

# Optimal Siting and Sizing of Distributed Energy Storage Systems via Alternating Direction Method of Multipliers

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**Abstract**—Energy Storage Systems (ESSs) has an important role in Active Distribution Networks (ADNs). Within this context this paper focuses on the problem of ESSs optimal siting and sizing. Following similar approaches already proposed by the Authors, this paper uses a multi-objective procedure to account various ancillary services that can be provided by ESSs. The proposed procedure takes into account the voltage support and network losses minimization along with minimization of the cost of energy from external grid. For the case of large-scale problems, accounting networks with large number of nodes and scenarios, the selection of the solution methodology is a non-trivial problem. In this respect, the paper proposes and discusses the applicability of the Alternative Direction Method of Multipliers in order to provide an efficient algorithm for large-scale networks that also provide a solution to the optimality aspect. A real large-scale network with real profiles of load and distributed photovoltaic generation is used as the case study to analyze the effectiveness of the proposed methodology.

**Keywords**— Alternating direction method of multipliers, active distribution networks, energy storage, optimal planning.

## NUMENCLATURE

### Parameters:

$P_r(t)$	Energy price at time $t$ and scenario $Sc$
$InvC_p, InvC_E$	Investment costs of ESS power rating and energy reservoir
$E_{max}^{ESS}, E_{min}^{ESS}$	Maximum/minimum allowed SOC of ESSs
$P_{j,Sc}^D(t), Q_{j,Sc}^D(t)$	Active/reactive load at node $j$ , time $t$ , and scenario $Sc$
$P_{j,Sc}^{DG}(t)$	Non-dispatchable DG production at node $j$ , time $t$ , and scenario $Sc$
$x_{ij}, r_{ij}(r_l)$	Longitudinal reactance / resistance of the line between nodes $i$ and $j$ (line $l$ )
$b_j^{sh}$	Total susceptance of the lines connected to node $j$
$\rho$	Penalty parameter
$W_{EP}, W_{vol}, W_{loss}$	Weighting coefficients of different terms in objective function
$C_{inv}^{Tot}, E_{inv,i}^{Tot}$	Total maximum capacity of ESS power rating/reservoir that can

$r_{ESS}$	installed in the whole network Resistive loss factor of ESS
$f_{ij}^{max}$	Maximum limit of squared current flow rating between buses $i$ , and $j$
$v^{max}, v^{min}$	Maximum/Minimum limits of squared network nodal voltages

### Variables:

$C_{inv,i}^{ESS}, E_{inv,i}^{ESS}$	ESSs power rating and energy reservoir capacity variables at node $i$ (installation cost minimization sub-problem)
$C_{op,i}^{ESS}, E_{op,i}^{ESS}$	ESSs power rating and energy reservoir capacity variables at node $i$ (operation cost minimization sub-problem)
$C_{op}^{ESS}, E_{op}^{ESS}, C_{inv}^{ESS}, E_{inv}^{ESS}$	Vectors of ESSs power rating/energy reservoir capacity (installation cost minimization/operation cost minimization sub-problem)
$V_{i,Sc}(t)$	Voltage at node $i$ , time $t$ , and scenario $Sc$
$F_{l,Sc}(t)$	Current flow on line $l$ , at time $t$ , and scenario $Sc$
$E_{i,Sc}^{ESS}(t)$	Energy stored in ESS at node $i$ , at time $t$ and scenario $Sc$
$E^{Ex}(t)$	Energy flow from substation transformer at time $t$ and scenario $Sc$
$P_{i,Sc}^{ESS}(t)$	Active power consumed/produced by ESS at node $i$ , time $t$ , and scenario $Sc$
$Q_{i,Sc}^{ESS}(t)$	Reactive power produced by ESS at node $i$ , time $t$ , and scenario $Sc$
$P_{ij,Sc}(t), Q_{ij,Sc}(t)$	Active/reactive line power flows between nodes $i$ and $j$ at time $t$ , and scenario $Sc$
$f_{ij,Sc}^2(t)$	Square of current flow on line $l$ , at time $t$ , and scenario $Sc$
$Q_{ij,Sc}^{sh}(t)$	Reactive power produced by shunt admittance on line $ij$ at time

$v_{i,Sc}(t)$   $t$ , scenario  $Sc$   
 Square of voltage at node  $i$  at  
 time  $t$ , and scenario  $Sc$

*Indices:*

$Sc$  Index of scenarios  
 $i, j$  Index of nodes  
 $l, ij$  Index of lines

## 1 INTRODUCTION

Active Distribution Networks (ADNs) are changing significantly by integrating new technologies that aims at improving their level of control. Energy Storage Systems (ESSs) have an important role in this context [1]. Indeed, they have the ability to be indirectly used to control the network providing several services like peak load shaving, supplementing renewable resources, and, as a consequence, postpone investments needed for network reinforcements (e.g., [2], [3]). They are also capable of providing network ancillary services like voltage and frequency control supports, indirectly control line congestions and, as a consequence, can be used for network losses reduction [4-6]. In this respect, one of the main problems associated to the use of ESSs in ADNs is to find their best location and size in order to maximize their actions.

In this context, several works have been done related to optimal planning of ESSs in ADNs. This issue has been addressed in both microgrids and ADNs. The Authors of [7] have proposed the use of Genetic Algorithm (GA) to find the optimal capacities of ESSs with an objective to minimize the operation costs of the targeted microgrid. A methodology to site and size different types of ESSs in a microgrid context is proposed in [8]. A GA algorithm is used to find the best solution to maximize the total net present value. A methodology for optimal siting and sizing of ESSs in a medium voltage distribution network, with the goal of decreasing wind energy curtailment and minimizing annual cost of the electricity, is presented in [9]. A hybrid GA, sequential quadratic programming algorithm is proposed in [10] to size and site DGs, energy storage and reactive power compensation systems. The goals of the planning problem are the minimization of the total network losses and the operation cost. The Authors of [6] have presented a hybrid method of dynamic programming with GA to find the best siting, rating and control strategy of ESSs, in order to minimize the overall investments and network costs (energy cost and losses). A cost-benefit analysis methodology is presented in [11] to find the best sizing and siting of ESSs in distribution networks. The goal of the optimization is to maximize the Distribution Network Operator (DNO) profits from energy transactions, investment and operation cost savings. The planning of ESSs connected to transmission networks is also investigated in literature (e.g., [12], [13]). In [12] the optimal planning of ESSs in a network with renewable and uncertain energy production resources is presented. The objective of the optimization is to minimize the operation and investment costs of energy storage devices. The application of ESSs in optimal allocation of wind capacity related to distant wind farms is investigated in [13]. The methodology simultaneously optimizes the wind power

capacity of each site, its ESS components and the required transmission connection capacity.

A limitation of the above-listed papers is represented by the fact they have not accounted in the problem formulation the ancillary services (e.g., voltage control) that ESSs can provide to ADNs. In [4] a specific algorithm for assessing the optimal siting of ESSs to maximize their contribution to voltage control was proposed. Voltage sensitivity coefficients, as a function of the nodal power injections, were used to linearize the objective function of the problem and some of the constraints. The augmented problem of optimal allocation of ESSs in ADNs with a multi-objective (i.e., loss, energy cost, and voltage deviation minimizations) is investigated in [14] by using a hybrid approach of GA and non-linear programming. Although the approach proposed in [14] provides satisfactory results, it is computationally expensive and the global optimal solution is not guaranteed due to the non-convexity of the problem and the use of the GA heuristic approach. As a matter of fact, the computational inefficiency of this approach resulted into limiting the possibility of solving large-scale problems characterized by: (i) networks with large number of nodes and (ii) multiple scenarios related to load and renewable resources profiles (i.e., seasonal variability and yearly evolutions). In [15] an SOCP formulation of the optimal power flow (OPF) is used to formulate the problem of the optimal siting and sizing of ESSs in ADNs. It considers both technical and economical goals. However, as expected, the size of the problem increases drastically with the increase of both network size and number of scenarios. As a consequence, a dedicated decomposition method might be required. These drawbacks have motivated this contribution. Indeed, long-term optimal planning problems are normally large-scale ones since they should include a reasonable number of scenarios to address the variations and uncertainties of various parameters. As known, decomposition methods can be used to decompose a large-scale problem into smaller ones and have been used for several power system problems (e.g., [16], [17]).

In this paper we propose to use the Alternating Direction Method of Multipliers (ADMM) [18-19] to break down the original problem and have a distributed parallel convex optimization. The Second Order Cone programming (SOCP) OPF approach of [20] is adapted to formulate the problem of the optimal siting and sizing of ESSs in ADNs in order to obtain a convex problem. In this respect, it is worth mentioning that the convex formulation of ESSs planning is a peculiar aspect of the problem that has not sufficiently treated in the literature. The proposed approach also accounts for a non-simplified power flow in which the reactive power associated to shunt admittances of lines/cables is accounted. Additionally, the ESSs are accurately modeled in terms of efficiency and state-of-charge (*SoC*). Also their interfaces to the AC grid are represented by means of active and reactive power capability limits. The targeted problem is formulated as a multi-objective one including voltage deviations, network losses, in addition to investment and operation cost minimizations.

The rest of the paper is organized as follows: Section II describes the problem and provides its formulation. Section III explains the proposed methodology to breakdown and solve the problem. An application example, referring to a real network

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configuration with real data, is presented in Section IV. Section V concludes the paper with final remarks concerning the applicability of the proposed procedure.

## 2 PROBLEM DESCRIPTION

The context of the problem refers to an ADN with the presence of non-dispatchable DGs. The objective is to find the best locations and sizes of a limited amount of ESSs where the limitation applies to the total DNO ESS investment. As anticipated, the problem accounts for two main objectives; i) minimization of the investment costs associated to ESSs installation, ii) minimization of a virtual cost that accounts for the network operation including both technical and economical costs. It should be noted that the charge/discharge cycles and the *SoC* level of ESSs are not considered in the operation costs. This is because these costs have been indirectly included in the problem by considering the ESSs limited lifetime (i.e., 10 years). The objective function is as in (1). It includes the investment cost of the ESSs and the operation cost of the network. The objective of the operation cost considers different terms (i) voltage deviation minimization, (ii) minimization of the cost of energy from the external grid, (iii) and total network losses minimization. The constraints of the problem are modeled by (2-19).

$$Obj = \quad (1)$$

$$W_{EP} \{ (C_{inv,i}^{ESS} InvC_P) + (E_{inv,i}^{ESS} InvC_E) \} +$$

$$\sum_{Sc} [\sum_t \{ \sum_i (W_{vol} |V_{i,Sc}^2(t) - 1|)$$

$$W_{loss} (r_i F_{i,Sc}^2(t)) + W_{EP} E^{Ex}(t) Pr(t) ]$$

$$E_{i,Sc}^{ESS}(t+1) = E_{i,Sc}^{ESS}(t) + P_{i,Sc}^{ESS}(t)t - L_{i,Sc}^{ESS}(t)t \quad (2)$$

$$E_{min}^{ESS} E_{op}^{ESS} \leq E_{i,Sc}^{ESS}(t) \leq E_{op}^{ESS} E_{max}^{ESS} \quad (3)$$

$$0 \leq P_{i,Sc}^{ESS}(t) \leq C_{op,i}^{ESS} \quad (4)$$

$$E_{op,i}^{ESS} - E_{inv,i}^{ESS} = 0 \quad (5)$$

$$C_{op,i}^{ESS} - C_{inv,i}^{ESS} = 0 \quad (6)$$

$$\sum_t E_{inv,i}^{ESS} \leq E_{inv}^{Tot} \quad (7)$$

$$\sum_i C_{inv,i}^{ESS} \leq C_{inv}^{Tot} \quad (8)$$

$$0 \leq C_{inv,i}^{ESS} \leq C_{inv,max}^{ESS} \quad (9)$$

$$0 \leq E_{inv,i}^{ESS} \leq E_{inv,max}^{ESS} \quad (10)$$

$$(P_{i,Sc}^{ESS}(t))^2 + (Q_{i,Sc}^{ESS}(t))^2 \leq (C_{op,i}^{ESS})^2 \quad (11)$$

$$L_{i,Sc}^{ESS}(t) \geq r_{ESS} (P_{i,Sc}^{ESS}(t))^2 + (Q_{i,Sc}^{ESS}(t))^2 \quad (12)$$

$$P_{ij,Sc}(t) = \sum_{k:(j,k) \in C} (P_{jk,Sc}(t)) + r_{ij} f_{ij,Sc}(t) + P_{j,Sc}^D(t) - P_{j,Sc}^{DG}(t) - P_{j,Sc}^{ESS}(t) \quad (13)$$

$$Q_{ij,Sc}(t) = \sum_{k:(j,k) \in C} (Q_{jk,Sc}(t)) + x_{ij} f_{ij,Sc}(t) - Q_{ij,Sc}^{sh}(t) + Q_{j,Sc}^D(t) - Q_{j,Sc}^{ESS}(t) \quad (14)$$

$$f_{ij,Sc}(t) \geq \frac{(P_{ij,Sc}(t))^2 + (Q_{ij,Sc}(t))^2}{v_{i,Sc}(t)} \quad (15)$$

$$f_{ij,Sc}(t) \leq f_{ij}^{max} \quad (16)$$

$$v_{j,Sc}(t) = v_{i,Sc}(t) - 2(r_{ij} P_{ij,Sc}(t) + x_{ij} Q_{ij,Sc}(t)) \quad (17)$$

$$+ (r_{ij}^2 + x_{ij}^2) \left( \frac{(P_{ij,Sc}(t))^2 + (Q_{ij,Sc}(t))^2}{v_{i,Sc}(t)} \right)$$

$$v_{i,Sc}^{min} \leq v_{i,Sc}(t) \leq v_{i,Sc}^{max} \quad (18)$$

$$Q_{j,Sc}^{sh}(t) = v_{j,Sc}(t) b_j^{sh} \quad (19)$$

The energy stored in the ESSs in each hour is dependent on the previous *SoC* and the amount of energy stored/withdrawn from its reservoir: equation (2) models this constraint. Equations (3) and (4) show the capacity constraints of ESSs power rating and energy reservoir. The limitation of the investment budgets related to the ESSs installation are represented by (7) and (8). Constraints (9)-(11) represent the maximum capacity of ESS power rating and energy reservoir that can be installed on each particular node. The capability curve of the ESSs is accounted by (11). It is worth observing that this constraint is piece-wise linearized in order to preserve the convexity of the whole optimization problem [15]<sup>1</sup>. The resistive loss of ESSs is modeled by equation (12). Constraints (13-19) define the security constraints associated to the network operation. The active and reactive load balances are modeled by (13) and (14) respectively. The feeders current flow limits are modeled by (15)<sup>2</sup> and (16) and, the voltage limits are defined by (17)<sup>3</sup> and (18). Finally, equation (19) account for the amount of reactive power related to the shunt impedance of the lines.

## 3 SOLUTION METHODOLOGY

### A. Summary about the alternating direction method of multipliers [18]

The inherent large-scale nature of the problem lays in the fact that it should cover a reasonable numbers of scenarios in order to obtain a solution accounting for a sufficiently large set of variations of the considered parameters. One of the most common approaches is to breakdown the problem into smaller ones. By following this idea, we propose to use the ADMM to breakdown the original problem and obtain a distributed parallel convex optimization.

As known, the ADMM is a powerful and well-suited method for decentralized convex optimization. The peculiarity of the ADMM is that it uses a decomposition-coordination procedure in order to find the solution to a large global problem by solving small local sub-problems in parallel. It uses the

<sup>1</sup> The linearization of the constraints of the problem has been already addressed by the Authors in [15].

<sup>2</sup> This constraint is the relaxed version of its original formulation composed by an equality instead of an inequality. This relaxation is exact since the current flow is minimized, in the objective function, by accounting the losses.

<sup>3</sup> The last term in (17) is very small compared to the other two ones. Therefore, it is neglected (e.g., [19]).

benefits of dual decomposition and augmented Lagrangian methods [18-19]. In the following, the ADMM is briefly described.

Suppose an optimization problem with the form represented by (20) and (21) where  $f$ ,  $g$ ,  $\zeta$ , and  $\zeta$  are convex. The  $f$  and  $g$  are independent from each other except they are linked by the constraint (21).

$$\underset{x,z}{\text{minimize}} \quad f(x) + h(z) \quad (20)$$

$$\text{subject to} \quad \text{dom } f = \{x | x \in \zeta\}$$

$$\text{dom } h = \{z | z \in \zeta\}$$

$$Ax + Bz = c \quad (21)$$

Where  $A \in \mathbb{R}^{p \times n}$ ,  $B \in \mathbb{R}^{p \times m}$  and  $c \in \mathbb{R}^p$  when the variables  $x \in \mathbb{R}^n$  and  $z \in \mathbb{R}^m$ . The augmented Lagrangian of this problem with respect to constraint (21) has the form as shown in (22).

$$L_\rho(x, y, z) = f(x) + h(z) + y^T (Ax + Bz - c) \quad (22)$$

$$+ \left(\frac{\rho}{2}\right) \|Ax + Bz - c\|_2^2$$

$$\text{subject to} \quad \text{dom } f = \{x | x \in \zeta\}$$

$$\text{dom } h = \{z | z \in \zeta\}$$

The ADMM procedure is deployed as follows. An iterative process between the three steps shown in equations (23-25) will result in the optimal solution. First, the augmented Lagrangian problem (22) will be minimized with  $z$  and  $y$  being fixed. Then the obtained  $x$  will be used in the minimization of (22) with  $x$ , and  $y$  being fixed. Finally, the dual multipliers will be updated as shown in (25) with the obtained  $x$  and  $y$  of the previous steps.

$$x^{k+1} := \underset{x}{\text{argmin}} L_\rho(x, y^k, z^k) \quad (23)$$

$$z^{k+1} := \underset{z}{\text{argmin}} L_\rho(x^{k+1}, y^k, z) \quad (24)$$

$$y^{k+1} := y^k + \rho(Ax^{k+1} + Bz^{k+1} - c) \quad (25)$$

This procedure will be continued until it converges to the global optimal solution.

### B. ADMM application to ESSs optimal planning

The ADMM is used here to decompose the installation-cost minimization problem from the one of the operation-cost minimization enabling a parallel formulation. The two problems are linked by a set of linear constraints. These constraints imply that the ESS capacities obtained in the first problem are identical to the ESSs capacities in the second one. The application of the ADMM to the ESS optimal planning problem is described in what follows. Functions  $f(x)$  and  $g(z)$  of (2) represent, in our case, the ESS investment cost and the operation cost functions respectively. The linking constraints are (6) and (7). They assure that the ESSs power rating and energy reservoir capacities should be the same for the two ADMM sub-problems. Therefore, the first step of the ADMM is formulated as shown in (26).

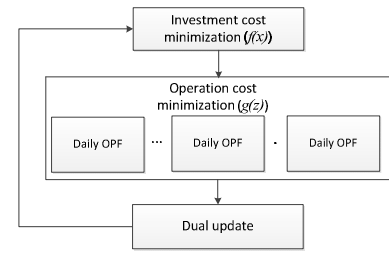


Fig. 1. ADMM procedure applied to the problem of ESS optimal planning.

The second step is represented by the operation-cost minimization for all the possible scenarios. In this step each scenario is accounted separately since they can be evaluated in parallel. The objective function of the problem is formulated as (27). The constraints of the problem are (2-6) and (11-19). In this step two more constraints are added to the first step in order to increase the convergence speed. In particular, we have assumed that the p.u. reservoir capacity of a generic ESS has to be greater than the corresponding p.u. power rating. Similarly, it is assumed that their p.u. power rating has to be greater than 0.15 of their corresponding p.u. reservoir capacity.

$$\text{minimize:} \quad (26)$$

$$C_{inv}^{ESS}, E_{inv}^{ESS}$$

$$W_{EP} [(C_{inv}^{ESS} Inv C_P^{ESS}) + (E_{inv}^{ESS} Inv C_E^{ESS})] +$$

$$\sum_{Sc} \left\{ y_{Sc}^T \left[ \begin{array}{c} C_{op}^{ESS,k} \\ E_{op}^{ESS,k} \end{array} \right] - \left[ \begin{array}{c} C_{inv}^{ESS} \\ E_{inv}^{ESS} \end{array} \right] \right\} +$$

$$\left( \frac{\rho}{2} \right) \left\| \left[ \begin{array}{c} C_{op}^{ESS,k} \\ E_{op}^{ESS,k} \end{array} \right] - \left[ \begin{array}{c} C_{inv}^{ESS} \\ E_{inv}^{ESS} \end{array} \right] \right\|_2^2$$

Subject to: (7-10)

$$\text{minimize:} \quad (27)$$

$$\sum_{Sc} \{ \sum_t \{$$

$$\sum_i (W_{vol} |V_{i,Sc,t}^2 - 1| + W_{loss} (r_i F_{i,Sc,t}^2)) + W_{EP} E_t^{Ex} Pr_t$$

$$W_{Cur} \sum_t (P_{i,Sc,t}^{Cur}) \} \}$$

$$\sum_{Sc} \left\{ y_{Sc}^T \left[ \begin{array}{c} C_{op}^{ESS} \\ E_{op}^{ESS} \end{array} \right] - \left[ \begin{array}{c} C_{inv}^{ESS,k+1} \\ E_{inv}^{ESS,k+1} \end{array} \right] \right\} +$$

$$\left( \frac{\rho}{2} \right) \left\| \left[ \begin{array}{c} C_{op}^{ESS} \\ E_{op}^{ESS} \end{array} \right] - \left[ \begin{array}{c} C_{inv}^{ESS,k+1} \\ E_{inv}^{ESS,k+1} \end{array} \right] \right\|_2^2$$

The last step is the dual is updated as shown in (28).

$$y_{Sc}^{k+1} = y_{Sc}^k + \rho \left( \left[ \begin{array}{c} C_{op}^{ESS,k+1} \\ E_{op}^{ESS,k+1} \end{array} \right] - \left[ \begin{array}{c} C_{inv}^{ESS,k+1} \\ E_{inv}^{ESS,k+1} \end{array} \right] \right) \quad (28)$$

These steps will be iterated until the solver converges to an optimal solution. The scheme of the proposed ADMM-based procedure is shown in Fig. 1.

#### 4 SIMULATION RESULTS

A real distribution network located in the southwest of Switzerland has been used as a case study (see Fig. 2). The network contains 287 nodes and is characterized by non-negligible amount of PV installations with a total capacity of 0.85 MW. A DG unit, with constant production of 1.5 MW is also connected to the grid in correspondence of node #247. Nodes where PVs and the above/mentioned DG unit are connected are shown in Fig. 2. The simulation is done for four weeks of the year: one in spring, one in summer, one in fall, and one in the winter. Experimentally measured loads and generation profiles for this specific grid are considered with a discretization time step of 15 minutes. Total active power load and PV profiles of these four time periods are shown in Fig. 3 and 4 respectively. The load is distributed between the feeders as shown in Table 1. As it can be seen from Fig. 4, the PV production is negligible in the winter period. The energy price profiles of these weeks are shown in Fig. 5. The weighting coefficients of the elements composing the objective function are: voltage deviation  $W_{vol}=0.61$ , total network loss  $W_{loss}=0.12$ , energy cost from external grid  $W_{ep}=0.27$ . These values have been inferred using the Analytic Hierarchy Process (AHP) [21].

The total maximum ESSs power rating and reservoir capacities were assumed to be equal 4 MW and 6 MWh respectively. The investment costs for ESSs capacity rating and energy reservoir are assumed to be 100 CHF/kW and 150 CHF/kWh respectively (these values are annualized and downscaled for 4 weeks; the lifetime of ESSs is assumed to be 10 years and the annual interest rate is assumed to be 4%). The ADMM penalty parameter  $\rho$  has been assumed equal to 1. The voltage minimization term in the objective function is activated when the voltage exhibits a deviation from the rated value larger than +/-2%.

The obtained optimal sites and sizes of ESSs are shown in Table II. As it can be seen, 3 nodes are selected to install ESSs. All the selected nodes are close to the largest loads. The ESS with the highest capacity is located on the feeder #2 located in correspondence of the highest loaded part of the network (feeder #7 has the highest loading but it also has a DG that can supply the load). Table III shows the total amount of network losses and energy cost imported from the external grid in both cases, namely: without ESSs and with optimally located ESSs. Both these quantities exhibit a clear decrease in case optimally allocated ESSs are available in the network.

TABLE I. AVERAGE FEEDER LOADING IN THE FOUR CONSIDERED WEEKS

Feeder #	1	2	3	4	5	6	7	8
Starting node of the considered feeder (see Fig. 2)	105	113	121	127	138	168	207	249
Load share with respect to total network loading (%)	0.62	25.1	13.2	7	20	5	0.29	28

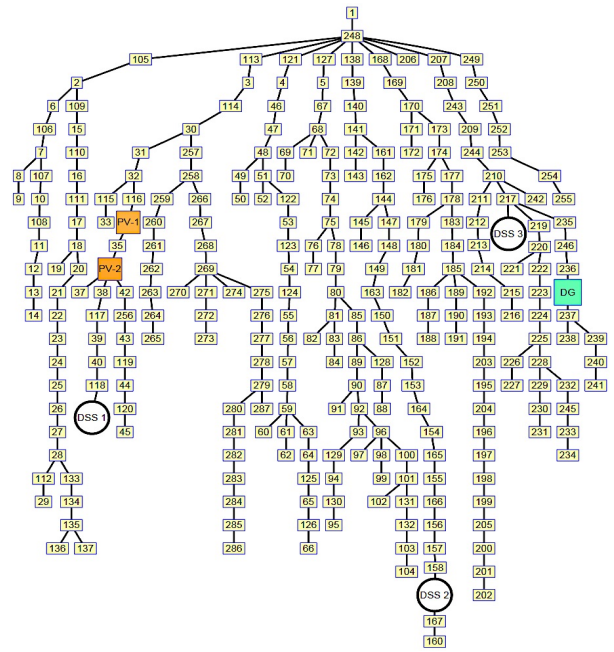


Fig. 2. The schematic of the test case study

TABLE II. OPTIMAL ESS SITE AND SIZE

ESS #	1	2	3
Power rating (MW)	1.7	0.5	1.125
Reservoir capacity (MWh)	3.15	1.1	1.7

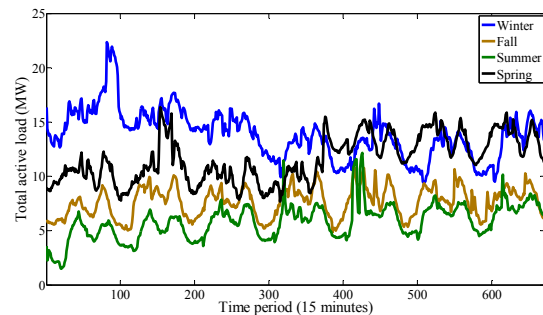


Fig. 3. Aggregated network loads: active-power profiles for the four considered weeks.

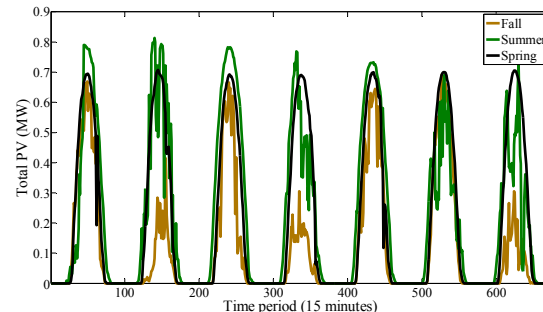


Fig. 4. Aggregated PV injections: active-power profiles for the four considered weeks.

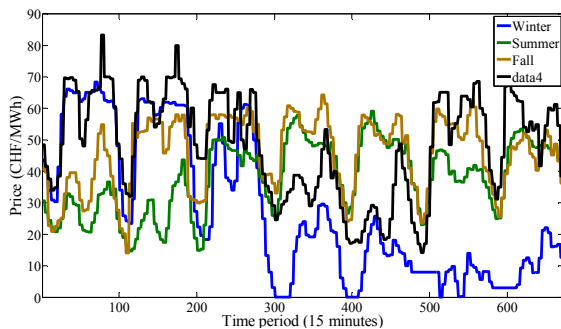


Fig. 5. Profiles of the electricity prices in the four considered weeks.

TABLE III. TOTAL NETWORK LOSSES AND THE COST OF ENERGY FROM EXTERNAL GRID

	Total network losses in the simulated weeks [MWh]	Total energy cost imported from the external grid in the simulated weeks (CHF)
<b>Optimal ESS siting and sizing</b>	127.1678	154931
<b>Without ESS</b>	138.5528	158893

Fig. 6 shows the Cumulative Distribution Function (CDF) of nodal voltages in both analyzed cases (i.e., with and without ESSs). It is evident that the presence of ESSs allows to largely improve the ADN quality-of-service with respect to voltage variations. In particular, the probability of occurrence of under-voltages below 0.98 p.u. has been entirely removed.

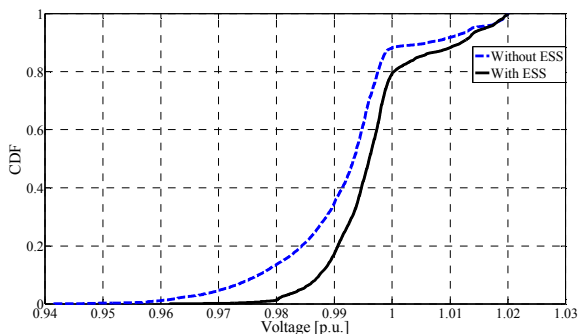


Fig. 6. The CDF of nodal voltages for the cases with and without optimal ESS siting and sizing

The *SoC* of all the ESSs in two days, one day in summer and one day in winter, are shown in Figs. 7 and 8 respectively. As it can be observed, the figures show how the constraint on the ESS *SoC* has been respected. Figs. 7 shows that, in the summer period, all of the ESSs follow the price profiles. This is due to fact that the loading of the system is low as well as the voltage deviations and the network losses. Fig. 8 shows that the ESSs *SoC* in the wintertime period is different. In particular, ESSs #1 and #4, located on feeders #2 and #5 respectively, are responding to the load profile. In this respect it is worth observing that these ESSs are located in the feeders characterized by the highest loading level with associated largest voltage variations. Thus, they tend to minimize the corresponding elements of the multi-objective function since

they have a larger priority. The other ESS, in addition to contributing to the voltage regulation, is also responding to electricity. Indeed, in the high price periods it provides energy. Indeed, this ESS is located in the feeder #7 where there is the DG that supplies power at constant rate. In view of the above considerations, it is evident how the proposed process is capable to locate each ESS by distinguishing their influences on: the network quality-of-service, the local energy balance and the network zone of influence.

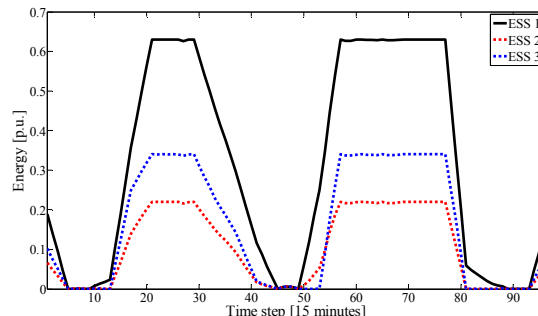


Fig. 7. *SoC* profiles of the ESSs in the summer period (Base value of energy is 5 MWh).

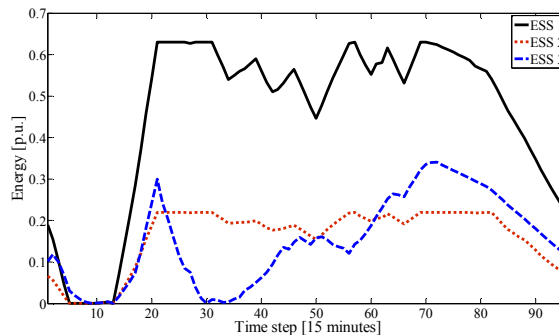


Fig. 8. *SoC* profiles of the ESSs in the winter period (Base value of energy is 5 MWh).

It should be noted that the case study investigated in this paper does not require for a load curtailment or network reinforcement. Therefore, these elements are not incorporated in the objective function. However, they can be straightforward incorporated as already discussed in [15]. It is also worth noting that the main objective investigated in this paper is the ESSs contribution to increase the ADN quality of service (i.e., compensate the voltage deviations). Therefore, this specific objective is characterized by the highest weight in the objective function. As a result, as it can be noted from Table III, the benefits resulting from the energy arbitrage cannot justify the ESS high capital cost alone.

As a final remark, it is important to point out that we used four weeks of the year as input data leading to have total number of 28 scenarios. However, increasing the number of scenarios will not be a limiting aspect since they will increase the number of separated daily AC-OPF that can be analyzed in parallel.

## 5 CONCLUSION

The paper has proposed a decomposition method based on

the ADMM methodology applied to the problem of optimal siting and sizing of ESSs in ADNs. The objective function of the problem accounts for different benefits of the ESSs. Indeed, the targeted problem has been formulated as a multi-objective one accounting for: voltage deviations, network losses, in addition to investment and operation cost minimizations.

Compared to other works already published by the Authors on the same subject, the paper has discussed the use of ADMM to propose an efficient procedure to solve large-scale problem accounting networks with realistic large number of nodes and scenarios. In this respect, after the description of the proposed planning procedure in terms of formulation and step-by-step process, the paper has discussed its application to the case of a real large-scale network with real profiles of load and distributed photovoltaic generation.

It is evident from the obtained results how the proposed process is capable to locate each ESS by distinguishing their influences on: the network quality-of-service, the local energy balance and the network zone of influence. It can be concluded that the proposed process can be used by DNOs to evaluate the possible use of ESSs as a valid alternative to the investments related to grid reinforcement or massive telecom infrastructure for direct DG control.

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