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Highlights

• We develop our models for product development cost and sales revenues. • We explicitly model diffusion dynamics. • We provide analytical results for the optimal frequency and parameter impacts. • For the extended model, we get a closed-form solution under special condition. • We prove the uniqueness of the optimal frequency under general conditions.

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Decision Support

On the optimal frequency of multiple generation product introductions

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ABSTRACT

This paper considers a firm that introduces multiple generations of a product to the market at regular intervals. We assume that the firm has only a single production generation in the market at any time. To maximize the total profit within a given planning horizon, the firm needs to decide the optimal frequency to introduce new product generations, taking into account the trade-off between sales revenues and product development costs. We model the sales quantity of each generation as a function of the technical decay and installed base effects. We analytically examine the optimal frequency for introducing new product generations as a function of these parameters.

1. Introduction

Products in competitive markets such as smart phones, tablets, computers, cameras, software, health and beauty products, and the like are usually offered as multiple generations. Various factors drive the development of successive product generations. First, the continuous and rapid technology improvements make it necessary to renew product generations frequently to stay competitive. Second, customers develop new needs over time. Third, in a relatively saturated market, new generation products can generate repeat purchases. For example, Elmer-DeWitt (2013) reports that “90 percent of iPhone 5S/5C buyers were upgrading from another version of the iPhone compared to 83 percent for the iPhone 5 launch and 73 percent for the iPhone 4S.” Erhun, Concalves, and Hopman (2007) point out that “managing the interplay between product generations can greatly increase the chances for success.” This is also supported by an empirical study across a wide range of industries in Morgan, Morgan, and Moore (2001), which shows that the introduction of multiple product generations is likely more profitable (26 percent higher) than a series of single-product generation introductions, and (40 percent higher) than a pure single-product generation introduction.

It appears that successive generations of many products are introduced in the market at regular time intervals. For example, Apple launched a new iPhone generation (around July–September) every year from 2007 to 2013. Likewise, between 2005 and 2013 a new generation of iPod Nano was introduced each September (except in 2011). Similarly, four generations of iPod touch were introduced each September from 2007 to 2010, and the fifth generation came to the market in October 2012. Moreover, in the automobile industry, Honda introduces a new generation of Accord each four to five years while Toyota brings a new generation of Lexus ES to the market circa every five years. This so-called time-pacing product development (PD) strategy has been widely recognized in the literature about other industries as well. Christensen (1997) shows that thanks to a time-pacing strategy, the medical technology company Medtronics was able to reduce uncertainty and improve the new PD process by eliminating requests for revisions to product features during the design process. Eisenhardt and Brown (1998) show that for rapidly shifting industries, a time-pacing PD strategy can improve the transition between new PD projects. Intel releases its chips with an approximately three-year cycle, and Morgan et al. (2001) point out that this strategy “allows it to profit from the investment it has made in developing and commercializing each generation while limiting competitions’ abilities to win sales”. Also, Souza, Bayus, and Wagner (2004) find that a time-pacing strategy “is not necessarily optimal, but generally does perform well under many conditions.” In this paper, we adopt the time-pacing PD strategy as a modeling assumption.

The process for phasing out an older product generation and introducing a new one in the market is called product rollover. A firm can choose one of two transition strategies during product rollover: phase-out transition or complete replacement. Using the phase-out strategy, old and new generations coexist in the market until sales of the old generation(s) drop to zero. Using the complete replacement strategy, a new generation product introduced in the market replaces in full the old generation product. These two strategies are also referred to as “dual-product roll” and “solo-product roll”,

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respectively (Billington, Lee, & Tang, 1998). In this paper, we assume that the firm adopts the complete replacement strategy. This assumption is supported: For example, Hewlett-Packard totally replaced DeskJet 500 printers with DeskJet 510 printers (Lim & Tang, 2006); Microsoft stops selling older software versions as soon as a new version is released; Google stopped selling Nexus 4 when launching Nexus 5 in September 2013, and so on. Consequently, the assumption of a complete replacement strategy is widely used in the literature (e.g., Arslan, Kachani, & Shmatov, 2009; Carrillo, 2005; Cohen, Eliashberg, & Ho, 1996, 2000).

We consider a firm that adopts a complete replacement strategy to introduce multiple generations of a product at regular time intervals within a given planning horizon. All product generations are assumed to be sold in the same geographical region and through the same channel. For each product generation, a PD cost is charged, and the sales quantity is related to the technical decay and the installed base effects. As technologies currently develop faster, the gap between the technology content of a certain product and the latest available technology increases over time. This gap precipitates the product gradually toward obsolescence and thus it loses its attractiveness to customers, we called this phenomenon "technical decay effect". We use the term "installed base effect" to refer to the combination of several social contagion effects: word-of-mouth, network effects, social preferences and observation learning (Narayanan & Nair, 2013). We consider diffusion dynamics by taking into account the installed base effect which allows the current sales rate to depend on the cumulative sales quantity.

The firm's objective is to maximize the sum of the profits of each product generation, which equals the sales revenue less the PD cost. To achieve the optimal total profit, it is important to decide on the optimal frequency of product introductions. If products are introduced too frequently, this may result in excessive PD costs. Moreover, as the time in the market is too short, each generation may experience poor sales, since there is insufficient time to build an installed base and reach peak sales. If a product generation stays in the market for too long, the technical decay effect may lead to a decrease in sales rate because customers are less willing to buy technically outdated products such as old generation computers with Intel 4004 chips for instance.

Our main contribution is to explicitly model diffusion dynamics and at the same time analytically study the optimal frequency of product introductions and its sensitivity to key model parameters. We model the PD cost based on the PD function in Druehl, Schmidt, and Souza (2009). To estimate product sales, we construct a primal sales model as a function of the various parameters mentioned above. We derive analytical results on the optimal frequency of product introductions and provide analytical sensitivity analysis of the impacts of different parameters on the optimal frequency and on the maximum total profit. Moreover, we extend our sales model, which allows a closed-form solution for the optimal frequency under some special conditions. We prove the uniqueness of the optimal frequency under general conditions. Finally we compare the sensitivity analyses between the primal and the extended sales models.

The rest of this paper is organized as follows. We review related literature in Section 2. In Section 3 we present the PD cost model, our primal sales model and the total profit function. In Section 4 we analyze the optimal product introduction frequency and parameter impacts. In Section 5, we present the extended sales model and analytical results. We conclude and discuss future research directions in Section 6. Proofs are provided in the Appendix. Proofs for Section 5 are provided as e-version due to the page limit.

2. Literature review

Our work is related to the literature on new product introduction (NPI). This literature has mainly focused on the product development and introduction of single product generation. Several papers consider multiple product generations and examine decisions during the product rollover as we do, by adopting "dual-product roll" or "solo-product roll" strategy (Billington et al., 1998).

Research focusing on single product generation introduction primarily studies the static trade-off between time-to-market and product performance (such as Bayus, 1997; Klastorin & Tsai, 2004; Krishnan & Ulrich, 2001; Savin & Terwiesch, 2005). Ozer and Uncu (2013) develop a dynamic decision-support tool to optimize the nested two-stage decisions on the time-to-market and product quantity for a component supplier. Ozer and Uncu (2015) extend their research to also integrate pricing and sales channels into decisions. Unlike their literature, the nature of our problem is such that multiple product generations are introduced to the market.

The research area of multiple generation products introduction can be classified into two steams according to the rollover strategies adopted. One stream assumes both old and new product generations to be sold during the transition period (dual-product roll). Studies in this stream consider the cannibalization effect or switch-over among old and new generations and address decision about time (e.g., Lim & Tang, 2006), price (e.g., Li & Graves, 2012), inventory quantity (e.g., Li, Graves, & Rosenfield, 2010), etc. Druehl et al. (2009) is the most closely related to our research. Both papers consider diffusion effect, adopt time-pacing strategy, examine the optimal pace of product introduction and analyze the parameter impacts. However, by adopting "dual-product roll" strategy and the Norton–Bass diffusion model, their model necessitates numerical approach due to the analytical complexity. Instead, under the "solo-product roll" assumption, our sales model keeps the analytical tractability, which differentiates the present paper from Druehl et al. (2009).

In the same vein as our research, another stream of the literature on multiple generation products introduction assumes a single generation in the market at any time (solo-product roll). Some papers examine product introduction decisions under competitive environment in a duopoly (e.g., Arslan et al., 2009; Cohen et al., 1996, 2000; Morgan et al., 2001; Souza, 2004; Souza et al., 2004), while others consider a monopoly as we do in our paper (e.g., Carrillo, 2005; Krankel, Duenyas, & Kapuscinski, 2006; Liu & Ozer, 2009; Wilhelm & Xu, 2002). Liu and Ozer (2009) is closely related to our work. We both show that the pace of technology evolution negatively impacts the firm's total profit, and a smaller product replacement cost encourages more product replacements. We model the relation between a product's profit and its performance gap (technical decay) in different ways; the product replacement cost in their model is fixed while our PD cost depends on the decision variable (product introduction frequency). More importantly, we consider the diffusion dynamics and explicitly discuss the impacts of diffusion speed and staff's specialization level on the optimal frequency and the total profit. However, unlike ours, they propose a model that helps a manager dynamically decide whether and when to adopt uncertain technological changes. Carrillo (2005) and Krankel et al. (2006) consider diffusion but they rely on numerical implementation and dynamic programming, respectively.

To the best of our knowledge, we are the first to analytically study the frequency of multiple generation product introductions while explicitly taking into account the diffusion effect. The diffusion effect has been widely observed in practice and extensively studied in the literature (Mahajan, Muller, & Bass, 1990; Meade & Islam, 2006). However, due to the analytical complexity of extant diffusion models for multiple generations (such as Mahajan & Muller, 1996; Norton & Bass, 1987), analytical results are not obtained by the literature of multiple generation product introduction considering the diffusion effect (such as Carrillo, 2005; Druehl et al., 2009; Krankel et al., 2006). We develop our sales model which considers diffusion and holds flexible shapes, and we provide analytical results for the optimal frequency and parameter impacts.

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3. Model

We consider a fixed planning horizon of length L (e.g., L months or years). We assume that the firm introduces a new product generation at constant time intervals T over the planning horizon L. Our model gives an explicit analytical expression of the optimal new product introduction frequency $n = \frac{L}{T}$, which is impacted by the PD cost and the cumulative sales of all product generations.

We use the following notations. All parameters are assumed to be positive.

Decision variable

n Frequency of new product introductions

Parameters

L Planning horizon
T Time between introduction of successive generations, $T = \frac{L}{n}$
t The time span since a product generation has been introduced, $0 \le t \le T$
$\lambda_i(t)$ Sales rate of product generation i after time t since its introduction in the market
N_i Sales quantity of product generation i
u Unit profit margin
a Sales rate scale parameter
β Technical decay effect parameter
γ Installed base effect parameter
D Scale parameter for PD cost curve
d First shape parameter for PD cost curve
f Second shape parameter for PD cost curve

Next we detail the analytical functions for the PD cost, the sales and the total profit.

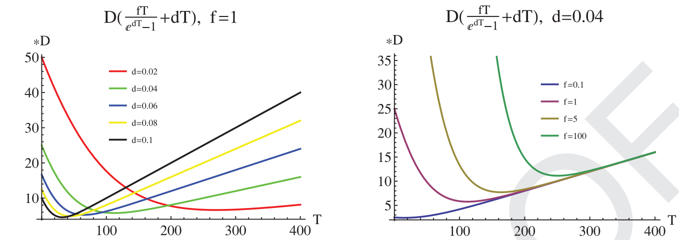
3.1. PD cost

We follow a standard assumption (Graves, 1989) that the trade-off between the PD cost for introducing a new product and its PD time is a "U-shaped" convex curve. That said, the PD cost grows when time is compressed as "crashing" the project requires more resource allocations such as training new team members. The PD cost also grows when the PD project is delayed because of decreasing motivation and additional setup cost as people move to other projects. This assumption is supported both empirically and theoretically in the literature (Bayus, 1997; Boehm, 1981; Graves, 1989).

Similar to Druehl et al. (2009), we assume all generations face the same PD cost curve and that the PD time per generation equals T. The "U-shaped" convex PD cost for each generation is given by

$$Cost(PD) = D\left(\frac{fT}{e^{dT}-1} + dT\right). \quad (1)$$

The parameter D represents the size of the overall development project, which may vary according to the industry, company and project. The parameter d can be interpreted as the staff's specialization level: highly specialized workers can finish the project within a shorter time span nevertheless it costs more to train and pay new workers (for PD project acceleration), as well as to switch them from other projects (due to PD project delay), that said the PD project is more cost sensitive with respect to time. Fig. 1(a) presents some samples of our PD cost curves associated with different values of d (given $f = 1$). We see that a higher d value corresponds to a steeper curve with a narrower bottom and a smaller optimal PD time (that associates with the minimum PD cost). The parameter f contributes to both the scale and the steepness of the PD cost, to allow more flexibility in fitting the shape of the PD cost curve. In Fig. 1(b) we show our PD cost curves for different values of f (with $d = 0.04$). We see that the value of f can be used to adjust the minimum cost as well as the associated time.



(a) PD cost curves with different values of d
(b) PD cost curves with different values of f

Fig. 1. Our product development cost curves.

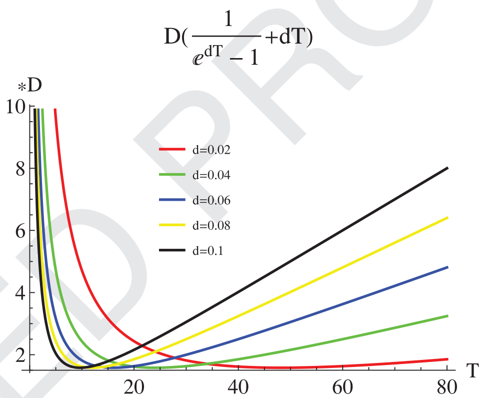


Fig. 2. PD cost curves of Druehl et al. (2009).

Our model is built based on the PD cost model of Druehl et al. (2009), which sets $fT = 1$:

$$Cost(PD) = D\left(\frac{1}{e^{dT}-1} + dT\right). \quad (2)$$

Fig. 2 presents the PD cost curves originated from Druehl et al. (2009) which uses the same values of d as in Fig. 1(a). We see that for a given shape parameter d, the PD curve in Fig. 2 is similar to that in Fig. 1(a), i.e., they both represent the empirically observed U-shape and a higher d value corresponds to a higher steepness of the convex PD curve. By setting $fT = 1$, all values of d yield the same PD cost minimum in their model. Our model provides more flexibility thanks to the additional parameter f. More importantly, it has more desirable mathematical properties as follows. We denote the sum of PD costs of n generations by $Cost(nPD)$. Given that $T = \frac{L}{n}$, we have:

$$Cost(nPD) = D * n * \left(\frac{fT}{e^{dT}-1} + dT\right) = D\left(\frac{fL}{e^{\frac{dL}{n}}-1} + dL\right). \quad (3)$$

Eq. (3) is an (increasing) convex function with respect to (WRT) n (see proof in Appendix A). The first order derivation of Eq. (3) WRT n is:

$$\frac{\partial Cost(nPD)}{\partial n} = \frac{DfL}{(e^{\frac{dL}{n}}-1)^2} e^{\frac{dL}{n}} \frac{dL}{n^2} \ge 0. \quad (4)$$

The first order derivation of n generations' PD cost using our model (Eq. (1)) is much simpler than that using Eq. (2) as a single-generation PD cost. This simplification helps to derive the explicit analytical expression of the optimal frequency n and the sensitivity analysis in Section 4. Moreover, it enables us to provide a closed-form solution of the optimal frequency in Section 5.

3.2. Sales

Note that the subscript i refers to the ith generation of new products introduced into the product market. We assume without loss of generality that the introduction of the ith generation is at time $(i-1)T$. We assume that the firm adopts complete replacement strategy. Let $\lambda_i(t)$ denote the sales rate of generation i at time t after its introduction

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$(0 \le t \le T)$, and let N_i denote the cumulative sales quantity of the ith generation through its product life cycle, we have $N_i = \int_0^T \lambda_i(t)dt$. In the following we introduce our sales model which considers diffusion and technical decay effects.

Let a denote the sales rate scale parameter. We add a negative technical decay effect $-\beta e^{\alpha t}$ because today's technologies change fast, and over time a product may progressively lose attractiveness because it becomes obsolete. The technical decay effect is well recognized and modeled in different ways in the literature. For example, Li and Graves (2012) assume a decreasing customer preference for the old product during inter-generational product transition; Liu and Ozer (2009) assume that a product's profit rate is a decreasing function of the performance gap between its underlying technology and the latest technology in the market. Souza (2004) assumes that product attraction decreases with respect to product age. In addition, we consider the installed base effect by assuming that the sales rate is proportional to the prior cumulative sales quantity. Installed base effect has formed the basis for the extensive aggregate diffusion literature in Marketing (Bass, 1969; Mahajan et al., 1990). This literature treats the entire population of past adopters as the reference group for a representative agent's product adoption decision. Narayanan and Nair (2013) investigate the identification and estimation of causal installed base effect in a linear model. Through an empirical analysis, they find a statistically significant and positive installed base effect in the adoption of the Toyota Prius Hybrid car.

The sales rate of the first generation $(i = 1)$ is thus defined as: $\lambda_1(t) = a - \beta e^{\alpha t} + \gamma \int_0^t \lambda_1(\tau)d\tau$, where β and α are the linear and exponential coefficients of technical decay effect, respectively, and γ indicates the rate of installed base effect. All the parameters are assumed to be constant and positive for different generations. In order to avoid the exceptional case that at $t = 0$ the technical decay effect is already $-\beta$, we can consider the parameter a as the scale value of a potential sales rate plus β.

Appendix B demonstrates that

$$\begin{aligned}\lambda_1(t) &= a - \beta e^{\gamma t} + \gamma \int_0^t \lambda_1(\tau)d\tau \\ &= (a - \beta - \gamma\beta t)e^{\gamma t}.\end{aligned} \tag{5}$$

Note that by parameter correction, we have $\alpha = \gamma$ thus γ appears in the technical decay effect function. This can be understood as: in a given market, if the diffusion speed is faster (γ increases), the diffusion may approach completion earlier ($\beta e^{\gamma t}$ is bigger thus sales slower down earlier).

From Eq. (5), we obtain the sales quantity of the first generation:

$$\begin{aligned}N_1 &= \int_0^T \lambda_1(\tau)d\tau = \frac{1}{\gamma}[\lambda_1(T) - (a - \beta e^{\gamma T})] \\ &= \frac{1}{\gamma}[(a - \gamma\beta T)e^{\gamma T} - a].\end{aligned} \tag{6}$$

Similarly, for the second generation $(i = 2)$, by using the results of Eqs. (5) and (6), we obtain the formulas for the sales rate $\lambda_2(t)$:

$$\begin{aligned}\lambda_2(t) &= a - \beta e^{\gamma t} + \gamma \int_0^t \lambda_2(\tau)d\tau + \gamma N_1 \\ &= \{a + [(a - \gamma\beta T)e^{\gamma T} - a] - \beta - \gamma\beta t\}e^{\gamma t} \\ &= [(a - \gamma\beta T)e^{\gamma T} - \beta - \gamma\beta t]e^{\gamma t},\end{aligned} \tag{7}$$

and the cumulative sales quantity of the first two generations:

$$\begin{aligned}N_1 + N_2 &= \frac{1}{\gamma}\{[(a - \gamma\beta T)e^{\gamma T} - \beta - \gamma\beta T]e^{\gamma T} - (a - \beta e^{\gamma T})\} \\ &= \frac{1}{\gamma}\{[(a - \gamma\beta T)e^{\gamma T} - \gamma\beta T]e^{\gamma T} - a\}.\end{aligned}$$

From Eq. (7) we can see that the sales rate is proportional to the cumulative sales quantity of both the current and previous generations. On the one hand, this is consistent with the "word-of-mouth effect" of the current generation in the Bass model (Bass, 1969) and the Norton–Bass model (Norton & Bass, 1987). On the other hand, we also take into account an installed base effect from previous generations, which can be interpreted as the social contagion effects between product generations. Or for consumers of very old generation products, if the internal influence or the social contagion effects are relatively small, the installed base effect between generations can be interpreted as including the number of consumers who renew their product (switching or repeat purchasing). This effect is not considered in the multi-generation Norton–Bass model (Norton & Bass, 1987), but represents the Apple example (Elmer-DeWitt, 2013) in the introduction very well.

For the jth generation, we give the general formulas of the sales rate $\lambda_j(t)$ and the cumulative sales quantity of the first j generations $\sum_{i=1}^{j} N_i$ as follows:

$$\lambda_j(t) = \left[a e^{\gamma(j-1)T} - \sum_{i=2}^{j} (\gamma\beta T) e^{\gamma(i-1)T} - \beta - \gamma\beta t \right] e^{\gamma t}, \tag{8}$$

$$\sum_{i=1}^{j} N_i = \frac{1}{\gamma}\left\{ \left(a - \frac{\gamma\beta T e^{\gamma T}}{e^{\gamma T} - 1} \right)(e^{\gamma j T} - 1) \right\}. \tag{9}$$

For any given generation j, we can also show the sales quantity expression for this generation as:

$$\begin{aligned}N_j &= \sum_{i=1}^{j} N_i - \sum_{i=1}^{j-1} N_i \\ &= \frac{1}{\gamma}\left(a - \frac{\gamma\beta T e^{\gamma T}}{e^{\gamma T} - 1} \right)(e^{\gamma j T} - e^{\gamma(j-1)T}) \\ &= \frac{1}{\gamma}[a(e^{\gamma T} - 1) - \gamma\beta T e^{\gamma T}]e^{\gamma(j-1)T}.\end{aligned}$$

The shape of our sales rate function is quite flexible. By adjusting the parameters a, γ and β, it is possible to plot different curve shapes. In Fig. 5(a), (c) and (e) (in Appendix G) we present some examples of our first generation sales rate curves.

In order to guarantee a positive sales rate, we have to assume that $\gamma\beta T \le a - \beta$. This assumption limits the maximum length of each generation, which is consistent with practice. If a product remains in the market for too long without renewal, it may become obsolete over time because of the technical decay. Thus it loses its attractiveness in the market (Souza, 2004), especially if there is strong competition.

Proposition 1. *If $\gamma\beta T \le a - \beta$, then $\lambda_i(t) \ge 0$ and $\lambda_{i+1}(t) \ge \lambda_i(t)$, $\forall 1 \le i \le n - 1, 0 \le t \le T$.*

Proposition 1 shows that the sales rate grows with successive generations. This is consistent with empirical results and the classic Norton–Bass Model (Norton & Bass, 1987). In Figs. 6 and 7 (in Appendix H) we present some examples of the first four generations' sales rates with different installed base effect levels ($\gamma = 0.3$ and 0.5, respectively). We can see that by adjusting the interplay among parameters a, β, γ and the scale of T, our model can represent the subsequent generations' sales rates growing with flexible shapes.

Proposition 2. *Given that $T = \frac{L}{n}$, let $y(n) = \sum_{i=1}^{n} N_i$ denote the cumulative sales quantity for the strategy of frequency n, $y(n)$ is strictly concave WRT n.*

Proposition 2 shows that introducing too few or too many product generations may diminish the cumulative sales quantity. For the former, sales are lost due to the technical decay effect; for the latter, each generation lacks the time to build the installed base to increase the sales.

The concavity of the cumulative sales quantity is a very useful property of our sales model. Because Druehl et al. (2009) use the Norton–Bass model to describe sales, they have to search for the optimal solution numerically because of the analytical complexity. Thanks to the concavity of our total sales quantity, we can provide an analytical expression of the optimal frequency of new generation introductions in Section 4.

In the NPI literature, for the sales rate of each product generation, some researchers such as Druehl et al. (2009) use the Bass diffusion model (Bass, 1969; Norton & Bass, 1987), others assume that the demand rate is constant over time (e.g., Cohen et al., 1996; Morgan et al., 2001), and still others develop new sales rate models as a function of price and/or reference price (e.g., Arslan et al., 2009; Lim & Tang, 2006), etc. In this section, we have developed a sales rate model by taking into account the technical decay and the diffusion effects. The shape of our sales rate function is flexible. More importantly, we prove the concavity of the cumulative sales quantity.

3.3. Total profit

The firm's objective is to maximize total profit, which results from the difference between the net revenues (cumulative sales quantity of all generations multiplied by its per-unit profit margin) and the total PD cost. Assume the unit profit margin u is constant over generations. Let $\Pi(n)$ denote the total profit over the whole planning horizon. We have:

$$\Pi(n) = uy(n) - \text{Cost}(nPD) = \frac{u}{\gamma}\left(a - \frac{\beta\frac{\gamma L}{n}e^{\frac{\gamma L}{n}}}{e^{\frac{\gamma L}{n}} - 1}\right)(e^{\gamma L} - 1) - D\left(\frac{fL}{e^{\frac{dL}{n}} - 1} + dL\right).$$

In this paper, we assume a constant unit profit margin u for all generations. In the literature, Morgan et al. (2001) and Krankel et al. (2006) also assume constant product margin across product generations. We give a discussion about cases where the profit margin increases or decreases over generations in Section 4.

In our model, we do not take the discount rate into account. In fact, Druehl et al. (2009) use more than 2000 scenarios to perform a detailed sensitivity analysis on the discount rate, and they conclude that "it does not significantly impact the optimal time between product introductions."

4. Optimal solution and impact of product development environment

In this section, we derive the optimal frequency of new product introductions and analyze the impacts of different parameters on the optimal frequency and on the maximum total profit.

Recall that $\text{Cost}(nPD)$ is convex and $y(n)$ is concave WRT n (as discussed in Sections 3.1 and 3.2, respectively), it is straightforward that:

Proposition 3. *Given the constant profit margin u, $\Pi(n)$ is a concave function WRT n. Let $G(n)$ denote the first order derivation of $\Pi(n)$ WRT n. We have:*

$$G(n) = \frac{\partial \Pi(n)}{\partial n} = u\beta(e^{\gamma L} - 1)\frac{(\frac{L}{n^2}e^{\frac{\gamma L}{n}})(e^{\frac{\gamma L}{n}} - 1 - \frac{\gamma L}{n})}{(e^{\frac{\gamma L}{n}} - 1)^2} - \frac{DfL}{(e^{\frac{dL}{n}} - 1)^2}e^{\frac{dL}{n}}\frac{dL}{n^2}.$$

The optimal solution n^* is thus the unique value (if it exists) which satisfies the first order condition (FOC) $G(n^*) = 0$. The optimal (integer) number of product generations to introduce is the ceiling or the floor of n^*.

We provide in below the impacts of all the parameters (concerning profit margin, sales, PD cost and planning horizon length) on the optimal frequency n^*.

Corollary 1.

(I) *The value n^* increases WRT unit profit margin u.*
(II) *Concerning the sales parameters, the value n^* increases WRT the technical decay effect β and the installed base effect γ; the sales rate scale parameter a has no impact on n^*.*
(III) *Concerning on the PD cost parameters, the value n^* increases WRT the first shape parameter d, decreases WRT the scale parameter D and the second shape parameter f.*
(IV) *The value n^* increases WRT the planning horizon length L.*

Intuitively, a higher margin per unit sold allows the firm to introduce more product generations because sales revenues are much greater than PD costs. Analytically, both the total sales quantity (concave) function and the n generations' PD cost (convex) function increase WRT n, and the optimal n corresponds to the intersection point of the sales revenue curve and the n generations' PD cost curve. If the margin increases, the sales revenue curve moves up, and its intersection point with the increasing PD cost curve corresponds to a bigger n^*.

For a given generation, a stronger technical decay effect β reduces the demand rate more quickly. Thus the firm would choose to introduce another generation when β is large. The installed base effect parameter γ in our model can be interpreted as a combination of the diffusion process parameter and the growth rate in the Norton–Bass model. Our analytical results for the installed base effect γ are also reflected in the numerical finding in Druehl et al. (2009) about their diffusion process parameter $(p + q)$ and their growth rate (g), which have a positive impact on product introduction frequency. The part $\frac{ua(e^{\gamma L}-1)}{\gamma}$ of the total profit can be considered as "potential fixed revenue," the sales rate scale parameter a does not influence the optimal number of product generations.

A larger scale value D leads to a higher PD cost per generation. It is thus intuitive that the firm tends to introduce fewer product generations when D is large. In terms of the shape parameter d, when it grows, the PD cost increases more sharply, which encourages the firm to speed up the new generation introduction. Both these analytical results are in line with the numerical findings in Druehl et al. (2009) about the impacts of D and d on n^*. For the second shape parameter f, a larger f brings a higher PD cost (see Fig. 1(b)) and it thus has a negative impact on n^*.

Due to the technical decay effect, the firm tends to introduce more product generations for a longer planning horizon. It is thus to be expected that n^* increases WRT the planning horizon length.

Now we analyze the parameters' impacts on the maximum total profit $\Pi(n^*)$.

Corollary 2.

(I) *The maximum total profit $\Pi(n^*)$ is increases WRT unit profit margin u.*
(II) *Concerning the sales parameters, $\Pi\ (n^*)$ decreases WRT the technical decay effect β, increases WRT the sales rate scale parameter a.*
(III) *Concerning on the PD cost parameters, $\Pi(n^*)$ decreases WRT the scale parameter D and the second shape parameter f, and is concave WRT the first shape parameter d.*
(IV) *$\Pi(n^*)$ increases WRT the planning horizon length L.*

If the unit profit margin decreases, even if the firm cuts its PD costs by introducing fewer product generations, it is still likely that the total profit will decrease. The maximum total profit decreases when the technical decay is more rapid. There are two reasons for this: More product generations lead to higher n generations' PD cost; at the same time, the sales quantity (sales revenue) decreases due to a faster technical decay. As a result, the total profit goes down. The maximum total profit increases with respect to the scale parameter a.

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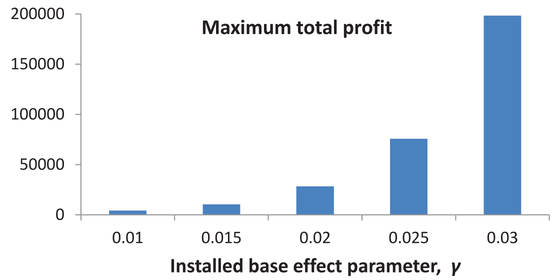

Fig. 3. Maximum total profit WRT installed base effect parameter γ.

This is obvious, because a bigger scale parameter a means a higher sales quantity when all other parameters stay the same.

Concerning the impacts of the PD cost parameters D and f, a larger value of D or f brings a higher PD cost, and thus has a negative impact on the optimal profit. The total profit is concave WRT d, which indicates that under a certain product development condition, there exists a staff's specialization level which is the most appropriate for a specific projet.

The result in (IV) is straightforward. Unless total profit increases with L, the firm will stop development and sales at a certain time.

Due to the analytical complexity, we numerically analyze the impact of the installed base effect parameter γ on the maximum total profit. We adopt the planning horizon length of Druehl et al. (2009): $L = 200$ months; the planning horizon is about 16 years. Without loss of generality, we consider the following parameter setting: $a = 14$, $u = 4$, $\beta = 10$, $D = 190$, $d = 0.02$ and $f = 0.08$. We consider five possible values of factor γ: 0.01, 0.015, 0.02, 0.025, 0.3. For each factor level, we compute the corresponding optimal value of n and the associated total profit. The results are presented in Fig. 3 where we can see that the maximum total profit increases with a higher installed base effect parameter γ. As mentioned before, the installed base effect parameter γ in our model can be interpreted as a combination of the diffusion process parameter and the growth rate in the Norton-Bass model. Our result is in keeping with the findings in Druehl et al. (2009) about these parameters ($p + q$ and g): as sales rise, the total profit increases.

We also numerically study the average yearly profit and the product introduction pace (i.e. average yearly product introduction frequency) with respect to L. We consider five possible values of L: 160, 180, 200, 220, 240. Similarly, for each value of L, we compute the corresponding n^* and the associated $\Pi(n^*)$. The results are presented in Fig. 4 where the left vertical axis corresponds to the average yearly profit $\Pi(n^*)/L$, and the right vertical axis corresponds to the product introduction pace n^*/L. We see that both values increase when L increases. Since the firm introduces more product generations for a longer planning horizon, and since the sales rate grows with successive generations (as discussed in Proposition 1), the average sales rate per year increases. The increased average sales revenue is greater than the increase in PD costs, consequently average profit per year increases. This accelerates the frequency of product introductions, and thus the yearly pace of product introduction increases for a longer planning horizon.

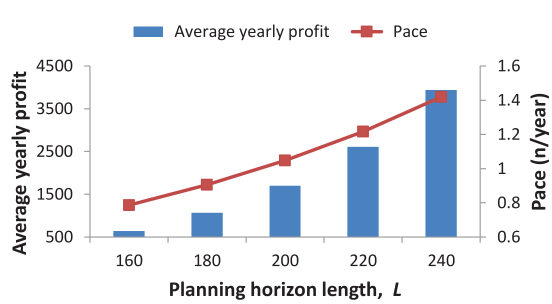

Fig. 4. Average yearly profit and product introduction pace WRT time length.

In this paper, we assume that the profit margin remains constant for the whole planning horizon. For cases where the profit margin increases or decreases over time, we also numerically examine the performance of our model. We find that when the profit margin decreases across generations and the sales rate scale parameter a is large, the sales revenues go down because of margin decrease, then increase thanks to the installed base effect. As a consequence, the total profit function does not remain concave with respect to n and we can no longer use the FOC to find n^*.

5. Extended sales model

In this section, we extend our sales functions presented in Section 3.2 into more general formulas. We keep all assumptions about the sales function in Section 3.2, except that for the technical decay effect, we add a linear effect $-\mu t$ in addition to the exponential effect $-\beta e^{\gamma t}$. The additional linear technical decay effect $-\mu t$ is a technicality which allows us to obtain a closed-form optimal solution under some special conditions.

We now present the functions of the sales rate and total sales quantity. For the first generation ($i = 1, 0 \le t' \le T, t = t' = 0$), the sales rate is:

$$\lambda_1(t) = a - \mu t - \beta e^{\gamma t} + \gamma \int_0^t \lambda_1(\tau) d\tau = \frac{\mu}{\gamma} + \left(a - \beta - \frac{\mu}{\gamma} - \gamma \beta t\right) e^{\gamma t}. \tag{10}$$

We can see that if $\mu = 0$, Eq. (10) equals Eq. (5).

From (10), the sales quantity at the end of time T is:

$$N_1 = \int_0^T \lambda_1(\tau) d\tau = \frac{1}{\gamma}[\lambda_1(T) - (a - \mu T - \beta e^{\gamma T})] = \frac{1}{\gamma}\left[\left(a - \beta - \frac{\mu}{\gamma} - \gamma \beta T\right) e^{\gamma T} - \left(a - \frac{\mu}{\gamma}\right) + \mu T + \beta e^{\gamma T}\right] = \frac{1}{\gamma}\left[\left(a - \frac{\mu}{\gamma} - \gamma \beta T\right) e^{\gamma T} - \left(a - \frac{\mu}{\gamma}\right) + \mu T\right].$$

For the jth generation, the general form of the sales rate $\lambda_j(t)$ is:

$$\lambda_j(t) = \frac{\mu}{\gamma} + \left[\left(a - \frac{\mu}{\gamma}\right) e^{\gamma (j-1)T} + \sum_{i=2}^{j} \mu t e^{\gamma (i-2)T} - \sum_{i=2}^{j} (\gamma \beta T) e^{\gamma (i-1)T} - \beta - \gamma \beta t\right] e^{\gamma t}. \tag{11}$$

The cumulative sales quantity for the first j generations $y(j)$ is:

$$y(j) = \frac{1}{\gamma}\left\{\left(a - \frac{\mu}{\gamma}\right)(e^{\gamma j T} - 1) - \frac{\gamma \beta T e^{\gamma T}}{e^{\gamma T} - 1}(e^{\gamma j T} - 1) + \mu T \frac{e^{\gamma L} - 1}{e^{\gamma T} - 1}\right\} = \frac{(a - \frac{\mu}{\gamma})}{\gamma}(e^{\gamma L} - 1) - \beta (e^{\gamma L} - 1)\frac{\frac{L}{j} e^{\frac{\gamma L}{j}}}{e^{\frac{\gamma L}{j}} - 1} + \frac{\mu}{\gamma}(e^{\gamma L} - 1)\frac{\frac{L}{j}}{e^{\frac{\gamma L}{j}} - 1}. \tag{12}$$

As mentioned above, the only difference between the primal and extended sales models is that the latter uses an additional linear function for the technical decay effect. Fig. 5 (in Appendix G) gives some examples of the first generation sales rates for the primal and extended models. Let $a - \beta = 1.8$, $\gamma \beta = 0.09$, we consider three different values of γ: 0.02, 0.18, 0.5 and three different values of μ: 0.1, 0.054, 0.29.

Table 1
The effects of different parameters on n_*.

Parameter model	The primal model	The extended model		
		$\beta > \frac{\mu}{\gamma}$	$\beta = \frac{\mu}{\gamma}$	$\beta < \frac{\mu}{\gamma}$
Profit Margin				
Profit Margin u	+	+	+	− +
Sales				
Technical decay effect β	+	+	+	− +
Installed base effect γ	+	+	+	− +
Sales rate scale parameter a	♯	♯	♯	♯
PD cost				
The first shape parameter d	+	+	+	− +
The second shape parameter f	−	−	−	+ −
The scale parameter D	−	−	−	+ −
Planning horizon length				
The planning horizon length L	+	+	+	− +

+ : positive effect; − : negative effect; ♯: no effect; + −: first positive then negative effect; − +: first negative then positive effect.

We can see that: for both models, depending on the parameter setting, the sales rates can be different shapes; and the shapes of the sales rates of the two models can be very similar. Given the same parameter setting, the sales rate of the extended model attenuates faster than that of the primal model because of the stronger technical decay effect. Intuitively, the sales rate of the extended model has more flexibility in terms of its shape thanks to an additional parameter μ. It can be used to describe the sales rate of a wider range of industries by adjusting all the parameters.

As in Section 4, the total profit over the planning horizon is:

$$\Pi(n) = uy(n) - Cost(nPD).$$

We denote the first order derivation of $\Pi(n)$ with respect to n by $G(n)$.

Proposition 4. *There is at most a unique value of $n^* \in [1, +\infty)$ that satisfies the FOC $G(n^*) = 0$. If $\beta = \frac{\mu}{\gamma}$,*

$$n^* = \frac{dL}{ln(\sqrt{\frac{z^2}{4} + z + 1} + \frac{z}{2})} \quad \text{with } z = \frac{DfdL}{u\beta(e^{\gamma L} - 1)}. \tag{13}$$

Note that if there is no value of $n \in [1, +\infty)$ that satisfies $G(n) = 0$, then the optimal n^* should be one of the two extreme points. Since for a fixed L, n^* cannot be infinity, it follows that $n^* = 1$. The proofs of Proposition 4 is available in Appendices I (in e-version).

Table 1 gives the associated sensitivity analyses of the primal sales model and the three cases of the extended sales model. It shows the effect on n^* of each of the parameters (concerning profit margin, sales function, PD cost and planning horizon length). We can see that the effects of different parameters on n^* associated with the primal sales model are exactly the same as those associated with the extended sales model with $\beta \geq \frac{\mu}{\gamma}$. For the case $\beta < \frac{\mu}{\gamma}$ in the extended sales model, the effects of all parameters reverse their directions once, because in this case, $\frac{\partial \Pi(n)}{\partial n}$ first increases then decreases with respect to n (Please see proof in Appendix J in e-version).

6. Conclusion

In this paper, we examine the optimal frequency of new generation product introductions assuming complete replacement and time-pacing strategies. We construct a new PD cost function based on the one in Druehl et al. (2009) and develop a primal sales quantity model by taking into account technical decay and diffusion effects. We analytically determine the optimal frequency of new generation product introductions, and provide an analytical study on the impacts of various parameters on the optimal frequency and on the maximum total profit. An extension based on our primal sales model is presented. This extended sales model enables us to obtain a closed-form solution for the optimal frequency under a special condition, and to prove the uniqueness of the solution for general conditions. We also provide a comparison between the two sales models in the associated sensitivity analysis. This is the first paper (to the best of our knowledge) to explicitly model diffusion dynamics and provide analytical results.

We have analytically shown that fast industrial technology evolution speeds up the product generation introduction, we thus expect companies in the electronics industry to have more frequent introductions than those in the sports equipment or health product industries. We also analytically demonstrate that fast industrial technology evolution may reduce the firm's total profit. For example, in the late 1980s, the computer industry suffered from a significant profit reduction while experiencing a fast pace of technology evolution (Lewis, 1989). In addition, we find that the diffusion speed positively impacts the product introduction frequency. In a given market, the diffusion process approaches completion and sales slow down earlier if the diffusion speed is higher, thus the firms tend to more frequently introduce new product generations. Thanks to the big diffusion effect, the cumulative sales quantity is large and so is the total profit.

We also find that a smaller PD cost encourages more frequent product generation introductions, which may partially explain why electronic product companies such as Apple more frequently introduce new product generations than companies in the automobile industry such as Honda and Toyota, as discussed in the introduction. A smaller PD cost leads to higher total profit, thus it is in the firms' interest to reduce PD cost, especially in fast changing industries. Moreover, under a certain product development environment, we see that a well-chosen staff's specialization level can increase the total profit for a specific project, and a high specialization level allows the firm to more frequently introduce new product generations. A possible implication of our results can be that if a firm aims to increases its profit, it is not necessary to hire over specialized PD staff; however, if the firm aims to speed up the product introduction frequency and negatively impact its competitors, it is helpful to hire highly specialized PD staff.

The analysis in this paper can be extended in several directions. First, by decomposing the profit margin to the unit price minus the unit cost, and setting the sales rate as price sensitive, the profit function is concave as to the unit price (thus probably jointly concave with respect to the unit price and n). It would be interesting to include price as an additional decision variable and analytically compare the result with our model. Second, Fig. 4 shows that the optimal introduction pace increases with respect to L. Our model assumes that the firm introduces a new product generation at constant time intervals T. Further work may relax this assumption by assuming decreasing time intervals $Te^{s(i-1)}$ with $s < 0$, for example, and search for the optimal values of s and T. Third, we assume that the product transition follows the complete replacement strategy, whereby only one product generation exists in the market at any time. In reality, successive generations may coexist at the transition period. It would be of interest to formalize the phase-out transition in our setting, despite the increasing analytical complexity. Lastly, we consider a single firm without considering competition or customer behavior. Future work could take these factors into account.

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Appendix A. Proof that Cost (n PD) is convex WRT n

To prove the convexity of Eq. (3), given its first order derivation Eq. (4), we show that its second order derivation on n is positive:

$$\frac{\partial^2 \text{Cost}(n\text{PD})}{\partial n^2} = DfL\left[\frac{e^{\frac{dL}{n}}\frac{dL}{n^2}}{(e^{\frac{dL}{n}}-1)^2}\right]' = DfL\frac{e^{\frac{dL}{n}}\frac{dL}{n^3}[\frac{dL}{n}(e^{\frac{dL}{n}}+1)-2(e^{\frac{dL}{n}}-1)]}{(e^{\frac{dL}{n}}-1)^3} \ge 0.$$

Let $x = \frac{dL}{n}$. We have $\frac{dL}{n}(e^{\frac{dL}{n}}+1)-2(e^{\frac{dL}{n}}-1) = x(e^x+1)-2(e^x-1)$. If $g(x) = x(e^x+1)-2(e^x-1) \ge 0$, then $\frac{\partial^2 \text{Cost}(n\text{PD})}{\partial n^2} \ge 0$. Since $g(0) = 0$, if we can prove that $g'(x) = \frac{\partial g(x)}{\partial x} \ge 0, \forall x \ge 0$, then we have $g(x) \ge 0$, $\forall x \ge 0$.

$g'(x) = \frac{\partial g(x)}{\partial x} = xe^x + e^x + 1 - 2e^x = xe^x - e^x + 1$ and $g'(0) = 0$. $\frac{\partial g'(x)}{\partial x} = e^x + xe^x - e^x = xe^x \ge 0$ for $x \ge 0$. Thus $g'(x)$ increases with respect to x, $g'(x) \ge 0$ for $x \ge 0$. Consequently, $g(x) \ge 0$, $x \ge 0$. Proved.

Appendix B. Proof of the formulas $\lambda_1(t)$

We define the sales rate of the first generation by $\lambda_1(t) = a - \beta e^{\alpha t} + \gamma\int_0^t \lambda_1(\tau)d\tau$. Assume that $\lambda_1(t) = A + Bt + Ce^{Dt} + Ete^{Ft}$ with A, B, C, D, E and F as parameters to be determined, we have:

$$A + Bt + Ce^{Dt} + Ete^{Ft} = a - \beta e^{\alpha t} + \gamma\left\{At + \frac{B}{2}t^2 + \frac{C}{D}(e^{Dt}-1) + \frac{E}{F}\left[te^{Ft} - \frac{1}{F}(e^{Ft}-1)\right]\right\}. \tag{B.1}$$

It is straightforward that $A = B = 0$. Equation (B.1) holds if $t = 0$ thus $C = a - \beta$. From $Ete^{Ft} = \gamma\frac{E}{F}te^{Ft}$ we have $F = \gamma$. Substitute the values of C and F in $Ce^{Dt} = -\beta e^{\alpha t} + \gamma\frac{C}{D}e^{Dt} - \frac{E}{\gamma}e^{Ft}$ we can find two groups of possible values of (D, E): (1) $D = \gamma$, $E = -\gamma\beta$ with $\alpha = \gamma$; (2) $D = \alpha$, $E = 0$ with $\alpha = \gamma\frac{a-\beta}{a}$. With the parameters in (2), $\lambda_1(t)$ is monotone with respect to t. As we aim to model more complex sales rates, we choose the parameters in (1) thus $\lambda_1(t) = (a - \beta - \gamma\beta t)e^{\gamma t}$.

Appendix C. Proof of Proposition 1

If $\gamma\beta T \le a - \beta$, it is obvious that $\lambda_1(t) \ge 0$, $\forall t \le T$.

For $i = 2$, $\lambda_2(t) = [(a - \gamma\beta T)e^{\gamma T} - \beta - \gamma\beta t]e^{\gamma t}$. Given that $\gamma\beta T \le a - \beta$, we have:

$$\begin{aligned}&(a-\gamma\beta T)e^{\gamma T} - \beta - \gamma\beta T\\ &\ge (a-\gamma\beta T)e^{\gamma T} - a = a(e^{\gamma T}-1) - \gamma\beta Te^{\gamma T}\\ &\ge (\beta+\gamma\beta T)(e^{\gamma T}-1) - \gamma\beta Te^{\gamma T} = \beta(e^{\gamma T}-1-\gamma T) \ge 0,\end{aligned} \tag{C.1}$$

because $\beta \ge 0$ and $e^{\gamma T} - 1 - \gamma T \ge 0$ (using Taylor series). From (C.1) we also see that $(a - \gamma\beta T)e^{\gamma T} \ge a$, so $\lambda_2(t) \ge \lambda_1(t)$ is proved.

For $i \ge 2$, we now prove that $\lambda_{i+1}(t) \ge \lambda_i(t)$. From Eq. (8), this therefore proves that $ae^{\gamma iT} - \sum_{j=2}^{i+1}(\gamma\beta T)e^{\gamma(j-1)T} \ge ae^{\gamma(i-1)T} - \sum_{j=2}^{i}(\gamma\beta T)e^{\gamma(j-1)T}$. Equally,

$$\begin{aligned}ae^{\gamma iT} - \gamma\beta T\tfrac{e^{\gamma(i+1)T}-1}{e^{\gamma T}-1} &\ge ae^{\gamma(i-1)T} - \gamma\beta T\tfrac{e^{\gamma iT}-1}{e^{\gamma T}-1},\\ ae^{\gamma(i-1)T}(e^{\gamma T}-1) &\ge \gamma\beta T\tfrac{e^{\gamma iT}}{e^{\gamma T}-1}(e^{\gamma T}-1).\end{aligned}$$

From (C.1) we see that $a(e^{\gamma T}-1) - \gamma\beta Te^{\gamma T} \ge 0$. Proved.

Appendix D. Proof of Proposition 2

The first order derivation of $y(n)$ is

$$\frac{\partial y(n)}{\partial n} = \beta(e^{\gamma L}-1)\left\{\frac{(\frac{L}{n^2}e^{\frac{\gamma L}{n}})(e^{\frac{\gamma L}{n}}-1-\frac{\gamma L}{n})}{(e^{\frac{\gamma L}{n}}-1)^2}\right\} = \beta(e^{\gamma L}-1)g_1(n)g_2(n), \tag{D.1}$$

with $g_1(n) = \frac{\frac{L}{n^2}e^{\frac{\gamma L}{n}}}{e^{\frac{\gamma L}{n}}-1} \ge 0$, $g_2(n) = \frac{e^{\frac{\gamma L}{n}}-1-\frac{\gamma L}{n}}{e^{\frac{\gamma L}{n}}-1} \ge 0$. If we can prove that both functions $g_1(n)$ and $g_2(n)$ strictly decrease with respect to n, then $\frac{\partial y(n)}{\partial n}$ decreases with respect to n, consequently $y(n)$ is strict concave with respect to n.

For function g_1, we have $\frac{\partial g_1(n)}{\partial n} = \frac{e^{\frac{\gamma L}{n}}\frac{L}{n^3}}{(e^{\frac{\gamma L}{n}}-1)^2}(\frac{\gamma L}{n} - 2e^{\frac{\gamma L}{n}} + 2)$. We now prove that $\frac{\gamma L}{n} - 2e^{\frac{\gamma L}{n}} + 2 < 0$. Let $f(x) = x - 2e^x + 2$ with $x = \frac{\gamma L}{n}$. We have $f(0) = 0$ and $f'(x) = 1 - 2e^x < 0$, $\forall x > 0$. So we have $f(x) < 0$, $\forall x > 0$. Function $g_1(n)$ decreases with respect to n is proved.

For function g_2, we have $\frac{\partial g_2(n)}{\partial n} = -\frac{\frac{\gamma L}{n^2}}{(e^{\frac{\gamma L}{n}}-1)^2}[\frac{\gamma L}{n}e^{\frac{\gamma L}{n}} + 1 - e^{\frac{\gamma L}{n}}]$. We now prove that $\frac{\gamma L}{n}e^{\frac{\gamma L}{n}} + 1 - e^{\frac{\gamma L}{n}} > 0$. Let $f(x) = xe^x + 1 - e^x$ with $x = \frac{\gamma L}{n}$. We have $f(0) = 0$ and $f'(x) = e^x + xe^x - e^x > 0$, $\forall x > 0$. So we have $f(x) > 0$, $\forall x > 0$. Consequently function $g_2(n)$ strictly decreases with respect to n is proved.

Since both functions $g_1(n)$ and $g_2(n)$ strictly decrease with respect to n, their product $g_1(n) * g_2(n)$ strictly decreases with respect to n too. Then $\frac{\partial y(n)}{\partial n}$ strictly decreases with respect to n. The strict concavity of $y(n)$ with respect to n is proved.

Appendix E. Proof of Corollary 1

Recall that $G(n) = u\beta(e^{\gamma L}-1)\frac{(\frac{L}{n^2}e^{\frac{\gamma L}{n}})(e^{\frac{\gamma L}{n}}-1-\frac{\gamma L}{n})}{(e^{\frac{\gamma L}{n}}-1)^2} - \frac{DfL}{(e^{\frac{dL}{n}}-1)^2}e^{\frac{dL}{n}}\frac{dL}{n^2}$. For any parameter x, its impact on n^* (the implicit function $n^*(x)$ as a function of x) is given by the equation $G(n^*, x) = 0$ and $\frac{\partial n^*(x)}{\partial x} = -\frac{\frac{\partial G(n^*,x)}{\partial x}}{\frac{\partial G(n^*,x)}{\partial n^*}}$. Given the strict concavity of $\Pi(n)$, we have $\frac{\partial G(n^*,x)}{\partial n^*} < 0$.

(I) $\forall n$, $\frac{\partial G(n,u)}{\partial u} = \beta(e^{\gamma L}-1)\{\frac{(\frac{L}{n^2}e^{\frac{\gamma L}{n}})(e^{\frac{\gamma L}{n}}-1-\frac{\gamma L}{n})}{(e^{\frac{\gamma L}{n}}-1)^2}\} \ge 0$, thus $\frac{\partial n^*(u)}{\partial u} \ge 0$.

(II) Following the proof in (I), we have $\frac{\partial n^*(\beta)}{\partial\beta} \ge 0$. We now prove that $\frac{\partial G(n^*,\gamma)}{\partial\gamma} \ge 0$, i.e., $\frac{\partial n^*(\gamma)}{\partial\gamma} \ge 0$. Let $G(n^*, r) = (e^{\gamma L}-1)G_2(\gamma)$ with $G_2(\gamma) = \frac{e^{\frac{\gamma L}{n}}(e^{\frac{\gamma L}{n}}-1-\frac{\gamma L}{n})}{(e^{\frac{\gamma L}{n}}-1)^2} \ge 0$. First, it is obvious that $e^{\gamma L}-1$ increases with respect to γ. Second, for $G_2(\gamma)$ we have $\frac{\partial G_2(\gamma)}{\partial\gamma} = \frac{e^{\frac{\gamma L}{n}}\frac{L}{n}}{(e^{\frac{\gamma L}{n}}-1)^3}[\frac{\gamma L}{n}e^{\frac{\gamma L}{n}} + \frac{\gamma L}{n} - 2e^{\frac{\gamma L}{n}} + 2] \ge 0$. The reason is as follows: Let $g(x) = xe^x + x - 2e^x + 2$ with $x = \frac{\gamma L}{n}$. We have $g(0) = 0$; $g'(x) = xe^x - e^x + 1$ and $g'(0) = 0$; $g''(x) = xe^x \ge 0$, $\forall x \ge 0$. So $g'(x)$ increases with respect to x, $g'(x) \ge 0$, $\forall x \ge 0$. Consequently, $g(x)$ increases with respect to x, $g(x) \ge 0$, $\forall x \ge 0$. As a result, $G(n^*, \gamma)$ increases with respect to γ, we have $\frac{\partial G(n^*,\gamma)}{\partial\gamma} \ge 0$, thus $\frac{\partial n^*(\gamma)}{\partial\gamma} \ge 0$.

The sales rate scale parameter a does not show up in the function $G(n)$, therefore they have no effect on n^*.

(III) $\forall n$, $\frac{\partial G(n,d)}{\partial d} = \frac{e^{\frac{dL}{n}}}{(e^{\frac{dL}{n}}-1)^3}[e^{\frac{dL}{n}}\frac{dL}{n} + \frac{dL}{n} - e^{\frac{dL}{n}} + 1] \ge 0$. The reason is as follows: Let $g(x) = e^x x + x - e^x + 1$ with $x = \frac{dL}{n}$. We have $g(0) = 0$; $g'(x) = e^x x \ge 0$, $\forall x \ge 0$. Thus we have $g(x) \ge 0$, $\forall x \ge 0$. As a consequence, $\frac{\partial n^*(d)}{\partial d} = -\frac{\partial G(n^*,d)}{\partial d} / \frac{\partial G(n^*,d)}{\partial n^*} \ge 0$. It is straightforward that $\frac{\partial G(n^*,D)}{\partial D} \le 0$, $\frac{\partial G(n^*,f)}{\partial f} \le 0$. thus n^* decreases WRT D and f.

(IV) Let $G_A(n, L) = e^{\gamma L} - 1$, $G_B(n, L) = \frac{\frac{L}{n^2}e^{\frac{\gamma L}{n}}}{e^{\frac{\gamma L}{n}}-1}$, $G_C(n, L) = \frac{e^{\frac{\gamma L}{n}}-1-\frac{\gamma L}{n}}{e^{\frac{\gamma L}{n}}-1}$ and $G_D(n, L) = -\frac{DfL}{(e^{\frac{dL}{n}}-1)^2}e^{\frac{dL}{n}}\frac{dL}{n^2}$. We have $G_A(n, L)$, $G_B(n, L)$, $G_C(n, L) \ge 0$ and $G(n, L) = u\beta G_A(n, L)G_B(n, L)G_C(n, L) + G_D(n, L)$.

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Obviously, $G_A(n, L)$ increases WRT L. $G_B(n^*, L)$ also increases WRT L because $\frac{\partial G_B(n,L)}{\partial L} = \frac{e^{\frac{\gamma L}{n}}\frac{1}{n^2}}{(e^{\frac{\gamma L}{n}}-1)^2}[e^{\frac{\gamma L}{n}} - \frac{\gamma L}{n} - 1] \geq 0$. For $G_C(n, L)$, we have $\frac{\partial G_C(n,L)}{\partial L} = \frac{\frac{\gamma}{n}}{(e^{\frac{\gamma L}{n}}-1)^2}[\frac{\gamma L}{n}e^{\frac{\gamma L}{n}} - e^{\frac{\gamma L}{n}} + 1] \geq 0$ by using the result from proof of Proposition 2 that $\frac{\gamma L}{n}e^{\frac{\gamma L}{n}} - e^{\frac{\gamma L}{n}} + 1 \geq 0$. For $G_D(n^*, L)$, we have $\frac{\partial G_D(n^*,L)}{\partial L} = \frac{DdfLe^{\frac{dL}{n}}}{(e^{\frac{dL}{n}}-1)^3 n^2}[\frac{\gamma L}{n}e^{\frac{\gamma L}{n}} + \frac{\gamma L}{n} - 2e^{\frac{\gamma L}{n}} + 2] \geq 0$ by using the result from proof of Corollary 1 (II) that $\frac{\gamma L}{n}e^{\frac{\gamma L}{n}} + \frac{\gamma L}{n} - 2e^{\frac{\gamma L}{n}} + 2 \geq 0$.

As a result, $G(n^*, L)$ increases with respect to L, $\frac{\partial n^*(L)}{\partial L} = -\frac{\frac{\partial G(n^*,L)}{\partial L}}{\frac{\partial G(n^*,L)}{\partial n^*}} \geq 0$. Proved.

Appendix F. Proof of Corollary 2

Recall that $\Pi(n) = u\frac{1}{\gamma}\{(m - \frac{\gamma\beta Te^{\gamma T}}{e^{\gamma T}-1})(e^{\gamma nT} - 1)\} - D(\frac{fL}{e^{\frac{dL}{n}}-1} + dL)$. Because of the FOC in Proposition 3 ($\frac{\partial\Pi(n^*)}{\partial n^*} = 0$), the impact of any parameter x on $\Pi(n^*)$ is

$$\frac{\partial\Pi(n^*)}{\partial x} = \frac{\partial\Pi(n^*, x)}{\partial x} + \frac{\partial\Pi(n^*, x)}{\partial n^*}\frac{\partial n^*(x)}{\partial x} = \frac{\partial\Pi(n^*, x)}{\partial x},$$

where we write $\Pi(n^*, x)$ to express the total profit as a function of both n^* and x. The impacts of u, a, β, D, d and f on $\Pi(n^*)$ are straightforward. As for the impact of L, unless total profit increases with L, the firm will stop development and sales at a certain time.

Appendix G. Comparison of the first generation sales rate between the primal and extended sales models

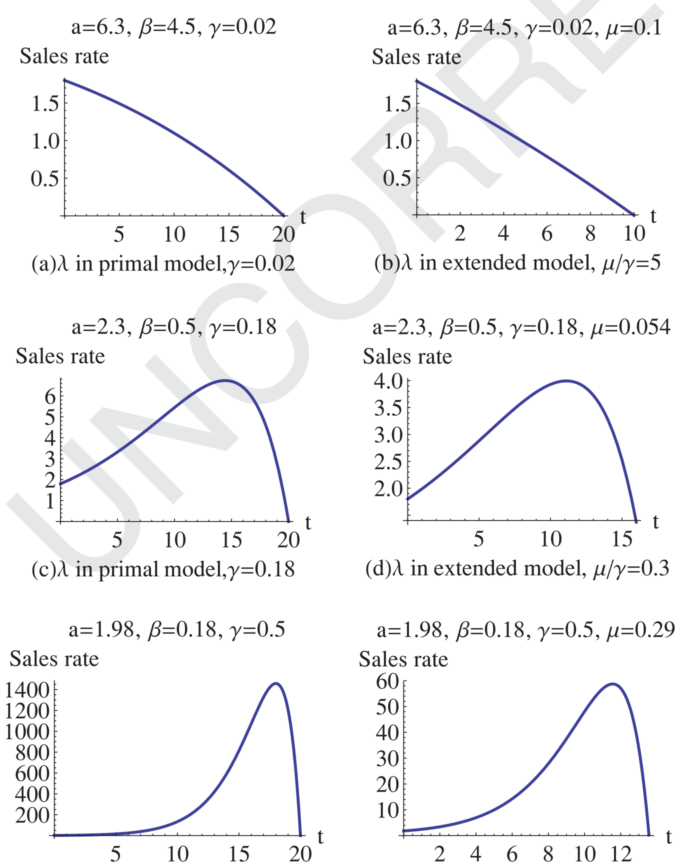

Fig. 5. Examples of sales rates for the primal and extended models.

Appendix H. Examples of successive generations sales rates

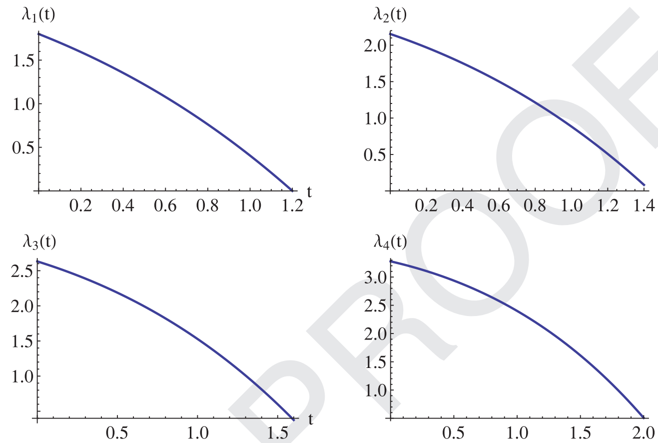

Fig. 6. Successive generations sales rates with $a = 6.8$, $\beta = 5$ and $\gamma = 0.3$.

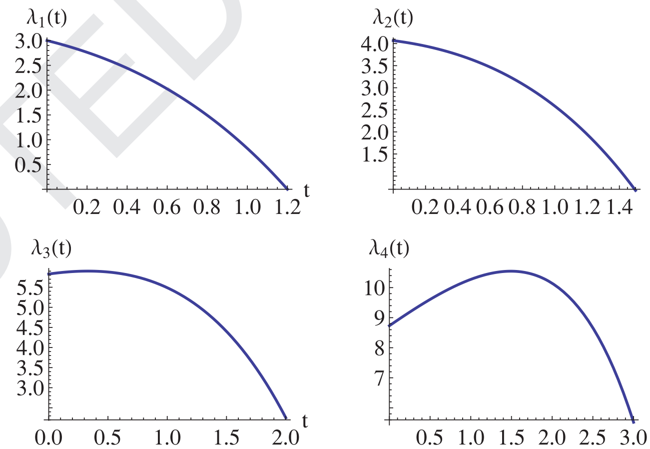

Fig. 7. Successive generations sales rates with $a = 8$, $\beta = 5$ and $\gamma = 0.5$.

Supplementary materials

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.ejor.2015.03.041.

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