Towards an integrated approach for demand forecasting and vehicle routing in recyclable waste collection

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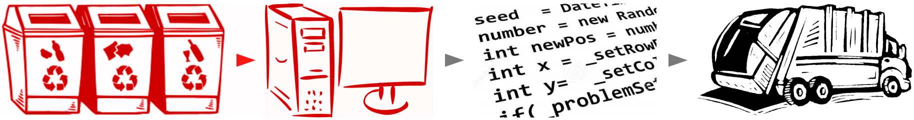

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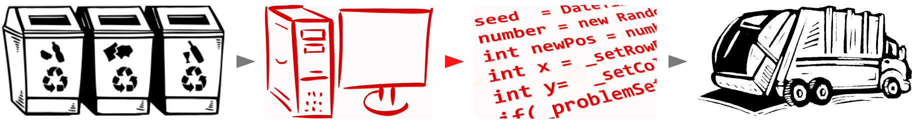

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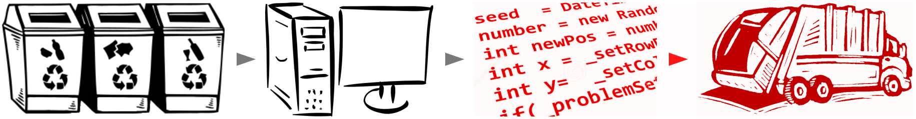

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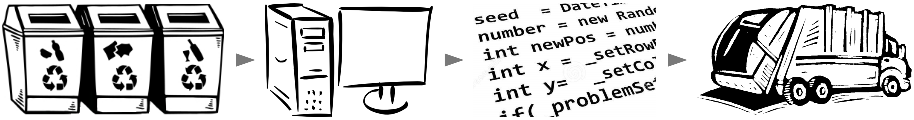
- Level data is used for container selection and vehicle routing, with tours *often planned several days in advance*
- Vehicles are dispatched to carry out the daily schedules produced by the routing algorithm
- Efficient waste collection thus depends on the ability to:
  - make **good forecasts** of the container levels at the time of collection
  - and **optimally route** the vehicles to serve the selected containers



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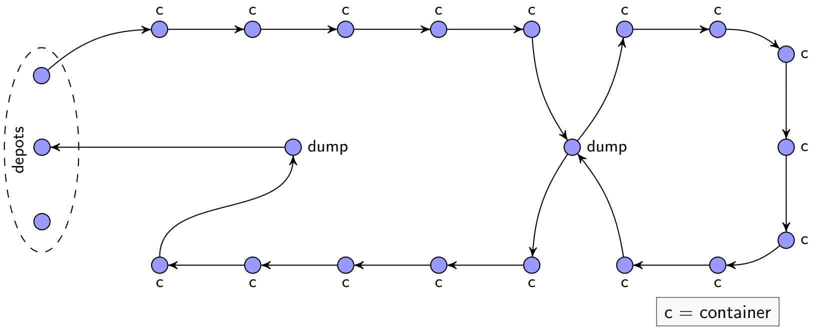
  - practiced in sparsely populated rural areas

- **There is a heterogeneous fixed fleet**
  - different volume and weight capacities, speeds, costs, etc...

Problem description

Figure 1: Tour illustration



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- VRP with intermediate facilities (VRP-IF):
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- Heterogeneous fixed fleet VRP:
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- Flexible assignment of depots:
  - Kek et al. (2008)

Contributions

- Integration of dynamic destination depot assignment into the VRP-IF
  - consideration of relocation costs
- Integration of heterogeneous fixed fleet into the VRP-IF
  - challenges posed by intermediate facility visits
- Benchmarking to several classes of simpler problems from the literature and state of practice:
  - E-VRPTW (modified from Schneider et al., 2014)
  - MDVRPI (Crevier et al., 2007)
  - optimal solutions, state of practice, etc...

Formulation

Sets

O'	$=$ set of origins	O''	$=$ set of destinations
D	$=$ set of dumps	P	$=$ set of containers
N	$= O' \cup O'' \cup D \cup P$	K	$=$ set of vehicles

Parameters

π_{ij}	$=$ length of edge (i, j)
α_{ijk}	$=$ 1 if edge (i, j) is accessible for vehicle k, 0 otherwise
τ_{ijk}	$=$ travel time of vehicle k on edge (i, j)
ε_i	$=$ service duration at point i
$[\lambda_i, \mu_i]$	$=$ time window lower and upper bound at point i
H	$=$ maximum tour duration
η	$=$ maximum continuous work limit after which a break is due
δ	$=$ break duration
ρ_i^v, ρ_i^w	$=$ volume and weight pickup quantity at point i
Ω_k^v, Ω_k^w	$=$ volume and weight capacity of vehicle k
ϕ_k	$=$ fixed cost of vehicle k
β_k	$=$ unit-distance running cost of vehicle k
θ_k	$=$ unit-time wage rate of vehicle k
Ψ	$=$ weight of relocation cost term

Formulation

Decision variables: binary

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ traverses edge } (i,j) \\ 0 & \text{otherwise} \end{cases}$$

$$z_{ijk} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are, respectively, the origin and destination of vehicle } k \\ 0 & \text{otherwise} \end{cases}$$

$$b_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ takes a break on edge } (i,j) \\ 0 & \text{otherwise} \end{cases}$$

$$y_k = \begin{cases} 1 & \text{if vehicle } k \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

Decision variables: continuous

S_{ik} = start-of-service time of vehicle k at point i

Q^v_{ik} = cumulative volume on vehicle k at point i

Q^w_{ik} = cumulative weight on vehicle k at point i

Formulation

$$\min \quad r = \sum_{k \in K} \left(\phi_k y_k + \beta_k \sum_{i \in N} \sum_{j \in N} \pi_{ij} x_{ijk} + \theta_k \left(\sum_{j \in O''_k} S_{jk} - \sum_{i \in O'_k} S_{ik} \right) \right) \tag{1}$$

$$+ \Psi \sum_{k \in K} \sum_{i \in O'_k} \sum_{j \in O''_k} \left(\beta_k \pi_{ji} + \theta_k \tau_{jik} \right) z_{ijk}$$

$$\text{s.t.} \quad \sum_{k \in K} \sum_{j \in D \cup P} x_{ijk} = 1, \qquad \forall i \in P \tag{2}$$

$$\sum_{i \in O'_k} \sum_{j \in N} x_{ijk} = y_k, \qquad \forall k \in K \tag{3}$$

$$\sum_{i \in D} \sum_{j \in O''_k} x_{ijk} = y_k, \qquad \forall k \in K \tag{4}$$

$$\sum_{i \in N} x_{ijk} = 0, \qquad \forall k \in K, j \in O' \cup (O'' \setminus O''_k) \tag{5}$$

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$$\sum_{i \in N:\, i \neq j} x_{ijk} = \sum_{i \in N:\, i \neq j} x_{jik}, \qquad \forall k \in K, j \in D \cup P \tag{7}$$

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$$x_{ijk} \leqslant \alpha_{ijk}, \qquad \forall k \in K, i \in O'_k \cup P \cup D, j \in P \cup D \cup O''_k \tag{9}$$

$$\rho_i^v \leqslant Q_{ik}^v \leqslant \Omega_k^v, \qquad \forall k \in K, i \in P \tag{10}$$

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$$S_{ik} + \varepsilon_i + \delta b_{ijk} + \tau_{ijk} \leqslant S_{jk} + M \left(1 - x_{ijk}\right), \qquad \forall k \in K, i \in O'_k \cup P \cup D, j \in P \cup D \cup O''_k \tag{16}$$

$$\lambda_i \sum_{j \in N} x_{ijk} \leqslant S_{ik}, \qquad \forall k \in K, i \in O'_k \cup P \cup D \tag{17}$$

$$S_{jk} \leqslant \mu_j \sum_{i \in N} x_{ijk}, \qquad \forall k \in K, j \in P \cup D \cup O''_k \tag{18}$$

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$$\text{s.t.} \left(S_{ik} - \sum_{m \in O'_k} S_{mk} \right) + \varepsilon_i - \eta \leqslant M\left(1 - b_{ijk}\right), \quad \forall k \in K, i \in O'_k \cup P \cup D, j \in P \cup D \cup O''_k \quad (20)$$

$$\eta - \left(S_{jk} - \sum_{m \in O'_k} S_{mk} \right) \leqslant M\left(1 - b_{ijk}\right), \quad \forall k \in K, i \in O'_k \cup P \cup D, j \in P \cup D \cup O''_k \quad (21)$$

$$b_{ijk} \leqslant x_{ijk}, \quad \forall k \in K, i, j \in N \quad (22)$$

$$\left(\sum_{j \in O''_k} S_{jk} - \sum_{i \in O'_k} S_{ik} \right) - \eta \leqslant (\mathrm{H} - \eta) \sum_{i \in N} \sum_{j \in N} b_{ijk}, \quad \forall k \in K \quad (23)$$

$$x_{ijk}, b_{ijk}, y_k \in \{0, 1\}, \quad \forall k \in K, i, j \in N \quad (24)$$

$$z_{ijk} \in \{0, 1\}, \quad \forall k \in K, i \in O', j \in O'' \quad (25)$$

$$Q^v_{ik}, Q^w_{ik}, S_{ik} \geqslant 0, \quad \forall k \in K, i \in N \quad (26)$$

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$$\left(\sum_{j \in O''_k} S_{jk} - \sum_{i \in O'_k} S_{ik} \right) - \eta \leqslant (\mathrm{H} - \eta) \sum_{i \in N} \sum_{j \in N} b_{ijk}, \quad \forall k \in K \quad (23)$$

$$x_{ijk}, b_{ijk}, y_k \in \{0, 1\}, \quad \forall k \in K, i, j \in N \quad (24)$$

$$z_{ijk} \in \{0, 1\}, \quad \forall k \in K, i \in O', j \in O'' \quad (25)$$

$$Q^v_{ik}, Q^w_{ik}, S_{ik} \geqslant 0, \quad \forall k \in K, i \in N \quad (26)$$

Solution methodology: Exact approach

- We apply variable fixing and valid inequalities
- Impossible traversals:

$$x_{iik} = 0, \qquad \forall k \in K, i \in N \tag{27}$$

$$x_{ijk} = 0, \qquad \forall k \in K, i \in O'_k, j \in D \cup O''_k \tag{28}$$

$$x_{ijk} = 0, \qquad \forall k \in K, i \in P, j \in O''_k \tag{29}$$

$$x_{ijk} = 0, \qquad \forall k \in K, i \in D, j \in D \colon i \neq j \tag{30}$$

- Time-window infeasible traversals:

$$x_{ijk} = 0, \qquad \forall k \in K, i \in O'_k \cup P \cup D, j \in P \cup D \cup O''_k : \lambda_i + \varepsilon_i + \tau_{ijk} > \mu_j \tag{31}$$

- Bounds on time:

$$\sum_{j \in O''_k} S_{jk} - \sum_{i \in O'_k} S_{ik} \geqslant \sum_{i \in N} \sum_{j \in N} x_{ijk} (\varepsilon_i + \tau_{ijk}), \qquad \forall k \in K \tag{32}$$

$$S_{ik} \leqslant \max_{m \in P} (\mu_m - \tau_{imk}) \, y_k, \qquad \forall k \in K, i \in O'_k \tag{33}$$

$$S_{jk} \geqslant \min_{m \in D} (\lambda_m + \varepsilon_m + \tau_{mjk}) \sum_{m \in D} x_{mjk}, \qquad \forall k \in K, j \in O''_k \tag{34}$$

Solution methodology: Exact approach

- Symmetry breaking for subsets K' of identical vehicles:

$$\sum_{i \in P} \sum_{j \in P \cup D} \rho_i^v x_{ijk'_g} \geqslant \sum_{i \in P} \sum_{j \in P \cup D} \rho_i^v x_{ijk'_{g+1}}, \qquad \forall g \in 1, \ldots, (|K'| - 1) \qquad (35)$$

- Symmetry breaking for replications of the same dump D':

$$\sum_{i \in P} i x_{ij'_g k} \leqslant \sum_{i \in P} i x_{ij'_{g+1} k}, \qquad \forall k \in K, g \in 1, \ldots, (|D'| - 1) \qquad (36)$$

- Bounds on dump visits:

$$\sum_{i \in P} x_{ijk} \leqslant 1, \qquad \forall k \in K, j \in D \qquad (37)$$

$$\sum_{i \in D} \sum_{j \in P} x_{ijk} \leqslant \min\left(|D| - 1, |P| - 1\right), \qquad \forall k \in K \qquad (38)$$

Solution methodology: Heuristic approach

- To solve instances of realistic size, we developed a heuristic algorithm
- It constructs a feasible initial solution using an insertion procedure
- It improves the initial solution through a multiple neighborhood search procedure admitting intermediate infeasibility with a dynamically evolving penalty

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- Periodically, we restart from the best feasible solution because feasibility may be hard to restore
- Periodically, we also reassign dump visits and evaluate vehicle reassignments because the fleet is heterogeneous and fixed

Results: Modified Schneider et al. (2014) instances

- 36 instances derived from the Solomon (1987) VRPTW instances
- 3 groups of 12 instances with 5, 10, and 15 customers
- Number of recharging stations: 2 to 8

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  - use 2 vehicle classes with different capacities, costs, and site dependencies
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- We compare the heuristic against the mathematical model
- For each instance, the heuristic is run 10 times

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		Heuristic			Solver on model with valid inequalities				Solver on model without valid inequalities			
Instance	Vehicles	Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improve-ment(%)	Objective	MIP Gap(%)	Runtime (s.)	Improve-ment(%)
c101C5	4	489.70	489.70	0.05	489.70	0.00	0.39	0.00	489.70	35.71	7200.01	0.00
c103C5	2	281.33	281.33	0.04	268.09	0.00	0.17	-4.94	268.09	0.00	4910.40	-4.94
c206C5	2	374.67	374.67	0.06	360.09	0.00	0.24	-4.05	360.09	0.00	90.27	-4.05
c208C5	2	343.20	343.20	0.06	343.20	0.00	0.49	0.00	343.20	38.57	7200.04	0.00
r104C5	1	182.81	182.81	0.02	182.81	0.00	0.04	0.00	182.81	0.00	1.90	0.00
r105C5	2	251.15	251.15	0.05	251.15	0.00	0.08	0.00	251.15	0.00	0.22	0.00
r202C5	1	176.52	176.52	0.02	176.52	0.00	0.05	0.00	176.52	0.00	2.67	0.00
r203C5	1	228.05	228.05	0.03	228.05	0.00	0.12	0.00	228.05	0.00	1504.85	0.00
rc105C5	2	327.19	327.19	0.05	327.19	0.00	0.15	0.00	327.19	25.27	7200.04	0.00
rc108C5	2	345.87	345.87	0.04	345.87	0.00	0.15	0.00	345.87	0.00	2069.22	0.00
rc204C5	1	223.17	223.17	0.09	223.17	0.00	0.17	0.00	223.17	0.00	1327.76	0.00
rc208C5	1	212.67	212.67	0.02	212.67	0.00	0.25	0.00	212.67	0.00	1156.35	0.00

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c101C10	6	846.10	846.10	0.61	837.13	0.00	5489.48	-1.07	846.10	77.21	7200.48	0.00
c104C10	3	456.86	456.86	0.43	456.86	0.00	37.28	0.00	456.86	53.05	7200.08	0.00
c202C10	4	549.74	549.74	0.42	549.74	18.44	7200.09	0.00	549.74	67.71	7200.31	0.00
c205C10	4	568.92	568.92	0.58	568.58	0.00	2788.37	-0.06	568.92	64.77	7200.11	0.00
r102C10	3	391.14	391.14	0.40	391.14	0.00	158.70	0.00	391.14	47.69	7200.16	0.00
r103C10	2	288.67	288.67	0.50	288.67	0.00	18.39	0.00	288.67	43.72	7200.04	0.00
r201C10	2	310.16	310.16	0.45	310.16	0.00	45.22	0.00	310.16	43.64	7200.46	0.00
r203C10	2	329.78	329.78	1.13	329.78	0.00	5757.28	0.00	329.78	47.26	7200.08	0.00
rc102C10	3	534.75	534.75	0.40	534.75	0.00	6.25	0.00	534.75	38.77	7200.09	0.00
rc108C10	2	429.79	429.79	0.42	429.79	0.00	6.94	0.00	429.79	25.30	7200.09	0.00
rc201C10	2	502.45	502.45	0.40	499.88	0.00	147.43	-0.51	502.45	58.48	7200.09	0.00
rc205C10	2	428.80	428.80	0.45	421.36	0.00	26.00	-1.77	428.80	40.52	7201.11	0.00

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c103C15	5	823.82	823.82	0.92	823.82	34.38	7200.18	0.00	823.82	73.45	7200.84	0.00
c106C15	5	653.46	653.46	0.69	653.46	17.67	7200.19	0.00	653.46	63.86	7200.07	0.00
c202C15	6	932.30	932.30	0.77	932.30	36.39	7200.23	0.00	932.30	68.58	7200.51	0.00
c208C15	5	725.23	725.23	1.55	725.23	25.75	7200.17	0.00	725.23	68.69	7200.38	0.00
r102C15	5	678.40	678.40	0.83	678.40	27.89	7200.17	0.00	678.40	64.94	7200.22	0.00
r105C15	3	462.52	462.52	0.70	462.52	0.00	56.82	0.00	462.52	53.50	7200.10	0.00
r202C15	3	528.59	535.08	1.41	528.59	30.25	7200.11	0.00	528.59	54.05	7200.95	0.00
r209C15	2	369.29	371.60	1.26	369.29	7.10	7200.11	0.00	369.29	37.62	7201.49	0.00
rc103C15	3	556.87	556.87	0.83	556.87	16.41	7200.10	0.00	556.87	58.12	7200.06	0.00
rc108C15	3	510.41	511.03	1.19	510.41	3.47	7200.07	0.00	510.41	49.31	7200.14	0.00
rc202C15	3	601.71	601.71	1.30	598.83	27.55	7200.18	-0.48	601.71	58.77	7200.24	0.00
rc204C15	2	421.54	422.22	5.67	421.54	25.44	7201.01	0.00	421.54	49.29	7201.67	0.00
Average		453.82	454.10	0.66	452.43	7.52	2803.97	-0.36	453.05	39.11	5707.60	-0.25

Results: Modified Schneider et al. (2014) instances

Table 1: Heuristic vs solver on modified Schneider et al. (2014) instances

Instance	Vehicles	Heuristic			Solver on model with valid inequalities				Solver on model without valid inequalities			
		Best	Average	Runtime avg(s.)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)	Objective	MIP Gap(%)	Runtime (s.)	Improvement(%)
c103C15	5	823.82	823.82	0.92	823.82	34.38	7200.18	0.00	823.82	73.45	7200.84	0.00
c106C15	5	653.46	653.46	0.69	653.46	17.67	7200.19	0.00	653.46	63.86	7200.07	0.00
c202C15	6	932.30	932.30	0.77	932.30	36.39	7200.23	0.00	932.30	68.58	7200.51	0.00
c208C15	5	725.23	725.23	1.55	725.23	25.75	7200.17	0.00	725.23	68.69	7200.38	0.00
r102C15	5	678.40	678.40	0.83	678.40	27.89	7200.17	0.00	678.40	64.94	7200.22	0.00
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Results: Crevier et al. (2007) instances

- 22 instances, with a limited homogeneous fleet stationed at one depot
- All depots can act as intermediate facilities
- BKS by Hemmelmayr et al. (2013)
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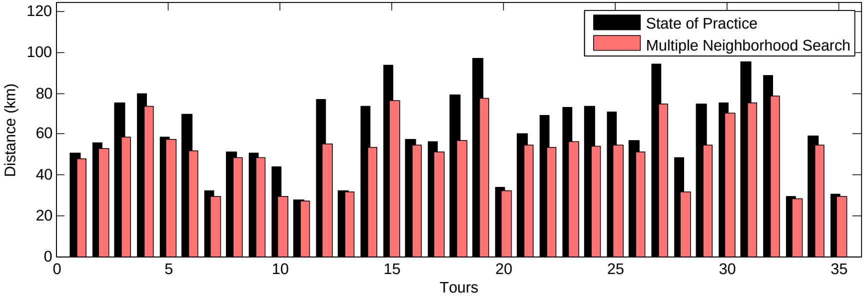
- Optimizing the home depot and the destination depot, we obtain:
  - **1.37%** average savings over 10 runs
  - **2.54%** savings in the best case

Results: Comparison to the state of practice

- 35 tours planned by specialized software for the canton of Geneva
- 7 to 38 containers per tour, up to 4 dump visits per tour
- MNS heuristic improves tours by **1.73%** to **34.91%**, on avg **14.75%**
- Extrapolating annually, cost reductions of at least **USD 300'000**

Figure 2: Comparison to the state of practice (average of 10 runs per tour)



Contents

State of the art

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  - Inventory levels in pharmacies (Nolz et al., 2011, 2014)
  - Recyclable materials from old cars (Krikke et al., 2008)
  - Charity donation banks (McLeod et al., 2013)
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  - Charity donation banks (McLeod et al., 2013)
  - Waste container levels (Johansson, 2006; Faccio et al., 2011; Mes, 2012; Mes et al., 2014)
- Contribution:
  - Operational level forecasting rather than critical levels
  - Estimated and validated on real data, compared to most of the literature which uses simulated data

Methodology

- Let n_{itk} denote the number of deposits in container i on day t of size q_k. The data generating process of the daily quantities is as follows:

$$Q_{it}^{\star} = \sum_{k=1}^{K} n_{itk} q_k \tag{39}$$

- Let $n_{itk} \xrightarrow{\text{iid}} \mathcal{P}\left(\lambda_{itk}\right)$ and have a probability π_{itk}. Then we obtain:

$$\mathbb{E}\left(Q_{it}^{\star}\right) = \sum_{k=1}^{K} q_k \lambda_{itk} \pi_{itk} \tag{40}$$

- We minimize the sum of squared differences between observed Q_{it} and expected $\mathbb{E}\left(Q_{it}^{\star}\right)$ over all containers N and days T:

$$\min_{\boldsymbol{\lambda},\boldsymbol{\pi}} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(Q_{it} - \sum_{k=1}^{K} q_k \lambda_{itk} \pi_{itk}\right)^2 \tag{41}$$

assuming strict exogeneity

Methodology

- Given vectors of covariates $\mathbf{x}_{it}$ and $\mathbf{z}_{it}$ and vectors of parameters $\boldsymbol{\beta}_k$ and $\boldsymbol{\gamma}_k$, we define Poisson rates and logit-type probabilities:

$$\lambda_{itk}(\boldsymbol{\theta}) = \exp\left(\mathbf{x}_{it}^{\mathrm{T}}\boldsymbol{\beta}_k\right) \tag{42}$$

$$\pi_{itk}(\boldsymbol{\theta}) = \frac{\exp\left(\mathbf{z}_{it}^{\mathrm{T}}\boldsymbol{\gamma}_k\right)}{\sum_{j=1}^{K}\exp\left(\mathbf{z}_{it}^{\mathrm{T}}\boldsymbol{\gamma}_j\right)} \tag{43}$$

- Then, in compact form, the minimization problem writes as:

$$\min_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} \sum_{i=1}^{N}\sum_{t=1}^{T}\left(Q_{it} - \sum_{k=1}^{K}\frac{\exp\left(\mathbf{x}_{it}^{\mathrm{T}}\boldsymbol{\beta}_k + \mathbf{z}_{it}^{\mathrm{T}}\boldsymbol{\gamma}_k + \ln(q_k)\right)}{\sum_{j=1}^{K}\exp\left(\mathbf{z}_{it}^{\mathrm{T}}\boldsymbol{\gamma}_j\right)}\right)^2 \tag{44}$$

- $\boldsymbol{\Theta} := (\boldsymbol{\beta}_k, \boldsymbol{\gamma}_k : \forall k)$, and $\boldsymbol{\gamma}_{k^\star} = \mathbf{0}$ for one arbitrarily chosen $k^\star$
- We will refer to this minimization problem as the *mixture model*

Methodology

- In case of only one deposit quantity, it degenerates to a pseudo-count data process:

$$\min_{\boldsymbol{\theta}\in\boldsymbol{\Theta}} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(Q_{it} - \exp\left(\mathbf{x}_{it}^{\mathsf{T}}\beta + \ln(q)\right)\right)^2 \qquad (45)$$

- We will refer to this minimization problem as the *simple model*

Data

- 36 containers for PET in the canton of Geneva with capacity of 3040 or 3100 liters
- Balanced panel covering March to June, 2014 (122 days), which brings the total number of observations to 4392

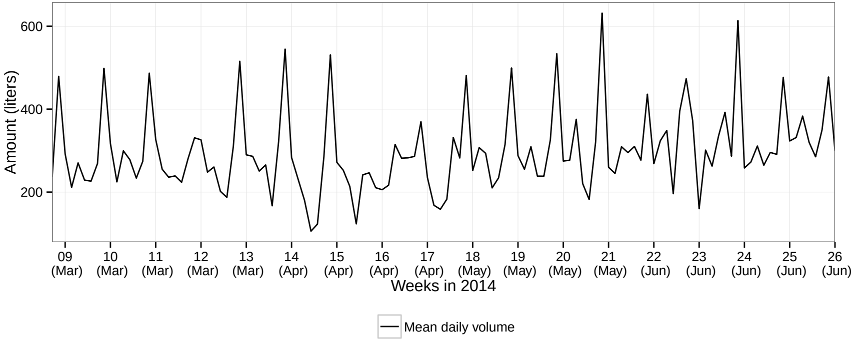
Data

- 36 containers for PET in the canton of Geneva with capacity of 3040 or 3100 liters
- Balanced panel covering March to June, 2014 (122 days), which brings the total number of observations to 4392
- The final sample excludes unreliable level data (removed after visual inspection)
- Missing data is linearly interpolated for the values of Q_{it}

Seasonality pattern

- Waste generation exhibits strong weekly seasonality
- Peaks are observed during the weekends
- There also appear to be longer-term effects for months

Figure 3: Mean daily volume deposited in the containers



Covariates

- Based on the above observations, we use the following covariates
- They are all used both for $\mathbf{x}_{it}$ (rates) and $\mathbf{z}_{it}$ (probabilities)

Table 2: Table of covariates

Variable	Type
Container fixed effect	dummy
Day of the week	dummy
Month	dummy
Minimum temperature in Celsius	continuous
Precipitation in mm	continuous
Pressure in hPa	continuous
Wind speed in kmph	continuous

Evaluating the fits

- Coefficient of determination

$$R^2 = 1 - \frac{SS_{\text{res}}}{SS_{\text{tot}}} \tag{46}$$

  with higher values for a better model
- Akaike information criterion (AIC):

$$\text{AIC} = \left(\frac{SS_{\text{res}}}{\#obs}\right) \exp\left(\frac{2 * \#params}{\#obs}\right) \tag{47}$$

  with lower values for a better model. The exponential penalizes model complexity
- SS_{res} is the residual sum of squares
- SS_{tot} is the total sum of squares

Estimation on full sample

- Mixture model: R^2 of **0.341** (AIC **52900**) with 5L and 15L
- Simple model: R^2 of **0.300** (AIC **53700**) with 10L

Table 3: Estimated coefficients of mixture model

	$\hat{\beta}_1$ (5L)***	$\hat{\beta}_2$ (15L)***	$\hat{\gamma}_2$***
Minimum temperature in Celsius	1461.356	0.022	-0.037
Precipitation in mm	-0.821	-0.009	0.018
Pressure in hPa	-13.724	-0.001	0.010
Wind speed in kmph	7.580	-0.004	0.020
Monday	402.235	2.166	-9.693
Tuesday	1908.233	2.293	-9.977
Wednesday	-844.662	1.432	0.202
Thursday	1937.385	1.198	1.453
Friday	1876.162	1.239	4.419
Saturday	-6981.339	1.358	4.723
Sunday	1831.715	1.905	2.832
March	-27.136	2.955	-1.453
April	1071.406	2.746	-1.532
May	1689.979	2.988	-1.603
June	-2604.520	2.901	-1.452

Validation

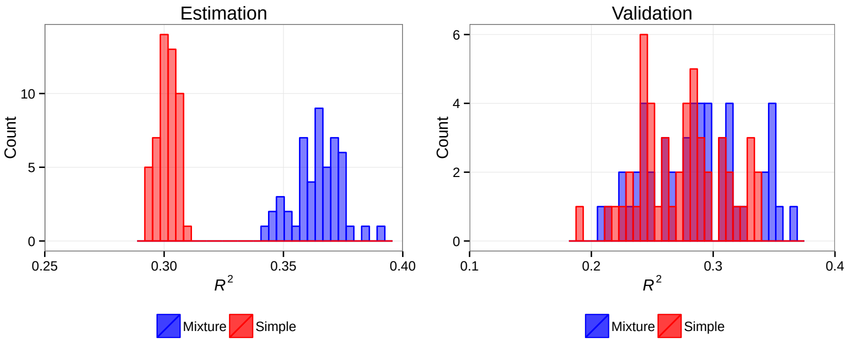
- 50 experiments
- The mixture and the simple model are estimated on a random sample of 90% of the panel
- They are validated on the remaining 10%
- In both cases the values are significantly different at 90% confidence level

Table 4: Mean R^2 for estimation and validation sets

	Mixture model mean R^2	Simple model mean R^2
Estimation	0.364 (AIC 51400)	0.302 (AIC 53600)
Validation	0.286	0.274

Validation

Figure 4: Histograms for estimation and validation samples



Contents

Inventory routing

- The inventory routing problem (IRP) is a complex logistical problem with three simultaneous decisions:
  - when to serve a customer
  - how much to deliver/collect
  - how to combine customers into vehicle tours

- Studied since the 80s, starting with the works of Bell et al. (1983), Golden et al. (1984), Dror and Ball (1987), Dror et al. (1985), Trudeau and Dror (1992), etc...

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- In many cases, the IRP is solved for a short planning horizon with the purpose of minimizing longer-term costs
- Therefore, an important ingredient is a link between the short term and the long term

Parameters

- Rolling horizon of H days, e.g. 1 or 2 weeks
- With each day h of the rolling horizon we associate a routing cost r_h
- With each container i on day h we associate:
  - an overflow probability p_{ih}
  - an overflow penalty δ
- We have the following trade-off:

lower routing cost higher overflow probability	↔	higher routing cost lower overflow probability

- Stochasticity in the problem is captured in the calculation of p_{ih}

Overflow probability

- Let L_{iT} denote the total quantity of container i on day T based on the sensor information I_{iT}. The overflow probability on a future day $h' \leqslant H$ is:

$$p_{ih'} = \mathbb{P}\left(L_{iT} + \sum\nolimits_{h=1}^{h'} Q_{i(T+h)} \geqslant C_i | I_{iT}\right) \tag{48}$$

- Our assumptions on the error terms:

$$Q_{it} = \mathbb{E}(Q^{\star}_{it}) + \varepsilon_{it}, \qquad \varepsilon_{it} \overset{iid}{\sim} \mathcal{N}(0, \sigma^2) \tag{49}$$

- Then (48) can be rewritten as:

$$p_{ih'} = \mathbb{P}\left(\sum_{h=1}^{h'} \varepsilon_{i(T+h)} \geqslant C_i - L_{iT} - \sum_{h=1}^{h'} \mathbb{E}\left(Q_{i(T+h)}\right) | I_{iT}\right) \tag{50}$$

Overflow probability

- Because the individual errors are iid normal, we have that:

$$\sum_{h=1}^{h'} \varepsilon_{i(T+h)} \overset{iid}{\sim} \mathcal{N}(0, h'\sigma^2) \tag{51}$$

- An estimate of the variance is given by:

$$\sigma^2 = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} (Q_{i,t} - \mathbb{E}(Q^{\star}_{i,t}))^2}{NT - \#\text{params}} \tag{52}$$

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- In the above formulas, all estimable quantities are replaced by their empirical counterparts

- Looking at (51) we recognize the value added of reestimating the forecast with renewed information I_{iT} every day.

Other considerations

- Route failure:
  - demand realizations may lead to a route failure
  - route failures can be evaluated probabilistically
  - the penalty is equal to the cost of visit to the nearest dump

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- Route failure:
  - demand realizations may lead to a route failure
  - route failures can be evaluated probabilistically
  - the penalty is equal to the cost of visit to the nearest dump
- Realized overflows:
  - an overflowed container must be collected within 24h
  - if I_{it} indicates an overflow, the container is scheduled for immediate collection

Problem setup

- The problem is solved for a rolling horizon of H days

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- Decomposition scheme (Campbell and Savelsbergh, 2004):
  - solve an assignment problem over H with the objective of minimizing travel costs (e.g. approximated by TSP tour costs) and overflow penalties
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  - solve a multi-period VRP-IF for the first D days, $D \ll H$, including route failure probabilities
- The solution of the assignment problem over the first D days reflects the long-term effects, but is only a suggestion
- It is refined in the VRP-IF solution to include more of the short-term
- Moving the service of container i from day h' to h'' results in:
  - changes in the routing costs: $r_{h'}$ and $r_{h''}$
  - change in the penalty cost attribution: from $\delta p_{ih'}$ to $\delta p_{ih''}$
  - changes in the route failure costs

Contributions

- Our IRP is richer compared to similar problems in the literature
- It integrates real-time forecasting:
  - the existing literature focuses on known distributions with fixed parameters
  - in our case the rates are time-dependent and there is not a unique optimal service frequency
- Compared to similar decomposition schemes (e.g. Campbell and Savelsbergh, 2004), we integrate stochasticity and further cost components

Contents

Conclusion

- At the moment, the forecasting model can produce future levels, for which the routing problem is solved
- Future research will focus on:
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- Future research will focus on:
  - extending the forecasting model with more deposit sizes or a continuous deposit size distribution
  - implementing the integration of the forecasting model and the routing algorithm into an IRP
- The IRP will solve simultaneously the container selection problem based on forecast levels and the routing problem in a rolling horizon framework
- Once integrated at the partnering company, the available data will allow for extensive testing and results

Thank you.
Questions?

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