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Practical & Provably Secure Distance-Bounding

Ioana Boureanu [a] Aikaterini Mitrokotsa [b,*] and Serge Vaudenay [c]
[a] *Akamai Technology Limited,*
London, UK
E-mail: icarlson@akamai.com
[b] *Chalmers University of Technology,*
Gothenburg, Sweden
E-mail:aikmitr@chalmers.se
[c] *Ecole Polytechnique Fédérale de Lausanne (EPFL),*
Lausanne, Switzerland
E-mail:serge.vaudenay@epfl.ch

AbstractFrom contactless payments to remote car unlocking, many applications are vulnerable to relay attacks. Distance bounding protocols are the main practical countermeasure against these attacks. In this paper, we present a formal analysis of **SKI**, which recently emerged as the *first* family of lightweight and *provably secure* distance bounding protocols. More precisely, we explicate a general formalism for distance-bounding protocols, which lead to this practical and *provably secure* class of protocols (and it could lead to others). We prove that **SKI** and its variants are provably secure, even under the real-life setting of noisy communications, against the main types of relay attacks: distance-fraud and generalised versions of mafia- and terrorist-fraud. To attain resistance to terrorist-fraud, we reinforce the idea of using secret sharing, combined with the new notion of a *leakage scheme*. In view of resistance to generalised mafia-frauds (and terrorist-frauds), we present the notion of *circular-keying* for pseudorandom functions (PRFs); this notion models the employment of a PRF, with possible *linear reuse* of the key. We also identify the need of *PRF masking* to fix common mistakes in existing security proofs/claims. Finally, we enhance our design to guarantee resistance to terrorist-fraud in the presence of noise.

Keywords: distance-bounding, authentication, relay attacks, provable security, man-in-the-middle attacks

1. Introduction

Cryptography sees many applications in the world of smart-cards, from the more and more sophisticated NFC bankcards to the simpler RFID access cards. But the security protocols implied (e.g., protocols for ATM systems) are vulnerable to relay attacks or to more general forms of man-in-the-middle attacks. Relay attacks have already been mounted against bankcards [21]. In access control applications, it is not guaranteed that the card computing the responses to the reader's challenges is indeed the one

*Corresponding author. E-mail: aikmitr@chalmers.se

requiring access [30]. Similarly, car manufacturers use RFID protocols to unlock and even start their vehicles (see, e.g., [25]). However, these protocols may unfortunately be compromised by relaying [26]. The most interesting cryptographic solution to these threats seems to be based on distance-bounding [21].

In [12], Brands and Chaum introduced distance-bounding (DB) protocols, based on some original idea by Beth and Desmedt [6]. They are employed so that a prover may demonstrate his proximity to a verifier as well as authenticate this honest prover to the verifier.[1] In the literature covering such protocols, three main types of possible attacks have been distinguished. The first is *distance-fraud*, in which a prover tries to convince the verifier that he is closer than he really is. The second type of attack is *mafia-fraud* and involves three entities: an honest prover, an honest verifier, and an adversary. The adversary communicates with both the prover and the verifier and tries to demonstrate to the verifier that the prover is in the verifier's proximity although the prover is in reality far away from the verifier. Finally, the third type of attack is denoted as *terrorist-fraud*.[2] Here, the adversary has the same goal as in the mafia-fraud attack, but in this case the prover is dishonest and colludes with the adversary up to the non-disclosure of essential information, e.g., (parts of) secret keys, that may facilitate later impersonations of this prover.

Ad-hoc countermeasures protecting against one or several such attacks have sometimes been provided [1]. It has also been claimed [33] that DB protocols in their commonly known form cannot protect against all three frauds at a time. Unfortunately, these frauds have become even more dangerous through recent generalisations [18,22]. Nonetheless, DB protocols will most probably soon be implemented by car manufacturers or bank payment companies in their products, as platforms for such deployments arise [38]. In these contexts, security proofs and clear, solid security models become of paramount importance. However, unitary security models and respective compelling security proofs have not yet been formulated with respect to this class of protocols. In the following, we endeavour in overcoming this shortcoming, providing a comprehensive security model for distance-bounding protocols and constructing *practical* and provably secure protocols in the model herein.

More precisely, in this paper we provide a formal analysis, for **SKI** the *first* family of lightweight and *provably secure* distance bounding protocols[3] that was initially introduced in [10,11]. We give a detailed description of the employed formal communication model for distance-bounding protocols. We define formally what a distance-bounding protocol is and we provide formal definitions for the resistance of a distance bounding protocol against the main types of attacks: *distance-fraud* and generalised versions of *mafia-* and *terrorist-fraud*. Furthermore, we describe in detail **SKI** and its variants. We also provide a detailed design and security assessment that includes the formal proofs for **SKI**'s resistance against the main relay attacks. We should point out here that in papers [10,11] were **SKI** was initially introduced the definitions provided were informal while the security proofs were sketched.

[1] In this paper, we consider *authenticated* distance-bounding. Namely: protocols where both participants use a pre-established secret.

[2] The terms "mafia-fraud" and "terrorist-fraud" were introduced in 1988 by Desmedt [19]. Although confusion-prone, these are the ones still used in the literature.

[3] The SKI protocols were first presented at FSE'13. The name "SKI" comes from the first names of the authors: **S**erge, **K**aterina, **I**oana.

2. Related Work & State-of-the-Art

2.1. Distance Bounding – Informal and Semi-formal Approaches

In this section we provide details on the practical requirements (i.e. tolerance to noisy conditions) for secure distance bounding protocols, review the related work in distance bounding and finally discuss its connection to location based cryptography.

Tolerance to Noise. Since distance-bounding protocols operate under time-critical constraints and with rapid-bit exchanges, they are likely to be subject to noise, i.e., to noisy communication channels. So, these protocols often tolerate a few faulty iterations, in such a way that honest executions would succeed with high probability. Of course, noisy, rapid-bit exchanges are a reality of applied cryptographic protocols. However, many research results on DB assume noiseless conditions [1,45,17,13]. In this paper, noise will be taken into consideration in our security assessments.

DB Protocols and Attacks Amendments. Many DB protocols [32,34,39,46] consist of a data agreement phase or *initialisation phase* and a *distance-bounding* phase. The distance-bounding phase is time-critical and it normally imposes very fast computation, typically of less than a single clock cycle per round. (Light travels one meter within about three nanoseconds. So, every bit must be treated on the fly, upon arrival, with no delay, and there is no part for any time-consuming computation.) Nevertheless, even if the time-of-flight is critical, some DB protocols are not secure against terrorist-fraud: an attacker can find ingenious ways to collude with provers, defeating DB; an example of the sort is the terrorist-fraud, recently shown against the Bussard and Bagga [13,14]. Hancke and Kuhn [29], Munilla and Peinado [35], Kim and Avoine [33], and Reid *et al.* [39] proposed follow-ups of each others' schemes, addressing either a better protection against terrorist-fraud or mafia-fraud, or a better suitability to practice, or a more formal description, etc. In general, attempts to construct secure distance-bounding protocols such as [34,43,46] have been proven flawed [37,36]. In fact, Kim *et al.* state [33] that there is no DB protocol, which has one-bit challenges/responses per iteration in the distance-bounding phase, resisting all three attacks (i.e., distance-, mafia-, and terrorist-frauds) with a significant probability. In [11,10, Table 1], the popular distance-bounding protocols and their vulnerabilities as best-known up to that point (2013) are reported. That table shows a dire situation, so the question of *provable security* against all frauds mounted has been standing prominently. Since, two (classes of) protocols (one class in [10] and one protocol in [24]) which are provably secure have been published.

Moreover, more general attacks have been recently described. In [18], Cremers *et al.* described distance-hijacking as an extension of distance-fraud, yet as an attack that is close to terrorist-fraud at the same time; the fraud involves one dishonest, far-away prover and several honest provers, without the latter colluding with the former. Impersonation (a type of man-in-the-middle) is presented in [22]. In the current work, our threat model also incorporates these latter, powerful attacks.

In [2], a targeted protocol-analysis is carried on the TDB protocol by Avoine *et al.* They especially address the protection against terrorist-fraud for the Hancke and Kuhn protocol, using secret sharing schemes. However, [2] does not state the sound, (necessary and) sufficient assumptions for combating terrorist-fraud. This will be amended and taken further in this paper; we generalise the underlying idea of using a secret sharing scheme [2] and introduce a taxonomy of security-enforcing conditions (some of which are linked to secret sharing).

Recently, Hancke [28] observed that terrorist-frauds could also be mounted, by simply abusing the aforementioned, noise-tolerance property required from DB. Basically, a malicious prover could help an

adversary to answer most challenges and not leak to this adversary the secret key but only a noisy version of the secret key. Also, this leaked information is such that it does not give the adversary any significant advantage in later attacks onto the scheme, i.e., the coerced prover mounts a valid terrorist-fraud. Similar to TDB and the protocols herein, there is the recent protocol in [49]; however, unlike the protocols herein, the protocol in [49] does not resist these new terrorist-frauds in noisy conditions by Hancke [28]. As a matter of fact, all but two protocols allegedly resisting the classical terrorist-frauds as they were known before Hancke's observation would now collapse under terrorist-frauds executed in this new scenario of Hancke's (at least, cnf. to [11,10, Table 1]). The protocols left standing in front of this attack are the **SKI** protocols [11,10] to be studied herein and the Fischlin-Onete protocol [24].

Position-based cryptography & distance bounding. Position-based cryptography (PBC) [15] becomes possible through secure positioning (SP), which involves a set of verifiers ensuring that a given prover is indeed at some claimed position. In other words, in PBC a verifier within the network not only estimates the distance to another device but is also helped by, e.g., trusted base-stations that offer position-data for coordinate-triangulation in his final decisions. In SP, this assistance by, e.g., base-stations can happen repeatedly, to defend against malicious behaviour. This is not the case in DB, where the verifier is on his own, with his much simpler measurements at hand. However, distance-bounding protocols could potentially be used as building blocks for SP.

The model needed to achieve PBC bears similarities with the one to follow, yet distance-bounding is a weaker requirement than secure positioning. DB informally implies one prover proving to *one* verifier only that the former is close enough to the latter, using the time-of-flight of their exchanges. Thus, while the "geometry" needed for achieving distance-bounding is much simpler, the notion of time is of greater importance for distance-bounding.

2.2. State-of-the-art: Towards Provable DB Security

DB Formalisations. In [1], Avoine *et al.* give a complete but rather informal model for distance-bounding. Herein, we will refer to this line as to the *ABKLM model*. They define distance-bounding as the combination of authentication and distance-checking. They further carry on a tentative analysis of the Munilla-Peinado protocol [35]. As we will further discuss below, [1] does not clearly state the exact assumptions needed on the underlying primitives in order to achieve the alleged security.

So far, the most promising model for distance-bounding was presented recently by Dürholz *et al.* in [22]. We refer to it as the *DFKO model*. This model does not provide a clear communication model and its notions of time or distance are only implicit. It requires to specify protocols by explicitly distinguishing a lazy phase and a time-critical one. The DFKO model formalises the three classical types of frauds and an extra notion of *impersonation fraud*. The attackers are very specific, presented in terms of protocol session interleaving. Maybe due to this specificity or to their requirements which may be too strict, the model is too strong, fact admitted by its authors in [23]. In this model, certain insecurities (impersonation or terrorist-fraud) are hard-to-defend claims, leading to no convincing attack. Fischlin and Onete later proposed a secure protocol, proven secure in a new, clearer, game-based security model advanced at the same time. This recent protocol is discussed and compared with **SKI** in [48]; therein, it was observed that the resistance of [24] to terrorist fraud lowered the resistance to mafia fraud.

Security shortcomings in DB. Practical DB should also be attack-proof. But, from the above, one can conclude that provably secure DB is still in the making. When security is rarely attained/proved against one fraud, another resistance is diminished [48]. But, more seriously, some of the literature on distance-

bounding uses either unsupported claims of the form "if f is a PRF, then this protocol is secure against...". In fact, in the line of Boureanu *et al.* [7], it was proven, by the technique of *PRF programming*, that if PRFs exist, then these results are incorrect. When employed with some specific PRFs, the TDB [2] protocol, an enhancement of the Kim-Avoine protocol [22], Hancke and Kuhn's [29] protocol, Avoine and Tchamkerten's [3], Reid's *et al.* [39] protocol, and the Swiss-Knife [34] protocol, they were all shown to be indeed vulnerable to distance-fraud and/or man-in-the-middle attacks. The DB security claims recently disproven by Boureanu *et al.* [7] seem to come from a mis-use of PRF techniques: replacing a PRF (in security arguments) by a random function at a place where the adversary has access to the PRF key or at a place where the PRF key is simultaneously used at other places in the protocol. In a parallel line, [34] proved that many existing distance-bounding protocols are also subject to mafia-fraud. And, in [4], it is revealed that public-key techniques do not necessarily protect against terrorist-fraud. Also therein, a family of protocols is exposed to generalised mafia-fraud attacks. Finally, Hancke [28] shows that noisy communications and tolerance to them must also be addressed in the security analysis. So, the technicalities of the model to be presented herein, to assure a solid provable security framework, are of utmost importance.

2.3. Contributions

In the context of the shortcomings above, our main contribution is three-fold:

1. We present a formalism for distance-bounding, which includes a sound communication and adversarial model. In these latter models, we incorporate the notion of time-of-flight for distance-based communication.[4] We further formalise security against distance-fraud, man-in-the-middle (MiM) generalising mafia-frauds, and an enhanced version of terrorist-fraud that we call *collusion-fraud*. As practice dictates, our formalisations take noisy communications into account.
2. Mainly in the context of security against generalised mafia-frauds (when TF-resistance is also enforced), we introduce the concept of *circular-keying security* to extend the security of a pseudo-random function (PRF) f to its possible uses in maps of the form $y \mapsto L(x) + f_x(y)$, for a secret key x and a transformation L. We also introduce a *leakage scheme*, to resist to collusion frauds, and a *PRF masking* technique to address distance-fraud issues. These formal mechanisms come to counteract mistakes like those in proofs based on PRF-constructions, errors of the kind exposed by Boureanu *et al.* in [7], and by Hancke in [28].
3. We analyse variants of the **SKI** protocol [11,10], leading us to a provably secure, practical class of distance-bounding protocols. On the way to this, we formalise the DB-driven requirements of the **SKI** protocols' components. In addition to enjoying provable security, the **SKI** protocols offer competitive performance and practical security. Especially in terms of suitability to practice, **SKI** is one of the two DB protocols that resist terrorist-frauds in the presence of noise.

Note: *The **SKI** contribution was first presented at FSE'13 [11] and then at LIGHTSEC'13 [10]. On these both occasions, the protocols were presented without details on a formal model and without their corresponding security proofs. For instance, to justify some security bounds on the classical DB threats, the authors used reasoning related to the best conceivable attacks, but no provable security argument was included. Recently, at ISC'13 [9], a partial security model for **SKI** and partially developed security*

[4]Since every send/receive action in our model is subject to a maximal transmission speed, there is no distinction between a lazy phase and a time-critical one as in the DFKO model [22,23].

proofs were included. In turn, this present article comes with the full, formal *security model,* full *security proofs for all the **SKI** protocols, as well as all the details on the* tight*, provable bounds of **SKI**'s security. With respect to a somewhat similar manuscript [8], we add that the present article contains significant improvements and updates, both on the proofs and on the tightness of the security bounds. We put together and also* complete *all the pieces needed for the full picture of **SKI** and its provable security; in this way, this is also the first complete document to provide a recipe on how to design provably secure DB and/or how to (dis)prove the security of existing ones.*

3. Model for Distance-Bounding Protocols

We consider a multiparty setting where each participant U is modelled by a polynomially bounded interactive Turing machine (ITM), has a location loc_U, and where communication messages from a location to another take some time, depending on the distance to travel. Some participants may be corrupted. Some are set up with a pre-shared key. All algorithms are bounded to probabilistic polynomial-time (ppt).

As aforementioned, we model a generic two-party communication protocol by the interactive system run by ITMs [27]; we now fix the notations.[5] Consider two honest participants P and V, each running a predefined *algorithm* denoting its side of the interaction to take place. Along standard lines, a general communication is formalised via an *experiment*, generically denoted $exp = (P(x;r_P) \longleftrightarrow V(y;r_V))$, where $r_{\langle\cdot\rangle}$ are the random coins of the participants and x is an input of P and y is the input of V. In some cases, $x = y$ denoting a long-term shared secret. The experiment above can be "enlarged" with an adversary $\mathcal{A}_0$ who interferes in the communication, up to his abilities (which will be described below). This "enlargement" is hereby denoted as $(P(x;r_P) \longleftrightarrow \mathcal{A}_0(r_{\mathcal{A}}) \longleftrightarrow V(y;r_V))$. At the end of each experiment, participant V has an *output*, denoted, Out_V. The *view* of a participant on an experiment is the collection of all its initial inputs (including coins) and his incoming messages, i.e., the view of $\mathcal{A}_0$ above subsumes his "communication" with P and his "communication" with V. In the notation $(P(\ldots) \longleftrightarrow \mathcal{A}(\ldots) \longleftrightarrow V(\ldots))$, we may group several participants under the same symbolic name; e.g., we may group several (colluding) malicious participants encapsulated under a single $\mathcal{A}$ denomination.

Bound on the Distance. To our modelling, we add a fixed integer constant $\mathbb{B}$ denoting the *distance-bound*. It defines what it means to be "close-enough" to a verifier V. Hence, the output of a verifier is 1 if the responses authenticate the prover and his estimated[6] location is not further than $\mathbb{B}$ in the metric space.

3.1. The Crux of the DB model

The crux of proving the security of DB protocols lies in Lemma 3.1, stated below.

Informal Formulation of Lemma 3.1. By Lemma 3.1 below, we informally mean the following: if V sends a challenge c, then the answer r from a close-by participant $\mathcal{A}$ is locally computed by $\mathcal{A}$ itself. In other words, to compute r, the close-by $\mathcal{A}$ cannot get any online, real-time help dependent on the challenge c, not from any far-away participant. This is logical: getting distant help dependent on c

[5] We use standard notations for ITMs. Namely, random coins are separated from other inputs by a semicolon or omitted for simplicity. Inputs consist of the initial input and the variable number of incoming messages.

[6] This estimation is based on round-trip time, i.e., each response ought to be received before V has $2\mathbb{B}$ standby actions.

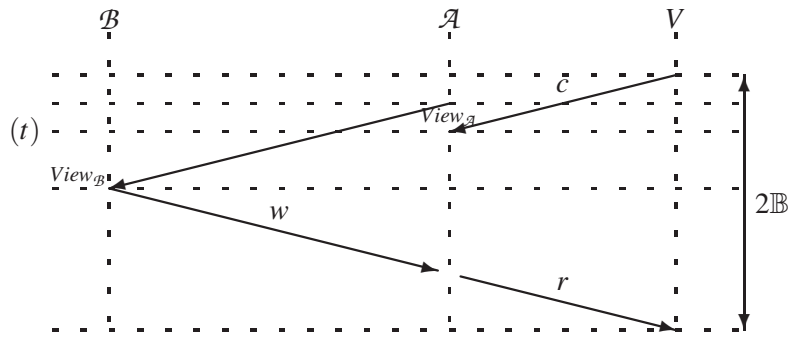

Figure 1. Adversarial Communication Flow Over Time

would mean that this challenge c travelled to that distant location, which in turn would mean failing the time/distance bound.

In more details, in computing such an answer, $\mathcal{A}$ would use two parts: 1). its own view gathered up to the arrival of c inclusively; 2). possible material that $\mathcal{A}$ may receive before $\mathcal{A}$ sends r out; all such material must be independent from c and from all the messages sent to $\mathcal{A}$ thereafter, even if it may come from far-away participants $\mathcal{B}$.

The Use of Lemma 3.1. We will use this lemma every time when a too-long-distance has an implication on the data-flow. We believe such a clear-cut formalisation eases the proofs. For instance, in the DFKO model [22], the implicitness of timed communications requires an effective distinction between a lazy and a time-critical phase in the runs of the protocols, which may in turn hinder the construction of clear security proofs. The DFKO model also requires to define exhaustively which data flows are allowed (the *tainted sessions*) for each security notion.

Another way to go about this would have been to introduce a full model in which such a lemma holds; in fact, we do so in Appendix A. Or, yet another way would have been to simply state the text of the lemma and take it axiomatically. Instead, we took the approach of enunciating it formally and proving it.

Lemma 3.1. *Assume an experiment $\mathcal{B}(z;r_{\mathcal{B}}) \leftrightarrow \mathcal{A}(u;r_{\mathcal{A}}) \leftrightarrow V(y;r_V)$ in which the verifier V plays a two-round protocol where he broadcasts a message c, then V receives a response r, and V accepts if r took at most time $2\mathbb{B}$ to arrive. In the experiment, $\mathcal{A}$ is the set of all participants which are within a distance up to $\mathbb{B}$ to V, and $\mathcal{B}$ is the set of all other participants. For each user U, we consider his partial view $View_U$ which includes all his input until just before the time when U can see the broadcast message c. We say that a message by U is* independent *from c if it is computed by U before this time (equivalently: if it is the result of applying U on $View_U$, or a prefix of it). There exists an algorithm $\mathcal{A}$ and a list w of messages independent from c such that if V accepts, then $r = \mathcal{A}(View_{\mathcal{A}}, c, w)$, where $View_{\mathcal{A}}$ is the list of all $View_A$, $A \in \mathcal{A}$.*

w.r.t. the model in Appendix A. We first assume a single participant in $\mathcal{A}$. Fig. 1 illustrates the communication flow. Let $(p;r_{\mathcal{A}})$ be the partial view such that $r = \mathcal{A}(p;r_{\mathcal{A}})$. Clearly, p can be written $p = (v,c,w)$ with $(v;r_{\mathcal{A}}) = View_{\mathcal{A}}$ and a list w of messages from $\mathcal{B}$ participants. If w includes a message m not independent from c, there is time for c to arrive to $\mathcal{B}$, to compute m, sent it to $\mathcal{A}$, compute r and sent it to V. Due to the distance between $\mathcal{B}$ and V, this is not the case. So, all messages in w are independent from c. This means that, in due time, $\mathcal{A}$ cannot get any help from $\mathcal{B}$ to answer to c.

With several participants in $\mathcal{A}$, there is one $A \in \mathcal{A}$ for which $r = A(v_A, c, w_A; r_A)$ and messages in w_A are either $\mathcal{A}$ messages, and can be written the same (recursively), or $\mathcal{B}$ messages which are independent from c. □

3.2. Formal Distance-Bounding

When modelling distance-bounding protocols, we consider provers, denoted by P and verifiers, denoted by V. We let $\mathcal{A}$ denote the adversary and P^* generally denote dishonest provers. We assume that provers have no output and verifiers output one bit Out_V denoting acceptance, i.e. $Out_V = 1$, or rejection, i.e., $Out_V = 0$ (e.g., privileges are granted or not). We proceed with the definition of a DB protocol.

Definition 3.1 (Distance-Bounding Protocols)**.** *A* distance-bounding (DB) protocol *is defined by a tuple* $(Gen, P, V, \mathbb{B})$*, where: 1. Gen is a randomised, key-generation algorithm such that* (x, y) *is the output*[7] *of* $Gen(1^s; r_k)$*, where* r_k *are the random coins of Gen and s is a security parameter; 2.* $P(x; r_P)$ *is a ppt. ITM running the algorithm of the prover with input x and random input* r_P*; 3.* $V(y; r_V)$ *is a ppt. ITM running the algorithm of the verifier with input y, and random input* r_V*; 4.* $\mathbb{B}$ *is a* distance-bound*. They must be such that the following two facts hold:*

- ***Termination****:* $(\forall s)(\forall R)(\forall r_k, r_V)(\forall loc_V)$ *if* $(\cdot, y) \leftarrow Gen(1^s; r_k)$ *and* $(R \longleftrightarrow V(y; r_V))$ *model the execution, it is the case that V halts in* $Poly(s)$ *computational steps, where R is any set of (unbounded) algorithms;*[8]
- p***-Completeness****:* $(\forall s)$ $(\forall loc_V, loc_P$ *such that* $d(loc_V, loc_P) \leq \mathbb{B})$ *we have*

$$\Pr_{r_k, r_P, r_V}\left[Out_V = 1 : \begin{array}{c}(x,y) \leftarrow Gen(1^s; r_k) \\ P(x; r_P) \longleftrightarrow V(y; r_V)\end{array}\right] \geq p.$$

Throughout, "$\Pr_r[\mathsf{event} : \mathsf{experiment}]$*" denotes the probability that an event takes place after the experiment has happened, taken on the set of random coins r underlying the experiment. The random variable associated to the event is defined via the experiment. Hence, we are not referring here to two events conditioning one another, but just to an experiment leading to the description of a random variable.*

DB Concurrency. Our model implicitly assumes *concurrency* involving participants not sharing the secret inputs amongst them. In security definitions, these extra participants are implicitly universally quantified. When several provers using the same input x appear in experiments, they will be explicitly mentioned. I.e., several instances of the same participant at different location and/or time.

3.3. DB Threats

The security requirements of DB protocols, i.e., the resistance to the different DB threats, are formalised in the definitions to follow. The parameters used therein, $\alpha, \beta, \gamma, \gamma$ are real numbers in the interval $[0, 1]$.

[7] We denote this output as $(x, y) \leftarrow Gen(1^s; r_k)$. For all protocols in this paper, there is just one common input, i.e., we assume $x = y$.

[8] In the above, only the termination of V is of interest, since it is only the verifier who has a meaningful output.

3.3.1. (Generalised) Distance-Fraud

Definition 3.2 (α-resistance to distance-fraud)**.** $(\forall s)$ $(\forall P^*)$ $(\forall loc_V$ *such that* $d(loc_V, loc_{P^*}) > \mathbb{B})$ $(\forall r_k)$*, we have*

$$\Pr_{r_V}\left[Out_V = 1 : \begin{array}{c}(x,y) \leftarrow Gen(1^s; r_k)\\ P^*(x) \longleftrightarrow V(y; r_V)\end{array}\right] \leq \alpha$$

where P^* is any (unbounded) dishonest prover. In a concurrent setting, we implicitly allow a polynomially bounded number of honest $P(x')$ and $V(y')$ close to $V(y)$ with independent (x', y').

Informal Explanation of Def. 3.2. The above definition states, in our modelling, the notion of resisting to distance-fraud: i.e., a participant P^* that is situated somewhere beyond the distance-bound should not succeed in making the verifier accept but with a very low probability hereby denoted by α.

Relation with Other Formalisms. In a 2-party setting, the above definition corresponds to the one of the ABKLM model [1]. When α is negligible, our security notion becomes equivalent to the one in the DFKO model [22].

Relation with Distance Hijacking [18]. Due to our concurrent setting, Def. 3.2 captures the notion of distance hijacking in [18], i.e., an experiment in which a dishonest far-away prover P^* may use several provers to get authenticated as one, honest P that is close to the verifier.

3.3.2. (Generalised) Mafia-Fraud

Definition 3.3 (β-resistance to MiM)**.** $(\forall s)(\forall m, \ell, z)$ *polynomially bounded,* $(\forall \mathcal{A}_1, \mathcal{A}_2)$ *polynomially bounded, for all locations such that* $d(loc_{P_j}, loc_V) > \mathbb{B}$*, where* $j \in \{m+1, \ldots, \ell\}$*, we have*

$$\Pr\left[Out_V = 1 : \begin{array}{c}(x,y) \leftarrow Gen(1^s)\\ P_1(x), \ldots, P_m(x) \longleftrightarrow \mathcal{A}_1 \longleftrightarrow V_1(y), \ldots, V_z(y)\\ P_{m+1}(x), \ldots, P_\ell(x) \longleftrightarrow \mathcal{A}_2(View_{\mathcal{A}_1}) \longleftrightarrow V(y)\end{array}\right] \leq \beta$$

over all random coins, where $View_{\mathcal{A}_1}$ is the final view of $\mathcal{A}_1$. In a concurrent setting, we implicitly allow a polynomially bounded number of $P(x')$, $P^*(x')$, and $V(y')$ with independent (x', y'), anywhere.

Informal Explanation of Def. 3.3. In man-in-the-middle (MiM) attacks or generalised mafia-frauds as above, we consider that during a learning phase*, the attacker interacts, in parallel, with $m \geq 0$ provers and $z \geq 0$ verifiers. Then —in the* attack phase*— the adversary tries to win in an experiment in front of a verifier which is far-away from $\ell - m \geq 0$ provers. (Using the notation $\mathcal{A}_1$ for the learning phase and $\mathcal{A}_2$ for the attack phase is just to show that the adversarial behaviours in these phases might be different. As the reader can notice, the attacker $\mathcal{A}_2$ shares the view/knowledge of $\mathcal{A}_1$.)*

By the learning phase, Def. 3.3 models practical threats. For instance, an attacker would have cloned several tags and would make them interact with several readers with which they are registered. From such a multi-party communication, the attacker can get potentially more benefits, in a shorter period of time. Of course, an attacker can in fact set up this learning phase as he pleases, to increase his gains. So, we can even imagine that he places prover-tags close to verifier-readers, even if being an active adversary between two neighbouring P and V is technically more challenging than interfering between two far-away parties. E.g., in this scenario, the adversary could interfere with the initial frequency synchronisation phase so that the $P \leftrightarrow \mathcal{A}$ and $\mathcal{A} \leftrightarrow V$ channels would become different (e.g., using different frequency bands) and P and V would not even be aware of the existence of the other channel.

In any case, note that the learning phase is not obligatory in our setting (m and z can be 0). Indeed, we further consider mafia-frauds as a specialisation of the above, where no learning phase is present. But, if and when a non-trivial learning phase is present, it renders a stronger threat model and proven resistance to such attacks entails better security.

Relations with Mafia-fraud and Other Frauds. The classical notion of mafia-fraud (the one from the ABKLM model [1]) corresponds to $m = z = 0$ (i.e., no learning phase), and $\ell = 1$.

The classical notion of impersonation for identification schemes corresponds to $\ell = m$ (i.e., there is no prover in the attack phase).

Relation with Other Formalisms. The DFKO model [22] of mafia-fraud already includes the above general extension since concurrent settings are implicit in the DFKO model.

Non-narrow Attackers. We will now describe a special type of (MiM) attackers, following a notion introduced in [47]. Thereby, a (MiM) attacker is *non-narrow* if he can learn the bit that the verifier outputs. A way in which this can be trivially formalised is by adding a return channel to the communication, here denoting that the verifier V sends Out_V as a final message, just before V halts. In real life this is the case, e.g., there is a LED on a door turning green denoting "access-granted" and turning red otherwise.

It is pertinent to formalise such an attacker: intruders learn obviously more information by looking also at whether the run was successful or not. Indeed, in the generalised MF presented in [4], it is this sort of return channel that facilitates the attacks. To avoid defining a new class of attacks (as done in the literature [47]), we define this as a property of the protocol.

Definition 3.4 (Non-narrow MiM)**.** *A distance-bounding protocol is called* non-narrow *if it terminates by V sending Out_V to P as his final message.*

3.3.3. (Generalised) Terrorist-Fraud

Definition 3.5 ((γ,γ')-resistance to collusion-fraud)**.** $(\forall s)(\forall P^*)$ $(\forall loc_{V_0}$ *such that* $d(loc_{V_0}, loc_{P^*}) > \mathbb{B})$ $(\forall \mathcal{A}^{\mathsf{CF}}$ *ppt.*$)$ *such that*

$$\Pr\left[Out_{V_0} = 1 : \begin{array}{c}(x,y) \leftarrow Gen(1^s)\\ P^*(x) \longleftrightarrow \mathcal{A}^{\mathsf{CF}} \longleftrightarrow V_0(y)\end{array}\right] \geq \gamma$$

over all random coins, there exists a (kind of)[9] *MiM attack $m, \ell, z, \mathcal{A}_1, \mathcal{A}_2, P_i, P_j, V_{i'}$ using P and P^* in the learning phase, such that*

$$\Pr\left[Out_V = 1 : \begin{array}{c}(x,y) \leftarrow Gen(1^s)\\ P_1^{(*)}(x), \ldots, P_m^{(*)}(x) \longleftrightarrow \mathcal{A}_1 \longleftrightarrow V_1(y), \ldots, V_z(y)\\ P_{m+1}(x), \ldots, P_\ell(x) \longleftrightarrow \mathcal{A}_2(View_{\mathcal{A}_1}) \longleftrightarrow V(y)\end{array}\right] \geq \gamma'$$

where P^* is any (unbounded) dishonest prover and $P^{(*)}$ runs either P or P^*. Following the MiM requirements, $d(loc_{P_j}, loc_V) > \mathbb{B}$, for all $j \in \{m+1, \ell\}$. In a concurrent setting, we implicitly allow a polynomially bounded number of $P(x')$, $P^*(x')$, and $V(y')$ with independent (x', y'), but no honest participant close to V_0.

[9] Def. 3.3 defines MiM attacks as using an honest $P(x)$. Here, we deviate a bit by introducing $P^*(x)$ as well.

Informal Explanation of Def. 3.5. Definition 3.5 expresses the following. Consider a prover P^*, situated far-away from V_0, who can help an adversary $\mathcal{A}^{\mathsf{CF}}$ located closer to V_0 pass a distance-bounding protocol. Then, a malicious adversary denoted as $(\mathcal{A}_1, \mathcal{A}_2)$ could run a successful MiM attack[10], "playing" with possibly multiple instances of $P^*(x)$ in the learning phase. In other words, a dishonest prover P^* cannot successfully collude with $\mathcal{A}^{\mathsf{CF}}$ without leaking some private information.

Note that collusion frauds are non-falsifiable. However, this is inherent to terrorist frauds.

Relation with Terrorist-fraud. Collusion-frauds are more general than terrorist-frauds. The classical notion of terrorist-fraud corresponds to a specialised case of Def. 3.5: the one where $m = z = \ell = 1$ and $\mathcal{A}_1$ runs just $\mathcal{A}^{\mathsf{CF}}$ in the learning phase. Put simply, in the classical terrorist-fraud, $\mathcal{A}^{\mathsf{CF}}$ gets information to directly impersonate the prover, whereas in Def. 3.5 we formalise the means to get this information via a learning phase.

Relation with Other Formalisms. In the ABKLM or DFKO models, only the specialised case of collusion frauds mentioned above, i.e., the traditional terrorist-fraud, is considered. In the DFKO model [22], the formalisation of terrorist-fraud further considers $p_A = \Pr[Out_{V_0} = 1]$, and $p_S = \Pr[Out_V = 1 | Out_{V_0} = 1]$. Following some results from [23], a protocol resists to terrorist-fraud if for every $\mathcal{A}^{\mathsf{CF}}$ there is a $\mathcal{A}_2$ such that $p_A \leq p_S$. However, we think that illustrating some $\mathcal{A}^{\mathsf{CF}}$ such that p_A is negligible but for no $\mathcal{A}_2$ we would have $p_A \leq p_S$ [23] is not a strong enough argument for insecurity. It rather shows that the definition from [22] is too strong. In our approach, we decided to characterise resistance herein through a pair of probabilities (γ, γ').

4. Practical and Secure Distance-Bounding Protocols

4.1. *SKI:* DESCRIPTION AND COMPLETENESS

At a high level, the protocol schema **SKI** is presented in Fig. 2. We use the parameters (s, q, n, k, t, t'), where s is the security parameter. The **SKI** protocols are built using a PRF (pseudorandom function), denoted $(f_x)_{x \in GF(q)^s}$, with q being a small power of prime. In the concrete examples in the main body of the paper, we employ $q = 2$, i.e., x, a are simply bitstrings as it is most practical. In the DB phase, n rounds are used, with $n \in \Omega(s)$. Then, **SKI** uses the value $f_x(N_P, N_V, L) \in GF(q)^{t'n}$, with nonces $N_P, N_V \in \{0,1\}^k$ and a mask $M \in GF(q)^{t'n}$, where $k \in \Omega(s)$. In the main proposal, $t' = 2$ is used, i.e., to keep the lightweight character. The element $a = (a_1, \ldots, a_n)$ with $a_i \in GF(q)^{t'}$ is established by V in the initialisation phase, and it is sent encrypted as $M := a + f_x(N_P, N_V, L)$, with $M \in GF(q)^{t'n}$, where $+$ denotes the $GF(q)$-vector addition. Similarly, V selects a random linear transformation L from a set[11] $\mathcal{L}$ which is specified by the **SKI** protocol instance and the parties compute $x' = L(x)$. Further, $c = (c_1, \ldots, c_n)$ is the challenge-vector with $c_i \in \{1, \ldots, t\}$, $r_i := F(c_i, a_i, x'_i)$ is the i-th response to the i-th challenge c_i, with $i \in \{1, \ldots, n\}$, $r_i \in GF(q)$ and F as specified below.[12] In other concrete proposals, $t = 3$, or $t = 2$ for the lighter version, are used. The protocol ends with a message Out_V denoting the output of the verifier (i.e., the success/failure of the protocol), to capture the notion of MiM attackers on a non-narrow protocol.

SKI Instances. We first depict $\mathbf{SKI_{pro}}$ through Fig. 3.

[10] In practice, $\mathcal{A}^{\mathsf{MiM}}$ and $\mathcal{A}^{\mathsf{CF}}$ represent the same adversarial party; we simply differentiate to show that different algorithms/attack-strategies may be involved.

[11] The $\mathcal{L}$ set will be later introduced as a *leakage scheme*; its purpose is to leak $L(x)$ in the case of a collusion-fraud/terrorist-fraud.

[12] This will be called the *F-scheme* and it will incorporate requirements towards (generalised) DF-, TF- and MF-resistance.

Verifier V		**Prover** P
$x; r_V$	$x \in_U GF(q)^s$	$x; r_P$
	Initialisation phase	
	$\xleftarrow{N_P}$	Use r_P to generate a nonce $N_P \in \{0,1\}^k$
Use r_V to generate $a \in GF(q)^{t'n}$, a transformation $L \in \mathcal{L}$ and a nonce $N_V \in \{0,1\}^k$	$\xrightarrow{M,L,N_V}$	
do $M := a + f_x(N_P, N_V, L)$ do $x' := L(x)$, with $x' \in GF(q)^n$		do $a := M - f_x(N_P, N_V, L)$; do $x' = L(x)$, with $x' \in GF(q)^n$
	Distance-bounding phase for $i = 1$ to n	
Use r_V to generate $c_i \in \{1,\ldots,t\}$		
Start Clock	$\xrightarrow{c_i}$	if $c_i \notin \{1,\ldots,t\}$, halt
Stop Clock	$\xleftarrow{r_i}$	do $r_i := F(c_i, a_i, x'_i)$
verify the responses and that clocked-rounds $\leq 2\mathbb{B}$ for at least τ iterations	$\xrightarrow{Out_V}$	

Figure 2. The **SKI** schema of Distance-Bounding Protocols

In fact, in Boureanu *et al.* [11,10], several variants of **SKI** were proposed. We now concentrate on two variants of **SKI**:

- $\mathbf{SKI_{pro}}$ with $q = 2$, $t' = 2$, $t = 3$, with the response-function

$$F(1, a_i, x'_i) = (a_i)_1 \qquad F(2, a_i, x'_i) = (a_i)_2 \qquad F(3, a_i, x'_i) = x'_i + (a_i)_1 + (a_i)_2,$$

  where $(a_i)_j$ denotes the jth bit of a_i, with the transforms L_μ defined each from a vector $\mu \in GF(q)^s$ by

$$L_\mu(x) = (\mu \cdot x, \ldots, \mu \cdot x)$$

  i.e., n repetitions of the same bit $\mu \cdot x$, the dot product of μ and x over $GF(2)$.
- $\mathbf{SKI_{lite}}$ with $q = 2$, $t' = 2$, $t = 2$, with the response-function

$$F(1, a_i, x'_i) = (a_i)_1 \qquad F(2, a_i, x'_i) = (a_i)_2,$$

  with the transform-set $\mathcal{L} = \{\emptyset\}$.

Namely, note that $\mathbf{SKI_{lite}}$ never uses the $c_i = 3$ challenge, i.e., it never uses the part x' having to do directly with the secret key x in the DB responses. Each $\mathbf{SKI_{pro}}$ session uses a transform L_μ on x such

Verifier V		**Prover** P
$x; r_V$	$x \in_U \{0,1\}^s$	$x; r_P$
	Initialisation phase	
	$\xleftarrow{N_P}$	Use r_P to generate a nonce $N_P \in \{0,1\}^k$
Use r_V to generate $a \in \{0,1\}^{2n}$, $\mu \in \{0,1\}^s$ and a nonce $N_V \in \{0,1\}^k$	$\xrightarrow{M,L_\mu,N_V}$	
do $M := a \oplus f_x(N_P, N_V, L_\mu)$ do $x' := L_\mu(x)$, with $x' \in \{0,1\}^n$ with $L_\mu(x) = (\mu \cdot x, \ldots, \mu \cdot x)$		do $a := M \oplus f_x(N_P, N_V, L_\mu)$; do $x' = L_\mu(x)$, with $x' \in \{0,1\}^n$
	Distance-bounding phase for $i = 1$ to n	
Use r_V to generate $c_i \in \{1,2,3\}$		
Start Clock	$\xrightarrow{c_i}$	if $c_i \notin \{1,2,3\}$, halt
Stop Clock	$\xleftarrow{r_i}$	do $r_i := F(c_i, a_i, x'_i)$, where $F(1, a_i, x'_i) = (a_i)_1$, $F(2, a_i, x'_i) = (a_i)_2$, $F(3, a_i, x'_i) = x'_i \oplus (a_i)_1 \oplus (a_i)_2$.
verify the responses and that clocked-rounds $\leq 2\mathbb{B}$ for at least τ iterations	$\xrightarrow{Out_V}$	

Figure 3. The $\mathbf{SKI_{pro}}$ Distance-Bounding Protocol ($q = 2, t = 3, t' = 2$)

that on x' all coordinates are set to the scalar product between μ and x. Since $\mathbf{SKI_{lite}}$ never uses x', $\mathcal{L}$ can be left empty.

We note that both instances are efficient. Indeed, we could precompute the table of $F(\cdot, a_i, x'_i)$ and just do a table lookup to compute r_i from c_i. For $\mathbf{SKI_{pro}}$, this can be done with a circuit of only 7 NAND gates and depth 4. For $\mathbf{SKI_{lite}}$, 3 NAND gates and a depth of 2 are enough. The heavy computation lies in the f_x evaluation which occurs in a phase which is not time-critical. In practice, any reasonable PRF suffices as it can satisfy the circular-keying condition to be stated below.

However, in our design, we need the reuse of x for protection against terrorist-fraud and/or collusion-fraud. Along these lines, the $\mathbf{SKI_{lite}}$ protocols do not assume circular-keying security (as defined below), but the $\mathbf{SKI_{pro}}$ do.

In Appendix B, we consider other variants of **SKI** with different F-schemes (using, e.g., two-bit responses) which we still deem very practical.

SKI Completeness (in Noisy Communications). We would like to investigate the suitability of the selected parameters. More precisely, we verify for which parameters, **SKI** is in line with Definition 3.1, i.e., it definitely terminates, but the completeness bound can be "tuned".

Each (c_i, r_i) exchange is time-critical, so it is subject to errors. To address this, we introduce the probability p_{noise} of one response being erroneous (à la Hancke-Kuhn [29]). Then, the **SKI** protocol specifies that the verifier accepts only if the number of correct answers is at least τ, where τ is an extra parameter. The probability that at least τ responses out of n are correct is clearly given by:

$$B(n, \tau, 1 - p_{noise}) = \sum_{i=\tau}^{n} \binom{n}{i} (1 - p_{noise})^i p_{noise}^{n-i}$$

It is natural to choose τ (and other parameters) such that we operate with correct DB protocols, cnf. with Definition 3.1. I.e., the protocol is complete: honest communications succeed with high probability.

Lemma 4.1. *Let $\varepsilon > 0$. For $\tau \leq (1 - p_{\mathsf{noise}} - \varepsilon)n$, the **SKI** protocols are $(1 - e^{-2\varepsilon^2 n})$-complete.*

Proof. Due to the the Chernoff-Hoeffding bound [16,31], $\tau \leq (1 - p_{\mathsf{noise}} - \varepsilon)n$ implies $B(n, \tau, 1 - p_{noise}) \geq 1 - e^{-2\varepsilon^2 n}$. According to Definition 3.1, this makes the **SKI** protocols $(1 - e^{-2\varepsilon^2 n})$-complete. □

In practice, we may use a constant p_{noise} (i.e., hard-coded in the protocol implementation). This also entails employing τ as some parameter which is linear in terms of n. A detailed analysis of the optimal selection of this threshold τ is provided in [20].

4.2. *SKI:* SECURITY-DRIVEN DESIGN & SECURITY ASSESSMENT

In this subsection, we discuss the design choices that we made in order to render the instances of **SKI** provably secure.

PRF masking. Importantly, **SKI** applies a random mask M on the output of f_x to fix the problems raised in Boureanu *et al.* [7]. We call this *PRF masking*.

We introduce PRF masking to protect against the class of attacks in [7]. I.e., Without PRF masking, M is not used (or equivalently, M is always set to 0). Then we could construct [7] a PRF such that, e.g., for all x and N_V, the value of $f_x(x, N_V, L)$ is such that $F(c_i, a_i, x_i')$ does not depend on c_i. In this way, a malicious prover could set, e.g., $N_P = x$ and predict the answer $F(c_i, a_i, x_i')$ without having received the challenge c_i. Hence, he could mount a successful distance-fraud. By having the verifier decide a (thus, by masking the value of the PRF-instance $f_x(x, N_V, L)$), **SKI** enforces that the distribution of a cannot be influenced by a malicious prover.

***F*-scheme.** In our way to prove security, we need some notions related to the response-function F; these characterise the concept of *F-scheme*. At the same time, these concepts give the sufficient conditions to protect against *all* three frauds possible against the concrete **SKI** instances to follow. Such a characterisation is different from the approach in Avoine *et al.* [2], where a response-function based on secret sharing is proposed for the protection against terrorist-fraud *only*, but no formal justification was given to that end; also, the relation between the other frauds and the response-function was not addressed therein. Thus, we stress that using a secret sharing scheme in computing the responses may be too strong and/or insufficient to characterise the protection against frauds mounted onto DB protocols, and we amend this with Definition 4.1 and Definition 4.3.

Definition 4.1 (F-scheme)**.** *Let $t, t' \geq 2$. The response-function $F : \{1, \ldots, t\} \times GF(q)^{t'} \times GF(q) \to GF(q)$ gives an* F-scheme*, which is characterised as follows.*

– *We say that the F-scheme is* linear *if for all challenges c_i in their domain, the $F(c_i,\cdot,\cdot)$ function is a linear form over the $GF(q)$-vector space $GF(q)^{t'} \times GF(q)$ which is non-degenerate in the a_i component.*
– *We say the F-scheme is* pairwise uniform *if*

$$(\forall I \subsetneq \{1,\ldots,t\}, \#I \leq 2)(H(x'_i|F(c_i,a_i,x'_i)_{c_i\in I}) = H(x'_i)),$$

where $(a_i,x'_i) \in_U GF(q)^{t'} \times GF(q)$, #S denotes the cardinality of a set S, and H denotes the Shannon entropy.
– *We say the F-scheme is t-*leaking *if there exists a polynomial time algorithm E such that for all $(a_i,x'_i) \in GF(q)^{t'} \times GF(q)$, we have $E\big(F(1,a_i,x'_i),\ldots,F(t,a_i,x'_i)\big) = x'_i$.*
– *Let F_{a_i,x'_i} denote $F(\cdot,a_i,x'_i)$. We say that the F-scheme is* σ-bounded *if for any $x'_i \in GF(q)$, we have $\mathbb{E}_{a_i}\left(\max_y\left(\#(F^{-1}_{a_i,x'_i}(y))\right)\right) \leq \sigma$, where $x' \in GF(q)$ and the expected-value is $\mathbb{E}$ taken over $a_i \in GF(q)^{t'}$.*

Informal Explanation of Def. 4.1. The pairwise uniformity and the t-leaking property of the F-scheme say that knowing the complete table of the response-function F for a given c_i leaks x'_i, yet knowing only up to 2 entries challenge-response in this table discloses no information about x'_i.

The σ-boundedness of the schemes says that the expected value (taken on the choice of the subsecrets a_i) of the largest preimage of the map $c_i \mapsto F(c_i,a_i,x'_i)$ is bounded by a constant σ. In simple words, it says that it should be hard to invert the response function.

We have $\frac{t}{q} \leq \sigma \leq t$ due to the pigeonhole principle, since $\sum_y \#(F^{-1}_{a_i,x'_i}(y)) = t$. Furthermore, $\sigma \geq 1$.

In relation with the definitions of the F-schemes above, we now prove the following lemma.

Lemma 4.2. *The F-scheme used in $\mathbf{SKI_{pro}}$ is linear, pairwise uniform, $\frac{9}{4}$-bounded, and t-leaking. The F-scheme used in $\mathbf{SKI_{lite}}$ is linear, pairwise uniform, $\frac{3}{2}$-bounded, but not t-leaking.*

This lemma extends to Lemma B.1 given and proven in Appendix B.

Leakage scheme. We can consider several sets $\mathcal{L}$ of transformations to be used in the PRF-instance, of the **SKI** initialisation phase. The idea of the set $\mathcal{L}$ is that, when leaking some noisy versions of $L(x)$ for some random $L \in \mathcal{L}$, the adversary can reconstruct x without noise.

More formally, we introduce the following notion.

Definition 4.2 (Leakage scheme)**.** *Let $\mathcal{L}$ be a set of linear functions from $GF(q)^s$ to $GF(q)^n$. Given $x \in GF(q)^s$ and a ppt. algorithm $e(x,L;r_e)$, we define an oracle $O_{\mathcal{L},x,e}$ producing a random pair $(L,e(x,L))$ with $L \in_U \mathcal{L}$. We say that $\mathcal{L}$ is a (T,u,p)-*leakage scheme *if there exists an oracle ppt. algorithm $\mathcal{A}^{\langle\cdot\rangle}$ limited to u queries, such that for all $x \in GF(q)^s$, for all ppt. e, $\Pr[\mathcal{A}^{O_{\mathcal{L},x,e}} = x|E] \geq p$, where E is the event that all queries return a value such that $d_H(e(x,L),L(x)) < T$, where d_H denotes the Hamming distance.*

Informal Explanation of Def. 4.2. Intuitively, this means that based on r values of L and a noisy $L(x)$, we can decode and return x.

We define $\mathcal{L}_{\mathsf{classic}} = \{L\}$, with only one transformation: the identity function L, i.e., $L(x) = x$. Unfortunately, this is not sufficient to add protection against collusion fraud due to Hancke [28]: given a constant θ, a malicious prover could select a vector e of Hamming weight $n - \tau + \theta$ and provide the full table of all $c_i \mapsto F(c_i,a_i,x_i)$ functions, only that some entries in the table had been changed. Namely, for each $i \in \{1,\ldots,n\}$ with $e_i = 1$, the dishonest prover flips $F(c_i,a_i,x_i)$ in this leaked table. Then, we would have

$\gamma = \left(1 - \frac{1}{t}\right)^{\theta}$, but this helped attacker can only reconstruct $x+e$. Using multiple coerced provers P^* will not reveal anything more, if the function $g(x)$ giving e is *deterministic* (i.e., then, several runs would have no randomised, adaptive choices of $g(x)$, coming from P^*'s). Depending on such functions g, and since $n-\tau$ is linear, recovering x takes exponential time. So, the value of $x+g(x)$ is not enough to run a MiM attack since we need x to evaluate f_x.

We consider the leakage scheme $\mathcal{L}_{\mathsf{bit}}$ of $\mathbf{SKI_{pro}}$, consisting of all L_μ transforms, where L_μ is defined from a vector $\mu \in GF(q)^s$ by

$$L_\mu(x) = (\mu \cdot x, \ldots, \mu \cdot x)$$

The following lemma is trivial.

Lemma 4.3. *$\mathcal{L}_{\mathsf{classic}}$ is a $(1,1,1)$-leakage scheme.*

Lemma 4.4. *For all constant $u > s$, $\mathcal{L}_{\mathsf{bit}}$ is a $(\frac{n}{2}, u, 1-q^{s-u})$-leakage scheme.*

Proof. $\mathcal{A}$ calls the oracle u times, then —by computing the majority— $\mathcal{A}$ deduces $\mu \cdot x$ whenever the Hamming distance to $L_\mu(x)$ of the returned vector is lower than $\frac{n}{2}$, for each of the obtained μ. After collecting u samples μ, they span the entire $\mathsf{GF}(q^s)$ vector space except with probability bounded by q^{s-u}. Then, we deduce x by solving a linear system. □

Circular-Keying Security. On our way to prove the security of the **SKI** protocols, we need and hereby introduce the notion of *security against circular-keying*. This notion of security will help protect against MiM, in the context in which the key x is used in the response-function to protect against TF. To attain provable security against MiM attackers, we take *secure* circular-keying as an extra assumption to the PRF $(f_x)_{x \in GF(q)^s}$ to handle the reuse of a fixed x outside of a PRF instance f_x.

Definition 4.3 (Circular-Keying)**.** *Let s be some security parameter, let b be a bit, let $q \geq 2$, let $m \in Poly(s)$, and let $x, \overline{x} \in GF(q)^s$ be two row-vectors. Let $(f_x)_{x \in GF(q)^s}$ be a family of (keyed) functions, e.g., $f_x : \{0,1\}^* \to GF(q)^m$. For an input y, the output $f_x(y)$ can be represented as a row-vector in $GF(q)^m$.*

We define an oracle $O_{f_x,\overline{x}}$ such that upon a query of form (y_i, A_i, B_i), with $A_i \in GF(q)^s$, $B_i \in GF(q)^m$, it answers $(A_i \cdot \overline{x}) + (B_i \cdot f_x(y_i))$. The game $Circ_{f_x,\overline{x}}$ of circular-keying *with an adversary $\mathcal{A}$ is described as follows: we set $b_{f_x,\overline{x}} := \mathcal{A}^{O_{f_x,\overline{x}}}$, where the queries (y_i, A_i, B_i) from $\mathcal{A}$ must follow the restriction that*

$$(\forall c_1, \ldots, c_k \in GF(q)) \left(\#\{y_i; c_i \neq 0\} = 1, \sum_{j=1}^{k} c_j B_j = 0 \Longrightarrow \sum_{j=1}^{k} c_j A_j = 0 \right).$$

*We say that the family of functions $(f_x)_{x \in GF(q)^s}$ is an (ε, C, Q)-*circular-PRF *if for any ppt. adversary $\mathcal{A}$ making Q queries and having complexity C, it is the case that $\Pr[b_{f_x,x} = b_{f^*,\overline{x}}] \leq \frac{1}{2} + \varepsilon$, where the probability is taken over the random coins of $\mathcal{A}$ and over the random selection of $x, \overline{x} \in GF(q)^s$ and the random function f^*.*

The condition on the queries means that for any set of queries with the same value y_i, any linear combination making B_j vanish makes A_j vanish at the same time. (Otherwise, we would trivially extract some information about $\overline{x}$ by linear combinations.)

We note that it is possible to create secure circular-keying in the random oracle model (ROM) [5]. This is a "sanity check" for our circular-keying notion. Indeed, any "reasonable" PRF should satisfy this constraint. Only special constructions would not. E.g., the ones based on PRF programming from [7].

Lemma 4.5. *Let $f_x(y) = H(x,y)$, where H is a random oracle, $x \in \{0,1\}^s$, and $y \in \{0,1\}^*$. Then, f is a $(T2^{-s}, T, Q)$-circular PRF for any T and Q.*

Proof. Let (y, A_i, B_i), $i \in 1,\ldots,k$, be some queries to $O_{f_{x,\overline{x}}}$ that share the same y, made by some $\mathcal{A}$, making no query to H. We define the matrices $A = (A_1 \cdots A_k)^T$ and $B = (B_1 \cdots B_k)^T$. Thus, $\mathcal{A}$ learns $A\overline{x} + BH(x,y)$. Now, w.l.o.g., assume that $\mathcal{A}$ multiplies $A\overline{x} + BH(x,y)$ to the left by a conveniently chosen, invertible matrix P, i.e., such that $PB = (I_p\ 0)^T$ where I_p is the identity matrix of rank p of B and 0 is a zero matrix block.

By taking $c = c'P$ with $c' = (0,\ldots,0,1,0\ldots,0)$, where 1 appears at some position j for any $j > p$, we have that $cB = 0$. Then, by circular keying, we have that $cA = 0$. Thus, all rows from positions beyond p, i.e., $p+1, p+2, \ldots$ "downwards" inside the matrix PA, are filled with zeroes. Thus, $\mathcal{A}$ learns $A'\overline{x} + H(x,y)$, where A' is the "upper-part" of PA, i.e., above the pth row. We have shown that $\mathcal{A}$ is equivalent to an adversary learning $A'\overline{x} + H(x,y)$ for some random matrix A'. So, we can replace $H(x,y)$ by something random and the advantage of the adversary $\mathcal{A}$ in this game would not change.

Now, in the random oracle model, $\mathcal{A}$ also queries H. We consider the hybrids of $\mathcal{A}$ in which the first queries to H are simulated and the hybrid stops before making the next query to H (there are up to T hybrids). We apply the previous argument to the hybrids to show that they cannot query H with x, except by guessing it with probability 2^{-s}. □

We proceed with inspecting the rest of the security requirements on these protocols.

Theorem 4.1. *The **SKI** protocols are secure distance-bounding protocols, i.e.,:*

- *A. If the F-scheme is linear and σ-bounded, if $(f_x)_{x \in GF(q)^n}$ is a (ε, nN, C)-circular PRF, then the **SKI** protocols offer α-resistance to distance-fraud, with $\alpha = B(n, \tau, \frac{\sigma}{t}) + \varepsilon$, for attacks limited to complexity C and N participants. So, we need $\frac{\tau}{n} > \frac{\sigma}{t}$ for security.*
- *B. If the F-scheme is linear and pairwise uniform, if $(f_x)_{x \in GF(q)^n}$ is a $(\varepsilon, n(\ell+z+1), C)$-circular PRF, if $\mathcal{L}$ is a set of linear mappings, the **SKI** protocols are β-resilient against (non-narrow) MiM attackers with parameters ℓ and z and a complexity bounded by C, $\beta = B(n, \tau, \frac{1}{t} + \frac{t-1}{t} \times \frac{1}{q}) + 2^{-k}\left(\frac{\ell(\ell-1)}{2} + \frac{z(z+1)}{2}\right) + \varepsilon$. So, we need $\frac{\tau}{n} > \frac{1}{t} + \frac{t-1}{t} \times \frac{1}{q}$ for security.*
- *B'. If the F-scheme is linear and pairwise uniform, if $(f_x)_{x \in GF(q)^n}$ is a $(\varepsilon, n(\ell+z+1), C)$-PRF, if the function $F(c_i, a_i, \cdot)$ is constant for each c_i, a_i, the **SKI** protocols are β-resilient against (non-narrow) MiM attackers with parameters ℓ and z and a complexity bounded by C, $\beta = B(n, \tau, \frac{1}{t} + \frac{t-1}{t} \times \frac{1}{q}) + 2^{-k}\left(\frac{\ell(\ell-1)}{2} + \frac{z(z+1)}{2}\right) + \varepsilon$. So, we need $\frac{\tau}{n} > \frac{1}{t} + \frac{t-1}{t} \times \frac{1}{q}$ for security.*
- *C. If the F-scheme is t-leaking, if $\mathcal{L}$ is a (T, u, p)-leakage scheme, for all $\theta \in]0,1[$, the **SKI** protocols offer $\left(\gamma, \gamma'\right)$-resistance to collusion-fraud, for $\gamma \geq B(T, T+\tau-n, \frac{t-1}{t})^{1-\theta}$, γ^{-1} is polynomially bounded, and $\gamma' = \left(1 - B(T, T+\tau-n, \frac{t-1}{t})^{\theta}\right)^u p$. So, we need $\frac{\tau}{n} > 1 - \frac{T}{tn}$ for security.*

The proof of Th. 4.1.B' is similar (and simplified) as the one of Th. 4.1.B. So, we prove the A, B, and C parts only.

In the noiseless case (i.e., with $p_{\mathsf{noise}} = 0$), we can work with $\tau = n$. Interestingly, we can then use $\mathcal{L} = \mathcal{L}_{\mathsf{classic}}$ and still have resistance to collusion fraud, with $\gamma \geq (\frac{t-1}{t})^{1-\theta}$ and $\gamma' = 1 - (\frac{t-1}{t})^{\theta}$.

Th. 4.1.A. For each key x' which is different from x and for which there is a $P(x')$ close to V (so, there is no $P^*(x')$ anywhere, due to the distance-fraud model), we apply the circular-PRF reduction. (Details as for why we can apply this reduction will appear in the proof of Th. 4.1.B.) We are losing a probability up to ε in this reduction.

We recall that if the F-scheme is linear, then $F(c_i,a_i,x'_i)$ must be non-degenerate in a_i. So, answers r_i coming from $P(x')$ instead of $P^*(x)$ are correct with probability $\frac{1}{t}$, since a_i is random, after the circular-PRF reduction.

If r_i now comes from P^*, due to Lemma 3.1, r_i must be a function independent from c_i. I.e., P^* must have $F(c_i,a_i,x'_i)$ ready, before c_i arrives from V. So, for any secret x and a, the probability to get one response right is given by $p_i = \Pr_{c_i\in\{1,\ldots,t\}}[r_i = F(c_i,a_i,x'_i)]$.

Thanks to PRF masking, the distribution of the a_i's is uniform. Namely, P^* cannot influence their distribution by selecting N_P maliciously.

To establish the probabilities p_i, consider the partitions I_j, $j\in\{1,\ldots,t\}$ as follows: for $i\in I_j$, the largest preimage of $F_{a_i,x'_i} : c_i \mapsto F(c_i,a_i,x'_i)$ has size j, i.e., $\max_y\left(\#(F^{-1}_{a_i,x'_i}(y)\right) = j$. Then, we are looking at the probability

$$P_j(x'_i) := \Pr_{a_i}\left[\max_y\left(\#(F^{-1}_{a_i,x'_i}(y))\right) = j\right],$$

where $\#(S)$ denotes the cardinality of a set S. Given x' fixed, each iteration has a probability to succeed equal to

$$\frac{P_1}{t} + \frac{2P_2}{t} + \cdots + \frac{tP_t}{t} = \frac{\sigma}{t}$$

So, the probability to win the experiment is bounded by $p = B(n,\tau,\frac{\sigma}{t})$. □

Tightness. The above result is tight as the following attack shows. It is thus the best distance fraud. We consider a malicious (far-away) prover who follows normally the initialisation phase. For the distance bounding phase, he anticipates the challenge c_i and sends the response r_i in advance so that it arrives on time. The response is chosen such that $\#F^{-1}_{a_i,x'_i}(r_i)$ is maximal. So, the probability that the verifier accepts is $B(n,\tau,\frac{\sigma}{t})$, which is negligibly close to α.

Th. 4.1.B. In the next, $P(\ldots)$ and $V(\ldots)$ respectively denote the algorithm/(part of the) protocol of a generic prover P and that of a generic verifier V, out of the ℓ provers and $z+1$ verifiers in this attack-game, run on specific parameters to be specified in-line. We herein denote V in the MiM-resistance definition as V_{z+1}.

We use the game-reduction methodology [42] to prove this lemma. Let $Game_0$ be the non-narrow MiM attack-game described in Definition 3.3 played by $\mathcal{A}$ against the honest parties in a **SKI** protocol.

Below we consider a prover P_j and a verifier V_k in an experiment, $j\in\{1,\ldots,\ell\}, k\in\{1,\ldots,z+1\}$. Let $(N_{P,j},\overline{M}_j,\overline{L}_j,\overline{N}_{V,j})$ be the values of the nonces (N_P,N_V), of the mask M, and of the transformation L that the prover P_j generates or sees respectively, and $(\overline{N}_{P,k},M_k,L_k,N_{V,k})$ be the values of the nonces (N_P,N_V), mask M, and transformation L that a verifier V_k generates or sees at his turn, $j\in\{1,\ldots,\ell\}, k\in\{1,\ldots,z+1\}$.

We apply a reduction by failure-event to prove that the game $Game_0$ is indistinguishable to the adversary $\mathcal{A}$ from a game $Game_1$ where no repetitions on $N_{P,j}$ or on $N_{V,k}$ happen for $j\in\{1,\ldots,\ell\}$,

$k \in \{1,\ldots,z+1\}$, i.e., there is no collision on the nonces generated by the provers and there is no collision on the nonces of the verifiers.

Assume that F is the event that at least a collision as above happens, i.e.,

$$F \equiv \Big(\bigvee_{0<i<j\leq \ell} (N_{P,i} = N_{P,j}) \Big) \bigvee \Big(\bigvee_{0<i'<j'\leq z+1} (N_{V,i'} = N_{V,j'}) \Big).$$

We want to have that, from the point of view of the adversary $\mathcal{A}$, $Game_0 \wedge \neg F \Leftrightarrow Game_1 \wedge \neg F \Leftrightarrow Game_1$. But,

$$\|Pr[A \text{ wins in } Game_0] - Pr[A \text{ wins in } Game_1]\| \leq \Pr[F].$$

Then, $\Pr[F] \leq 2^{-k}\Big(\frac{\ell(\ell-1)}{2} + \frac{z(z+1)}{2}\Big)$.

Since the F-scheme is linear, we can write $F(c_i, a_i, x'_i) = u_i(c_i)x'_i + (v_i(c_i) \cdot a_i)$ where $u_i(c_i) \in GF(q), v_i(c_i) \in GF(q)^{t'}$. Note that, in terms of i, the $(v_i(1),\ldots,v_i(t))$'s span independent linear spaces. In $Game_1$, each (N_P, N_V, L, i) tuple can be invoked only twice (with a prover and a verifier) by the adversary. The pairwise uniformity of the F-scheme implies that $yv_i(c_i) + y'v_i(c'_i) = 0$ implies $yu_i(c_i) + y'u_i(c'_i) = 0$ for all $c_i, c'_i \in \{1,\ldots,t\}$ and all $y, y' \in GF(q)$. So, we deduce that the condition to apply the circular-keying reduction is fulfilled. We can thus apply the circular-PRF reduction and reduce to $Game_2$, where $F(c_i, f_x(N_P, N_V, L)_i, x'_i)$ is replaced by $u_i(c_i)\tilde{x}_i + (v_i(c_i) \cdot f^*(N_P, N_V, L)_i)$, where f^* is a random function. This reduction has a probability loss of up to ε.

From here, we use a simple bridging step to say that the adversary $\mathcal{A}$ has virtually no advantage over $Game_2$ and a game $Game_3$, where the vector $a = f^*(N_P, N_V, L)$ is selected at random; we recall that this is the case since there is no repetition on N_P and f^* is a random function. The (N_P, N_V, L) triplet used by V in the attack phase can be used by only one P_j, in the attack phase as well, where $j \in \{m+1,\ldots,\ell\}$. We can simulate all other P's and V's based on a (simulated) random a. This reduces to an adversary making no use of the learning phase and using only P_j and V in the attack phase.

So, the probability p of $\mathcal{A}$ of succeeding in $Game_3$ is the probability that at least τ rounds have a correct r_i. Due to Lemma 3.1, r_i must be computed by $\mathcal{A}$ (and not P_j). Getting r_i correct for c_i can thus be attained in two distinct ways: 1. in the event $e1$ of guessing $c'_i = c_i$ and sending it beforehand to P_j and getting the correct response r_i, or 2. in the event $e2$ of simply guessing the correct answer r_i (for a challenge $c'_i \neq c_i$). So, $p = B(n, \tau, \Pr[e1] + \Pr[e2]) = B(n, \tau, \frac{1}{t} + \frac{t-1}{t} \times \frac{1}{q})$. □

Tightness. The above result is tight as the following attack shows. It is thus the best MiM attack. We consider an adversary who first relays the messages between the (far away) prover and the verifier during the initialisation phase. Then, he simulates a distance bounding phase with the prover to learn some $c_i \mapsto r_i$ relations. During the distance bounding phase with the verifier, either the challenge matches the learnt c_i, in which case he can answer r_i and pass with probability 1, or the challenge is different, in which case he can answer randomly and pass with probability $\frac{1}{q}$. The overall probability to pass one round is $\frac{1}{t} + (1 - \frac{1}{t})\frac{1}{q}$. So, the probability that the verifier accepts is $B(n, \tau, \frac{1}{t} + (1 - \frac{1}{t})\frac{1}{q})$, which is negligibly close to β.

Th. 4.1.C. Assume as per the requirement for resistance to collusion-fraud that there is an experiment $exp^{\mathsf{CF}} = (P^*(x) \longleftrightarrow \mathcal{A}^{\mathsf{CF}}(r_{\mathsf{CF}}) \longleftrightarrow V_0(y; r_{V_0}))$, with P^* a coerced prover who is far away from V_0 and that $\Pr_{r_{V_0}, r_{\mathsf{CF}}}[Out_{V_0} = 1] = \gamma$. Given some random $c_1,\ldots,c_n$ from the verifier, we define the random variable

$View_i$ as being the view of $\mathcal{A}^{\mathsf{CF}}$ before receiving c_i from V, and the random variable w_i being all the information that $\mathcal{A}^{\mathsf{CF}}$ has received from P^* before the time when sending out r_i would become critical (i.e., before it would be too late to send r_i on to V_0). This answer r_i done by $\mathcal{A}^{\mathsf{CF}}$ is formalised in Lemma 3.1. So, $r_i := \mathcal{A}^{\mathsf{CF}}(View_i \| c_i \| w_i)$.

Let C_i be the set of all possible c_i's on which the functions $\mathcal{A}^{\mathsf{CF}}(View_i \| \cdot \| w_i)$ and $F(.,a_i,x'_i)$ match (i.e., $\mathcal{A}^{\mathsf{CF}}$ answers correctly to the challenge c_i at round i). Let S be the set of i's such that $c_i \in C_i$ (i.e., $\mathcal{A}^{\mathsf{CF}}$ answers correctly at round i). Finally, let R be the set of i's such that $\#C_i = t$ (i.e., $\mathcal{A}^{\mathsf{CF}}$ answers correctly at round i whatever the challenge). I.e., $C_i = \{c \in \{1,\ldots,t\} \,|\, \mathcal{A}^{\mathsf{CF}}(View_i \| c \| w_i) = F(c,a_i,x'_i)\}$, $S = \{i \in \{1,\ldots,n\} \,|\, c_i \in C_i\}$, and $R = \{i \in \{1,\ldots,n\} \,|\, \#C_i = t\}$. The adversary $\mathcal{A}$ succeeds in exp^{CF} if $\#S \geq \tau$, i.e., if he can pass at least τ rounds, for the challenges that V_0 will fix in those rounds.

For terrorist-fraud resistance, we would also like that—in the second, MiM experiment—the adversary $\mathcal{A}_2$ can answer τ rounds (or more), no matter what the challenge, i.e., in this way, $\mathcal{A}$ could extract x and the TF would be invalid. In other words, we would like that $\#R$ is large, i.e. $\#R > n - T$ so that we can decode.

So, if we were to pick a set of challenges such that $\#S \geq \tau$ and $\#R \leq n - T$, we should select a good challenge (from no more than $t-1$ existing out of t), for at least $T + \tau - n$ rounds out of T. In other words, $\Pr[\#S \geq \tau, \#R \leq n - T] \leq B(T, T + \tau - n, \frac{t-1}{t})$. But, by the hypothesis, $\Pr[\#S \geq \tau] \geq \gamma$. So, we deduce immediately that $\Pr[\#R \leq n - T | \#S \geq \tau] \leq \gamma^{-1} B(T, T + \tau - n, \frac{t-1}{t})$. Therefore, $\Pr[\#R > n - T | \#S \geq \tau] \geq 1 - \gamma^{-1} B(T, T + \tau - n, \frac{t-1}{t})$.

We use $m = \ell = z = O(\gamma^{-1} r)$ (i.e., $\mathcal{A}_2$ will directly impersonate P to V after $\mathcal{A}_1$ ran m times the collusion fraud, with P^* and V). We define $\mathcal{A}_2$ such that, for each execution of the collusion fraud with P^* and V, it gets $View_i$, w_i. For each i, $\mathcal{A}_2$ computes the table $c \mapsto \mathcal{A}^{\mathsf{CF}}(View_i \| c \| w_i)$ and apply the t-leaking function E of the F-scheme on this table to obtain $y_i = E(c \mapsto \mathcal{A}^{\mathsf{CF}}(View_i \| c \| w_i))$. For each $i \in R$, the table matches the one of $c \mapsto F(c,a_i,x'_i)$ with $x' = L(x)$, and we have $y_i = x'_i$. So, $\mathcal{A}_2$ computes a vector y. If V accepts the proof, then y coincides with $L(x)$ on at least $n - T + 1$ positions, with a probability of at least $\rho := 1 - \gamma^{-1} B(T, T + \tau - n, \frac{t-1}{t})$. That is, after $O(\gamma^{-1})$ runs, $\mathcal{A}_2$ implements an oracle which produces a random $L \in \mathcal{L}$ and a y which has a Hamming distance to $L(x)$ up to $T - 1$.

By applying the leakage scheme decoder e on this oracle, with u samples, it can fully recover x, with probability at least $\rho^u p$: just obtain a list of possible values for x and isolate the good one based on the collected information. Then, by taking $\gamma = B(T, T + \tau - n, \frac{t-1}{t})^{1-\theta}$ and $\gamma' = \left(1 - B(T, T + \tau - n, \frac{t-1}{t})^{\theta}\right)^u p$, we obtain our result. □

Tightness. For our $\mathbf{SKI_{pro}}$ construction using $T = \frac{n}{2}$, the above result is tight as the following attack shows. It is thus the best collusion fraud. We assume there is a function g mapping the secret x and the leak function L_μ to a vector $e = g(x, L_\mu)$ with Hamming weight $\frac{n}{2}$. We consider a malicious prover who selects some challenges c_i^* at random. He runs the initialisation phase normally. Then, he sends to the adversary a table $c_i \mapsto r_i$ for each round i. For i such that $e_i = 0$, he gives the full table $c_i \mapsto F(c_i, a_i, x'_i)$. For i such that $e_1 = 1$, he gives the table, except for $c_i = c_i^*$, for which the correct response is flipped. The adversary uses the table to answer to each round. Due to the leakage property, the adversary learns $L_\mu(x) + e$ which is a vector with Hamming weight $\frac{n}{2}$, which leaks no information about $L_\mu(x)$. We can show more formally that this does not leak any useable information about x.

During the collusion fraud, the adversary passes each round such that $e_i = 0$. When $e_i = 1$, the probability to pass is $1 - \frac{1}{t}$, i.e. the probability that the challenge is not c_i^*. So, the overall probability to pass the protocol is $\gamma = B(\frac{n}{2}, \tau - \frac{n}{2}, 1 - \frac{1}{t})$, without leaking any useable information. Our result indicates that this is the largest γ we can achieve.

Thus, under the circumstances where protection against terrorist-fraud and/or collusion-fraud[13] is not of primary importance, one can use the proposed $\mathbf{SKI_{lite}}$ protocols, the security of which does not rely on the assumption of circular-keying security.

Following Lemma 4.2 and Th. 4.1, it is clear that the probabilities α and β to succeed respectively in a distance-fraud and MiM, against the **SKI** protocols are based on:

	$\mathbf{SKI_{pro}}$	$\mathbf{SKI_{lite}}$
α:	$B(n,\tau,\frac{3}{4})$	$B(n,\tau,\frac{3}{4})$
β:	$B(n,\tau,\frac{2}{3})$	$B(n,\tau,\frac{3}{4})$

SKI's parameters: Let $\varepsilon > 0$. Remember (from page 14, Lemma 4.1) that the **SKI** protocols are $(1 - e^{-2\varepsilon^2 n})$-complete if τ is at most $(1 - p_{\mathsf{noise}} - \varepsilon)n$.

According to the data in the table above, we must take $1 - p_{\mathsf{noise}} - \varepsilon \geq \frac{\tau}{n} \geq \frac{3}{4} + \varepsilon$ to make the above instances of **SKI** secure, with a failure probability bounded β by $e^{-2\varepsilon^2 n}$ (by the Chernoff-Hoeffding bound [16,31]).

By changing the F-scheme, we can decrease the value $\frac{3}{4}$ in α. For instance, using the Shamir secret sharing [41], we reduce it to $\frac{5}{8}$, as shown in Appendix B.

If we require TF-resistance (as per Th. 4.1.C), we also get a constraint of $\frac{\tau}{n} > \frac{5}{6} + \frac{\varepsilon}{2}$, similarly.

5. Summarising SKI's Contributions

The contributions of the **SKI** families of protocols is two-fold: *provable security* and *efficiency*.

Provable Security. As we discussed in subsection 2.1, most distance-bounding protocols, new or old, do not enjoy formal security proofs. On the contrary, most have been proven vulnerable to various attacks (see [11,10, Table 1]). The only two, recent protocols which amend this and come with a formal security model and adjacent security protocol are the **SKI** family here, and the Fischlin-Onete (FO) protocol [24]. Moreover, in this paper, we also discuss the tightness of our security proofs.

The two formalisms are different; the FO model is game-based, the current one being based on simpler experiments run by interactive Turing-machines. Throughout the paper, for each formulated definition where it was pertinent, we discussed the link with the FO model. We remind that the FO recent protocol is discussed and compared with **SKI** in [48]; therein, it was observed, e.g., that the resistance of [24] to terrorist fraud lowered the resistance to mafia fraud.

Efficiency. The **SKI** protocol is generally more efficient than the FO protocol. To see this, one has to set acceptable levels for the noise (e.g., 5%) and completeness (e.g., 99%), look at the necessary number of rounds to obtain different levels of resistance to the different frauds (i.e., see what n, implies which α, β, γ, etc). By doing so, one can see that, e.g., **SKI** offers the double of MiM-resistance (e.g., 2^{-20} as opposed to 2^{-10}) for the same number of rounds (e.g., 120). The more in-depth matter of protocol efficiency is however not the focus of this paper.

6. Conclusions

In this paper, we have specified distance-bounding protocols and their security requirements, i.e., resistance to (generalised) distance-fraud, man-in-the-middle, terrorist-fraud attacks, in a general formalism

[13] It is clear that these $\mathbf{SKI_{lite}}$ protocols do not protect against terrorist-fraud (given the F-scheme used inside them).

for modelling location-driven security protocols developed herein. We also proposed the formal proofs for a provably secure class of practical protocols for distance-bounding, by identifying the requirements on the building blocks (i.e., the F-scheme, the leakage scheme, PRF masking, and the circular-keying security). Thus, these protocols are practical, efficient and provably secure against all frauds and their generalisations, even in noisy conditions. As a by-product, we introduced (at least) a new security notion, i.e., circular-keying for pseudorandom functions (PRFs); this models the employment of a PRF, with possible linear reuse of the key.

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Appendix

A. A Communication Model

We introduce a model for distance-bounding protocols. We first specify the main ideas at a high-level and then, in Section A.2, we formalise our communication and our threat model.

A.1. General Communication Principles

We impose the following **gold principles**: 1. participants have one location; 2. messages travelling one unit of distance between two locations require one time-unit for delivery; 3. messages under transmission are broadcast and become readable at a location when they physically reach its proximity. We now explain the above in more depth and add some extra specifications.

A participant has a physical location, modelled as a centre of a sphere with the radius of one distance-unit. A sender S who wants to send a message to a receiver R just broadcasts the message, setting R as the aimed "delivery address". Every time-unit, a message sent by S moves from the sphere centred on S to another sphere with a radius augmented by one unit (see Fig. 4). Participant R can read the message as soon as the growing sphere on which the message is travelling includes R.

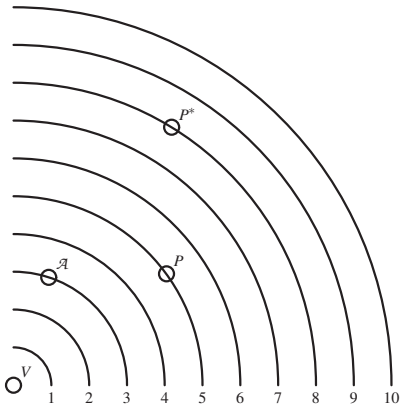

Figure 4. Sketch of Message-Transmitting Model: A message send by V is broadcast and travels at one unit of distance per clock cycle. Assume P is the purported recipient. However, $\mathcal{A}$ can read the message two clock cycles before P, whereas P^* must wait three clock cycles more than P before the message reaches him.

Honest participants are supposed to read only the messages for which they are the purported recipient.

There is no implicit authentication: received messages may have been previously sent by any participant.

The adversary can change the destination to himself (so that the legitimate receiver does not read the corresponding message).

In the following, we give further, more formal explanations on these, as well as on time-increments and the communication model.

A.2. Computation and Communication Models

Formalised Participants. Each participant in the protocol is formally described by an interactive Turing machine (ITM). The ITMs we use in this formalisation have the following tapes: an input tape, a random tape, an incoming communication tape, an outgoing communication tape, a read/write working tape, and an output tape. Each machine has an assigned algorithm, which describes the behaviour of that participant in the protocol to model. As suggested, each participant U has a location denoted by loc_U in a metric space, where d is the distance-function of this metric space (i.e., there is a distance-unit and the classical requirements to measure distances). The distance is assumed to measure the time-of-flight of messages between two locations (i.e., as if messages were travelling uniformly at a speed of one distance-unit per time-unit). At this stage, the reader can refer to Fermat's principle [40] for the notion of time-of-flight.

The time-of-flight is further described by a global counter called *clock*. This *clock* is incremented at certain execution-points, as the communication model will explain below. We underline that the complexity of the machines is measured in the number of computational[14] steps and it is **not** linked to this notion of time-of-flight. Thus, we assume that all computation of (parts of) messages is instantaneous (in terms of the ticks of the *clock*). Only other actions, e.g., sending a message from one location to another, have a time-duration on the *clock*.

Also, there is a global system-recall called *history*. The tuples stored in this register are of the form

$$(\mathsf{message}, \mathsf{locationOfOrigin}, \mathsf{timeOfSending}, \mathsf{destination})$$

i.e., a message that has been sent, from some initiator-origin, departing at some time and being aimed at some participant.

Communication. In the following, we assume that the network is asynchronous. We consider insecure and noisy channels. However, the adversary receives messages with no noise.[15] In addition to this, and for simplicity, protocol messages which are not "time-critical" (as clearly explained later) can be assumed to be noiseless, or equivalently, that participants use a computations overhead for error correction. All channels employed in this model are timed, i.e., by the (units of) global counter *clock*. As aforementioned, we assume that all communication happens through a broadcast anonymous channel.

All machines have communication-related actions of three types: *send*, *standby* and *halt*. If a machine does a *halt* action, then its execution is terminated. Before halting, the machines write their output on the output tape. If a machine M performs the action $send(m,P)$, this denotes that the message m is aimed at a participant P. Namely, the message m is written on the outgoing communication tape of the sending machine M and the tuple $(m, loc_M, clock\_value, P)$ is added to the *history*, where *clock_value* is the value of the *clock* register at the time of this sending by M. After some sendings or simply at some point, the machine will do a *standby* action: i.e., the machine waits for a reactivation. When all participants are in a "hanging" state (e.g., some in standby, some halted), the global counter *clock* is incremented by a unit and the participants standing by are reactivated.

Let *clock_value* be the current value of the global *clock* register. For each tuple in the global *history* of the form $(m, loc_M, time\_sent, dest)$, if $d(loc_P, loc_M) \leq (clock\_value - time\_sent)$ then the participant

[14] We will still consider "time complexity", namely polynomial versus non-polynomial computational complexity, but it does not relate to the notion of time-of-flight that we refer to in this section.

[15] This is due to adversaries using a more elaborate equipment.

P *can* read[16] the content m of its incoming communication tape. However, an honest participant P will not read m if $dest \neq P$.

Adversary. An adversary $\mathcal{A}$ is modelled by an ITM of the above kind, i.e., he is part of the system as described, he has a location, etc. Moreover, an adversary $\mathcal{A}$ has the following abilities: 1. reading messages for which he is not necessarily the intended recipient; 2. corrupting the channel between any two participants S and R (i.e., upon corruption, for an action $send(m,R)$ done by S, the system performs the action $send(m,\mathcal{A})$ instead, re-aiming the message m to $\mathcal{A}$); 3. sending his own messages to different participants. An adversary is not able to: modify sent messages.

If the adversary $\mathcal{A}$ could modify a flying message sent by S to R before R could actually read it, this would implement a super-fast channel contradicting our gold principles. We could then design the following trivial (but unrealistic) distance-fraud. The malicious prover can send a random response before receiving the challenge, wait to receive the verifier's challenge and use this super-fast channel to modify his own flying response when it has not reached the verifier yet. Clearly, any sent message could thuswise be used as a "carrier" to send messages faster than allowed by our gold principles. Instead of modifying a message far along its course, $\mathcal{A}$ can change the destination from R to $\mathcal{A}$ and may send another message to R. We believe this does not decrease the capabilities compared to practice since adversaries can still carry out man-in-the-middle attacks.

Similarly, the action 2 has a restriction: a message m sent by S in the past, present, or future is blocked by corrupting the channel, unless it would reach R before a message which would have been sent by $\mathcal{A}$ to R at the corruption time.

Also, the adversary has no control over the global counter $clock$. This is normal, since the counter $clock$ simply models time passing, as we know it. However, the adversary is the first to be activated after each increment of the $clock$ (i.e., as he may, e.g., corrupt a channel before a new message is sent on it).

B. SKI Variants

Our F-scheme can be instantiated to produce different **SKI** protocols, some arguably more practical/secure than others. In the main body of this paper, we presented a version that is in-line with the existing literature in the field, i.e., one-bit responses and a set of values for challenges of small cardinality, e.g., 3. Irrespective of this alignment with the state-of-the-art, the practicality of today's RFID/NFC cards goes beyond one-bit responses [44]. Moreover, pre-computation tables can be used.

As formalised above, to attain security, the idea behind such an F-scheme is that it should be a secret sharing scheme in which the response to the $t > 2$ challenges in round i reveals the component x_i of the secret, but the answers to only 2 of these challenges (e.g., one from the prover and another indirectly leaked by the verifier, e.g., within a non-narrow MiM attack) do not reveal x_i. Namely, we will consider two generic such response-functions in which the ith response ($1 \leq i \leq n$) is produced as follows:

$$\mathbf{F}_{\mathbf{shamir}}(c_i,a_i,x_i) = x_i + (a_i)_1\overline{c}_i + (a_i)_2{\overline{c}_i}^2 + \ldots + (a_i)_{t-1}{\overline{c}_i}^{t-1}$$

[16]This formalises the discussion in page 24 about broadcasting and reading messages when the intended recipients are on the correct spheres.

where $x_i \in GF(q)$, $q \geq 4$, $c_i \in \{1,\ldots,t\}$ is mapped to $\overline{c}_i \in GF(q)^*$ by an arbitrary injective mapping, $(a_i)_j \in GF(q), j \in \{1,\ldots,t-1\}$;

$$\mathbf{F_{xor}}(c_i,a_i,x_i) = x_i 1_{c_i=t} + (a_i)_1 1_{c_i\in\{t,1\}} + \ldots + (a_i)_{t-1} 1_{c_i\in\{t,t-1\}}$$

where $c_i \in \{1,\ldots,t\}$, $x_i \in GF(q)$, $q \geq 2$, $(a_i)_j \in GF(q)$, $j \in \{1,\ldots,t-1\}$, and 1_R is 1 if R is true and 0 otherwise.

Note that the function $\mathbf{F_{xor}}$ has been invoked in the main body of this paper to define $\mathbf{SKI_{pro}}$ and $\mathbf{SKI_{lite}}$. We give two more variants of it, $\mathbf{SKI_{shamir}}$ and $\mathbf{SKI_4}$.

In our numerical studies, we actually look at three specific F-schemes dictated by the functions above, giving three specific **SKI** protocols as follows:

- $\mathbf{SKI_{shamir}}$: defined by $\mathcal{L} = \mathcal{L}_{\mathsf{bit}}$, and the response-function $\mathbf{F_{shamir}}$ above, with $q = 4$, $t = 3$, $t' = 2$, i.e., $F(c_i,a_i,x_i) = x_i + (a_i)_1\overline{c}_i + (a_i)_2\overline{c}_i^2$, with $x_i,(a_i)_1,(a_i)_2 \in GF(4)$ and $\overline{c}_i \in GF(4)^*$;
- $\mathbf{SKI_{pro}}$: defined by $\mathcal{L} = \mathcal{L}_{\mathsf{bit}}$, and the response-function $\mathbf{F_{xor}}$ above, with $q = 2$, $t = 3$, $t' = 2$, i.e., $F(c_i,a_i,x_i) = (a_i)_{c_i}$ for $c_i \in \{1,2\}$ and $F(3,a_i,x_i) = x_i + (a_i)_1 + (a_i)_2$, with $(a_i)_1,(a_i)_2,x_i \in GF(2)$;
- $\mathbf{SKI_4}$: defined by $\mathcal{L} = \mathcal{L}_{\mathsf{bit}}$, and the response-function $\mathbf{F_{xor}}$ above, with $q = 2$, $t = 4$, $t' = 3$, i.e., $F(c_i,a_i,x_i) = (a_i)_{c_i}$ for $c_i \in \{1,2,3\}$ and $F(4,a_i,x_i) = x_i + (a_i)_1 + (a_i)_2 + (a_i)_3$, with $(a_i)_1,(a_i)_2,(a_i)_3,x_i \in GF(2)$;
- $\mathbf{SKI_{lite}}$: defined by a variant of response-function $\mathbf{F_{xor}}$ above (not depending on x_i), with $q = 2$, $t = t' = 2$, i.e., $F(c_i,a_i,x_i) = (a_i)_{c_i}$ for $c_i \in \{1,2\}$, with $(a_i)_1,(a_i)_2 \in GF(2)$. Since x' is not used, $\mathcal{L}$ can be let empty.

In relation with the definitions of the F-schemes and protocols above, we prove the following lemma.

Lemma B.1. *The F-schemes used in $\mathbf{SKI_{shamir}}$, $\mathbf{SKI_{pro}}$ and $\mathbf{SKI_4}$ are linear, pairwise uniform, t-leaking. The F-scheme used in $\mathbf{SKI_{lite}}$ is linear, pairwise uniform and not t-leaking.*

- ***Lemma B.1.1:*** The F-scheme used in $\mathbf{SKI_{shamir}}$ is $\frac{15}{8}$-bounded.
- ***Lemma B.1.2:*** The F-scheme used in $\mathbf{SKI_{pro}}$ is $\frac{9}{4}$-bounded.
- ***Lemma B.1.3:*** The F-scheme used in $\mathbf{SKI_4}$ is 3-bounded.
- ***Lemma B.1.4:*** The F-scheme used in $\mathbf{SKI_{lite}}$ is $\frac{3}{2}$-bounded.

Following Lemma B.1 and Th. 4.1, it is clear that the probabilities α and β to succeed respectively in distance-frauds and in MiMs, against the **SKI** protocols are:

	$\mathbf{SKI_{shamir}}$	$\mathbf{SKI_{pro}}$	$\mathbf{SKI_4}$	$\mathbf{SKI_{lite}}$
α:	$B(n,\tau,\frac{5}{8})$	$B(n,\tau,\frac{3}{4})$	$B(n,\tau,\frac{3}{4})$	$B(n,\tau,\frac{3}{4})$
β:	$B(n,\tau,\frac{1}{2})$	$B(n,\tau,\frac{2}{3})$	$B(n,\tau,\frac{5}{8})$	$B(n,\tau,\frac{3}{4})$

Proof. The first three properties (i.e, linearity, pairwise uniformity, t-leaking property) follow easily from the respective definitions of the three functions.

For the property of σ-boundedness, we will carry the proof using the notation

$$P_j(x_i) := \Pr_{a_i}\left[\max_y \left(\#(F^{-1}_{a_i,x_i}(y))\right) = j\right]$$

for $F_{a_i,x_i} : c_i \mapsto F(c_i,a_i,x_i)$. We will compute the bound σ as $\max_{x_i}\sum_{j=1}^{t} jP_j(x_i)$. We recall that $P_j(x_i) = 0$ for $j < \frac{t}{q}$.

We start by proving Lemma B.1.1, i.e., the response-function F that gives the ith response as $F(c_i,a_i,x_i) = x_i + (a_i)_1\bar{c}_i + (a_i)_2\bar{c}_i^2$, with $x_i,(a_i)_1,(a_i)_2 \in GF(4)$ and $\bar{c}_i \in GF(4)^*$ is the mapped of the challenge $c_i \in \{1,\ldots,t\}$.

We can show that:

$$\max_y \left(\#(F^{-1}_{a_i,x_i}(y))\right) = 1 \Leftrightarrow (a_i)_2 = 0 \text{ and } (a_i)_1 \neq 0$$

$$\max_y \left(\#(F^{-1}_{a_i,x_i}(y))\right) = 2 \Leftrightarrow (a_i)_2 \neq 0$$

$$\max_y \left(\#(F^{-1}_{a_i,x_i}(y))\right) = 3 \Leftrightarrow (a_i)_2 = (a_i)_1 = 0$$

So, for a component x_i in the secret vector x as per above, it holds that:

$$P_1(x_i) = \frac{3}{16}, \quad P_2(x_i) = \frac{3}{4}, \quad P_3(x_i) = \frac{1}{16}.$$

Thus, $\sigma = 1 \times \frac{3}{16} + 2 \times \frac{3}{4} + 3 \times \frac{1}{16} = \frac{15}{8}$. This ends the proof of Lemma B.1.1.

We now proceed to proving Lemma B.1.2, i.e., the response-function F that gives the ith response as $F(c_i,a_i,x_i) = (a_i)_{c_i}$ for $c_i \in \{1,2\}$ and $F(3,a_i,x_i) = x_i + (a_i)_1 + (a_i)_2$, with $(a_i)_1,(a_i)_2,x_i \in GF(2)$.

Following a similar calculation as above, we have:

$$\max_y \left(\#(F^{-1}_{a_i,x_i}(y))\right) = 3 \Leftrightarrow (a_i)_1 = (a_i)_2 = x_i, \text{ thus } P_3(x_i) = \frac{1}{4}.$$

For $j < \frac{t}{q}, P_j(x_i) = 0$, so since $1 < \frac{3}{2}$ we have that $P_1(x_i) = 0$. So, $P_2(x_i) = 1 - P_3(x_i) = \frac{3}{4}$. Thus, $\sigma = (2 \times \frac{3}{4} + 3 \times \frac{1}{4}) = \frac{9}{4}$. This ends the proof of Lemma B.1.2.

We now proceed to proving Lemma B.1.3, i.e., the response-function F that gives $F(c_i,a_i,x_i) = (a_i)_{c_i}$ for $c_i \in \{1,2,3\}$ and $F(4,a_i,x_i) = x_i + (a_i)_1 + (a_i)_2 + (a_i)_3$, with $(a_i)_1,(a_i)_2,(a_i)_3,x_i \in GF(2)$. For $j < \frac{t}{q}, P_j(x_i) = 0$, so since $1 < \frac{4}{2}$, $P_1(x_i) = 0$.

If $x_i = 0$ we have:

$$\max_y \left(\#(F^{-1}_{a_i,x_i}(y))\right) = 4 \Leftrightarrow (a_i)_1 = (a_i)_2 = (a_i)_3, \text{ thus } P_4(x_i) = \frac{1}{4}.$$

We have that $\max_y \left(\#(F^{-1}_{a_i,x_i}(y))\right) = 3$ is impossible, i.e., $P_3(x_i) = 0$. So, $P_2(x_i) = 1 - P_4(x_i) = \frac{3}{4}$. Finally, $\left(4 \times \frac{1}{4} + 2 \times \frac{3}{4}\right) = \frac{5}{2}$.

If $x_i = 1$, then $\max_y \left(\#(F^{-1}_{a_i,x_i}(y))\right) = 4$ or 2 are impossible, i.e, $P_4(x_i) = 0$. Thus, for $x_i = 1$ we have $\max_y \left(\#(F^{-1}_{a_i,x_i}(y))\right) = 3$. We conclude that $\sigma = \max\left\{\frac{5}{2},3\right\} = 3$. This ends the proof of Lemma B.1.3.

The proof of Lemma B.1.4 is along the same lines as in the above, especially as in Lemma B.1.2. □