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CALCULATION OF AZIMUTHAL MAGNETIC FORCES IN THE TCV TOKAMAK

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Introduction

The poloidal and vertical fields in a tokamak will interact with the current in the toroidal field coils to produce a twisting force on the coils, which must be counteracted by some form of support structure. The purpose of these calculations is to estimate the size and effect of these forces.

The magnetic field in question is produced by combined effects of the Ohmic heating coil and the field shaping coils (Fig. 1). These currents are symmetric about the mid-plane of the machine and thus only half of the machine need be considered.

The toroidal field coil consists of 24 rectangular coils of copper, each comprising 2 turns. At the centre these coils are wedge shaped and are held together by pressure from the outside, to form a self supporting rigid vault {1, 2}. The vault thus consists of laminations of alternately copper and glass-epoxy. Therefore, each coil is supported by the self supporting vault at the centre and by a support frame at the outside connected to the outer corners of the coil {2}.

The effect of the interaction is to produce a force that twists the coils about the r axis (Fig. 1). Considering the toroidal coil as a whole it can be seen that the force will be translated into a tangential force acting at the outer corners of the coil and a torque acting on the centre vault. It is important to calculate these forces so that an adequate support structure can be designed and also to ensure that the internal stresses in the coils (particularly in the vault) do not exceed those allowable for the materials in question.

Calculations

The distribution of force/unit length around the coils was calculated using the usual force relation

$$\underline{dF} = I\, \underline{d\ell} \times \underline{B}$$

where $dF/d\ell$ = Force/unit length

I = Current

$\underline{B}$ = local Magnetic Field.

As the coils are constructed from bars of copper with a cross-section of 18x3.5 cm, there will be a spatial variation of the magnetic field over the cross-section of the bar. This variation is taken into account by dividing the current into 3 filaments distributed over the breadth of the conductor. The actual variation of the magnetic field is calculated using the program ECOIL which simply integrates around each toroidal current to calculate the field at a given point. The vertical conductors that form the vault are wedge shaped in cross-section and therefore the current will depend on radius; the three current filaments in these conductors were adjusted, in size, accordingly.

The most problematical point in the analysis was to determine a valid current distribution around the corners of the coil (i.e. the shaded area A in Fig. 1). In the final calculations the distribution shown in Fig. 2 was used; that is, six current filaments flowing in a circular path from one conductor to the next. However, various configurations were tried and it was found that the resulting force was relatively insensitive to the current distribution, despite the fact that the magnetic field in this region is very strong (Fig. 3).

As the coils are rectangular, the current paths are always parallel to the axes (assuming a right cylindrical coordinate system). The force on a radial (i.e. horizontal) current will be given by

$$dF = - Id\ell B_Z \quad ,$$

and that acting on a vertical component will be given by

$$dF = Id\ell B_R \quad .$$

Figure 3 shows the distribution of the magnetic field components perpendicular to the current paths in two components of the coil. The variation of the axial component B_Z along the radius is shown in Fig. 3A and the variation of the radial component B_R along the vertical current path in Fig. 3B. Since the force is directly proportional to the magnetic field, these curves also indicate the variation of the force/unit length along these current paths.

From Fig. 3 it can be seen that the force (and hence the torque) on the vault (Fig. 3B) has a distinct maximum towards the top of the vault. However, the force on the vault due to the $B_Z I_R$ interaction (Fig. 3A) is some factor 7 greater than the forces due to the $B_R I_Z$ interactions and furthermore it is of opposite sign. Therefore it is justifiable to make the assumption that the torque acting on the vault is applied purely at the ends. This assumption greatly simplifies the estimation of the stresses in the vault.

Results

The pressure on the support at the outer edge of the coil (Force $\underline{F}_2$, Fig. 4) was calculated from the force distribution in the horizontal component of the coil (Section C. Fig. 4) and the vertical component (Section D) using the rigid vault and the r axis, respectively, as pivots. In this way, the force was found to be

$$\underline{F}_2 = 0.3 \text{ Tonnes / turns}$$

giving a total torque $\underline{T}_2 = 15.5$ Tonne m.

It should be noted that the main component of this force comes from the horizontal current (C) as the magnetic field in the region D is very weak.

In a similar manner the total force acting on the inside corner (Area A) was calculated using the force distributions along the section (B and C). This yields a force of

$$\underline{F}_1 = 1.5 \text{ Tonnes / turn}$$

or a total torque of $\underline{T}_1 = 10.1$ Tonne m.

The fact that the force acting on the vault is greater than that acting at the outer edge has already been indicated by the magnetic field profiles of Fig. 3.

The shear stress in the vault can be calculated as a function of radius, from the formula {5}

$$\text{Stress} = \frac{T}{J} r$$

where J is the cross-sectional moment of inertia $J = \int_s r^2 ds$.

Thus for our case

$$\text{Stress} = 2.13 \times 10^{-3}\ r$$

which leads to a maximum stress, at the outer edge of the vault, of

$$\text{Maximum Shear Stress} = 0.5\ \text{kgm/mm}^2\ .$$

This value is well within the generally accepted values for epoxy impregnated fibre-glass or copper {1}. Under the assumption that this torque is applied purely at the ends of the vault, then the stress will be approximately uniform over the length of the column.

The strain on the vault has been calculated using two different methods. Fistly, we use the homogeneous structure approximation. In this method, it is assumed that the vault is composed of a homogeneous mixture of epoxy-glass and copper, in the ratio of the fractions of the total cross-section made of epoxy and copper, with an effective Young's modulus {1} of

$$E = f_c E_c + f_e E_e$$

where E_c = Young's modulus of copper
f_c = fraction of total cross-section made of copper
E_e = Young's modulus of epoxy
f_e = fraction of total cross-section made of copper .

A Poisson's ratio of $\sigma = 0.3$ was assumed.

Using a conservative value of $E_e = 100$ kgm/mm^2 {4} and $f_c = 10.5\%$ (assumes 2mm layers of epoxy between the conductors) an effective Young's modulus of

$$E = 1.2 \times 10^4\ \text{kgm/mm}^2$$

was obtained for the homogeneous vault.

The strain on a hollow cylinder is given by the expression {3}

$$\theta = T2\ell/\pi G(R_o^4 - R_1^4)$$

where θ = Angle of displacement R_o = outer radius
T = Applied torque R_1 = inner radius
ℓ = length
G = shear modulus.

Using the torque calculated previously (T = 10.1 Tonne m) and substituting the values appropriate to the vault into this expression, a value of

$$\theta = 3.66 \times 10^{-4} \text{ radians}$$

is obtained for the angular strain about the z axis. This corresponds to a movement of

$$D = 0.09 \text{ mm}$$

at the outer edge of the vault (i.e. at r = 235mm), see Fig. 5.

In the second method it is assumed that each bar is independent, such that the vault is made up of a system of wedge shaped copper bars with an air gap between each. This air gap corresponds to the thickness of the epoxy that would normally be there. In this situation it can be shown that the total torque about the central axis can be divided into components acting on the separate bars. Thus each bar in the vault is twisted about its cross-sectional centre of mass by a torque of magnitude

$$t = T/n \, ,$$

t = torque on a single bar about the cross-sectional centre of mass
T = total torque on the vault
n = number of bars in the vault.

It is now possible to calculate the strain on each bar and consequently the resulting strain on the entire vault. The strain on an individual bar is given by

$$\theta = \frac{t}{GJ} \qquad \{6\}$$

where θ = length of the bar

G = shear modulus

J = moment of inertia.

The value of J about the centre of mass was evaluated as outlined in the appendix and was found to be

$$J = 7.81 \times 10^{6} mm^{4}.$$

Once again substituting in the appropriate values of the parameters the angular strain on the vault was calculated to be

$$\theta = 4.38 \times 10^{-3},$$

which corresponds to a movement of

$$D = 1.03 \text{ mm}$$

at the outer edge of the vault.

Conclusions

The calculated stresses that occur in the toroidal field coils due to the interaction between the poloidal fields and the toroidal coil currents are within the allowable limits of the structure. The force $\underline{F}_2$ at the outer edge is easily supported by an external frame {2} and the stress on the vault due to $\underline{F}_1$ is within accepted limits for the materials used {1}.

The results of the two strain calculations differ by about a factor of 10, the homogeneous case indicating a greater rigidity than the independent bar model. In the interpretation of these results it must be remembered that this is a "worst case" calculation for the following reasons :

1. It represents the case when there is zero plasma current (e.g. disruption). Since the plasma current flows in the opposite sense to the other toroidal currents, it will decrease the effective magnetic fields that cause these forces.

2. The pressure acting on the area A (Fig. 1) due to the horizontal component (C) will be reduced as some fraction of this force must be absorbed in the bending of the horizontal bar to allow for movement of the vault, assuming that the outer section of the coil is held rigid.

3. The second method of calculation assumed that the bars can move independently. If the core is potted in epoxy then this movement will be restricted and consequently the rigidity will be improved. In reality the strain on the vault will lie between the values given by the two calculations.

As a result, it appears that the strain on the central vault of the Tokamak will be acceptably within the elastic limits of the structure, meaning that no major supplementary support structure will be necessary for the vault.

APPENDIX   Calculation of J

The moment of intertia of a given, homogeneous area about the origin of an arbitrary coordinate system is given by

$$J_o = \int_s r^2 ds$$

where s = area, r = distance from origin to ds. Using the coordinate system shown in Fig. 6, the moment of inertia of the wedge shaped area (shaded portion) about the origin is given by :

$$J_o = 2\int_0^{b_1} \int_0^{a(1-y/b)} (x^2 + y^2)\, dx\, dy$$

which can be evaluated in terms of the parameters a, b, a_1 and b_1. To calculate the moment of inertia about the centre of mass, the following translation formula is used

$$J \text{ (centre of mass)} = J_o - R^2 A. \qquad \{5\}$$

R = distance from origin to centre of mass

A = area of the cross-section.

REFERENCES

{1} R. Aymar et al. CEA Report : EUR-CEA-FC-824
"Torus II - Technical description of the design proposal" (1976)

{2} R. Gruber et al., "The TCV Tokamak Project" (1977) - CRPP

{3} H.O. Knesser, "Physik", Springer Verlag (1966)

{4} "Properties of Araldite", Ciba-Geigy Publication No. 31 464/f

{5} E. Meissner and H. Ziegler, "Mechanik I", Verlag Birkhäuser, Basel (1948)

FIGURES

1. Cross-section of the Tokamak showing the positions of the Ohmic heating coils and the field shaping coil relative to the toroidal field coil.

2. Toroidal field coil showing the current paths used in the computations.

3. A Variation of the axial magnetic field B_z along the radius at z = 80cm (i.e. along the horizontal current path).

   B Variation of the radial magnetic field B_R along the z direction at r = 14 cm (i.e. up the inside vertical current path).

4. Schematic diagram of a toroidal field coil showing forces.

5. Diagram of shear strain in a cylinder.

6. Diagram of the cross-section of one of the vertical bars that constitute the central vault.

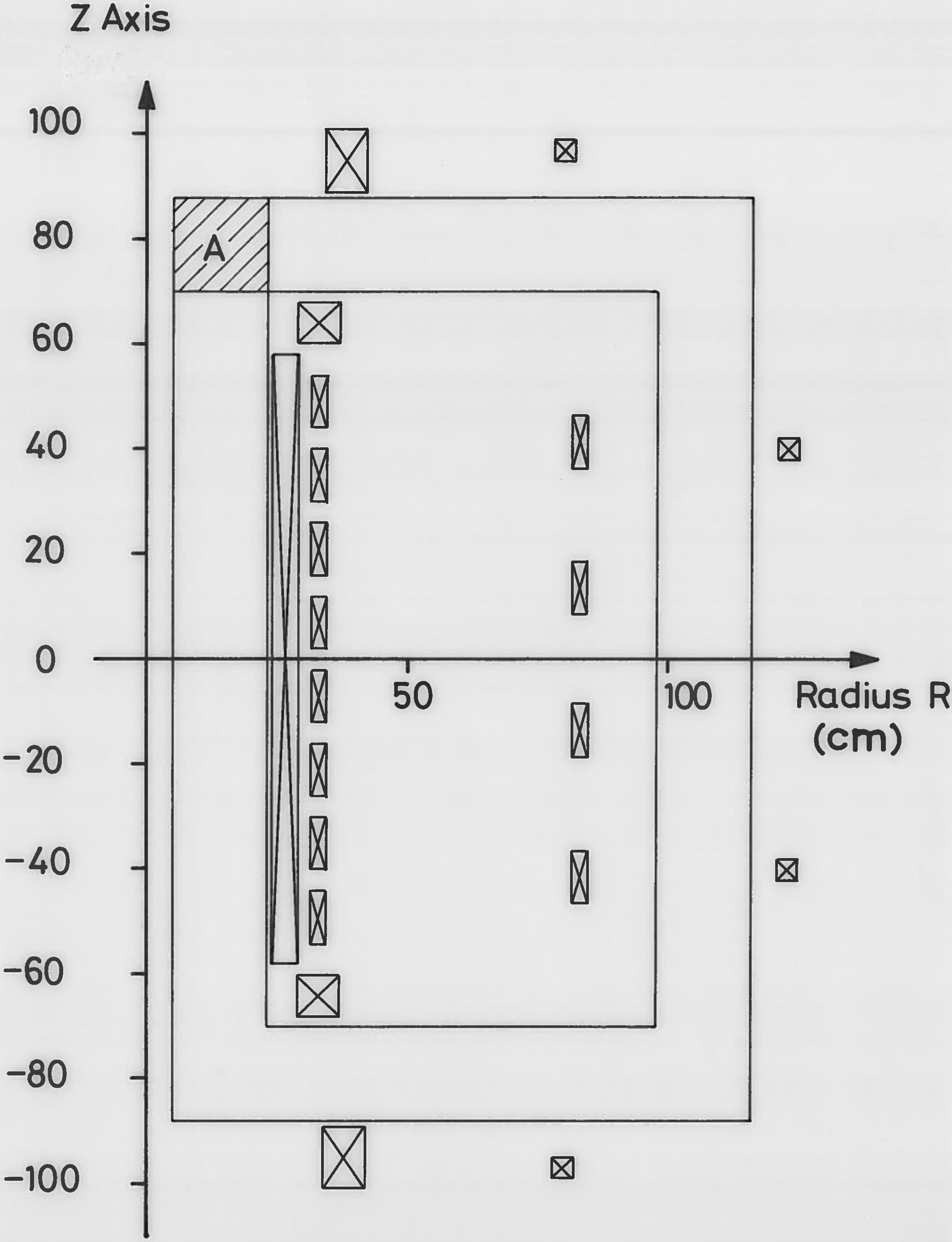


Figure 1

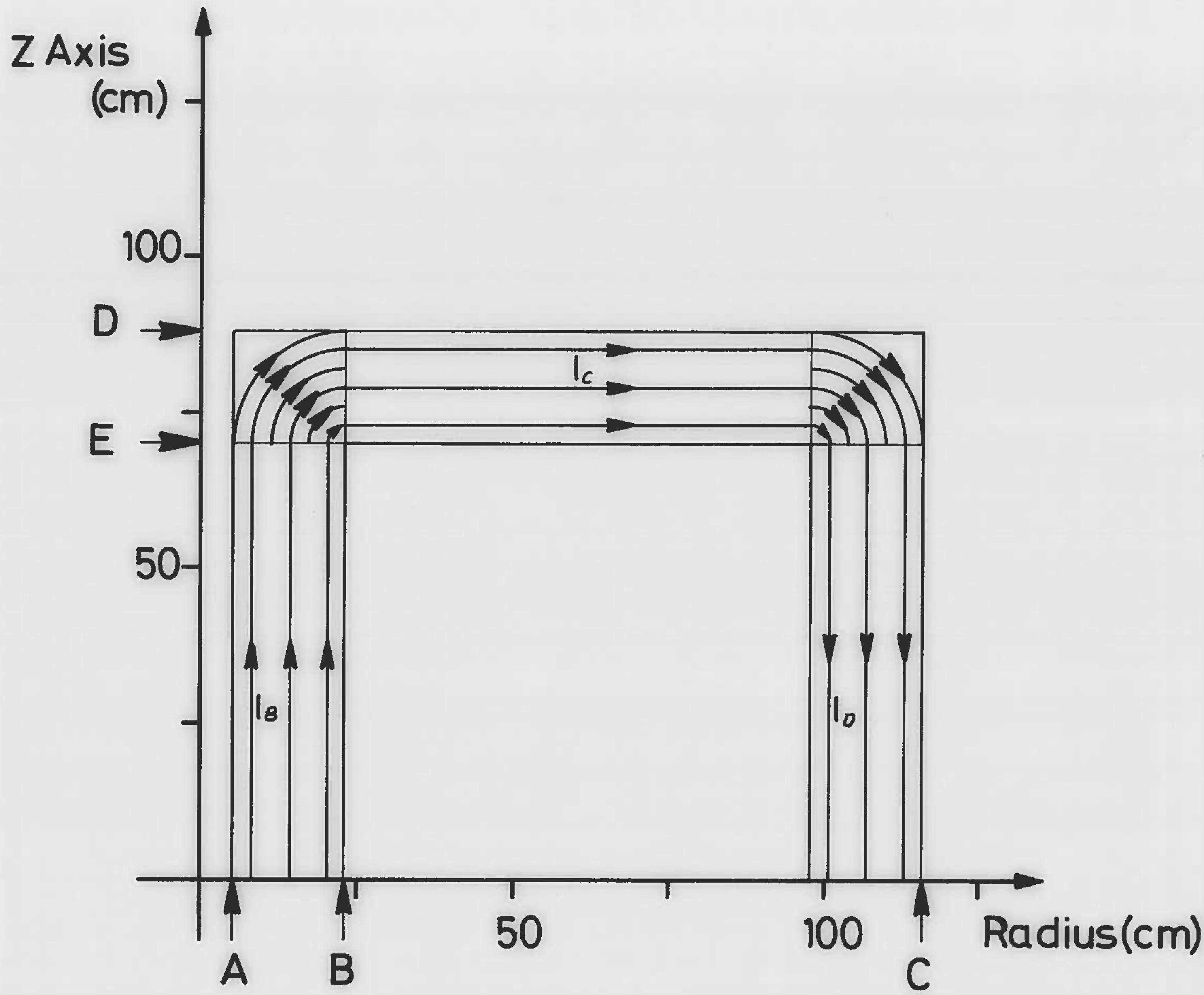
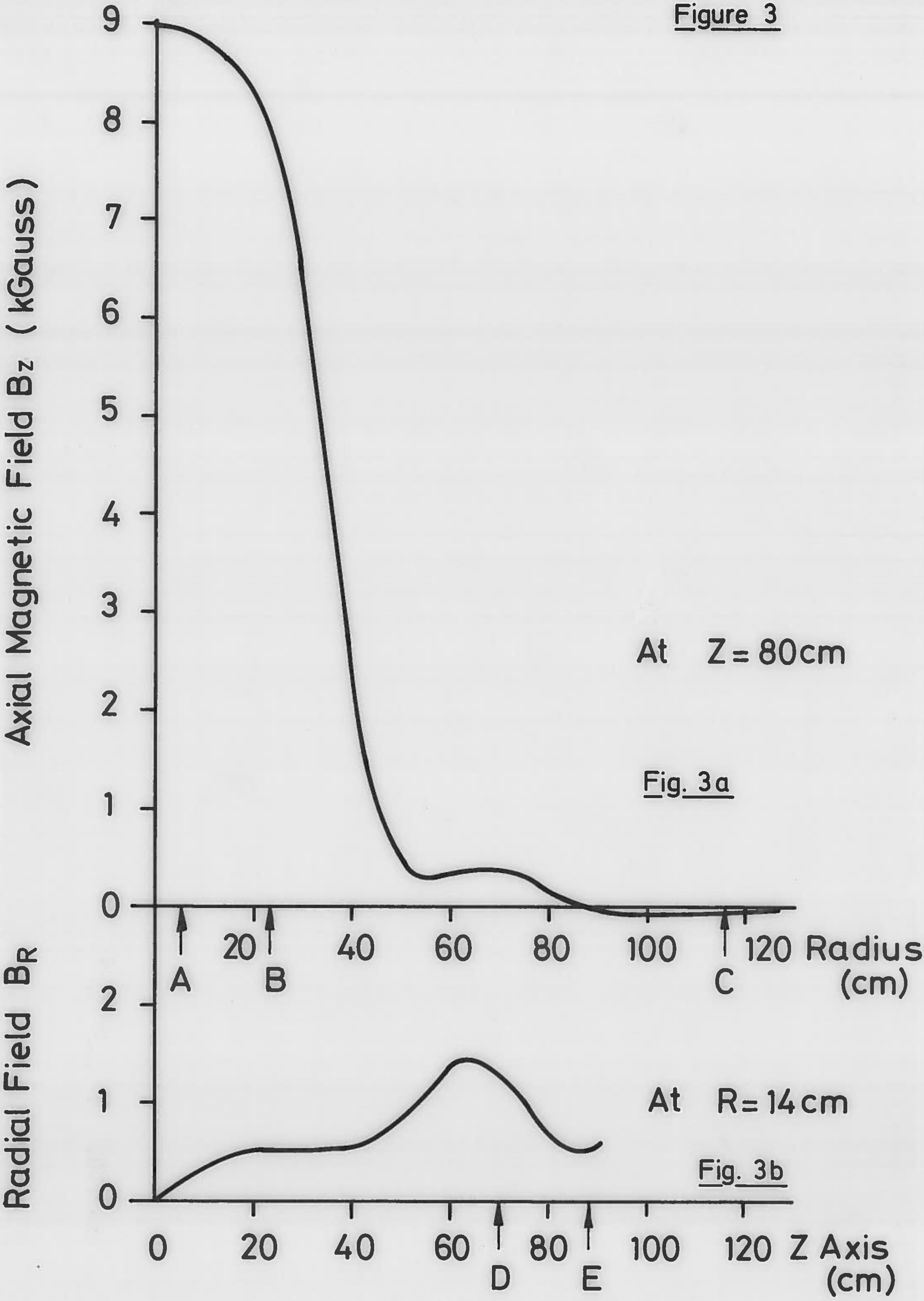
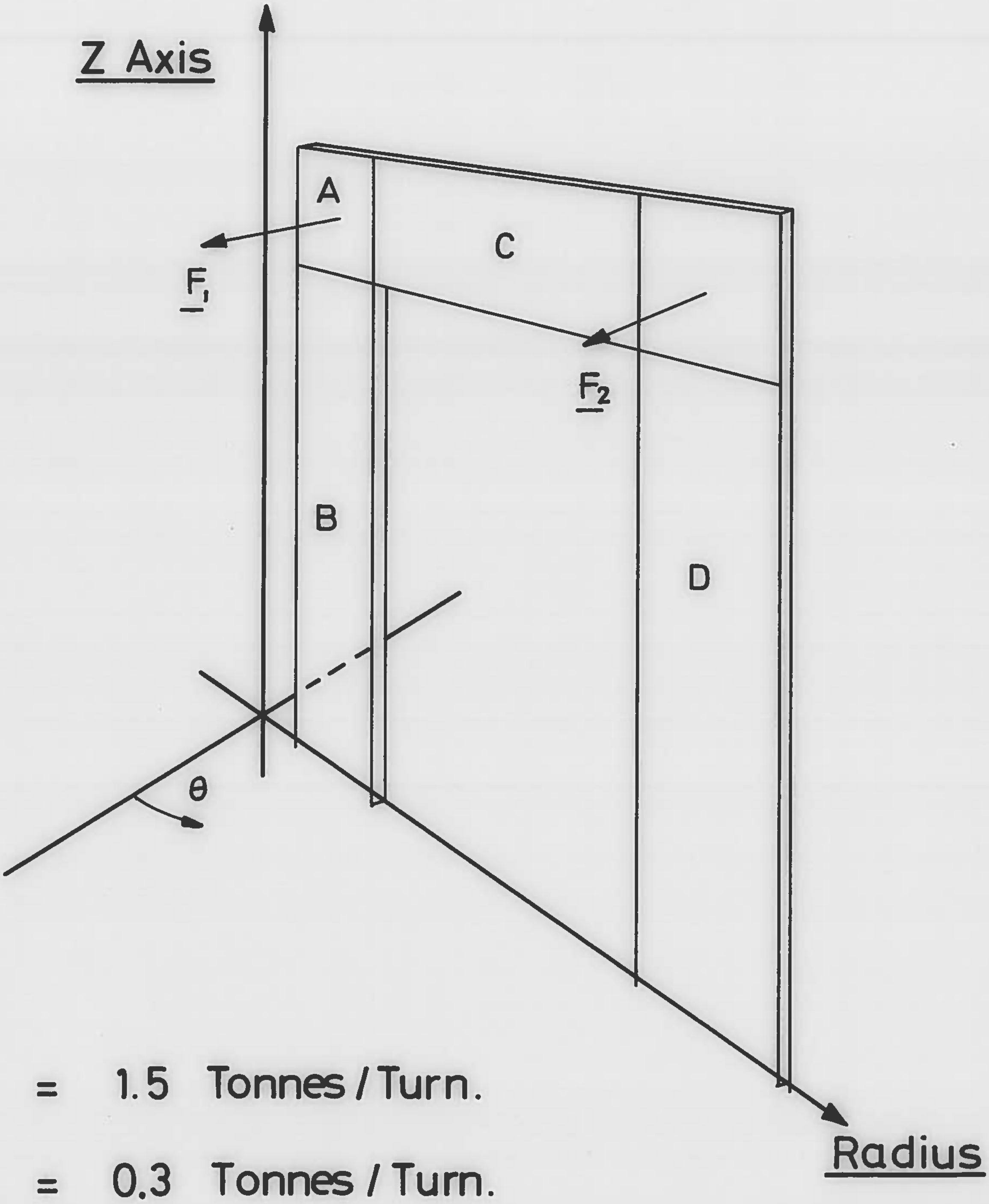


Figure 2

Figure 3



Axial Magnetic Field Bz (kGauss)
9
8
7
6
5
4
3
2
1
0
At Z = 80cm
Fig. 3a
20
40
60
80
100
120 Radius (cm)
A
B
C
Radial Field BR
2
1
0
At R = 14cm
Fig. 3b
0
20
40
60
80
100
120 Z Axis (cm)
D
E

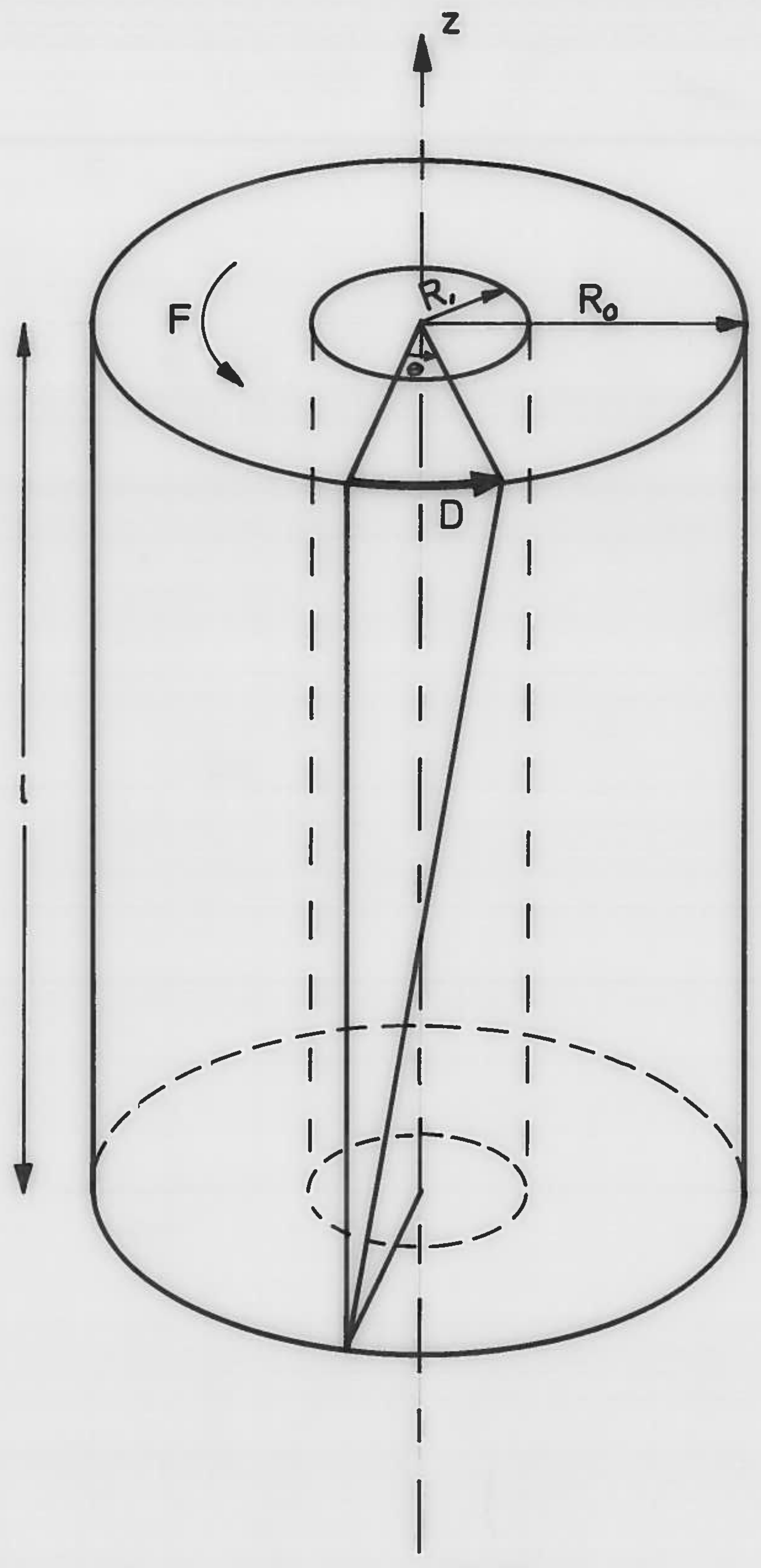


F_1 = 1.5 Tonnes / Turn.

F_2 = 0.3 Tonnes / Turn.

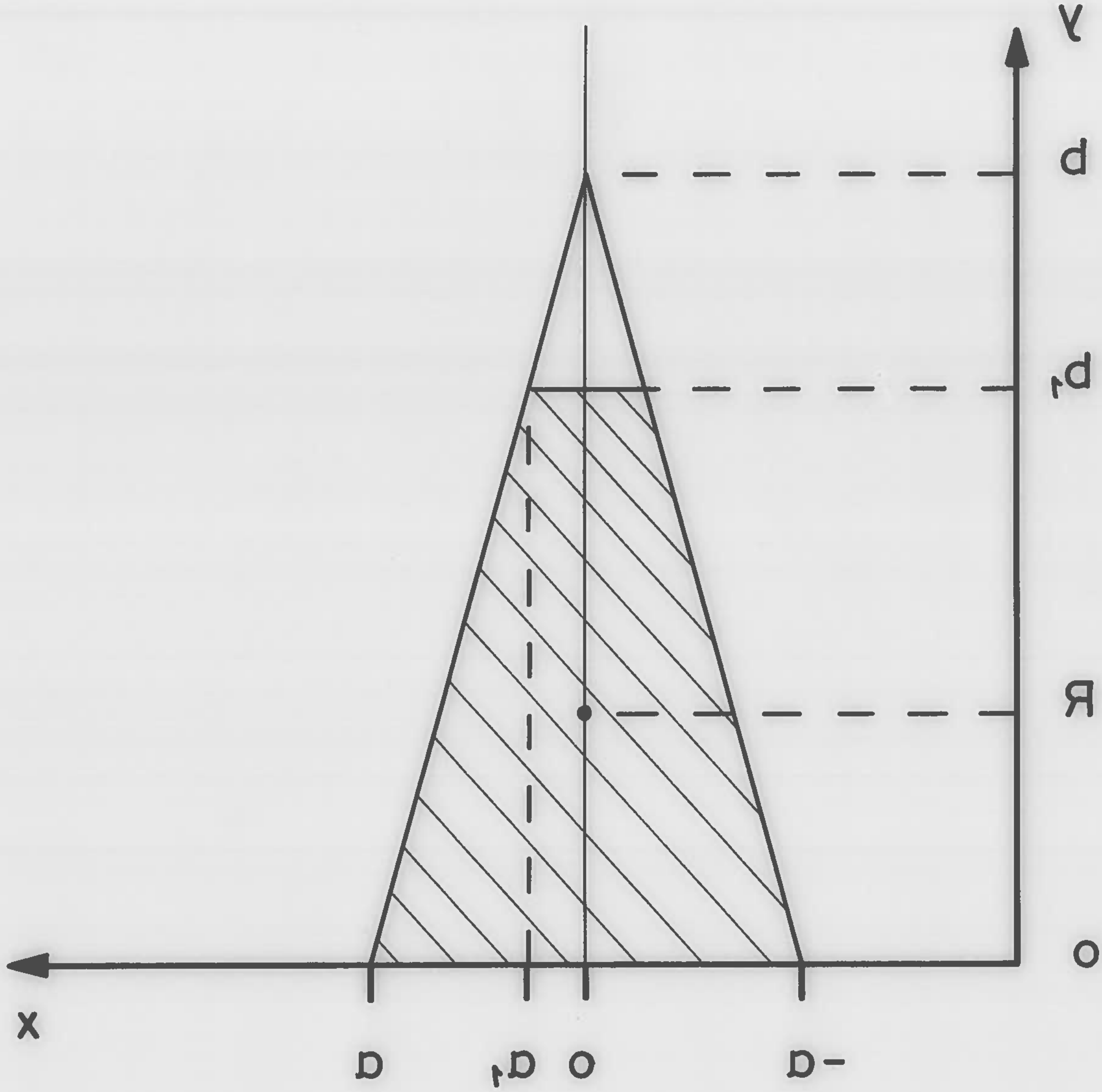
Cylindrical Coordinates (R, θ, Z)

Figure 4



$$D = R_o\Theta = R_o \frac{2lT}{\pi G(R_o^4 - R_1^4)}$$

Figure 5



$b = 235$ mm $\quad a = 14$ mm

$b_1 = 185$ mm $\quad a_1 = 2.98$ mm

$R = 72.5$ mm (Center of mass)

Figure 6