H_∞ Smith Predictor Design for Time-Delayed MIMO Systems via Convex Optimization

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***Abstract*— A new method for robust fixed-order H_∞ controller design for uncertain time-delayed MIMO systems is presented. It is shown that the H_∞ robust performance condition can be represented by a set of convex constraints with respect to the parameters of a linearly parameterized primary controller in the Smith predictor structure. Therefore, the parameters of the primary controller can be obtained by convex optimization. The proposed method will be applied to stable MIMO models with uncertain dead-time and with multimodel and frequency-dependent uncertainty. The performance of this method is illustrated by simulation examples of industrial processes.**

I. Introduction

Many dynamical systems in the industry possess unavoidable time delays. These delays can be caused by accumulation of time lags for dynamic systems interconnected in series, transportation delay or measurement delay [1]. Time delays in control loops can cause significant complications in modern industrial applications. The rapid development in data and communication network technologies has caused a need for real-time data processing [2]. The first time-delay compensation method was proposed in the late 1950s by [3]. This method is known as the Smith Predictor (SP), and it has become one of the most widely implemented control schemes used to regulate industrial systems with time delays.

The SP, however, is somewhat limited in its performance, since an accurate model of the system is generally required for satisfactory operation. In certain circumstances, small modeling errors may lead to undesirable performance, where the system can become unstable. For this reason, research efforts have been focused on robustness issues of the SP.

Many researchers are interested in the optimal control of dead-time systems, especially H_∞ control, i.e., to find a controller to internally stabilize the system and to minimize the H_∞-norm of an associated transfer function. Many relevant results have been presented in this framework using modified versions of the SP. See, for instance, [4], [5] and [6]. Recently, the single-input-single-output (SISO) SP has been extended and generalized for multiple-input-multiple-output (MIMO) systems. In [7], a structured uncertainty approach was implemented for SP's with diagonal delay matrices. This method, however, does not consider general and distinct time delays for each element of the plant transfer matrix. A diagonal H_2 optimal controller for non-square plants is designed by factorization methods in [8]. In [9], a generalized predictive control (GPC) method is implemented on MIMO SP systems with multiple delays. These control techniques, although efficient, are quite complex from both the design and implementation perspective.

There are a wide variety of industrial applications that involve MIMO processes with time delays, and it is of practical interest to develop robust control techniques for such systems. The proposed control scheme is based on the ideas presented in [10] for SP design of SISO systems and in [11] for designing decoupling MIMO controllers. However, in this paper, the SP design method for computing H_∞ controllers for SISO models is extended to MIMO SP's with process plants that possess uncertain time delays. A convex optimization approach is implemented to design a linearly parameterized primary controller in a SP structure for a MIMO system with uncertain time delays.

This paper is organized as follows: In Section (II), the class of models, controllers and control objectives are defined. Section (III) will discuss the control design methodology and stability conditions of the proposed method for the MIMO Smith predictor. This methodology is based on the convex constraints in the Nyquist diagram. Section (IV) will demonstrate the effectiveness of the proposed method by considering several case studies from industrial processes. Finally the concluding remarks are given in Section (V).

II. Problem Formulation

In this section, the SP for MIMO systems with generalized time delays is investigated. For notation purposes, bold face characters will represent transfer function matrices.

A. *Class of models*

Let n_o and n_i represent the number of outputs and the number of inputs of a system, respectively. The set of LTI-MIMO stable strictly proper models with multiplicative uncertainty and uncertain time delays can be defined as follows:

$$\mathcal{P} = \{\mathbf{P}_c(s)[\mathbf{I} + \Delta_c(s)\mathbf{W}_{2_c}(s)]; \quad c = 1, \ldots, m\} \qquad (1)$$

where each element in $\mathbf{P}_c(s)$ possesses a time delay that can vary over a range of specified values, and $\mathbf{W}_{2_c}$ is a transfer function matrix that represents the multiplicative input uncertainty of the system. $\Delta_c(s)$ represents the unknown stable transfer function matrix satisfying $\|\Delta_c\|_\infty < 1$. For simplicity, one model from the set $\mathcal{P}$ will be investigated, and the subscript c will be omitted. The uncertain $n_o \times n_i$

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time delayed plant has the following form:

$$\mathbf{P}(s) = \begin{bmatrix} G_{11}(s)e^{-\tau_{11}s} & \cdots & G_{1n_i}(s)e^{-\tau_{1n_i}s} \\ \vdots & \ddots & \vdots \\ G_{n_o1}(s)e^{-\tau_{n_o1}s} & \cdots & G_{n_on_i}(s)e^{-\tau_{n_on_i}s} \end{bmatrix} \quad (2)$$

where $G_{qp}(s)$ is a strictly proper delay-free transfer function, and τ_{qp} is the uncertain time-delay of the process for $p = 1, \ldots, n_i$ and $q = 1, \ldots, n_o$. Note that τ_{qp} is a set that is composed of elements τ_{qp_i} for $i = 1, \ldots, l$ and belongs in the domain $\{\tau_{qp} \in \mathbb{R} : \tau_{qp_i} > 0 \ \forall \{p, q, i\}\}$.

B. Class of controllers

As stated in [11], an $n_i \times n_o$ matrix can be formed to represent the controller $\mathbf{C}(s, \rho)$. The elements of $\mathbf{C}(s, \rho)$ will possess linearly parameterized elements

$$C_{pq}(s, \rho) = \rho_{pq}^T \phi_{pq}(s) \quad (3)$$

where ρ_{pq}^T is a vector of parameters, and $\phi_{pq}(s)$ is a vector of stable transfer functions chosen from a set of orthogonal basis functions. The non-diagonal elements of $\mathbf{C}(s, \rho)$ strive to decouple the system, while the diagonal elements aim to control the single-loop subsystems. The main purpose of parameterizing the controller in this manner is due to the fact that the components of the open loop transfer function can be written as a linear function of the control parameters ρ,

$$\rho = [\rho_{11}, \ldots, \rho_{1_{n_i}}, \ldots, \rho_{n_o1}, \ldots, \rho_{n_on_i}] \quad (4)$$

C. Design specifications

Fig. 1 displays the SP for the MIMO case, where $\mathbf{G}_n(s)$ is an $n_o \times n_i$ nominal delay-free transfer function matrix with elements $G_{qp}(s)$, and $\mathbf{P}_n(s)$ is an $n_o \times n_i$ nominal transfer function matrix that includes the nominal values of the time delays, which is comprised of elements $G_{qp}(s)e^{-\zeta_{qp}s}$ (where ζ_{qp} represents the qp-th nominal time delay). Both $\mathbf{Y}(s)$ and $\mathbf{R}(s)$ are $n_o \times 1$ column vectors that possess elements $y_q(s)$ and $r_q(s)$, respectively. The transfer function from the inputs of $\mathbf{C}(s)$ to $\mathbf{Y}_p(s)$ will represent the open-loop transfer function,

$$\mathbf{L}(s) = [\mathbf{P}(s) + \mathbf{H}(s)]\mathbf{C}(s) \quad (5)$$

where $\mathbf{H}(s) = \mathbf{G}_n(s) - \mathbf{P}_n(s)$. Notice that if $\mathbf{P}(s) = \mathbf{P}_n(s)$, then $\mathbf{L}(s) = \mathbf{G}_n(s)\mathbf{C}(s)$. Since the class of controllers to be designed for this system are linearly parameterized, the elements of the controller $\mathbf{C}(s)$ will actually be a linear function of the controller parameters ρ. Therefore, $\mathbf{C}(s)$ will be represented as $\mathbf{C}(s, \rho)$ with elements $C_{pq}(s, \rho)$, as asserted in (3).

The transfer function from the output disturbance $\mathbf{D}(s)$ to $\mathbf{Y}(s)$ is the output sensitivity function $\mathbf{S}(s, \rho)$, while the transfer function from $\mathbf{R}(s)$ to $\mathbf{Y}(s)$ is the complementary sensitivity function $\mathbf{T}(s, \rho)$:

$$\begin{aligned} \mathbf{S}(s, \rho) &= [\mathbf{I} + \mathbf{H}(s)\mathbf{C}(s, \rho)]\mathbf{Z}^{-1}(s, \rho) \\ \mathbf{T}(s, \rho) &= \mathbf{P}(s)\mathbf{C}(s, \rho)\mathbf{Z}^{-1}(s, \rho) \end{aligned} \quad (6)$$

where $\mathbf{Z}(s, \rho) = [\mathbf{I} + \mathbf{L}(s, \rho)]$. The objective is to determine the controller $\mathbf{C}(s, \rho)$ that will guarantee the robust performance and robust stability of the closed-loop SP system.

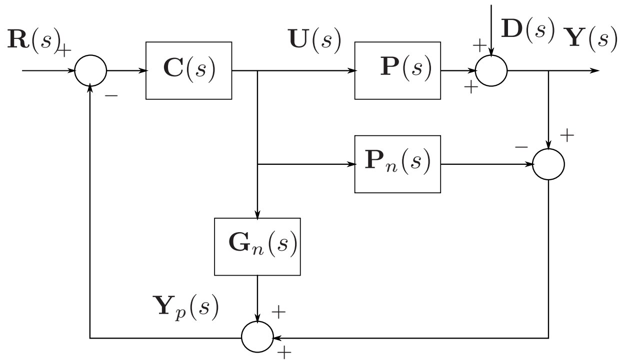

Fig. 1. MIMO representation of the Smith Predictor

III. Proposed method

It is well known that if a SISO model is described by unstructured multiplicative uncertainty, and possesses both robustness and performance weighing functions W_1 and W_2, then the necessary and sufficient condition for robust performance is given by [12]:

$$\||W_1S| + |W_2T|\|_\infty < 1 \quad (7)$$

where S and T are the sensitivity and complementary sensitivity functions of a SISO system, respectively.

For the moment, assume the case when a closed-loop MIMO system is fully decoupled. Then the MIMO sensitivity and complementary sensitivity functions can essentially be treated as functions containing independent SISO subsystems. Thus it is judicious to define $\mathbf{W}_1(s)$ as a diagonal filter with diagonal elements W_{1_q} and a diagonal filter $\mathbf{W}_2(s)$ with diagonal elements W_{2_q} representing, respectively, the nominal performance and multiplicative uncertainty for the SISO subsystems. This rationalization leads to the following theorem:

Theorem 1 *Let $M_{qq}(j\omega, \rho)$ represent the diagonal elements of $\mathbf{H}(j\omega)\mathbf{C}(j\omega, \rho)$ and $N_{qq}(j\omega, \rho)$ represent the diagonal elements of $\mathbf{P}(j\omega)\mathbf{C}(j\omega, \rho)$. Suppose that $\mathbf{S}(s, \rho)$ and $\mathbf{T}(s, \rho)$ in (6) are diagonal transfer function matrices (the closed-loop system is fully decoupled). Then the linearly parameterized controller in (3) will guarantee the closed-loop stability of the system and satisfy the following robust performance criterion:*

$$\||W_{1_q}(j\omega)S_{qq}(j\omega, \rho)| + |W_{2_q}(j\omega)T_{qq}(j\omega, \rho)|\|_\infty < 1 \quad \textit{for } q = 1, \ldots, n_o \quad (8)$$

if

$$\begin{aligned} \{|W_{1_q}(j\omega)[1 + M_{qq}(j\omega, \rho)]| + |W_{2_q}(j\omega)N_{qq}(j\omega, \rho)|\} \\ \times|1 + L_{D_q}(j\omega)| - \Psi_q(j\omega, \rho) < 0 \\ \forall\omega \ \textit{for } q = 1, \ldots, n_o \end{aligned} \quad (9)$$

where

$$\Psi_q(j\omega, \rho) = R_e\{[1 + L_{D_q}^*(j\omega)][1 + L_{qq}(j\omega, \rho)]\}$$

and S_{qq} and T_{qq} are the q-th diagonal elements of $\mathbf{S}(s,\rho)$ and $\mathbf{T}(s,\rho)$, respectively. $L_{D_q}(s)$ is the q-th diagonal element of a diagonal transfer function matrix $\mathbf{L}_D(s)$ that contains strictly proper transfer functions which do not encircle the critical point, and $L^*_{D_q}$ is its complex conjugate.

Proof: If the closed-loop MIMO system is fully decoupled, then the MIMO sensitivity and complementary sensitivity functions can be considered as systems containing independent SISO systems. Since the real part of a complex number is less than or equal to its magnitude, we have

$$\begin{aligned} &Re\{[1+L^*_{D_q}(j\omega)][1+L_{qq}(j\omega,\rho)]\} \\ &\quad \le |[1+L^*_D(j\omega)][1+L_{qq}(j\omega,\rho)]| \end{aligned} \tag{10}$$

Then, by combining (10) and (9) (and noting that $|1+L_{D_q}| = |1+L^*_{D_q}|$), one obtains

$$\begin{aligned} &\left|W_{1_q}(1+M_{qq}(j\omega,\rho))\right| + \left|W_{2_q}N_{qq}(j\omega,\rho)\right| \\ &\qquad -|1+L_{qq}((j\omega,\rho))| < 0 \\ &\qquad \forall\omega \ \text{ for } q=1,\dots,n_o \end{aligned} \tag{11}$$

The above equation can be rearranged and expressed as follows:

$$\begin{aligned} &\frac{|W_{1_q}(1+M_{qq}(j\omega,\rho))| + |W_{2_q}N_{qq}(j\omega,\rho)|}{|1+L_{qq}((j\omega,\rho)|} < 1 \\ &\qquad \forall\omega \ \text{ for } q=1,\dots,n_o \end{aligned} \tag{12}$$

Since M_{qq} and N_{qq} are the $q-th$ diagonal elements of $\mathbf{H}(s)\mathbf{C}(s,\rho)$ and $\mathbf{P}(s)\mathbf{C}(s,\rho)$ in (6), respectively, it can be seen that (12) leads directly to (8). ■

In order to fully decouple the MIMO system, a controller must be designed such that the off-diagonal elements of the open-loop transfer function matrix are equal to zero. The proposed method will be to define a diagonal open-loop transfer function matrix $\mathbf{L}_D(s)$, where the diagonal elements satisfy the desired performance for single loop systems. Therefore, by minimizing the objective function $\|\mathbf{L}(s,\rho)-\mathbf{L}_D(s)\|_2^2$, a controller can be designed to simultaneously minimize the magnitudes of the off-diagonal elements of $\mathbf{L}(s,\rho)$ and drive the diagonal elements to be approximately equal to $L_{D_q}(s)$.

However, the resulting controller will stabilize the closed-loop system only if it is fully decoupled. In practice, with a finite order controller, it is not always possible to make the off-diagonal elements of $\mathbf{L}(j\omega,\rho)$ equal to zero. In this case, the generalized Nyquist stability criterion should be used to guarantee the stability of the MIMO system. According to this theorem, the eigenvalues of the open-loop transfer function (5) should not encircle the critical point. However, these eigenvalues are non-convex functions of the linear control parameters, which complicates the design process. A possible solution to this problem is to implement the Gershgorin band theorem in order to approximate the eigenvalues of $\mathbf{L}(j\omega,\rho)$. The Gershgorin bands represent disks centered at the diagonal elements of a matrix that include the eigenvalues. For the open-loop transfer matrix $\mathbf{L}(j\omega,\rho)$, the radius of these disks are computed by:

$$r_q(\omega,\rho) = \sum_{p=1,p\neq q}^{n_o} |L_{qp}(j\omega,\rho)| \tag{13}$$

where $L_{qp}(j\omega,\rho)$ represents the qp-th element of $\mathbf{L}(j\omega,\rho)$. Note that $r_q(\omega,\rho)$ is convex with respect to the control parameter ρ. The closed-loop stability of the MIMO system is guaranteed if these disks do not encircle the critical point. This precondition leads to the following theorem:

Theorem 2 *Given the open loop transfer function matrix $\mathbf{L}(j\omega,\rho)$, the linearly parameterized controller (3) stabilizes the closed-loop system if*

$$\begin{aligned} &|r_q(j\omega,\rho)[1+L_{D_q}(j\omega)]| - \Psi_q(\rho,\omega) < 0 \\ &\qquad \forall\omega \ \textit{ for } q=1,\dots,n_o \end{aligned} \tag{14}$$

Proof: By combining the constraint in (14) and (10) (and noting that $|1+L_{D_q}| = |1+L^*_{D_q}|$), one obtains

$$\begin{aligned} &|r_q(j\omega,\rho)| < |1+L_{qq}(j\omega,\rho)| \\ &\qquad \forall\omega \ \text{ for } q=1,\dots,n_o \end{aligned} \tag{15}$$

The constraint in (15) guarantees that the disk with radius $r_q(j\omega,\rho)$ centered at $L_{qq}(j\omega,\rho)$ does not encircle the critical point $(-1+j0)$, and thus the system remains stable for all ω. ■

A. Primary controller design

In designing the controller $\mathbf{C}(s,\rho)$ for the MIMO SP, one must consider all of the possible combinations of the uncertain delay parameters τ_{qp}. Suppose that the cardinality of τ_{qp} is β_{qp}. Then the total number of possible combinations that must be considered in the design of the controller is given by the rule of product,

$$\begin{aligned} &m = \prod \beta_{qp} \\ &\forall \ q=1,\dots,n_o; \ p=1,\dots,n_i \end{aligned} \tag{16}$$

If the number of uncertainties are equal for each τ_{qp} (i.e., $\beta_{qp} = \beta_{pq} = \beta \ \forall \ \{p,q\}$), then the total number of combinations will be $m = \beta^{n_o n_i}$. By combining the constraints presented in Theorem 1 and Theorem 2, one can define the following optimization problem for the multimodel system:

$$\min_{\rho} \sum_{c=1}^{m}\sum_{k=1}^{N} \|\mathbf{L}_c(j\omega_k,\rho) - \mathbf{L}_{D_c}(j\omega_k)\|_F$$

$$\begin{aligned} &\text{Subject to:} \\ &|r_{q_c}(j\omega_k,\rho)[1+L_{D_{q_c}}(j\omega_k)]| - \Psi_{q_c}(\rho,\omega_k) < 0 \\ &\{|W_{1_{q_c}}(j\omega_k)[1+M_{qq_c}(j\omega_k,\rho)]| + \\ &|W_{2_{q_c}}(j\omega_k)N_{qq_c}(j\omega_k,\rho)|\}|1+L_{D_{q_c}}(j\omega_k)| \\ &-\Psi_{q_c}(\rho,\omega_k) < 0 \\ &\text{for } k=1,\dots,N; \ q=1,\dots,n_o; \ c=1,\dots,m \end{aligned} \tag{17}$$

where

$$\Psi_{q_c}(j\omega_k,\rho) = R_e\{[1+L^*_{D_{q_c}}(j\omega_k)][1+L_{qq_c}(j\omega_k,\rho)]\}$$

$$M_{qq_c}(j\omega_k,\rho) = \sum_{z=1}^{n_o} G_{qz_c}(j\omega_k)(1-e^{-j\omega_k\zeta_{qz_c}})C_{zq_c}(j\omega_k,\rho)$$

$$N_{qq_c}(j\omega_k,\rho) = \sum_{z=1}^{n_o} P_{qz_c}(j\omega_k)C_{zq_c}(j\omega_k,\rho)$$

and $\|\cdot\|_F$ is the Frobenius norm. The objective function in (17), which is an approximation of the 2-norm, is convex with respect to the controller parameters ρ. Note that the first inequality shows that the Gershgorin bands do not encircle the critical point and so the MIMO system remains stable even if it is not fully decoupled. The second inequality guarantees the robust performance for the SISO subsystems of the decoupled MIMO system.

IV. Industrial Case Studies

The following examples will demonstrate the effectiveness of the proposed method for several industrial processes proposed in literature.

A. Case 1 - SP with fixed time delays

In [11], the proposed method was applied to a unity feedback MIMO system with fixed time delays. The plant model is represented by a 2×2 interactive chemical process which is used in industrial applications, and was defined as:

$$\mathbf{P}(s) = \begin{bmatrix} G_{11}(s)e^{-6s} & G_{12}(s)e^{-10s} \\ G_{21}(s)e^{-12s} & G_{22}(s)e^{-8s} \end{bmatrix} = \begin{bmatrix} \dfrac{10e^{-6s}}{8s+1} & \dfrac{5e^{-10s}}{30s+1} \\ \dfrac{-8e^{-12s}}{40s+1} & \dfrac{2e^{-8s}}{10s+1} \end{bmatrix} \quad (18)$$

where the time scale is defined in minutes. The elements $G_{qp}(s)$ for $q=1,2$ and $p=1,2$ represent the strictly proper delay-free transfer functions in $\mathbf{G}_n(s)$. A relative-gain-array (RGA) analysis confirms that the system is not diagonally dominant.

Since the time delay parameters are fixed for this process, the nominal time-delayed plant model $\mathbf{P}_n(s)$ will be chosen to be equal to $\mathbf{P}(s)$. In this manner, the open loop transfer function will be $\mathbf{L}(s,\rho) = \mathbf{G}_n(s)\mathbf{C}(s,\rho)$. The performance and uncertainty filters chosen for this case will be identical to those in [11],

$$W_{1_q} = 0.5 \quad , \quad W_{2_q} = 0.5\left(\frac{2s+1}{s+1}\right) \qquad q=1,2 \quad (19)$$

For comparative purposes, a PI MIMO controller will be designed for this process. Thus the linearly parameterized controller will posses the following matrix form:

$$\mathbf{C}(s,\rho) = \begin{bmatrix} [\rho_{11_1} \ \ \rho_{11_2}]\phi^T(s) & [\rho_{12_1} \ \ \rho_{12_2}]\phi^T(s) \\ [\rho_{21_1} \ \ \rho_{21_2}]\phi^T(s) & [\rho_{22_1} \ \ \rho_{22_2}]\phi^T(s) \end{bmatrix} \quad (20)$$

where $\phi(s) = [1 \quad 1/s]$. Additionally, the desired diagonal open-loop transfer function $\mathbf{L}_D(s)$ will be chosen as simple integrators with time constants equal to 30 minutes (i.e., $\mathbf{L}_D(s) = (1/30s)\mathbf{I}$). The optimization problem in (17) can now be solved by repeating the stability constraints for each ω_k. The frequency grid will be chosen to be between 10^{-2} and 10 rad/min with $N=150$ equally spaced points. The PI MIMO controller obtained from optimization is:

$$\mathbf{C}(s) = \begin{bmatrix} \dfrac{0.03289s+0.001272}{s} & \dfrac{-0.03511s-0.00311}{s} \\ \dfrac{0.05056s+0.004511}{s} & \dfrac{0.2128s+0.006133}{s} \end{bmatrix}$$

Fig. 2 displays the closed loop response of the system with the controller obtained in [11] and with the controller obtained with the SP. It can be seen that the controller for the SP produces no overshoot and asymptotically decouples the system much faster. Note that if the time constant of the desired open-loop transfer function matrix is decreased to 5 minutes, the rise and settling time of the system response is significantly improved.

B. Case 2 - SP with uncertain time delays

The proposed optimization problem will now be applied to an uncertain time-delayed MIMO SP. Consider the 2×2 plant process $\mathbf{P}(s)$ (i.e., $c=1$) that was analyzed in Case (1). The time delays for this plant will now possess uncertain values that will belong to a set. This plant will now be represented as follows:

$$\mathbf{P}(s) = \begin{bmatrix} \dfrac{10e^{-\tau_{11}s}}{8s+1} & \dfrac{5e^{-\tau_{12}s}}{30s+1} \\ \dfrac{-8e^{-\tau_{21}s}}{40s+1} & \dfrac{2e^{-\tau_{22}s}}{10s+1} \end{bmatrix} \quad (21)$$

where the time delays τ_{qp} possess values in the sets:

$$\tau_{11} = \{3,9\} \quad \tau_{12} = \{7,13\} \quad \tau_{21} = \{9,15\} \quad \tau_{22} = \{5,11\} \quad (22)$$

The nominal model is the same as defined in (18). Again, the elements $G_{qp}(s)$ for $q=1,2$ and $p=1,2$ represent the strictly proper delay-free transfer functions in $\mathbf{G}_n(s)$. The performance and uncertainty filters chosen for this example will be identical to those in section (IV-A). The desired diagonal open-loop transfer function $\mathbf{L}_D(s)$ will be chosen as simple integrators with time constants equal to 7 minutes (i.e., $\mathbf{L}_D(s) = (1/7s)\mathbf{I}$).

For simplicity, a PI controller will be designed for this process. Note that in designing this controller, all possible combinations of the uncertainties in (22) must be considered. Therefore, since $\beta_{qp} = 2 \ \forall \ \{p,q\}$, there will be a total of $m = 2^4$ possible cases to consider. The optimization problem in (17) can now be solved by repeating the stability constraints for each combination of the uncertainties in (22). The frequency grid will be chosen to be between 10^{-2} and 10 rad/min with $N = 150$ equally spaced points. The PI MIMO controller obtained from the optimization problem is:

$$\mathbf{C}(s) = \begin{bmatrix} \dfrac{0.06234s+0.001464}{s} & \dfrac{-0.04803s-0.005408}{s} \\ \dfrac{0.1585s+0.0168}{s} & \dfrac{0.3113s+0.005995}{s} \end{bmatrix}$$

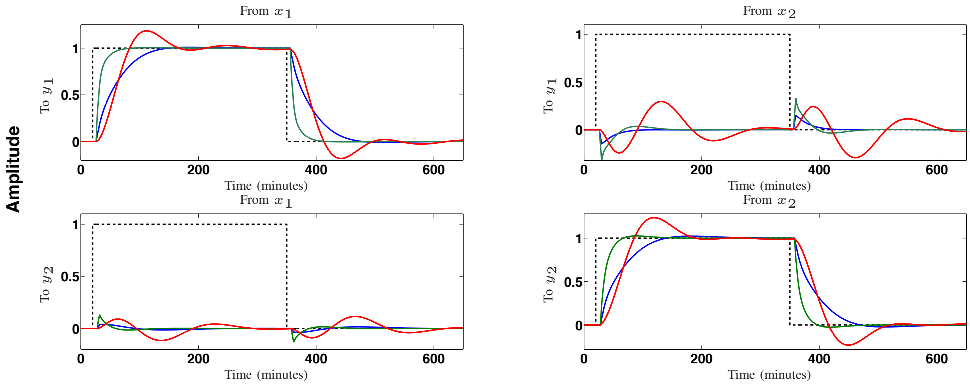

Fig. 2. Closed loop comparison between time delayed MIMO system with unity feedback and time delayed MIMO SP: unit step reference signal (black, dash), response with system proposed in [11] (red, solid), response with SP and with $diag\,(\mathbf{L}_D(s)) = \frac{1}{30s}$ (blue, solid), response with SP and with $diag\,(\mathbf{L}_D(s)) = \frac{1}{5s}$ (green, solid).

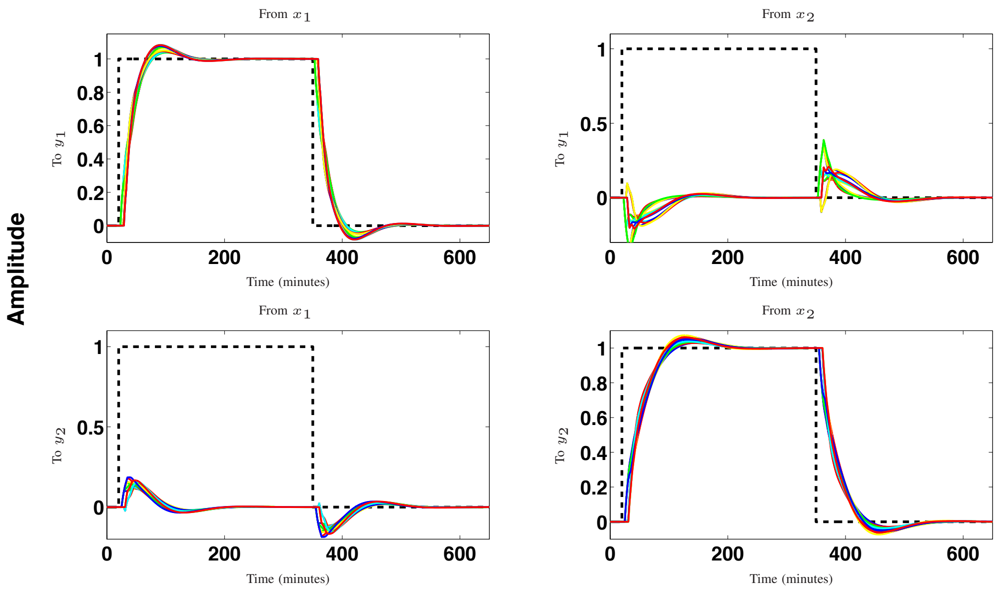

Fig. 3. MIMO response to a unit step input: reference signal (black,dash), the remaining $\Omega = 16$ closed-loop responses are for all possible combinations of the time delay parameters in (22).

Fig. 3 displays the closed-loop MIMO response to a step input. Notice that with this controller, the MIMO process achieves robust performance while simultaneously decoupling the system. The Gershgorin bands are depicted in Fig. 4 for the system possessing the largest delay time uncertainty ($\tau_{11} = 9$, $\tau_{12} = 13$, $\tau_{21} = 15$, $\tau_{22} = 11$). The red and blue bands possess a radius of $|r_q(j\omega_k)|$ for $q = 1, 2$ and $k = 1, \ldots, N$. Notice how the Gershgorin bands never intersect with the performance filter centered at $(-1 + j0)$. This proves that the MIMO system is stable, robust, and satisfies the optimization criterion in (17).

C. Case 3 - The Shell control problem

The multivariable heavy oil fractionator (known as the Shell process) is a highly coupled system which is predominantly used in petrochemical processes. Efficient control methods are essential for attaining viable production rates, minimizing energy consumption, and reducing the overall operating costs. These types of systems are difficult to control for two reasons: the system interactions are strong, and the large time delays that are inherent to the system dynamics.

Consider the 2×3 industrial Shell problem in [13],

$$\mathbf{P}_n(s) = \begin{bmatrix} G_{11}(s)e^{-81s} & G_{12}(s)e^{-84s} & G_{13}(s)e^{-81s} \\ G_{21}(s)e^{-54s} & G_{22}(s)e^{-42s} & G_{32}(s)e^{-45s} \end{bmatrix}$$
$$= \begin{bmatrix} \dfrac{4.05e^{-81s}}{50s+1} & \dfrac{1.77e^{-84s}}{60s+1} & \dfrac{5.88e^{-81s}}{50s+1} \\ \dfrac{5.39e^{-54s}}{50s+1} & \dfrac{5.72e^{-42s}}{60s+1} & \dfrac{6.9e^{-45s}}{40s+1} \end{bmatrix} \tag{23}$$

where the time scale is defined in minutes. Note that (23) is represented as the nominal model of the process. It should be noted that the controller outputs $[u_1(t)\ u_2(t)\ u_3(t)]^T$ should

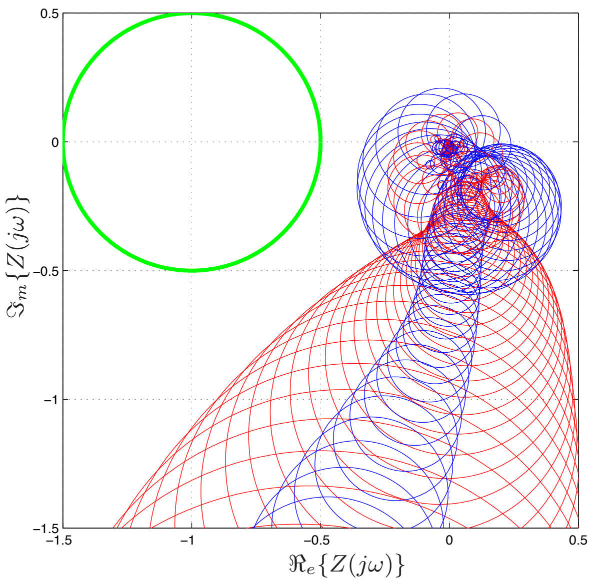

Fig. 4. Gershgorin bands centered at L_{qq} with the largest time delay combination in (22): performance filter with $|W_{1_q}| = 0.5$ (green circle), Gershgorin bands corresponding to $q = 1$ (blue circles), Gershgorin bands corresponding to $q = 2$ (red circles). Note that $Z(j\omega)$ is simply the complex number representation of each circle in the plot.

be within the saturation bounds of the physical system : $[-0.5\,,\,0.5]$ (see [14]). The elements $G_{qp}(s)$ for $q = 1, 2$ and $p = 1, 2, 3$ represent the strictly proper delay-free transfer functions in $\mathbf{G}_n(s)$. Now consider the case when the time delays are varied to $+20\%$ of their nominal values shown in (23). As with the previous example, the plant $\mathbf{P}(s)$ can be represented as a system with uncertain time delays. Since $\beta_{qp} = 2\ \forall\ \{p, q\}$, there will be a total of $m = 2^6 = 64$ possible cases to consider.

For comparative purposes, a PI controller will be designed for this process. Thus the controller $\mathbf{C}(s, \rho)$ will be a 3×2 transfer function matrix with $n = 12$ optimization parameters ρ. The frequency grid will be chosen to be between 10^{-4} and 10 rad/min with $N = 200$ equally spaced points (since the frequencies of interest of the open-loop system lie within this range). The desired diagonal open-loop transfer function matrix will be chosen as simple integrators with bandwidths that are approximately 20% greater than the open-loop system bandwidths (i.e., $\mathbf{L}_D(s) = ({}^{1}\!/_{35s})\mathbf{I}$). By solving the optimization problem in (17) for each combination of the uncertainties (i.e., $\{\tau_{11}, \ldots, \tau_{qp}\}\ \forall\ \{p, q\}$ where $\tau_{qp} \in \{\zeta_{qp}, 1.2\zeta_{qp}\}$), one obtains the following PI controller

$$\mathbf{C}(s) = \begin{bmatrix} \dfrac{0.2053s + 0.004997}{s} & \dfrac{-0.01315s - 0.00146}{s} \\ \dfrac{-0.6735s - 0.01008}{s} & \dfrac{0.4977s + 0.008098}{s} \\ \dfrac{0.2839s + 0.004451}{s} & \dfrac{-0.1041s - 0.001432}{s} \end{bmatrix}$$

Fig. 5 displays the closed-loop step response of the SP for the nominal delay case, while Fig. 6 displays the response with the worst case delay (the case where $\tau_{qp} = 1.2\zeta_{qp}\ \forall\ \{p, q\}$).

From Fig. 5 and Fig. 6, it can be observed that the purposed method in this paper produces improved SISO subsystem performance with minimal overshoot. In addition, the decoupling transients are significantly reduced for both the nominal and worst case output responses. Fig. 7 displays the controller outputs of the system.

V. Conclusion

This paper has proposed a new method for computing multivariable SP controllers with H_∞ performance. The method is based on a convex approximation of the H_∞ robust performance criterion in the Nyquist diagram. This approximation relies on the choice of a desired open-loop transfer function $\mathbf{L}_D$ for the dead-time free model of the plant. With a linearly parameterized controller, one possesses the flexibility to design PI, PID, or higher order controllers for a system. For the industrial processes considered in this paper, the proposed method has been proven to be robust; H_∞ performance was achieved for MIMO systems with both multiplicative and time delay uncertainties. The solution to the optimization problem generates a controller such that a system becomes decoupled and simultaneously optimizes the single-loop performances of the SISO subsystems.

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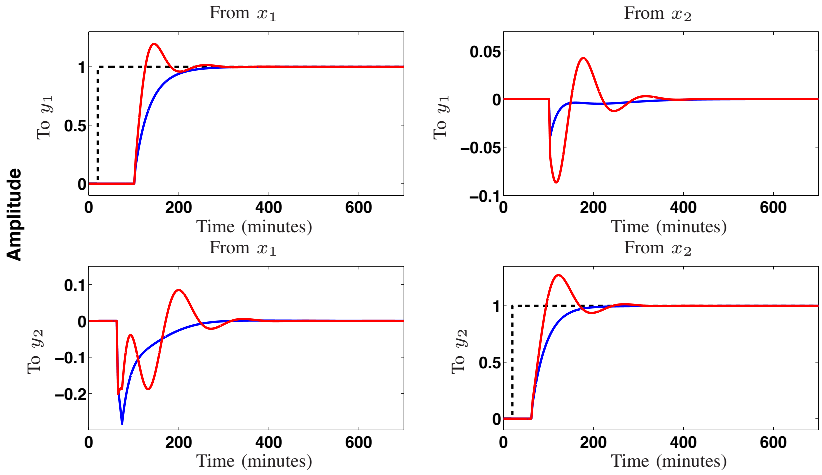

Fig. 5. MIMO SP closed-loop response to a unit step input with $\tau_{qp} = \zeta_{qp}\ \forall\ \{p, q\}$: reference signal (black,dash), output response with the proposed optimization method (blue, solid), output response with the proposed method in [13] (red, solid), which is based on the "squared down" method.

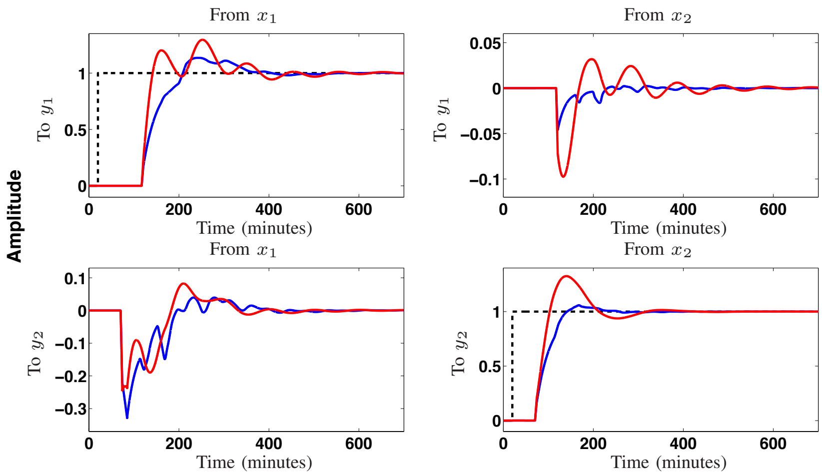

Fig. 6. MIMO SP closed-loop response to a unit step input with $\tau_{qp} = 1.2\zeta_{qp}\ \forall\ \{p, q\}$: reference signal (black,dash), output response with the proposed optimization method (blue, solid), output response with the proposed method in [13] (red, solid), which is based on the "squared down" method.

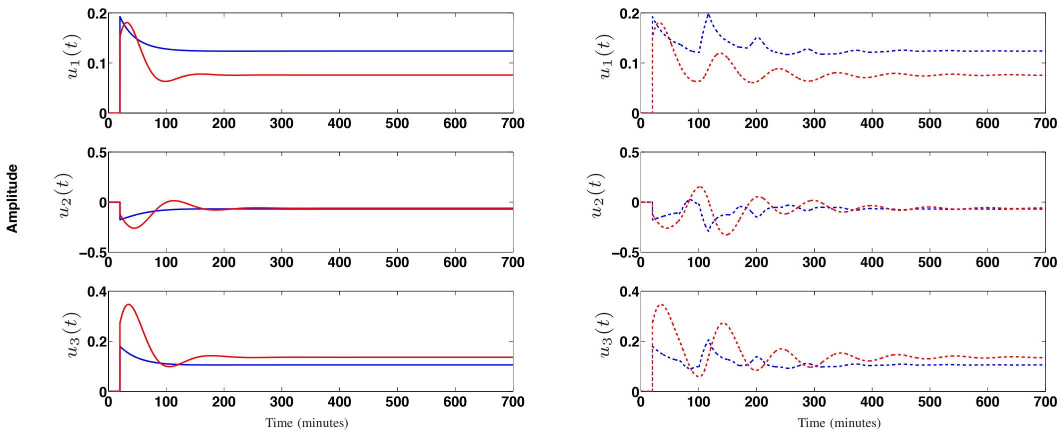

Fig. 7. MIMO SP controller output response to a unit step reference: Controller output response of proposed method with $\tau_{qp} = \zeta_{qp}$ (blue, solid), controller output response of "squared down" method in [13] with $\tau_{qp} = \zeta_{qp}$ (red, solid), controller output response of proposed method with $\tau_{qp} = 1.2\zeta_{qp}$ (blue, dash), controller output response of "squared down" method in [13] with $\tau_{qp} = 1.2\zeta_{qp}$ (red, dash)