## Sparse Modeling of Neural Network Posterior Probabilities for Exemplar-Based Speech Recognition

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We study automatic speech recognition by direct use of acoustic features (exemplars) without any assumption on the underlying stochastic process. The prior studies exploit spectral exemplars. In this work, we present the use of neural network sub-word posterior probabilities as exemplars. The space of sub-word observations is lowdimensional (e.g.  $\mathbb{R}^{K \times T}$ ) whereas the word transcription requires reconstructing a high-dimensional representation (e.g.  $\mathbb{R}^{L \times T}$ ,  $L \gg K$ ). Given the prior knowledge that for any given utterance, the word representation is highly sparse, we cast the speech recognition problem as sparse reconstruction of word posteriors given the compressed (low-dimensional) acoustic observation.

The sub-word units (phones) are denoted by  $\{q_k\}_{k=1}^K$ . Given an input spectral feature  $x_t$  at time t, a (deep) neural network [1] is used to estimate the posterior probabilities  $\{p(q_k|x_t)\}_{k=1}^K$ . The phone posterior probabilities is related to the word posterior probabilities  $p(w_l|x_t)$  through

$$p(q_k|x_t) = \sum_{l=1}^{L} p(q_k, w_l|x_t) = \sum_{l=1}^{L} p(q_k|w_l) p(w_l|x_t);$$
(1)

the last equality holds due to conditional independence of the acoustic observation and input speech given a superphone lexical unit such as word. Defining an over-complete dictionary **D** such that the atoms are exemplars obtained by conditioning the phone posteriors on a different linguistic unit  $w_l$ , we have

$$\underbrace{\begin{bmatrix} p(q_1|x_t)\\ p(q_2|x_t)\\ \vdots\\ p(q_K|x_t)\end{bmatrix}}_{z_t} = \underbrace{\begin{bmatrix} p(q_1|w_1) & \cdots & p(q_1|w_l) & \cdots & p(q_1|w_L)\\ p(q_2|w_1) & \cdots & p(q_2|w_l) & \cdots & p(q_2|w_L)\\ \vdots & \vdots\\ p(q_K|w_1) & \cdots & p(q_K|w_l) & \cdots & p(q_K|w_L) \end{bmatrix}}_{\text{Dictionary: } \mathbf{D} = [d_1 \dots d_l \dots d_L]} \times \underbrace{\begin{bmatrix} p(w_1|x_t) \\ \vdots\\ p(w_l|x_t) \\ \vdots\\ p(w_L|x_t) \end{bmatrix}}_{\alpha_t}$$
(2)

Construction of the dictionary as described in (2) requires modeling the subspaces of each word using the acoustic features in terms of phone posterior probabilities. To that end, we learn word-specific dictionaries such that each column of the dictionary in (2),  $d_l$  has a sparse representation stated as

$$\underbrace{\left[\begin{array}{c} p(q_{1}|w_{l})\\ p(q_{2}|w_{l})\\ \vdots\\ p(q_{K}|w_{l})\\ \vdots\\ p(q_{K}|w_{l})\\ \end{array}\right]}_{d_{l}} = \underbrace{\left[\begin{array}{c} p(q_{1}|sw_{1}^{w_{1}}) & \cdots & p(q_{1}|sw_{s}^{w_{l}}) & \cdots & p(q_{1}|sw_{s}^{w_{l}})\\ p(q_{2}|sw_{1}^{w_{1}}) & \cdots & p(q_{2}|sw_{s}^{w_{l}}) & \cdots & p(q_{2}|sw_{s}^{w_{l}})\\ \vdots\\ p(q_{K}|sw_{1}^{w_{1}}) & \cdots & p(q_{K}|sw_{s}^{w_{l}}) & \cdots & p(q_{K}|sw_{s}^{w_{l}})\\ \vdots\\ p(q_{K}|sw_{1}^{w_{1}}) & \cdots & p(q_{K}|sw_{s}^{w_{1}}) & \cdots & p(q_{K}|sw_{s}^{w_{l}})\\ \end{array}\right]}_{Word manifold modeling dictionary:D_{w_{l}}} \times \begin{bmatrix} p(sw_{1}^{w_{1}}|w_{l})\\ \vdots\\ p(sw_{s}^{w_{l}}|w_{l})\\ \vdots\\ p(sw_{s}^{w_{l}}|w_{l})\\ \end{array}\right]$$

where  $sw_s^{w_l}$  denotes the  $s^{th}$  sub-word unit of the word  $w_l$ ,  $S_{w_l}$  represents the total number of (over-complete) "bases" to model the sub-space of word  $w_l$ .

Equations (2) and (3) lead us to an intuitive and natural representation for continuous speech in terms of posterior features and word-to-subword hierarchical dictionaries. Thereby, the posterior-based sparse modeling dictionary is obtained as  $\mathbf{D} = [\mathbf{D}_{w_1} \cdots \mathbf{D}_{w_l} \cdots \mathbf{D}_{w_L}]$ . The dictionary  $\mathbf{D}$ , has an internal partitioning defined by the boundaries of individual sub-dictionaries  $\mathbf{D}_{w_l}$ . Ideally, an input posterior feature  $z_t$  belonging to a realization of word  $w_l$ , when sparse decoded using the dictionary above will have a sparse representation  $\alpha_t$  such that only the atoms corresponding to the subdictionary  $\mathbf{D}_{w_l}$ , denoted as  $\alpha_t^{w_l}$ , will have non-zero values.  $\alpha_t^{w_l}$  is expressed as

A sequence of posterior features  $\mathbf{Z} = [z_1, ..., z_t]$ , extracted from an utterance of word  $w_l$ , will have a hierarchical group structure underlying the sparse representation  $\mathbf{A} = [\alpha_1, ..., \alpha_t]$  where all the coefficients tend to collaborate in time to activate a higher level group corresponding to  $w_l$ . This collaborative hierarchical structure (depicted in Figure 1) is leveraged using the C-HiLasso algorithm [2] to obtain the sparse representation  $\alpha_t$  as

$$\alpha_t = \min_{\alpha} \frac{1}{2} \|\mathbf{Z} - \mathbf{D}\mathbf{A}\|_F^2 + \lambda_2 \psi_{\mathcal{G}}(\mathbf{A}) + \lambda_1 \sum_{t=1}^T \|\alpha\|_1 \quad (5)$$

where  $\psi_{\mathcal{G}}$  is group Lasso regularizer defined as  $\psi_{\mathcal{G}}(\alpha_t) = \sum_{G \in \mathcal{G}} \|\alpha_{[G]}\|_2$ . The posterior probability  $p(w_l|x_t)$  for a word  $w_l$  is estimated as  $p(w_l|x_t) := \|\alpha_t^{w_l}\|_1 / \|\alpha_t\|_1$ .

For speech recognition, frame-level word-posterior probabilities  $p(w_l|x_t)$ 's are used to obtain the maximum-aposteriori word recognition through

$$w_{\text{recognized}} := \arg \max_{w_l} p(w_l | \mathbf{X}) = \arg \max_{w_l} \prod_{t=1}^T p(w_l | x_t)$$
(6)

where  $\mathbf{X} = [x_1 \dots x_T]$ . The potential of the proposed approach is demonstrated on isolated word (Phonebook corpus [3]) and continuous speech (Numbers corpus [4]) recognition tasks. The results are listed in Table I. For dictionary learning and sparse decoding, online dictionary algorithm [5] and lasso solver [6] were used respectively.



Fig. 1: Given a sequence of acoustic features in  $\mathbf{Z}$ , the sparse representation matrix  $\mathbf{A}$  will have a block structure associated to the word-specific dictionaries where the inner block coefficients are sparse. This collaborative hierarchical sparsity structure is exploited in [2] to devise an efficient C-HiLasso algorithm for sparse recovery.

#	Task	Accuracy
1	Isolated Word (Phonebook-75 words)	97.8%
2	Isolated Word (Phonebook-600 words)	93.2%
3	Connected Digit (Numbers)	87.5%

TABLE I: Results for Isolated Word and Connected Digit Recognition using sparse modeling. Accuracies in case of Connected Digit are given by (100 - WER), where WER is word error rate obtained by Levenshtein distance. The conventional spectral exemplars yield less than 50% accuracy in Isolated Word recognition and around 70% accuracy in Connected Digit recognition tasks.

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