underline{Y}_c^* \tanh^* \underline{\gamma} l + 1}$$ ONLY IF $\underline{Z}_c^* = \underline{Z}_c$.

$\therefore \underline{Z}_L{}^* = \underline{Z}_{Gl} \Leftrightarrow \underline{Z}_G = \underline{Z}_{lL}{}^*$ ONLY IF the characteristic impedance is real, i.e. a *lossless* line, as expected.
Therefore conjugate matching applies everywhere only for lossless line (as for lossless network also).



The maximum power fraction transferred across the line therefore depends only on the line and the load parameters, (because the matching network is now implicitly adjusted to give a conjugate match to the load seen through the line):

$$\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) = \frac{R_{eq}}{\mathrm{Re}[\underline{Z}_{lL}]} = \frac{R_{eq}}{R_{lL}} = \frac{R_L|\underline{Y}_c|^2}{\left|\underline{Z}_L\underline{Y}_c^{\,2}\sinh\underline{\gamma l} + \underline{Y}_c\cosh\underline{\gamma l}\right|^2} \Bigg/ \mathrm{Re}\left[\frac{\sinh\underline{\gamma l} + \underline{Z}_L\underline{Y}_c\cosh\underline{\gamma l}}{\underline{Z}_L\underline{Y}_c^{\,2}\sinh\underline{\gamma l} + \underline{Y}_c\cosh\underline{\gamma l}}\right]$$

$$\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) = R_L|\underline{Y}_c|^2 \Big/ \mathrm{Re}\left[\left(\sinh\underline{\gamma l} + \underline{Z}_L\underline{Y}_c\cosh\underline{\gamma l}\right)\left(\underline{Z}_L\underline{Y}_c^{\,2}\sinh\underline{\gamma l} + \underline{Y}_c\cosh\underline{\gamma l}\right)^*\right] \quad \otimes$$

• Again, note that if the line is lossless, this becomes:

$$\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right)_{lossless} = R_L Y_c / \mathrm{Re}\left[(j\sin kl + \underline{Z}_L Y_c\cos kl)(-\underline{Z}_L{}^* Y_c j\sin kl + \cos kl)\right] \text{ using } \sinh(jk) = j\sin(x)$$

$$= R_L Y_c / \mathrm{Re}\left[\underline{Z}_L{}^* Y_c\sin^2 kl + \underline{Z}_L Y_c\cos^2 kl + \text{imag}^y \text{ terms}\right] = R_L Y_c / \left[R_L Y_c\sin^2 kl + R_L Y_c\cos^2 kl\right] = 1$$

i.e. any load can be chosen to perfectly match the load transformed by a lossless transmission line (see Gardiol p208-9).

• Note if the line length $l \to 0$,

$$\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) \to \frac{R_L}{\mathrm{Re}[\underline{Z}_L]} = 1, \text{ as expected, because there are no line losses.}$$

• Note if $R_L \to 0$,

$$\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) \to 0, \text{ even though power is dissipated in the lossy line,}$$

because the load cannot dissipate any power (no resistance, purely reactive).

Experimentally, we always adjust the matching to operate in matched conditions at matching input. The power transfer to the load across the line will therefore depend only on the load and the line (and not on the generator and matchbox).

Generally, $\underbrace{\max}_{\text{wrt } Zg}\left(\frac{P_L}{P_M}\right) < 1$, because of losses in the transmission line.

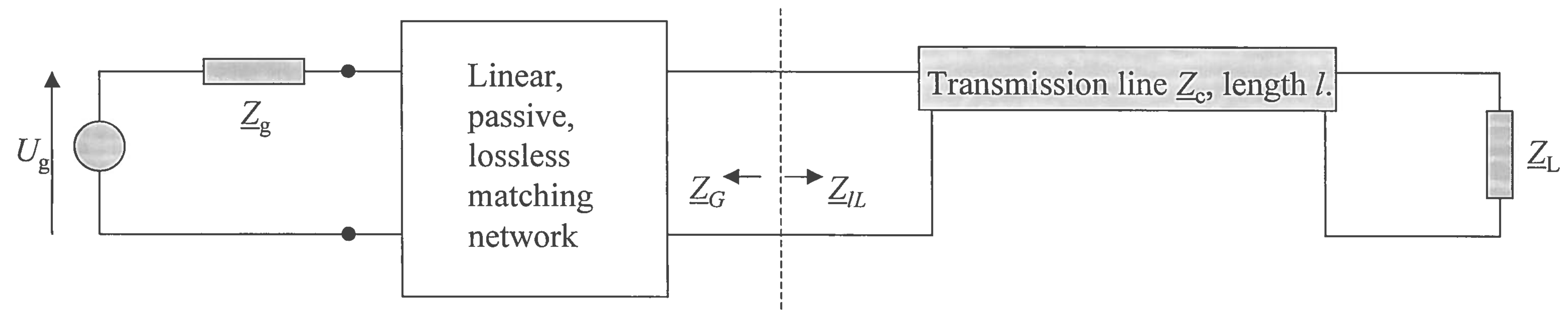
Our aim is to design the load and the line to increase the value of $\underbrace{\max}_{\text{wrt } Zg}\left(\frac{P_L}{P_M}\right)$ as close as possible to 1.

We aim to optimise power transfer across the line to the load; losses in the matchbox are not considered here.

Note P_M is the maximum power that the generator can deliver even though the values of $|U_g|^2$ and R_g are both transformed across the (lossless) matching box, i.e. $|U_G|^2/R_G = |U_g|^2/R_g = |U_g|^2/50$.

Ideally, for low matchbox losses, the transformation ratio $\mathrm{Re}[Z_{IL}] : Z_g$ would be as close to one as possible, i.e. ideally, $\mathrm{Re}[Z_{IL}] \cong 50\Omega$ (but this is unrealistic).

Note that if the generator *is* perfectly matched to a lossless match box, because it sees 50 Ω, then the generator is already conjugately matched to the line - transformed load. This is the condition for maximum power transfer to the load.



Find an **analytical expression** for the power transfer across the line in terms of line losses and the load impedance:

The line resistance and dielectric losses are << line inductance and capacitive impedances per unit length, respectively.

i.e. $R' \ll \omega L'$ and $G' \ll \omega C'$. Therefore we can expand the expression for $\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right)$ using :

$\sinh(\underline{\gamma l}) = \sinh(\alpha l + jkl) = j\sin(kl - j\alpha l)$, using $\sinh(jx) = j\sin(x)$, $\cong j\sin(kl) + \alpha l$; and

$\cosh(\underline{\gamma l}) = \cosh(\alpha l + jkl) = \cos(kl - j\alpha l)$, using $\cosh(jx) = \cos(x)$, $\cong \cos(kl) + j\alpha l$, for $\alpha l \ll 1$.

Also, $\underline{Y}_c = Y_c(1 + jH)$, $\therefore |\underline{Y}_c|^2 = Y_c^2(1 + H^2) \cong Y_c^2$ and $\underline{Y}_c^2 = Y_c^2(1 + jH^2) \cong Y_c^2(1 + j2H)$, since $H \ll 1$.

Substitute into ⊗ $\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) = R_L|\underline{Y}_c|^2 / \mathrm{Re}\left[\left(\sinh \underline{\gamma l} + \underline{Z}_L \underline{Y}_c \cosh \underline{\gamma l}\right)\left(\underline{Z}_L \underline{Y}_c{}^2 \sinh \underline{\gamma l} + \underline{Y}_c \cosh \underline{\gamma l}\right)^*\right]$,

$= R_L Y_c / \mathrm{Re}\left[\left(\sinh \underline{\gamma l} + \underline{Z}_L Y_c(1 + jH)\cosh \underline{\gamma l}\right)\left(\underline{Z}_L Y_c(1 + j2H)\sinh \underline{\gamma l} + (1 + jH)\cosh \underline{\gamma l}\right)^*\right]$.

Expand 1st bracket, ignoring terms with products of small quantities αl and H :

$\{\alpha l \cos(kl) + YR_L \cos(kl) - YX_L(\alpha l \sin(kl) + H\cos(kl))\} + j\{\sin(kl) + YX_L \cos(kl) + YR_L(\alpha l \sin(kl) + H\cos(kl))\}$.

Expand 2nd bracket likewise (without taking complex conjugate here) :

$\{R_L Y\alpha l \cos(kl) - R_L Y 2H \sin(kl) - YX_L \sin(kl) + \cos(kl)\}$

$+ j\{X_L Y\alpha l \cos(kl) - X_L Y 2H \sin(kl) + YR_L \sin(kl) + \alpha l \sin(kl) + H\cos(kl)\}$.

Re[denominator] = Re[1st bracket] · Re[2nd bracket] + Im[1st bracket] · Im[2nd bracket]

(since we did not take complex conjugate above).

What is the physical interpretation of small loss terms?

Small losses in the conductor $\Rightarrow \dfrac{R'}{2\omega L'} << 1$.

If dielectric losses are neglected for the moment, we have $\dfrac{R'}{2\omega L'} = \dfrac{\alpha}{k} = \dfrac{R'Y}{2k} << 1$ i.e. $\dfrac{R'}{2k} << Z_c$

• For valid binomial expansion of the initial expressions for $\underline{Y}$ and $\underline{\gamma}$:

Small losses $\Rightarrow \dfrac{R'\lambda}{4\pi} << Z_c$ i.e. <u>the resistance of a wavelength of line << the characteristic impedance.</u>

• For valid binomial expansion of the expression for power transfer fraction, we require $\alpha l << 1$:

Accurate expansion requires $\dfrac{R'l}{2} << Z_c$ i.e. <u>the resistance of the whole line << the characteristic impedance</u>

<u>In practice, the analytic expression is accurate for HDS line up to 30 m long.</u>

Note that $\dfrac{\alpha l}{Y} \cong \dfrac{R'l}{2} = \dfrac{\text{line resistance}}{2}$, if dielectric losses are neglected.

An analytical expression for the power transfer across the line in terms of line losses and the load impedance:
Collecting terms, we find :

$$\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right)=(R_L Y)/\left\{R_L Y+\alpha l\left(1+Y^2|\underline{Z}_L|^2\right)+H\sin(kl)\left[\left(1-Y^2|\underline{Z}_L|^2\right)\cos(kl)-2X_L Y\sin(kl)\right]\right\}.$$

where $\left(\frac{\alpha}{k}\right)=\left(\frac{R'}{2\omega L'}+\frac{G'}{2\omega C'}\right)$ and $H=\left(\frac{R'}{2\omega L'}-\frac{G'}{2\omega C'}\right)$ and $R' << \omega L'$ and $G' << \omega C'$.

Limiting cases :

- Note that $\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right)\rightarrow 1$ for $l\rightarrow 0$, and also for any lossless line (αl and $H\rightarrow 0$), as expected, since the load is conjugately matched to the source via a lossless connection.
- $\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right)\rightarrow 0$ as $R_L\rightarrow 0$ because there can be no dissipation in a purely reactive load.
- If the load is matched to the line characteristic impedance, i.e. if $\underline{Z}_L=R_L=1/Y$, then

$$\underbrace{\max}_{\text{Zg \& load}}\left(\frac{P_L}{P_M}\right)_{R_L=1/Y}=(R_L Y)/\left\{R_L Y+\alpha l\left(1+Y^2 R_L{}^2\right)\right\}=1/(1+2\alpha l)\cong(1-2\alpha l).$$ This is also the condition for a reflectionless match, for which all the power is forward power which decreases as $(\text{voltage})^2$ along the line, i.e.

$$\underbrace{\max}_{\text{Zg \& load}}\left(\frac{P_L}{P_M}\right)_{\text{reflnless}}=\exp(-2\alpha l)\cong(1-2\alpha l),$$ in agreement with the above expression. This is the best situation (PTO).

Generally, $\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right)<\underbrace{\max}_{\text{Zg \& load}}\left(\frac{P_L}{P_M}\right)_{\text{reflnless}}$ because of many reflections on the absorbing line for load/line mismatch.

We can show that the amplitude of the loss term oscillatory in (kl) is always smaller than the constant level loss term :

$$\frac{\text{Oscillatory term}}{\text{Constant term}} = \frac{H \sin(kl)\left[\left(1 - Y^2 |\underline{Z}_L|^2\right)\cos(kl) - 2X_L Y \sin(kl)\right]}{(\alpha/k)(kl)\left(1 + Y^2 |\underline{Z}_L|^2\right)}.$$

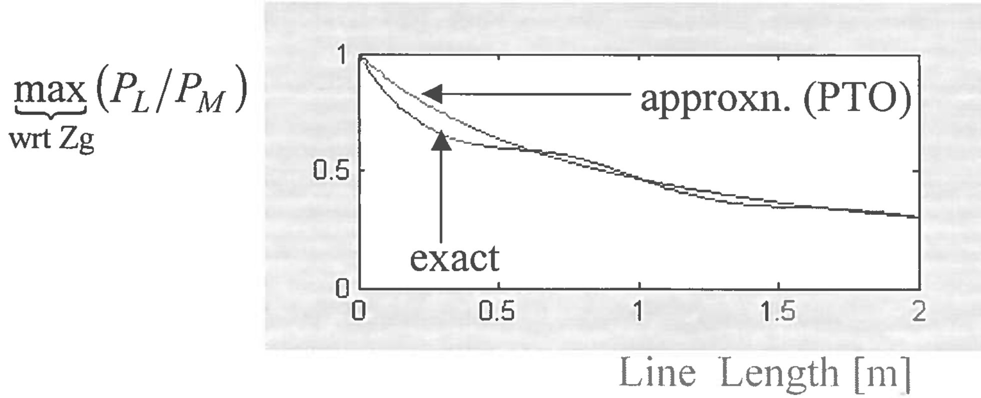
Note that (α/k) is the sum of loss terms, whereas H is their difference, therefore $H < (\alpha/k)$ always. Also, $\frac{\sin(kl)}{(kl)} \le 1$,

$$\therefore \frac{\text{Oscillatory term}}{\text{Constant term}} < \frac{\left[\left(1 - Y^2 |\underline{Z}_L|^2\right)\cos(kl) - 2X_L Y \sin(kl)\right]}{\left(1 + Y^2 |\underline{Z}_L|^2\right)}$$

, and this fraction is < 1 for all values of (kl) and $\frac{X_L}{|\underline{Z}_L|}$.

Therefore the loss term oscillatory in (kl) is always smaller than the constant level loss term, and so

$\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) \le 1$, for all conditions, as required, because P_M is the maximum deliverable power.



Simplification of power transfer fraction.

Ignoring the small-amplitude oscillations, the envelope of $\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right)$ simplifies further to:

$$\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) = (R_L Y) \Big/ \left\{R_L Y + \alpha l\left(1 + Y^2|\underline{Z}_L|^2\right)\right\} = \frac{R_L}{\left\{R_L + \frac{R'l}{2}\left(1 + Y^2|\underline{Z}_L|^2\right)\right\}},$$ by adjustment of the generator impedance.

To *further* maximise this with respect to the *load* impedance, set the load reactance to zero: $X_L = 0$, i.e. $|\underline{Z}_L|^2 \to R_L^2$.

$$\frac{\partial}{\partial R_L}\left[\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right)\right] = \frac{\partial}{\partial R_L}\left[R_L \Big/ \left\{R_L + \frac{R'l}{2}\left(1 + Y^2 R_L^2\right)\right\}\right] = 0,$$ gives a maximum value for $R_L Y = 1 \Rightarrow R_L = Z_c$.

This is a <u>conjugate</u> and <u>reflectionless</u> match, where $\underbrace{\max}_{\text{Zg \& load}}\left(\frac{P_L}{P_M}\right) = \frac{R_L}{R_L + R'l} = \frac{Z_c}{Z_c + R'l} \cong 1$ unless $l >> \lambda$.

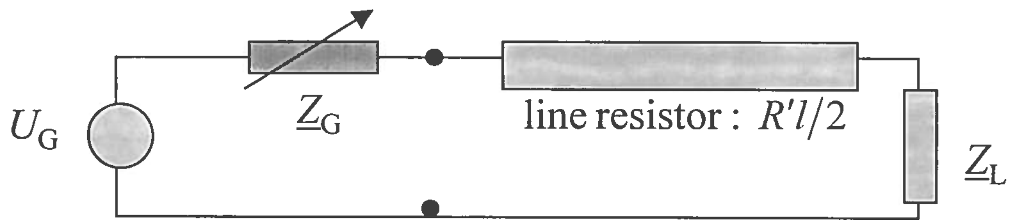
<u>This is exactly the same result as for the line replaced by its straightforward resistance $R'l$!</u>

Note: for $Y^2|\underline{Z}_L|^2 << 1$, we have

$$\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) = \frac{R_L}{R_L + \frac{R'l}{2}}$$

<u>This is the same result as for the line replaced by half of its resistance $R'l$.</u> For the case where $R_L < Z_c$,

we can use the last expression, because at reflectionless match, $R_L = Z_c >> R'l$ and so $\underbrace{\max}_{\text{Zg \& load}}\left(\frac{P_L}{P_M}\right) \cong 1$ in any case.



This is simpler than equivalent circuit models with T-sections!

Comparison of different expressions for power transfer fraction, for generator conjugately matched.

- Full expression (Gardiol p208):

$$\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) = R_L |\underline{Y}_c|^2 \Big/ \mathrm{Re}\left[\left(\sinh \underline{\gamma} l + \underline{Z}_L \underline{Y}_c \cosh \underline{\gamma} l\right)\left(\underline{Z}_L \underline{Y}_c^{\,2} \sinh \underline{\gamma} l + \underline{Y}_c \cosh \underline{\gamma} l\right)^*\right]$$

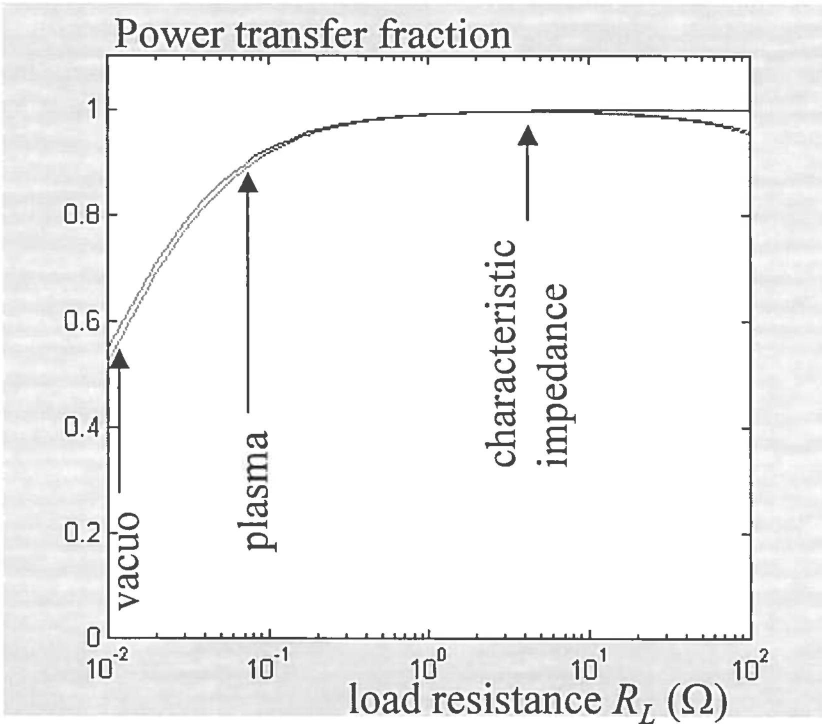
- Binomial expansion for small losses, without the oscillatory terms:

$$\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) = \frac{R_L}{\left\{R_L + \frac{R'l}{2}\left(1 + Y^2 |\underline{Z}_L|^2\right)\right\}} \cong \frac{R_L}{\left\{R_L + \frac{R'l}{2}\left(1 + Y^2 R_L^{\,2}\right)\right\}}$$ to a good approximation for $X_L < Z_c$

effect of transmission line

- Simplest expression, replacing the line by a resistor = half of the line resistance $R'l$:

$$\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) = \frac{R_L}{R_L + \frac{R'l}{2}}$$

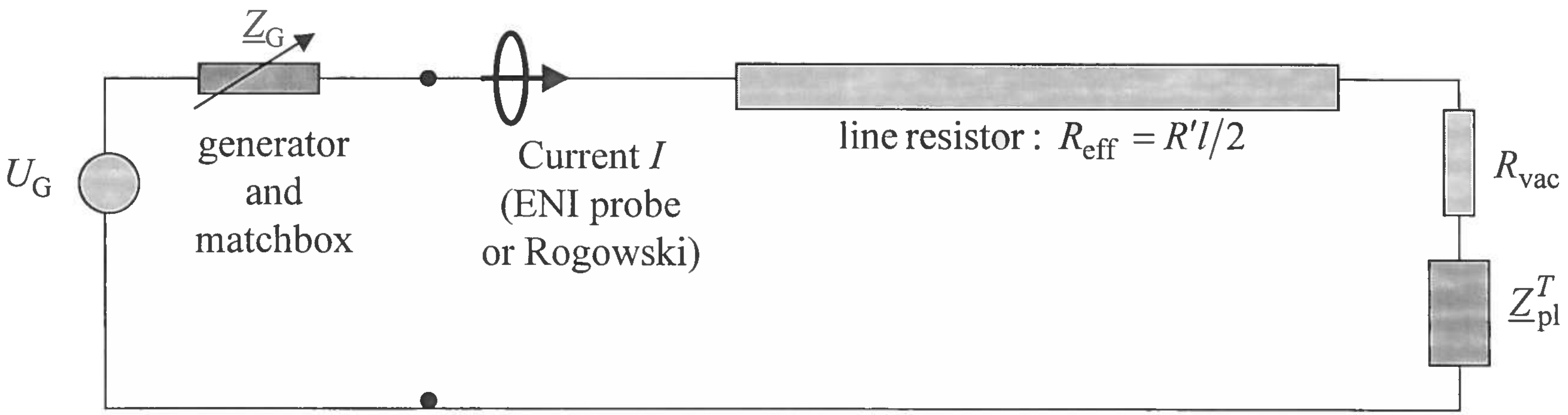


Note:
i)Efficiency increases strongly with load resistance

ii)Broad maximum for load = characteristic impedance

iii) Simplest expression is accurate for load < 10 ohms !!

Consequences for the subtractive power method



Power in vacuum circuit (after the matchbox) with no plasma $P_{vac} = \frac{1}{2}\mathrm{Re}[\underline{U}_{eni} \cdot \underline{I}_{eni}{}^*] = I_{rms}^2(R_{eff} + R_{vac})$.

Total input power (after the matchbox) with plasma :

$$P_{in} = \frac{1}{2}\mathrm{Re}[\underline{U}_{eni} \cdot \underline{I}_{eni}{}^*] = I_{rms}^2\left(R_{eff} + R_{vac}^*\right) + I_{rms}^2 \,\mathrm{Re}\left[\underline{Z}_{pl}^T\right] = I_{rms}^2\left(R_{eff} + R_{vac}^*\right) + P_{pl} = P_{vac}^* + P_{pl},$$

where R_{vac}^* is the modified reactor vacuum resistance due to the change in current distribution with plasma, PTO*

and $\underline{Z}_{pl}^T$ is the plasma impedance transformed by the reactor impedance.

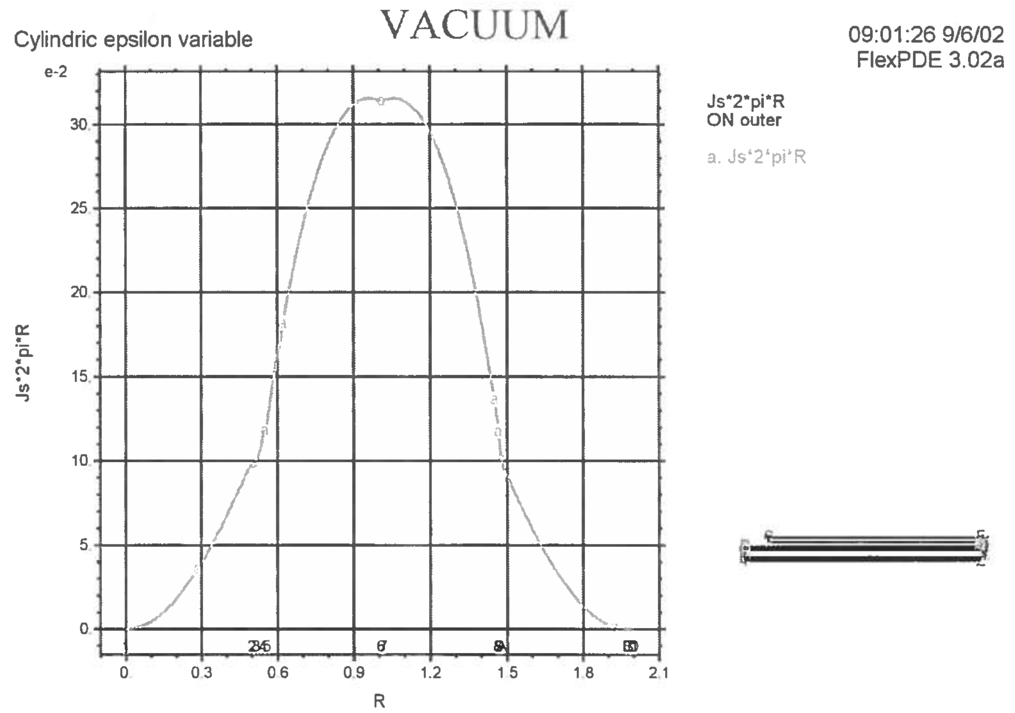
Conclusion : $(P_{in})_I \cong (P_{vac})_I + (P_{plasma})_I$

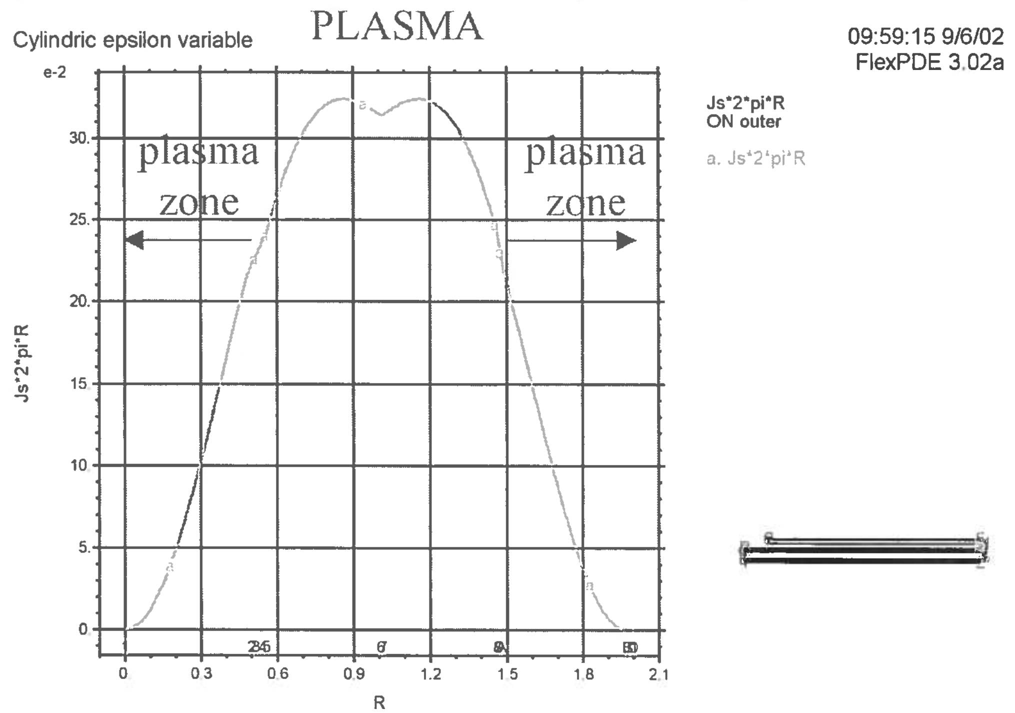
therefore the plasma power is the input power minus the vacuum power (measured for the same current magnitude), provided that the change in R_{vac} due to plasma is not too large (or accounted for) which is the proof of validity of the subtractive method for a plasma reactor connected by a transmission line.

*By how much does the 'vacuum resistance of the reactor' change in presence of plasma?

The 'vacuum resistance of the reactor' R_{vac} is due to the current flowing in the skin depth of the reactor metal. When the plasma is ignited, the current distribution changes in order to supply the plasma current. For a given input current, the 'vacuum resistance', and the power dissipated in it, has changed: see figures.
A FLEX simulation (LS) estimates that R_{vac} **increases by about 1/3 in presence of plasma.**

FLEX plots of surface-integrated current in the skin depth across the reactor geometry (100 MHz)





Final simplified power fraction appropriate to HDS reactor

Ignoring the small - amplitude oscillations, and for $Y^2|\underline{Z}_L|^2 << 1$ (as for the HDS reactor),

$$\underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) \cong \frac{R_L Y}{R_L Y + \alpha l} \cong \frac{R_L}{R_L + \frac{R'l}{2}} = \frac{1}{1 + \frac{R'l}{2R_L}} = \frac{1}{1 + \frac{\frac{1}{2}\text{line resistance}}{\text{load resistance}}} = \frac{1}{1 + \frac{1}{4R_L}\left(\frac{length}{width}\sqrt{\frac{\mu_0 \pi f}{\sigma}}\right)}.$$

<u>For efficient power transfer, we can recommend :</u>

a high load resistance (preferably similar to the line characteristic impedance),

a short, wide, high conductivity line (with low dielectric loss tangent).

Note : independent of height, ε_r and load reactance,

- BUT the width : height ratio of the line, ε_r and the load reactance must be low enough to keep Y small

so that $Y^2|\underline{Z}_L|^2 << 1$ is still satisfied.

- lower frequencies are good provided that the load is not capacitive, otherwise $|\underline{Z}_L|^2$ may become too large;

Physical parameters for HDS reactor

$\varepsilon_r = 2.1$ (teflon in reactor and stripline)

$f = 100$ MHz

$\lambda = 2.07$ m

$k = 3.04\ \text{m}^{-1}$

Reactor:

$X_L = 1/\omega C_L = -0.6\ \Omega \cong |Z_L|$

load phase angle $= -89°$

$\Rightarrow R_L \cong 0.01\ \Omega$

Stripline:

$width = 0.08$ m

$gap\ height = 0.003$ m

$length = 0.9\ \text{m} = 43.5\%$ of a wavelength

$\sigma = 38 \cdot 10^6\ \Omega^{-1}\text{m}^{-1}$

(can be 3x smaller for Al alloys).

Teflon loss tangent $\delta \cong 2 \cdot 10^{-4}$ (room temperature!)

Derived parameters

Characteristic impedance, $Z_{lossless} = 4.66\ \Omega$

Characteristic admittance, $Y_{lossless} = 0.215\ \Omega^{-1}$

Skin depth $\delta = 8\ \mu\text{m}$

Resistance per unit length $R' = 0.0206\ \Omega/\text{m}$

Resistance of the stripline $= 0.0185\ \Omega$

...we expect R_L to be less (at least in vacuum)

Inductance per unit length $L' = 22.5$ nH/m

$R'/2\omega L' = 7.3 \cdot 10^{-4} << 1$

$G'/2\omega C' = \tan\delta/2 = 1 \cdot 10^{-4} << 1$

$H = R'/2\omega L' - G'/2\omega C' = 6.3 \cdot 10^{-4}$

Complex admittance $Y_c = Y_{lossless}(1 + jH) = 0.215(1 + j6.3 \cdot 10^{-4})$

$\alpha/k = R'/2\omega L' + G'/2\omega C' = 8.3 \cdot 10^{-4}$

Attenuation per unit length, $\alpha = 0.0025\ \text{m}^{-1}$

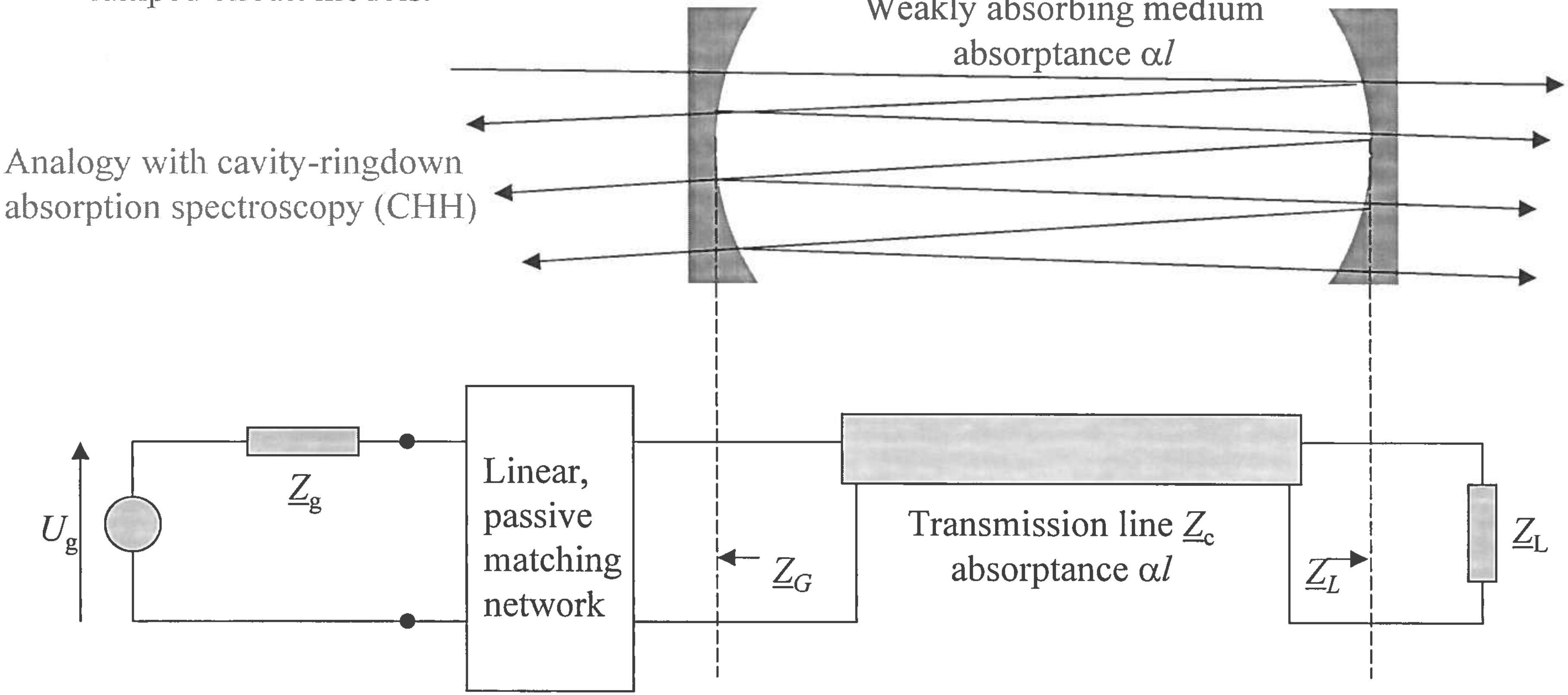
Propagation constant, $\underline{\gamma} = \alpha + jk = 0.0025 + j3.04$

Impedance to be matched = Load transformed by stripline $= Z_{lL} = 0.0264 - j2.776$, phase angle $= -89.4°$

$R_L Y = 0.00215 << 1$; $Y^2|\underline{Z}_L|^2 = 0.0166 << 1$; $\alpha l = 0.0022$; Single pass power transfer efficiency $= \exp(-2\alpha l) = \underline{99.6\%}$

Estimated power fraction transferred when matched $= \underbrace{\max}_{\text{wrt Zg}}\left(\frac{P_L}{P_M}\right) = \underline{51\%} \cong \frac{R_L}{R_L + \frac{R'l}{2}} = 52\%.$

The power loss is much greater than the power attentuation factor, $\exp(-2\alpha l)$, because the power is reflected back and forth many times along the weakly-absorbing line. In conjugate match, reflections are supposed to « help » the power transfer the load, but this is **not** helpful when there is an intermediate lossy transmission line. This analogue is missing from lumped circuit models.



The reflection coefficient of the mirrors is analagous to the reflection coefficients at the ends of the line :

Complex reflection coefficient $\underline{\rho}_{End} = \dfrac{\underline{Z}_{End} - \underline{Z}_c}{\underline{Z}_{End} + \underline{Z}_c}$ where $\underline{Z}_{End} = \underline{Z}_G$ and $\underline{Z}_L$ respectively.

The same equations as before can be obtained by summing the infinite series of the reflections.

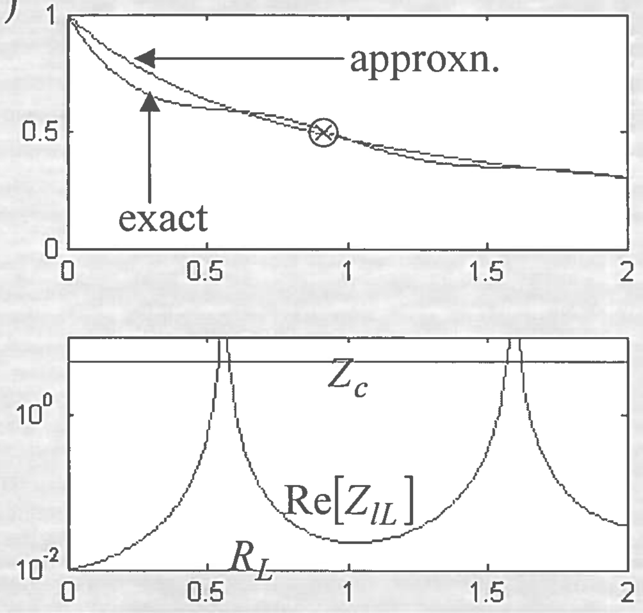
Effects of physical parameters on power transfer efficiency and line-transformed load resistance $\mathrm{Re}[Z_{lL}]$ which has to be transformed to 50 ohms by the matching network. See Matlab *PLPM.m*

$\underbrace{\max}_{\text{wrt Zg}}(P_L/P_M)$

vs

Line Length [m]

(Short is best. Improve matching condition?)

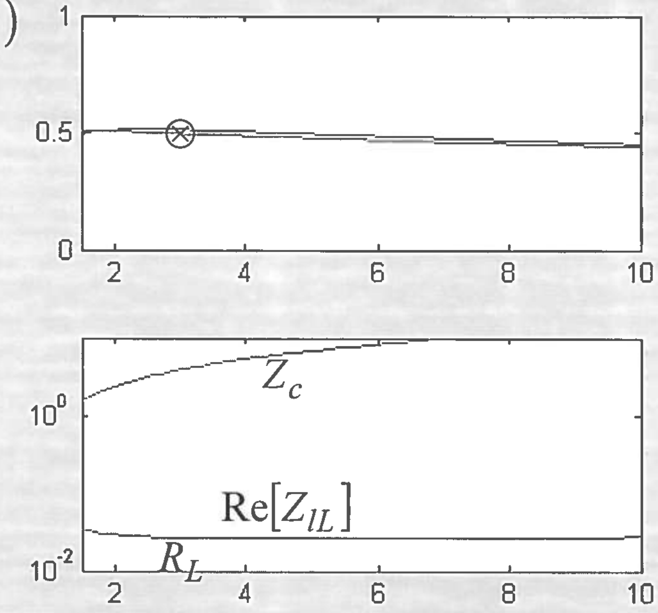


$\underbrace{\max}_{\text{wrt Zg}}(P_L/P_M)$

vs

Line Height [mm]

(No big effect)

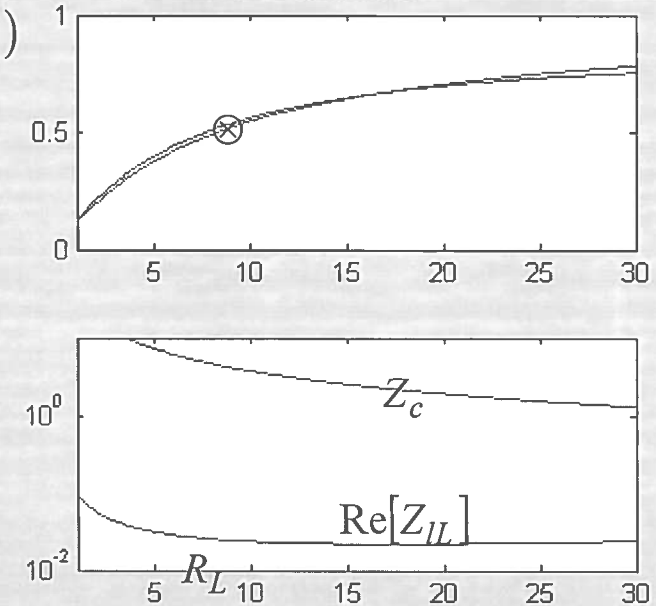


$\underbrace{\max}_{\text{wrt Zg}}(P_L/P_M)$

vs

Line Width [cm]

(Wide is better)

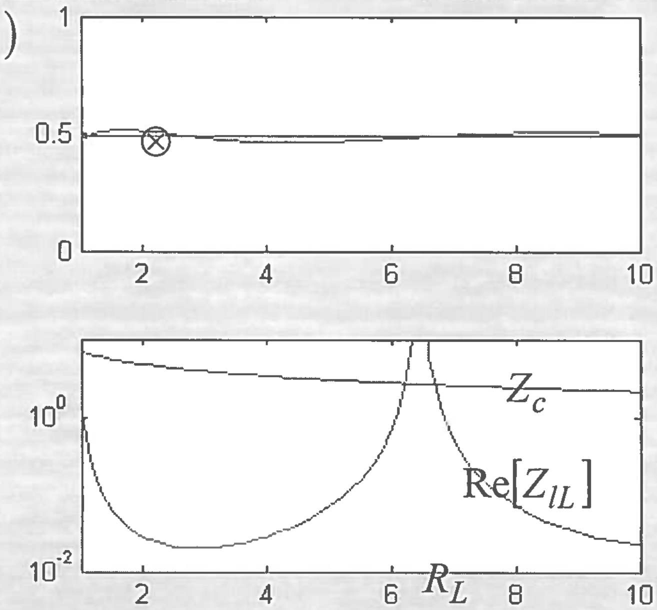


$\underbrace{\max}_{\text{wrt Zg}}(P_L/P_M)$

vs

Relative Permittivity

(Easier matching in vacuo?)



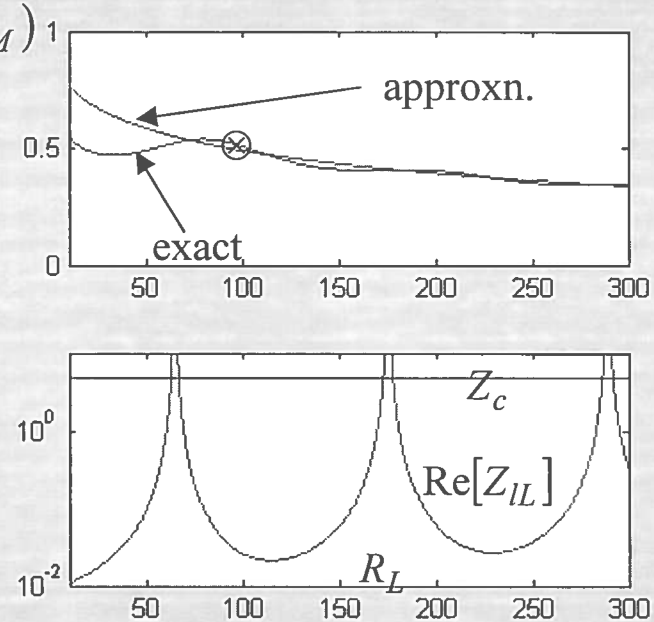
Effects of other physical parameters on power transfer efficiency. See Matlab *PLPM.m*

$\underbrace{\max}_{\text{wrt } Z_g}(P_L/P_M)$

vs

Frequency [MHz]

(possible to improve matching condition?)

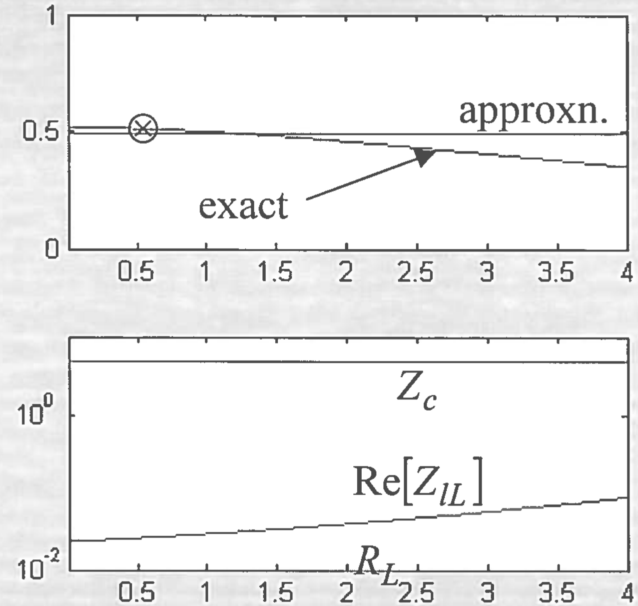


$\underbrace{\max}_{\text{wrt } Z_g}(P_L/P_M)$

vs

Load Reactance [ohms]

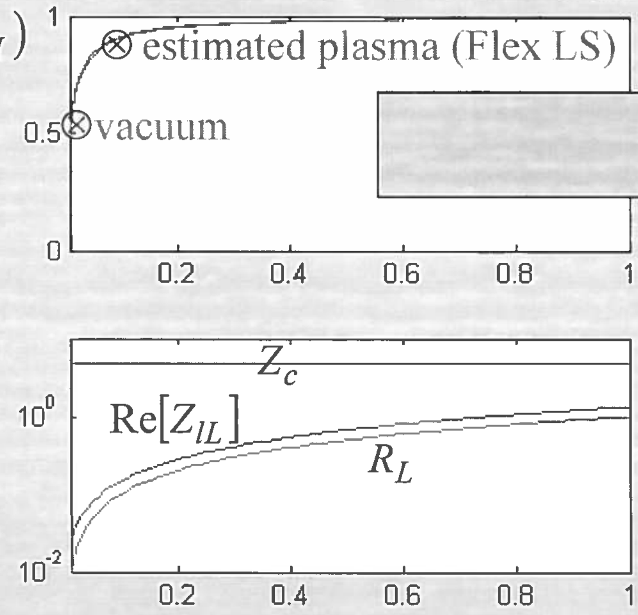
(Changing the load reactance has almost no effect!!)



$\underbrace{\max}_{\text{wrt } Z_g}(P_L/P_M)$

vs

Load Resistance [ohms]



Much higher efficiency as load becomes more resistive: Predict approx. 90% transmission for reactor-transformed plasma resistance of 0.1 Ω

=> losses are elsewhere? Connections?

How to improve power transmission?

<u>Reduce line losses :</u>

Is most loss due to poor mechanical contact between connectors? Use IR camera for local heating

All alloys of aluminium increase resistivity (by factor 2 to 3). I assumed the lowest resistiviy (pure aluminium).

Surface roughness increases resistivity (20% for 3μm, 80% for 10μm @100MHz using mwi.exe). I used smooth surface.

High conductivity coating (few microns)

Teflon dielectric losses are already 7 times smaller than conduction losses (already underestimated by factor 4 or so).

Wider stripline

<u>Increase the effective resistance of load :</u>

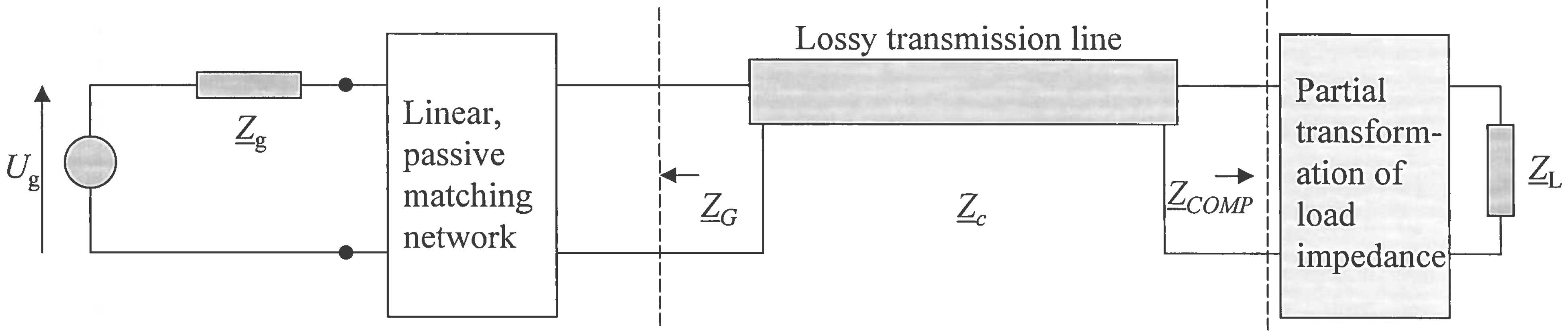
Use a compensation circuit

& /or decrease the characteristic impedance of the line,

ideally to make $R_L = 1/Y_c = Z_c$ = characteristic impedance of the line (ideally close to 50Ω to reduce matching losses).

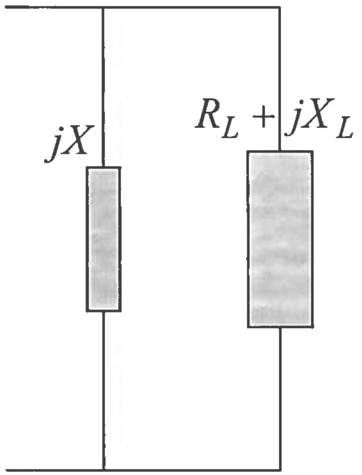
to obtain, in fact, a reflectionless match .

Simply cancelling out the reactive part of the reactor is not necessary nor very useful provided $Y^2|\underline{Z}_L|^2 << 1$ remains true.



Increase the effective load resistance by transforming it upwards (use a stub?). Power will circulate in the compensation section which is shorter, therefore less losses (cascaded matching probably has less total losses).

Consider a purely reactive shunt at the reactor: power reflections confined to very short connectors.



Load impedance $\underline{Z}_L = R_L + jX_L$, with $X_L >> R_L$; and shunt impedance $= jX$

$$\text{Transformed load admittance} = \frac{1}{jX} + \frac{1}{R_L + jX_L}$$

$$\text{Transformed load impedance} = \frac{R_L X^2 + j\left\{XR_L^2 + XX_L\left(X + X_L\right)\right\}}{R_L^2 + \left(X + X_L\right)^2}$$

For example, if we choose $X = -X_L$, the transformed load impedance $= X_L^2/R_L + jX_L$

which has strongly increased the resistive component from R_L to X_L^2/R_L;

the reactance X_L is unchanged;

and so the phase angle decreases from $\arctan\left(\frac{X_L}{R_L}\right) \cong \pm 90°$, to $\arctan\left(\frac{R_L}{X_L}\right) \cong 0°$.

What could this shunt reactance be? A stub?

A section of short - circuited (lossless) line is a pure reactance (Bleaney p275):

$Z_{stub} = jZ_{c(stub)} \tan(kz)$ where $Z_{c(stub)}$ is the characteristic impedance of the stub, length z, wavenumber k in the stub.

For $z < \lambda/4$, the stub is an inductance.

This has several advantages for transforming the reactor capacitance:

1) the stub can be short even with no dielectric (<< 75cm), by choice of its characteristic impedance:

$Z_{c(stub)} \cong Z_0 \cdot thickness/\left(2 \cdot width \cdot \sqrt{\varepsilon_r}\right)$.

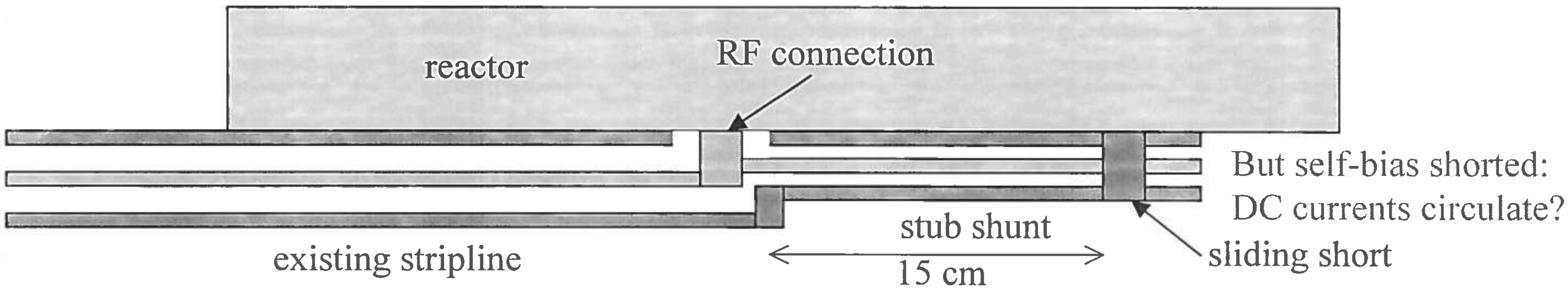
2) the short - circuited stub length is well - defined (unlike open stubs) and does not radiate.

3) the stub can be tuned by adjusting its length (with a sliding short, easy for no dielectric).

Example of a stub shunt for transforming the reactor impedance and increasing power transfer (but we lose the self-bias, and maybe cause DC circulating current in asymmetric reactors)

For 10 cm wide stripline with 1 mm spacing, and no dielectric insert,

$Z_{c(stub)} \cong 1.88\Omega$ and the calculated stub length $\cong 15$ cm for reflectionless matching.



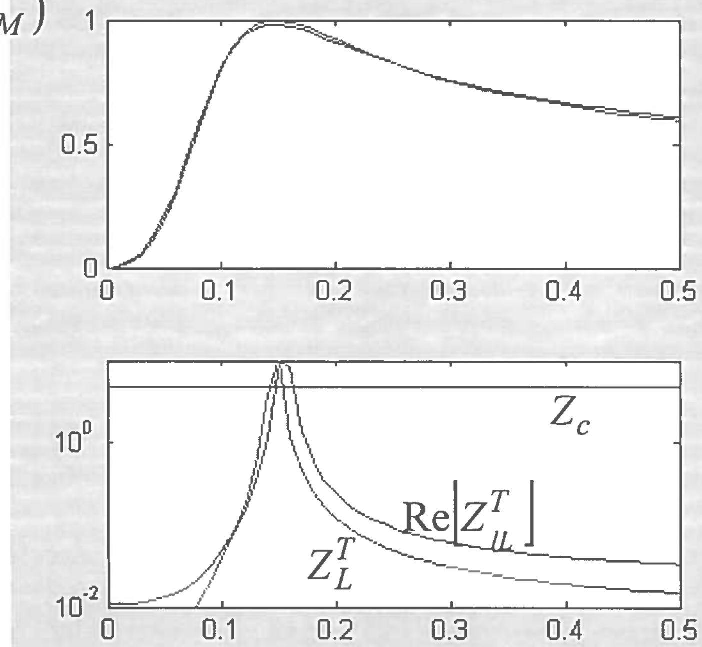
For $R_L = 0.01\,\Omega$ (vacuum)

$\underbrace{\max}_{\text{wrt Zg}}(P_L/P_M)$

vs

Shunt stub Length [m]

Excellent efficiency (and easier matching)

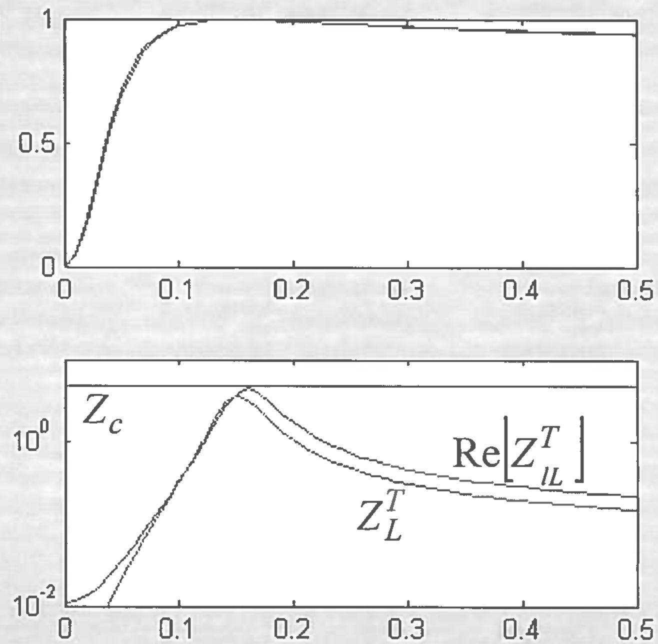


For $R_L = 0.1\,\Omega$ (plasma...) optimum stub length unchanged

$\underbrace{\max}_{\text{wrt Zg}}(P_L/P_M)$

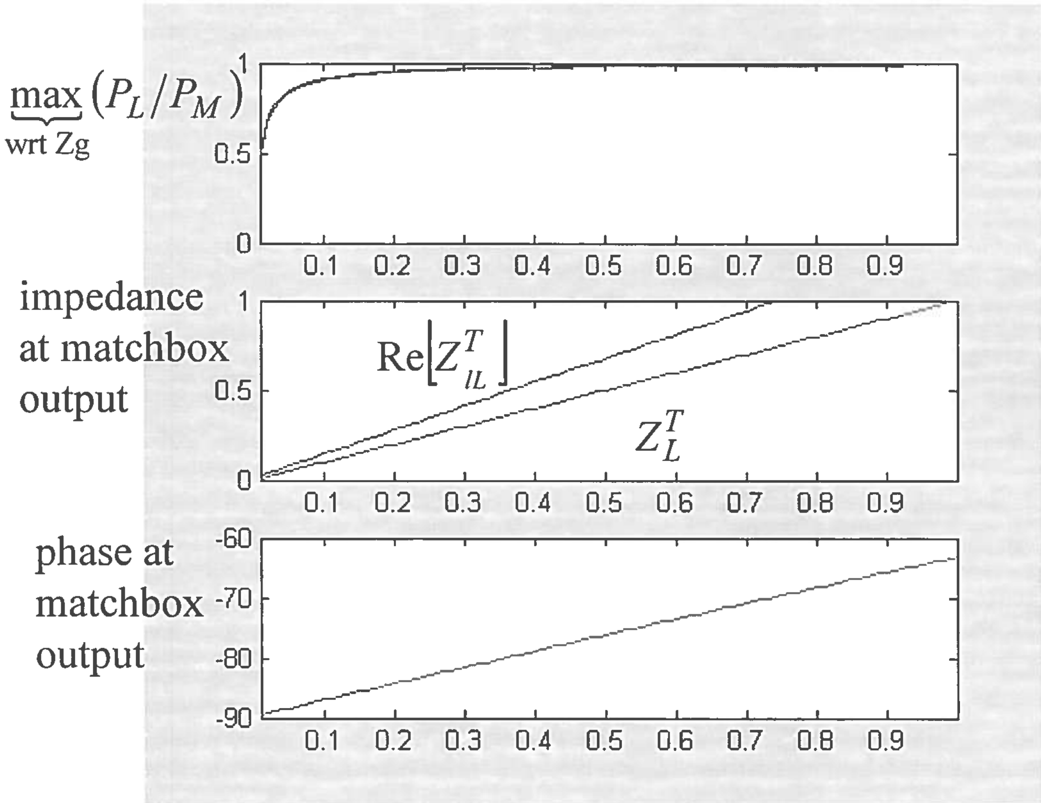
vs

Shunt stub Length [m]

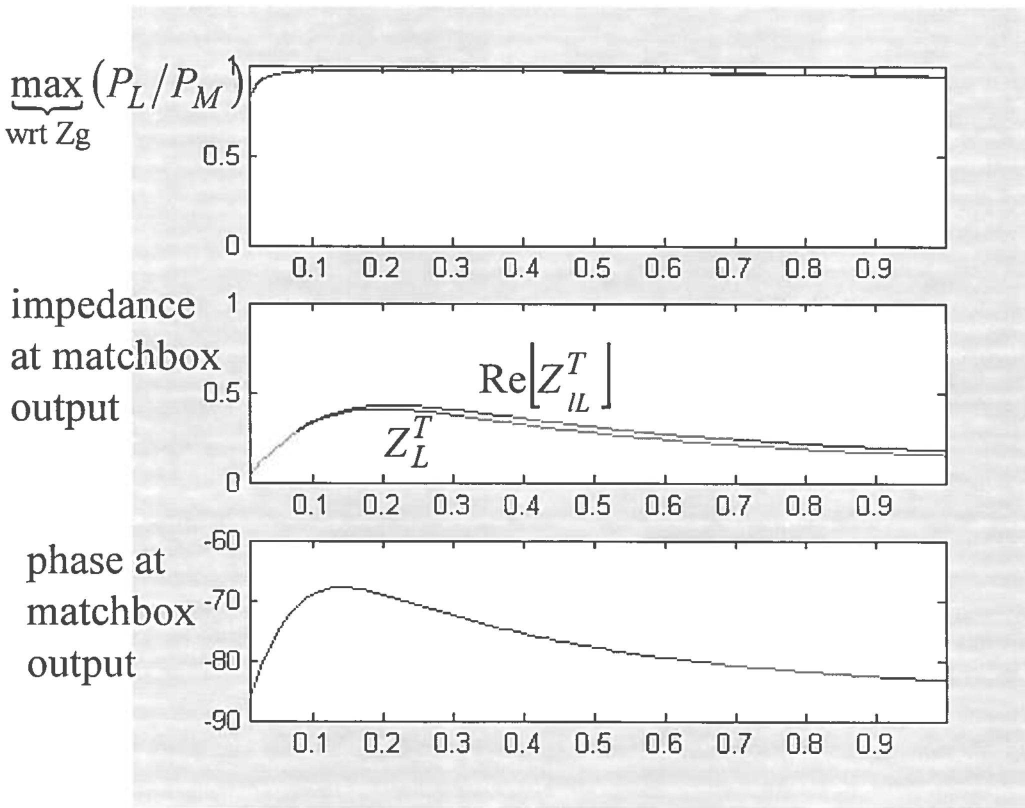


Example of a matching variation as the load resistance is varied due to plasma...

... with no shunt



... with 0.1 m shunt



hypothetical load resistance variation =0.01 to 1 ohms

CONCLUSIONS

- **The power transmission efficiency via a transmission line to a load can be expressed very simply as the ratio of the (load resistance) :(load resistance+half of line resistance) for an arbitrary length of line.**

- **The subtractive method for plasma power estimation is valid for constant current conditions.**

- **A good way to improve power transfer efficiency is to partially compensate the reactive load with a stub shunt.**

- **Edge and fringe field effects are negligible in calculating the stripline attenuation coefft. and line resistance.**