DECEMBER 1979 INT 97/79

A MODEL FOR THE ION-ACOUSTIC TURBULENCE EXCITED BY A CONSTANT CURRENT

J. Vaclavik and K. Appert

Centre de Recherches en Physique des Plasmas

ECOLE POLYTECHNIQUE FEDERALE DE LAUSANNE

1. INTRODUCTION

In our previous work {1,2} it was shown that non-resonant (adiabatic) wave-particle interaction plays an important role in the dynamics of the ion-acoustic turbulence excited by a constant current. At the same time, it was argued that quasilinear equations which include this type of interaction are amenable to a numerical procedure only if particle distribution functions are made one-dimensional in some manner. Up to now we have used so-called "brute-force" methods {2}. The experience gained with these allows us to propose new, more-consistent model equations which also comprise spontaneous and wave-wave-particle effects.

First, we formulate general equations which consistently take into account all the processes mentioned above and show that they are conservative. These equations are then simplified assuming that the particle distribution functions have special forms.

../2

2. GENERAL EQUATIONS AND CONSERVATION LAWS

All quantities will be given in dimensionless units; the units of time, length, distribution function, spectrum, electric field and temperature are, respectively, ω_{pe}^{-1}, λ_{Do}, $n(m_e/T_{eo})^{s/2}$, $4\,\Pi\, n\, T_{eo}\, \lambda_{Do}^{s}$, $(4\,\Pi\, n\, T_{eo})^{1/2}$ and T_{eo}.

Here s(=2,3) is the dimension of the system considered, T_{eo} is the initial electron temperature and the other notations are standard. Throughout we shall use the convention

$$\sum_{\vec{k}} \equiv \int_{(k_z>0)} \frac{d^s\vec{k}}{(2\pi)^s} \quad .$$

The plasma under consideration is assumed to be collisionless, uniform and nonmagnetized. The electrons are hot ($T_e \gg T_i$) and drift, with respect to the ions, along the z-axis. The kinetic equation for the particle distribution function f_j of species j (= e,i) can then be given in the form

$$\frac{\partial f_j}{\partial t} + \alpha_j \mu_j E \frac{\partial f_j}{\partial v_z} = \mu_j^2 \frac{\partial}{\partial \vec{v}} \cdot \left(\overleftrightarrow{D}_j \cdot \frac{\partial f_j}{\partial \vec{v}} + \overleftrightarrow{D}_{Nt} \cdot \frac{\partial^2 f_j}{\partial \vec{v}\, \partial t} \right) \qquad (1)$$

$$+ \mu_j \frac{\partial}{\partial \vec{v}} \cdot \left(\vec{F} f_j \right) ,$$

where

$$\alpha_j = \begin{cases} -1 \\ 1 \end{cases} , \quad \mu_j = \begin{cases} 1 & j=e \\ \mu & j=i \end{cases} ; \quad \mu = \frac{m_e}{m_i} , \tag{2}$$

and E is the electric field associated with the current.

The diffusion tensor $\overleftrightarrow{D}_j$ consists of a number of parts

$$\overleftrightarrow{D}_j = \overleftrightarrow{D}_R + \overleftrightarrow{D}_N + \overleftrightarrow{D}_{N\omega} + \overleftrightarrow{D}_A \, \delta_{j,i} , \tag{3}$$

where

$$\overleftrightarrow{D}_R = \sum_{\vec{k}} \frac{\vec{k}\vec{k}}{k^2} I_{\vec{k}} \, 2\pi \, \delta(\omega_{\vec{k}} - \vec{k}\cdot\vec{v}) \tag{4}$$

and

$$\overleftrightarrow{D}_N = \sum_{\vec{k}} \frac{\vec{k}\vec{k}}{k^2} \frac{B_{\vec{k}}}{(\omega_{\vec{k}} - \vec{k}\cdot\vec{v})^2} , \quad \overleftrightarrow{D}_{N\omega} = \sum_{\vec{k}} \frac{\vec{k}\vec{k}}{k^2} I_{\vec{k}} \frac{\partial \omega_{\vec{k}}}{\partial t} \frac{\partial}{\partial \omega_{\vec{k}}} \frac{1}{(\omega_{\vec{k}} - \vec{k}\cdot\vec{v})^2} \tag{5}$$

represent the resonant and adiabatic wave-particle interactions, respectively, and

$$\overleftrightarrow{D}_A = \pi T_i \mu \sum_{\vec{k},\vec{k}'} \frac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} \frac{\partial \varepsilon'(\vec{k}',\omega_{\vec{k}'})}{\partial \omega_{\vec{k}'}} \frac{(\vec{k}\cdot\vec{k}')^2 [\vec{k},\vec{k}']^2}{(kk')^2} \tag{6}$$

$$\times I_{\vec{k}} I_{\vec{k}'} \frac{(\vec{k}-\vec{k}')(\vec{k}-\vec{k}')}{(\vec{k}-\vec{k}')^2} \delta(\omega_{\vec{k}} - \omega_{\vec{k}'} - (\vec{k}-\vec{k}')\cdot\vec{v})$$

corresponds to the wave-wave scattering off the ions. The tensor $\overleftrightarrow{D}_{Nt}$, which takes into account a part of the adiabatic interaction, and the dynamical friction vector $\vec{F}$, corresponding to the spontaneous emission of plasmons, are given by

$$\overleftrightarrow{D}_{Nt} = \sum_{\vec{k}} \frac{\vec{k}\vec{k}}{k^2} \frac{2 I_{\vec{k}}}{(\omega_{\vec{k}} - \vec{k}\cdot\vec{v})^2} , \tag{7}$$

$$\vec{F} = 2\pi g \sum_{\vec{k}} \frac{\vec{k}}{k^2 \dfrac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}}} \delta(\omega_{\vec{k}} - \vec{k}\cdot\vec{v}), \tag{8}$$

where $g = (\lambda^s_{Do} n)^{-1}$ is the plasma parameter.

The spectrum $I_{\vec{k}}$ satisfies the wave kinetic equation

$$\frac{\partial I_{\vec{k}}}{\partial t} = B_{\vec{k}} \equiv I_{\vec{k}} \Big(2\gamma_{\vec{k}} + \sum_{\vec{k}'} A_{\vec{k},\vec{k}'} I_{\vec{k}'} + C_{\vec{k}} \Big) + S_{\vec{k}} , \tag{9}$$

where $\gamma_{\vec{k}}$ is the quasilinear growth (damping) rate

$$\gamma_{\vec{k}} = - \frac{\varepsilon''(\vec{k},\omega_{\vec{k}})}{\dfrac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}}} , \quad \varepsilon''(\vec{k},\omega_{\vec{k}}) = \sum_j \chi''_j(\vec{k},\omega_{\vec{k}}), \tag{10}$$

$$\varepsilon'(\vec{k},\omega_{\vec{k}}) = 1 + \sum_j \chi'_j(\vec{k},\omega_{\vec{k}}) \equiv 0, \tag{11}$$

$$\chi''_j(\vec{k},\omega_{\vec{k}}) = - \frac{\pi \mu_j}{k^2} \int d^s\vec{v}\, \vec{k}\cdot \frac{\partial f_j}{\partial \vec{v}} \delta(\omega_{\vec{k}} - \vec{k}\cdot\vec{v}), \tag{12}$$

$$\chi_j'(\vec{k},\omega_{\vec{k}}) = \frac{\mu_j}{k^2}\int d^s\vec{v}\ \frac{\vec{k}\cdot\frac{\partial f_j}{\partial\vec{v}}}{\omega_{\vec{k}}-\vec{k}\cdot\vec{v}} \quad . \tag{13}$$

The interaction kernel $A_{\vec{k},\vec{k}'}$ represents the wave-wave scattering off the ions

$$A_{\vec{k},\vec{k}'} = -2\mu T_i \frac{(\vec{k}\cdot\vec{k}')^2[\vec{k},\vec{k}']^2}{(kk')^2}\frac{\partial\varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial\omega_{\vec{k}}}\chi_i''(\vec{k}-\vec{k}',\omega_{\vec{k}}-\omega_{\vec{k}'}), \tag{14}$$

the expression

$$S_{\vec{k}} = \sum_j S_{j\vec{k}}\ ,\quad S_{j\vec{k}} = \frac{2\pi g}{k^2\left(\frac{\partial\varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial\omega_{\vec{k}}}\right)^2}\int d^s\vec{v}\, f_j\,\delta(\omega_{\vec{k}}-\vec{k}\cdot\vec{v}) \tag{15}$$

is the source term due to the spontaneous emission of plasmons and the term $C_{\vec{k}}$ is the counterpart of the quantities $\overleftrightarrow{D}_{Nt}$ and $\overleftrightarrow{D}_{N\omega}$

$$C_{\vec{k}} = -\frac{1}{\frac{\partial\varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial\omega_{\vec{k}}}}\left(2\frac{\partial^2\varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial t\,\partial\omega_{\vec{k}}} + \frac{\partial^2\varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial\omega_{\vec{k}}^2}\frac{\partial\omega_{\vec{k}}}{\partial t}\right). \tag{16}$$

Momentum conservation

Let us define the drift velocity of species j by the relation

$$v_{d_j} = \int v_z f_j\, d^s\vec{v}\ . \tag{17}$$

On multiplying Eq. (1) by v_z and integrating over $d^s\vec{v}$ we obtain

$$\dot{v}_{dj} - \alpha_j \mu_j E = \mu_j \sum_{\vec{k}} k_z \Bigg\{ \Bigg(2\chi_j''(\vec{k},\omega_{\vec{k}}) + 2\,\frac{\partial^2 \chi_j'(\vec{k},\omega_{\vec{k}})}{\partial t\,\partial \omega_{\vec{k}}} + \frac{\partial^2 \chi_j'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}^2}\,\frac{\partial \omega_{\vec{k}}}{\partial t} \Bigg) I_{\vec{k}} + \frac{\partial \chi_j'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} B_{\vec{k}} - \frac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} S_{j\vec{k}} - \delta_{j,i}\,\frac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} I_{\vec{k}} \sum_{\vec{k}'} A_{\vec{k},\vec{k}'} I_{\vec{k}'} \Bigg\}, \tag{18}$$

where Eqs. (12) - (15) have been used. Further, multiplying Eq. (18) by μ_j^{-1} and summing over the species we finally find

$$\frac{d}{dt} \sum_j \frac{v_{dj}}{\mu_j} = 0, \tag{19}$$

where Eqs. (9), (10) and (16) have been used.

If we choose v_{de} = const. and $v_{di}(t = 0) = 0$, Eq. (19) implies $v_{di} \equiv 0$. We then have

$$E = \sum_{\vec{k}} k_z \Bigg\{ \Bigg(2\chi_e''(\vec{k},\omega_{\vec{k}}) + 2\,\frac{\partial^2 \chi_e'(\vec{k},\omega_{\vec{k}})}{\partial t\,\partial \omega_{\vec{k}}} + \frac{\partial^2 \chi_e'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}^2}\,\frac{\partial \omega_{\vec{k}}}{\partial t} \Bigg) \times I_{\vec{k}} + \frac{\partial \chi_e'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} B_{\vec{k}} - \frac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} S_{e\vec{k}} \Bigg\} \equiv -\nu^* v_{de}, \tag{20}$$

where ν^* is the turbulent collison frequency (anomalous resistivity).

Energy conservation

Les us define the temperature of species j by the relation

$$T_j = \frac{1}{s\mu_j} \int (\vec{v} - v_{dj}\vec{e}_z)^2 f_j \, d^s\vec{v} \, . \tag{21}$$

On multiplying Eq. (1) by $(\vec{v} - v_{dj}\vec{e}_z)^2/(2\mu_j)$ and integrating over $d^s\vec{v}$ we obtain

$$\begin{aligned} \frac{s}{2}\dot{T}_j = \sum_{\vec{k}} \Bigg\{ (\omega_{\vec{k}} - k_z v_{dj}) \Bigg[\Bigg(2\chi''_j(\vec{k},\omega_{\vec{k}}) + 2\frac{\partial^2 \chi'_j(\vec{k},\omega_{\vec{k}})}{\partial t \, \partial \omega_{\vec{k}}} \\ + \frac{\partial^2 \chi'_j(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}^2} \frac{\partial \omega_{\vec{k}}}{\partial t} \Bigg) I_{\vec{k}} + \frac{\partial \chi'_j(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} B_{\vec{k}} \\ - \frac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} S_{j\vec{k}} - \delta_{j,i} \frac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} I_{\vec{k}} \sum_{\vec{k}'} A_{\vec{k},\vec{k}'} I_{\vec{k}'} \Bigg] \\ + \chi'_j(\vec{k},\omega_{\vec{k}}) B_{\vec{k}} + 2\Bigg(\frac{\partial \chi'_j(\vec{k},\omega_{\vec{k}})}{\partial t} + \frac{\partial \chi'_j(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} \\ \times \frac{\partial \omega_{\vec{k}}}{\partial t} \Bigg) I_{\vec{k}} \Bigg\} \, . \end{aligned} \tag{22}$$

Eq. (22), summed over the species, implies

$$\frac{s}{2}\sum_j \dot{T}_j + \frac{d}{dt}\sum_{\vec{k}} I_{\vec{k}} = -\sum_{\vec{k}} k_z \sum_j v_{d_j}\Big\{2\chi_j''(\vec{k},\omega_{\vec{k}})I_{\vec{k}} + \frac{\partial \chi_j'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} B_{\vec{k}} - \frac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} S_{j\vec{k}} - \delta_{ji}\frac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} \times I_{\vec{k}} \sum_{\vec{k}'} A_{\vec{k},\vec{k}'} I_{\vec{k}'} + \Big(2\frac{\partial^2 \chi_j'(\vec{k},\omega_{\vec{k}})}{\partial t\,\partial \omega_{\vec{k}}} + \frac{\partial^2 \chi_j'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}^2} \times \frac{\partial \omega_{\vec{k}}}{\partial t}\Big) I_{\vec{k}}\Big\}, \tag{23}$$

where Eqs. (9) and (11) have been used. On combining Eqs. (18) and (23) we finally find

$$\frac{d}{dt}\Big\{\sum_j\Big(\frac{s}{2}T_j + \frac{1}{2\mu_j}v_{d_j}^2\Big) + \sum_{\vec{k}} I_{\vec{k}}\Big\} = \sum_j \alpha_j v_{d_j} E \equiv JE, \tag{24}$$

where J is the current and $\sum_{\vec{k}} I_{\vec{k}}$ is the total electrostatic energy density.

In the case where v_{de} = const. and v_{di} = 0 we have

$$\frac{d}{dt}\left\{ \frac{s}{2} \sum_{j} T_{j} + \sum_{\vec{k}} I_{\vec{k}} \right\} = \nu^{*} v_{de}^{2} \ . \tag{25}$$

3. MODEL

We now assume $f_j = f_j(|\vec{v} - v_{dj}\vec{e}_z|) \equiv f_j(v)$.

On integrating Eq. (1) over the ignorable variables (the polar angles in the velocity space) we find

$$\frac{\partial f_j}{\partial t} = \frac{\mu_j^2}{v^{s-1}} \frac{\partial}{\partial v}\left(D_j^{(s)} \frac{\partial f_j}{\partial v} + D_{jNt}^{(s)} \frac{\partial^2 f_j}{\partial v \partial t}\right) + \frac{\mu_j}{v^{s-1}} \frac{\partial}{\partial v}\left(F_j^{(s)} f_j\right), \tag{26}$$

where

$$D_{jR}^{(2)} = \sum_{\vec{k}} 2 I_{\vec{k}} \left(\frac{\omega_{\vec{k}j}}{k}\right)^2 \frac{H(kv - |\omega_{\vec{k}j}|)}{v\left[(kv)^2 - \omega_{\vec{k}j}^2\right]^{1/2}} \ , \tag{27}$$

$$D_{jN}^{(2)} = \sum_{\vec{k}} \frac{B_{\vec{k}}}{k^2 v} \left\{ 1 + |\omega_{\vec{k}j}| \frac{2(kv)^2 - \omega_{\vec{k}j}^2}{\left[\omega_{\vec{k}j}^2 - (kv)^2\right]^{3/2}} H(|\omega_{\vec{k}j}| - kv) \right\}, \tag{28}$$

$$D^{(2)}_{jN\omega} = -\sum_{\vec{k}} I_{\vec{k}} \frac{\partial \omega_{\vec{k}}}{\partial t} \frac{v\left(\omega^2_{\vec{k}j} + 2(kv)^2\right)}{\left[\omega^2_{\vec{k}j} - (kv)^2\right]^{5/2}} \operatorname{sign}(\omega_{\vec{k}j}) H(|\omega_{\vec{k}j}| - kv), \tag{29}$$

$$D^{(2)}_{jNt} = \sum_{\vec{k}} \frac{2 I_{\vec{k}}}{k^2 v} \left\{1 + |\omega_{\vec{k}j}| \frac{2(kv)^2 - \omega^2_{\vec{k}j}}{\left[\omega^2_{\vec{k}j} - (kv)^2\right]^{3/2}} H(|\omega_{\vec{k}j}| - kv)\right\}, \tag{30}$$

$$D^{(2)}_{A} = T_i \mu \sum_{\vec{k},\vec{k}'} \frac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} \frac{\partial \varepsilon'(\vec{k}',\omega_{\vec{k}'})}{\partial \omega_{\vec{k}'}} \frac{(\vec{k}\cdot\vec{k}')^2 [\vec{k},\vec{k}']^2}{(kk')^2}$$

$$\times \frac{(\omega_{\vec{k}j} - \omega_{\vec{k}'j})^2}{(\vec{k}-\vec{k}')^2} \frac{H\left(|\vec{k}-\vec{k}'|v - |\omega_{\vec{k}j} - \omega_{\vec{k}'j}|\right)}{v\left[(\vec{k}-\vec{k}')^2 v^2 - (\omega_{\vec{k}j} - \omega_{\vec{k}'j})^2\right]^{1/2}} I_{\vec{k}} I_{\vec{k}'}, \tag{31}$$

$$F^{(2)}_{j} = 2g \sum_{\vec{k}} \frac{\omega_{\vec{k}j}}{k^2 \frac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}}} \frac{H(kv - |\omega_{\vec{k}j}|)}{\left[(kv)^2 - \omega^2_{\vec{k}j}\right]^{1/2}}; \tag{32}$$

$$D^{(3)}_{jR} = \sum_{\vec{k}} \mathcal{K} I_{\vec{k}} \frac{\omega^2_{\vec{k}j}}{k^3 v} H(kv - |\omega_{\vec{k}j}|), \tag{33}$$

$$D_{jN}^{(3)} = \sum_{\vec{k}} \frac{B_{\vec{k}}}{k^2} \left\{ 1 + \frac{\omega_{\vec{k}j}}{kv} \lg \left| \frac{kv - \omega_{\vec{k}j}}{kv + \omega_{\vec{k}j}} \right| + \frac{\omega_{\vec{k}j}^2}{\omega_{\vec{k}j}^2 - (kv)^2} \right\}, \tag{34}$$

$$D_{jN\omega}^{(3)} = \sum_{\vec{k}} I_{\vec{k}} \frac{\partial \omega_{\vec{k}}}{\partial t} \frac{1}{k^2} \left\{ \frac{1}{kv} \lg \left| \frac{kv - \omega_{\vec{k}j}}{kv + \omega_{\vec{k}j}} \right| + \frac{2\omega_{\vec{k}j}\left(\omega_{\vec{k}j}^2 - 2(kv)^2\right)}{\left(\omega_{\vec{k}j}^2 - (kv)^2\right)^2} \right\}, \tag{35}$$

$$D_{jNt}^{(3)} = \sum_{\vec{k}} \frac{2 I_{\vec{k}}}{k^2} \left\{ 1 + \frac{\omega_{\vec{k}j}}{kv} \lg \left| \frac{kv - \omega_{\vec{k}j}}{kv + \omega_{\vec{k}j}} \right| + \frac{\omega_{\vec{k}j}^2}{\omega_{\vec{k}j}^2 - (kv)^2} \right\}, \tag{36}$$

$$D_{A}^{(3)} = \frac{\tilde{\pi}}{2} T_{i}\mu \sum_{\vec{k},\vec{k}'} \frac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} \frac{\partial \varepsilon'(\vec{k}',\omega_{\vec{k}'})}{\partial \omega_{\vec{k}'}} \frac{(\vec{k}\cdot\vec{k}')^2 [\vec{k},\vec{k}']^2}{(kk')^2}$$

$$\times I_{\vec{k}} I_{\vec{k}'} \frac{(\omega_{\vec{k}j} - \omega_{\vec{k}'j})^2}{v\,|\vec{k}-\vec{k}'|^3} H\left(|\vec{k}-\vec{k}'|v - |\omega_{\vec{k}j} - \omega_{\vec{k}'j}|\right), \tag{37}$$

$$F_{j}^{(3)} = \tilde{\pi} g \sum_{\vec{k}} \frac{\omega_{\vec{k}j}}{k^3 \dfrac{\partial \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}}} H\left(kv - |\omega_{\vec{k}j}|\right), \tag{38}$$

$$\omega_{\vec{k}_j} = \omega_{\vec{k}} - k_z v_{d_j} \ , \quad H(x) = \begin{cases} 1 & x > 0 \\ 0 & x < 0 \ . \end{cases} \tag{39}$$

The same procedure as that used above, applied to Eqs. (12), (13) and (15) yields

$$\chi_j^{\prime(2)}(\vec{k},\omega_{\vec{k}}) = \frac{2\tilde{\pi}\mu_j}{k^2}\int_0^\infty \frac{\partial f_j}{\partial v}\,dv\left\{\frac{|\omega_{\vec{k}_j}|}{[\omega^2_{\vec{k}_j}-(kv)^2]^{1/2}}H(|\omega_{\vec{k}_j}|-kv)-1\right\}, \tag{40}$$

$$\chi_j^{\prime\prime(2)}(\vec{k},\omega_{\vec{k}}) = -\frac{2\tilde{\pi}\mu_j}{k^2}\int_0^\infty \frac{\partial f_j}{\partial v}\,dv\,\frac{\omega_{\vec{k}_j}}{[(kv)^2-\omega^2_{\vec{k}_j}]^{1/2}}H(kv-|\omega_{\vec{k}_j}|), \tag{41}$$

$$S^{(2)}_{j\vec{k}} = \frac{4\tilde{\pi}g}{k^2\left(\dfrac{\partial\varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial\omega_{\vec{k}}}\right)^2}\int_0^\infty f_j\,v\,dv\,\frac{H(kv-|\omega_{\vec{k}_j}|)}{[(kv)^2-\omega^2_{\vec{k}_j}]^{1/2}} \ ; \tag{42}$$

$$\chi_j^{\prime(3)}(\vec{k},\omega_{\vec{k}}) = -\frac{2\tilde{\pi}\mu_j}{k^2}\int_0^\infty \frac{\partial f_j}{\partial v}\,dv\left\{2v + \frac{\omega_{\vec{k}_j}}{k}\lg\left|\frac{kv-\omega_{\vec{k}_j}}{kv+\omega_{\vec{k}_j}}\right|\right\}, \tag{43}$$

../13

$$\chi_j''^{(3)}(\vec{k},\omega_{\vec{k}}) = \frac{2\pi^2\mu_j}{k^3}\,\omega_{\vec{k}j}\,f_j\left(v = \frac{|\omega_{\vec{k}j}|}{k}\right), \tag{44}$$

$$S_{j\vec{k}}^{(3)} = \frac{4\pi^2 g}{k^3\left(\dfrac{\partial\varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial\omega_{\vec{k}}}\right)^2}\int_0^\infty f_j\, v\, dv\, H\left(kv - |\omega_{\vec{k}j}|\right). \tag{45}$$

In the case where v_{de} = const. and $v_{di} = 0$ Eqs. (40) and (43) may be approximated by

$$\chi_i'(\vec{k},\omega_{\vec{k}}) = -\frac{\mu}{\omega_{\vec{k}}^2}\quad, \tag{46}$$

$$\chi_e'(\vec{k},\omega_{\vec{k}}) = \frac{1}{k^2 T_e}\left\{1 + \frac{2\omega_{\vec{k}} k_z v_{de}}{k^2 T_e} - \frac{(k_z v_{de})^2}{k^2 T_e}\right\}. \tag{47}$$

Equation (11) then implies

$$\omega_{\vec{k}} = (\mu T_e)^{1/2} k\left\{1 + k^2 T_e - \frac{(k_z v_{de})^2}{k^2 T_e}\right\}^{-1/2}. \tag{48}$$

Moreover, it follows from Eqs. (46) and (47) that

$$\frac{\partial \chi_i'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} = \frac{2\mu}{\omega_{\vec{k}}^3} , \tag{49}$$

$$\frac{\partial \chi_e'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} = \frac{2 k_z v_{de}}{(k^2 T_e)^2} . \tag{50}$$

For a small v_{de} such that $(k_z v_{de})^2 << k^2 T_e$ we can approximately set

$$\chi_e'(\vec{k},\omega_{\vec{k}}) = \frac{1}{k^2 T_e} , \quad \frac{\partial \chi_e'(\vec{k},\omega_{\vec{k}})}{\partial \omega_{\vec{k}}} = 0 , \quad \omega_{\vec{k}} = \frac{(\mu T_e)^{1/2} k}{(1 + k^2 T_e)^{1/2}} . \tag{51}$$

We then have

$$\frac{\partial^2 \chi_j'(\vec{k},\omega_{\vec{k}})}{\partial t \, \partial \omega_{\vec{k}}} = \frac{\partial^2 \varepsilon'(\vec{k},\omega_{\vec{k}})}{\partial t \, \partial \omega_{\vec{k}}} = 0 . \tag{52}$$

Consistent with Eq. (52) we can dispense with Eqs. (30) and (36) (in the electron kinetic equation also Eq. (29) and (35)), and Eq. (16) is reduced to

$$C_{\vec{k}} = \frac{3}{\omega_{\vec{k}}} \frac{\partial \omega_{\vec{k}}}{\partial t} = \frac{3}{2} \frac{1}{1+k^2 T_e} \frac{\dot{T}_e}{T_e} \quad . \tag{53}$$

Also the expression for the electric field (20) is simplified to

$$E = 2 \sum_{\vec{k}} k_z \left(I_{\vec{k}} \chi_e''(\vec{k}, \omega_{\vec{k}}) - \frac{\mu}{\omega_{\vec{k}}^3} S_{e\vec{k}} \right) \tag{54}$$

and Eq. (22) implies

$$\frac{s}{2} \dot{T}_e = \sum_{\vec{k}} \left\{ (\omega_{\vec{k}} - k_z v_{de}) 2 \left(I_{\vec{k}} \chi_e''(\vec{k}, \omega_{\vec{k}}) - \frac{\mu}{\omega_{\vec{k}}^3} S_{e\vec{k}} \right) + \frac{B_{\vec{k}}}{k^2 T_e} \right\} . \tag{55}$$

The ion temperature may be determined from Eq. (25).

Equations (9) and (26) are to be solved with the initial conditions

$$f_e(t=0) = \frac{1}{(2\pi)^{s/2}} \exp\left\{ -\frac{v^2}{2} \right\} ,$$

$$f_i(t=0) = \frac{1}{(2\pi \mu T_{i0})^{s/2}} \exp\left\{ -\frac{v^2}{2\mu T_{i0}} \right\} ,$$

$$I_{\vec{k}}(t=0) = \text{const.}$$

REFERENCES

{1} K. Appert, R. Bingham, J. Vaclavik and E.S. Weibel, Proc. 9th Europ. Conf. on Controlled Fusion and Plasma Physics, Oxford (1979), 64.

{2} K. Appert and J. Vaclavik, LRP 161/79 (to be published).