Multichannel ECG Analysis Using VPW-FRI

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***Abstract*—In this paper, we present an application of Variable Pulse Width Finite Rate of Innovation (VPW-FRI) in dealing with multichannel Electrocardiogram (ECG) data using a common annihilator. By extending the conventional FRI model to include additional parameters such as pulse width and asymmetry, VPW-FRI has been able to deal with a more general class of pulses. The common annihilator, which is introduced in the annihilating filter step, shows a common support in multichannel ECG data, which provides interesting possibilities in compression. A model based de-noising method will be presented which is fast and non-iterative. Also, an application to detect QRS complexes in ECG signals will be demonstrated. The results will show the robustness of the common annihilator and the QRS detection even in the presence of noise.**

I. Introduction

The concept of sampling and reconstructing signals at the rate of innovation was first presented by *Vetterli et al.* [1]. They showed that non band-limited classes of signals such as streams of Diracs had a finite number of degrees of freedom and could be completely defined by their location and amplitude parameters. These classes of signals were termed Finite Rate of Innovation (FRI) signals. These FRI signals could be sampled minimally at the rate of innovation and perfectly reconstructed.

Variable Pulse Width FRI (VPW-FRI) was developed by *Quick et al.* [2] as an extension of the traditional FRI method in that it added two additional parameters, namely the pulse width and asymmetry, to the model. This allows it some flexibility in dealing with pulses of various forms and widens the scope of its application. It does this by considering roots which fall inside the unit circle as compared to traditional FRI where the roots lie on the unit circle.

The generalisation of Diracs in VPW-FRI allowed it to be used successfully in compression of Electrocardiogram (ECG) signals [2], [3] where the P, QRS and T waveforms could be represented by pulses of varying amplitude, width and asymmetry. This allowed for a compression scheme which only requires 7 pulses per beat, with 4 parameters per pulse, which is far below the Nyquist rate of around $200-250$Hz at which most devices record ECG signals.

Other methods have also been used for compression such as compressed sensing [7], wavelet methods [8] and finite rate of innovation [4]. The FRI method in [4] divides the ECG signal into two parts. The QRS is modelled as a non-uniform linear spline while the remainder of the signal is considered a residual signal which is sampled at a low rate of $15Hz$. The difference here is that VPW-FRI considers each waveform, P, QRS and T, as a pulse and parameterizes them accordingly. Using VPW-FRI allows for a much lower number of samples and higher compression ratio.

In this paper, we demonstrate a multichannel approach to calculating the location parameter. To achieve this, a common annihilator is used in the reconstruction step to derive the locations. This aids in the compression of the multichannel signal and has potential applications such as QRS detection which we will also present.

Also, VPW-FRI has de-noising capabilities. This is achieved through a model based de-noising method [5] which is fast and non-iterative. This is its main advantage especially when compared to Cadzow [2], [6] denoising which is iterative and requires oversampling. Most importantly, de-noising is done without affecting the morphology of the pulses which is especially important when clinicians examine an ECG recording.

This paper is organised as follows. Section II will present some background on FRI theory followed by an explanation of VPW-FRI. Section III will demonstrate the multichannel VPW-FRI approach. This will be followed by Section IV where an application of VPW FRI in ECG wave detection will be shown. The 12 lead ECG data used and the results will be presented in Section V. Finally, conclusions will be drawn and some thoughts on future work will constitute Section VI.

II. Variable Pulse Width Finite Rate of Innovation

Since VPW-FRI is an extension of the original FRI theory, we will present a short description of FRI theory followed by the changes in the VPW-FRI algorithm.

A. FRI

A stream of K Diracs with period τ is defined by

$$x(t) = \sum_{k=0}^{K-1} b_k \delta(t - t_k) \tag{1}$$

$$= \sum_{m\in\mathbb{Z}} \underbrace{\frac{1}{\tau} \sum_{k=0}^{K-1} b_k \underbrace{e^{-i(2\pi m t_k)/\tau}}_{u_k^m}}_{X[m]} e^{i(2\pi m t)/\tau} \tag{2}$$

where Eq. (2) is the Poisson Summation Formula derivation of Eq. (1). The signal is then sampled uniformly. The samples, y_n are defined by

$$y_n = \langle h_b(t-nT), x(t)\rangle, \quad n = 0, ..., N-1 \quad (3)$$
$$= \sum_{m=-M}^{M} X[m] e^{i(2\pi mnT/\tau)}, \quad (4)$$

where T represents the sampling period, N is the number of samples, $B \geq \frac{2K}{\tau}$, $M = \lfloor B\tau/2 \rfloor$ and the sampling kernel $h_b(t) = Bsinc(Bt)$.

In the reconstruction step, the annihilating filter [1] in Eq. (5) is used to retrieve the u_k values

$$\begin{bmatrix} X[-1] & \dots & X[-K] \\ X[0] & \dots & X[-K+1] \\ \vdots & \ddots & \vdots \\ X[K-2] & \dots & X[-1] \end{bmatrix} \cdot \begin{bmatrix} A[1] \\ A[2] \\ \vdots \\ A[K] \end{bmatrix} = 0, \quad (5)$$

where $A[k]$ represents the annihilating filter coefficients.

A common way of solving for A would be to find the minimal right singular vector of the Toeplitz matrix in Eq. (5). Since the filter coefficients are of the form

$$A(z) = \sum_{k=0}^{K} A[k] z^{-k} = \prod_{k=0}^{K-1} (1 - u_k z^{-1}), \quad (6)$$

the roots of the filter coefficients would correspond to u_k, defined in Eq. (2), and the locations, t_k can be calculated directly from u_k.

The amplitudes, b_k, can be resolved using a Vandermonde system of equations [1].

B. Sampling and Reconstruction of VPW-FRI signals

The Dirac model can be generalised with the addition of width and asymmetry parameters. This can be used to expand FRI theory by interpreting the u_k and $X[m]$ coefficients differently. The u_k values are defined as

$$u_k = e^{-2\pi(a_k + it_k)/\tau}, \quad a_k \geq 0 \quad (7)$$

where a_k is the width parameter. The $X[m]$ coefficients are defined as

$$X[m] = X^{(1)}[m] + X^{(2)}[m], \quad (8)$$

where

$$X^{(1)}[m] = \sum_{k=0}^{K-1} c_k e^{-2\pi(a_k|m| + it_k m)/\tau} \quad (9)$$

and

$$X^{(2)}[m] = -\sum_{k=0}^{K-1} d_k sgn(m) e^{-2\pi(a_k|m| + it_k m)/\tau}. \quad (10)$$

The $X^{(2)}[m]$ coefficients are the Hibert transform of $X^{(1)}[m]$ and the spectra of $X^{(1)}[m]$ and $X^{(2)}[m]$ are symmetric.

The same annihilating filter in Eq. (5) can be used. For stability, the annihilating filter roots which lie within the unit circle are admitted and those which lie outside are rejected. The t_k and a_k parameters can be retrieved from the roots of the annihilating filter coefficients.

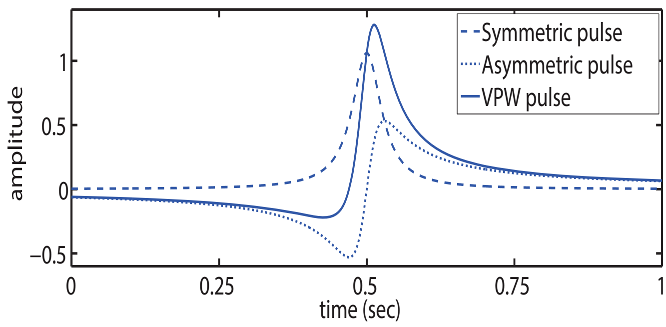

Fig. 1. Symmetric and asymmetric components of a VPW-FRI pulse

The $\{c_k\}_{k=0}^{K-1}$ and $\{d_k\}_{k=0}^{K-1}$ coefficients, which are the real and imaginary part of b_k respectively, can be solved using the Vandermonde system [1] over the complex numbers as compared to the original FRI theory where it is solved over the real numbers.

The continuous-time signal, $x(t)$ can be recovered by applying the inverse Fourier Transform,

$$x(t) = \sum_{k=0}^{K=1} x_k(t) \quad (11)$$
$$= \sum_{k=0}^{K-1} \sum_{n\in\mathbb{Z}} c_k \frac{a_k}{\pi(a_k^2 + (t - t_k - n\tau)^2)}$$
$$+ \sum_{k=0}^{K-1} \sum_{n\in\mathbb{Z}} d_k \frac{t - t_k - n\pi}{\pi(a_k^2 + (t - t_k - n\tau)^2)} .$$

An alternate formula for $x_k(t)$ that avoids the infinite sum is given by:

$$x_k(t) = \frac{c_k}{\tau} \frac{1 - |z_t|^2}{(1 - z_t)(1 - z_t^*)} + \frac{d_k}{\tau} \frac{2\Im\{z_t\}}{(1 - z_t)(1 - z_t^*)} \quad (12)$$

where $z_t = e^{2\pi(-a_k + i(t - t_k))/\tau}$. As can be seen in Equation (11) and in Fig. 1, the VPW pulse consists of a symmetric and asymmetric pulse. The symmetric pulse is a Cauchy-Lorentz function and the asymmetric pulse is the Hilbert Transform of the symmetric pulse.

III. VPW-FRI ON MULTICHANNEL DATA

When dealing with multichannel data where the pulses occur at the same locations across all the channels, multi lead ECG for example, it would make sense to compute the locations for all the channels simultaneously rather than for each channel individually. This is achieved using a common annihilator which is the main mechanism that allows VPW-FRI to handle multichannel data.

In FRI theory, the annihilating filter would be where the u_k values are determined. However, it only deals with single channel information. Therefore by modifying the input to the annihilating filter, we can create a common annihilator for all the input channels. This can be achieved [11] by stacking the Toeplitz matrices of each channel vertically and applying the annihilating filter,

$$\begin{bmatrix} X_1 \\ X_2 \\ \vdots \\ X_M \end{bmatrix} \cdot \begin{bmatrix} A[1] \\ A[2] \\ \vdots \\ A[K] \end{bmatrix} = 0. \tag{13}$$

where $\{X_m\}_{m=1}^{M}$ represents the Toeplitz matrices of the M channels and $\{A[k]\}_{k=1}^{K}$ represents the annihilating filter coefficients similar to Eq. (6). The roots of the annihilating filter would yield the common locations of the pulses across all the channels.

A model based de-noising technique was implemented in this paper which is based on the subspace based approach presented in [5]. From Eq. (13), V is used to estimate the noiseless signal. Therefore,

$$\overline{V} = \underline{V} \cdot \Phi_H, \tag{14}$$

where $\overline{(\cdot)}$ and $\underline{(\cdot)}$ denote the operation of omitting the first and last row of $(\cdot)$, respectively. The conjugates of the eigenvalues of Φ_H will yield the roots of the annihilating filter and not the filter coefficients as seen in Eq. (5). For a detailed proof, please refer to [5].

The u_k values retrieved from the roots of the annihilating filter can be used to calculate the locations, t_k, for the pulses in all the channels.

This offers an interesting perspective especially when considering the physiology and the way the heart's electrical signals are recorded. The denominator of the filter describes the common activities such as time of arrival of the electrical vectors at the electrodes while the numerator captures the morphological information of the pulse. This could be studied further especially when developing automated diagnostic or wave detection tools.

IV. QRS Detection

One application of VPW-FRI, besides sampling and reconstruction, is QRS detection in ECG signals. Paired with the common annihilator method presented in Section III, this method of QRS detection is workable even in noisy signals.

The QRS complexes present the sharpest transition out of all the ECG waveforms. Hence, in Eq. (13), the highest values of $diag(S)$ would correspond to the QRS complexes due to the fact that it has the highest energy out of all the pulses. This can also be seen in the roots of the annihilating filter, as the roots closest to the unit circle would represent the QRS complexes though the distinction is not as clear. If we represent $A_n = \{S_{1,1}, S_{2,2}, \ldots, S_{N-1,N-1}\}$ and $B_n = \{S_{2,2}, S_{3,3}, \ldots, S_{N,N}\}$,

$$E_n = A_n / B_n, \quad n = 1, \ldots, N-1. \tag{15}$$

Then the number of QRS complexes can be found by thresholding E_n. Empirically, this threshold was found to be 1.2.

By only keeping the subspace associated with these QRS complexes, the QRS pulses can be accurately identified.

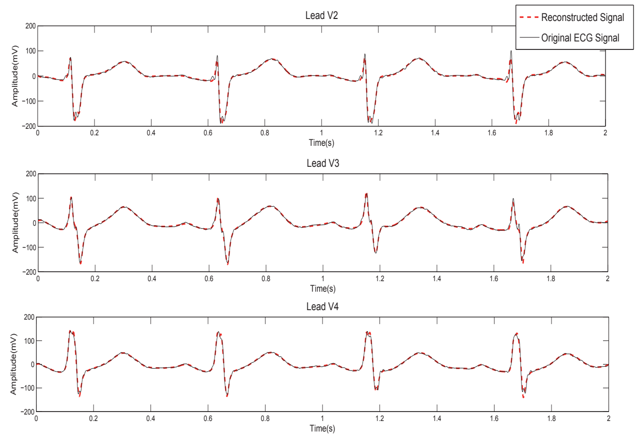

Fig. 2. Reconstructed signal for ECG leads V2-V4

V. Results

In this section, the ECG data that was used to generate results will be introduced. This will be followed by results from the VPW-FRI, common annihilator and QRS detection.

A. Data

The data used was 12 lead Stress ECG data recordings from Tan Tock Seng Hospital, Singapore. The subjects were patients who were undergoing treadmill ECG tests as recommended by their physician. All subjects voluntarily signed an agreement to have their anonymised data used for research purposes. The test conducted were under the conditions of the BRUCE protocol [9] which is a stress ECG protocol where the incline and speed of the treadmill are increased at intervals of 3mins.

The data was collected using the GE Marquette CASE Stress System with the T2100 treadmill. This data, collected from 6 patients, varied in length from 12 mins to 20 mins long depending on the patient's fitness level , cardiac health and the discretion of the physician. The leads recorded are I, II, III, aVR, aVL, aVF, V1, V2, V3, V4, V5 and V6. A simple and concise write up about the leads and their significance can be found in [10]. The data is sampled at $200Hz$.

B. Results

The reconstruction error of VPW-FRI used in this paper is the Signal to Residue Ratio (SRR) which is defined as

$$SRR = 10log\left(\frac{\sum_{n=0}^{N-1} x[n]^2}{\sum_{n=0}^{N-1}(x[n] - \hat{x}[n])^2}\right). \tag{16}$$

A hundred segments of data, each $2s$ long, were used to evaluate the performance of the reconstruction. Section V-A. One segment can be viewed in Fig. 2.

For the VPW-FRI with the common annihilator, the algorithm tested with a mean SRR of $\mu = 19.41dB$ with a standard deviation of $\sigma = 2.28dB$ as can be seen in Table I. The low standard deviation shows consistency in reconstructing all the channels using the common annihilator. The high SRR coupled with the low standard deviation also

TABLE I
SRR VALUES ACROSS ALL 12 ECG LEADS

Mean SRR	19.41
Standard Deviation	2.28
Minimum SRR	14.81
Maximum SRR	22.25

proves the theoretical prediction that the common annihilator would provide information on the common parameters of all the channels in ECG.

The segments run in this test were good quality signals as they were relatively free of noise. The purpose of this was to demonstrate the sampling and reconstruction ability of VPW-FRI. Seven pulses were used per QRS complex.

The de-noising capability of the algorithm is significant as can be seen in Fig. 3. Noise in the form of Additive Gaussian White Noise (AWGN) was added at an SNR of $10dB$ to simulate Electromyogram (EMG) or muscle noise. It should be noted that the Cadzow de-noising [2] performs similarly but is iterative and therefore more computationally intensive.

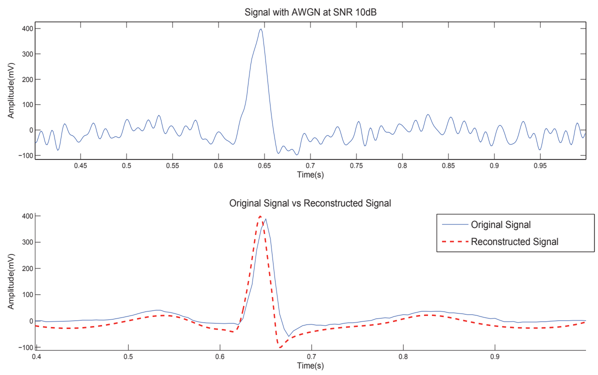

Fig. 3. Denoising on an ECG signal with AWGN at SNR 10dB

The QRS detection also tested well. Again, AWGN at SNR 0dB was added to test the robustness of the detection algorithm. The AWGN was added to all 12 channels. The QRS detector was then applied with only the pulse associated with the QRS being reconstructed. This was tested on the same 100 sets of signal used earlier in this section. A one second segment from lead II can be seen in Fig. 4. It was able to detect the number of QRS complexes and the locations perfectly on 97 of those segments. On the other 3 segments, it missed one QRS. However, when the SNR is raised to 5dB, it was able to detect all the QRS complexes in all the segments perfectly.

VI. CONCLUSION

The results demonstrate the robustness of the VPW-FRI method in compressing signals, in de-noising and also in wave detection. They also demonstrate that for the case of multichannel data, the ECG signals share a common support which translates to having a common denominator in the VPW-FRI model. This leads to additional opportunities for compression in the case of multichannel data. Future work

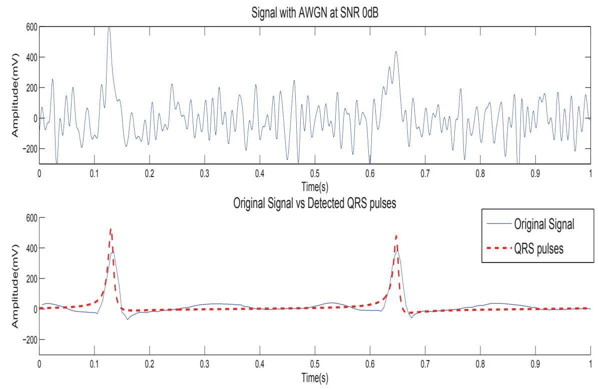

Fig. 4. QRS detection on ECG signal with AWGN at SNR 0dB

can be in the direction of application of VPW-FRI for feature detection in ECG as well as testing for compatibility with other biomedical signals.

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