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Effect of Power Randomization on Saturation Throughput of IEEE 802.11 WLAN

Saeid Sahraei and Farid Ashtiani, *IEEE Member*
Advanced Communications Research Institute (ACRI),
Department of Electrical Engineering, Sharif University of Technology
saeid_s_1366@yahoo.com, ashtianimt@sharif.edu

Abstract- **In this paper, we evaluate the saturation throughput for an IEEE 802.11 based wireless network considering capture effect at the receiver, while nodes transmit with random powers. In this respect, we consider a scenario consisting of a specific number of wireless nodes. Then, we derive the transmission as well as collision probabilities with respect to the perfect capture effect. In order to maximize the saturation throughput we set up an optimization problem and obtain how to compute optimum values for the probabilities corresponding to different power levels. By providing the numerical results, we deduce that power randomization may lead to a significant improvement in saturation throughput.**

Keywords-Power randomization, saturation throughput, capture.

I. INTRODUCTION

Wireless Local Area Networks (WLANs) play a key role in providing data services and more specifically internet connections. Undoubtedly, one of the very important attributes of WLANs reverts to its nice MAC protocol, i.e., IEEE 802.11 Distributed Coordinated Function (DCF) [1]-[2]. The importance of this protocol motivates the researchers focusing on other types of wireless networks, e.g., wireless mesh networks (WMNs), vehicular ad hoc networks (VANETs), wireless sensor networks (WSNs), etc. to modify it and standardize its newer versions for different situations, e.g., IEEE 802.11s, IEEE 802.11p, IEEE 802.11e, etc. Therefore, any endeavor in order to enhance the efficiency of IEEE 802.11 MAC protocol may lead to more efficient wireless networks in the future.

Up to now, many research works have been done to analyze 802.11 DCF and evaluate its performance metrics, e.g., throughput, delay, fairness, etc. [2]-[7]. And a few of the papers have been dedicated to improve the efficiency of the protocol in general [8] or in some special environments [9]. One of the important parameters that is not usually included in the previous analyses is the transmission power. However, some authors have considered it in a few research works [10]-[13].

In fact, in a wireless network, a random access MAC scheme may lead to more efficient usage of spectrum and network resources than the scheduled access ones. This is due to the topology dynamics, random nature of the arrival traffic, intermittent nature of the services, etc. The heart of IEEE 802.11 DCF is also CSMA/CA that is a well known random access MAC scheme. One of the natural side effects of random access is collision. Actually, in this type of channel access the wireless nodes contend to each other in order to capture the channel. And sometimes collision occurs. Obviously, collision leads to resources wastage that should be avoided as much as possible. Most of the analyses carried out on IEEE 802.11 MAC protocol is based on Protocol model [14] for modeling a successful packet transmission. That is, a transmitted packet can be successfully received by the receiver provided that there is not any other simultaneous transmitter in the vicinity of the receiver. In this model, we neglect some of the potential abilities of the receiver due to signal capturing in the physical layer. In another model, i.e., Physical model [14], a transmitted packet is received successfully provided that SINR at the receiver exceeds a threshold. Therefore, it appears that this model can be more practical and leads to higher throughput, because irrespective of possible simultaneous transmissions some packet receptions may be successful. Such a model is more useful when some factors naturally lead to different power levels at the receiver. Among these factors, channel propagation conditions and different transmission powers can be mentioned. In the first case, the transmission powers from different simultaneous transmitters are received differently because the propagation channels between the receiver and the transmitters are usually distinct and independent. And in the second case, the transmission powers can be set differently such that by estimating channel attenuation and some type of power control the desired signal powers reach the receiver.

Since one of the results of considering Physical model is that the collision status changes, its analysis is very useful. It is worth mentioning that some of previous works focused on this fact. The authors in [10] considered the effect of capture on capacity for the case of two power levels. The authors in [11] carried out a complete analysis of the unsaturated throughput of IEEE 802.11 MAC in the presence of non-ideal propagation environments and showed how throughput changes for different capture models. The authors in [13] have focused on service differentiation capability using different transmission power levels in an ideal propagation channel. Clearly, in the case of non-ideal propagation environments power control is necessary to satisfy the conditions. The authors in [12] have considered throughput and delay analysis of an ad hoc network based on IEEE 802.11, regarding the Physical model. They have considered spatial Poisson process for the wireless nodes distribution, and a simple propagation model, i.e., power-law attenuation without any fading and shadowing, in order to consider the effect of capture. On the other hand, some of the nice research works ([15]) have been done for simpler MAC scheme, i.e., slotted Aloha, that indicates power randomization is an effective factor in throughput enhancement.

In this paper, we focus on transmission power randomization as an effective factor in throughput increasing for IEEE 802.11 DCF. In fact, in a wireless local area network that all nodes are within the transmission range of each other, due to capture effect some of the collisions can be removed. However, it depends strongly on the transmission power levels as well as its distribution. In this respect, in our analysis we modify the well known analytical relations of [2] in order to include the effect of different transmission power levels. Then, we set up an optimization problem in order to determine the optimum probability distributions for a given set of power levels, to maximize saturation throughput. We focus on ideal propagation environment as well as perfect capture model. We will show how much we can increase the throughput by power randomization by several numerical examples.

Following this introduction, we briefly review IEEE 802.11 DCF and the analytical model in [2] in Section II. Then, we modify the analytical equations in order to include the effect of different power levels and capture effect, in Section III. In this section, we also prepare the optimization problem and provide its solution. In Section IV, we present the numerical results and conclude the paper in Section V.

II. Brief Review on IEEE 802.11 MAC Analytical Model

Up to now several modeling approaches have been proposed for IEEE 802.11 DCF MAC protocol. Although some of them are a bit different [4]-[5], most of them have been founded on the seminal work of Bianchi [2]. Therefore, in this section, we review the protocol as well as his modeling approach.

In a wireless network that all nodes are within the transmission range of each other and all nodes are in saturation status, i.e., each node has always a packet for transmission, a typical node monitors the channel before beginning the transmission process of a typical packet. Then, if it finds the channel idle for the duration of a Distributed Inter-Frame Space (DIFS), it sets a backoff window. The window is selected among the interval [0, 2^iW_0-1], randomly, where W_0 denotes the minimum contention window size and i is the backoff stage (between 0 and m). Then, a counter set on the selected backoff window, down-counts after each time slot. However, at each time slot the channel is monitored and if it is occupied by another transmission the counter freezes until the channel is sensed idle again for a DIFS. Therefore, the time slot may be extended. So, it is called as a *virtual time slot* that may have a length equal to a successful packet transmission, an unsuccessful packet transmission (in the case of collision), or an idle time slot. When the counter reaches zero, the packet is transmitted. If there is not any collision, the contention window size of the transmitter node is set on W_0 for the next packet transmission. Otherwise, the contention window size is doubled provided that $i \leq m$. If $i=m$ then the contention window size remains the same for all next retransmissions.

Bianchi in [2] has modeled the above transmission scenario by a two-dimensional discrete time Markov chain. In this model, the time between each two consecutive transitions of the counter equals a virtual time slot. By applying a simplified assumption that at each time slot, each node behaves independently, we will have the following equation for transmission probability at a typical time slot, obtained by solving the related global balance equations (GBE's) [2]:

$$\tau = \frac{2(1-2p)}{(1-2p)(w+1)+pw(1-(2p)^m)}, \qquad (1)$$

where p, m denote the collision probability and the number of backoff stages. On the other hand, according to Protocol model the collision probability for a typical transmitted packet equals as in the following:

$$p = 1-(1-\tau)^{n-1}, \qquad (2)$$

where n is the number of wireless nodes.

The normalized network throughput can be computed as in the following [2]:

$$S = \frac{n\tau(1-p)E(L)}{(1-\tau)^n\sigma + n\tau(1-p)T_s + (1-n\tau(1-p)-(1-\tau)^n)T_c}, \qquad (3)$$

where σ, T_s, and T_c are the duration of a typical idle time slot, the duration of a successful packet transmission, and the duration of an unsuccessful packet transmission (i.e., collision). In addition, $E(L)$ denotes the average packet size.

It is worth mentioning that there are two transmission modes; basic access mode (two-way handshaking) and RTS/CTS mode (four-way handshaking). According to [2], the above equations are true for both modes.

III. Power Randomization and Transmission Scenario

In this section, we describe the transmission scenario and the effect of power randomization. In this respect, we consider a single hop ad hoc network, i.e., all nodes are in the transmission range of each other. When a node wants to transmit a packet, it follows exactly IEEE 802.11 DCF protocol, described in the previous section. However, it selects a transmission power among several transmission power levels according to a random distribution and transmits the packet. The main goal of applying different transmission powers that lead naturally to different reception powers is providing the possibility of exploiting some of the collisions. In fact, when collided packets have different powers at the receiver, because of capture effect the most powerful received power may be dominant and be received correctly. This strongly depends on the receiver structure, the number of collided packets and difference among reception power levels.

In the case of perfect capture model that is considered in this paper, we have assumed there is the same propagation condition between transmitters and a typical receiver. Therefore, all transmission powers are received at a typical receiver with the same attenuation. This assumption is correct when all nodes are near to each other and the distances are approximately the same. Another scenario justifying this assumption is the case that the access point is the only receiver and there is a perfect power control for the transmitters such that the power control loop compensates the random attenuation and fading of the channel between each transmitter and the access point.

In the case of perfect capture model, if one of the received powers is higher than the other simultaneous transmissions during a collision, the strongest packet captures the receiver. Then such a collision is not considered as a destructive collision for the most powerful transmitter.

Now, we consider that a typical packet is transmitted. Since all nodes are similar, then the transmission probability at a typical slot, τ, is the same for all nodes. On the other hand, the collision probability for a typical packet transmission, p, can be computed as in the following:

$$p=\sum_{j=1}^{l}P_j\sum_{\bar{k}}\delta(\bar{k},j)\binom{n-1}{k_1,k_2,\cdots,k_l,k_0}\prod_{i=1}^{l}(P_i\tau)^{k_i}(1-\tau)^{k_0}\ ;$$

$$\delta(\bar{k},j)=\begin{cases}1;\exists i\ge j:k_i>0\\0;otherwise\end{cases},\bar{k}=(k_1,k_2,...,k_l,k_0)\quad ,\qquad(4)$$

where l is the number of transmission power levels. We have also assumed that k_i is the number of transmitters selecting i-th power level and k_0 denotes the non-transmitting nodes at that time slot. Moreover, P_i denotes the probability that a node selects i-th power level for transmitting a packet. In (4) we have considered perfect capture model such that if the transmitter selects j-th power level it encounters a collision if at least another node with a power level greater than or equal to the j-th power level transmits simultaneously. By some manipulations the collision probability for a typical transmitter, (4), can be simplified to the following equation:

$$p=\sum_{j=1}^{l}P_j\left(\left(\sum_{i=1}^{l}(P_i\tau)+(1-\tau)\right)^{n-1}-\left(\sum_{i=1}^{j-1}(P_i\tau)+(1-\tau)\right)^{n-1}\right)$$
$$=1-\sum_{j=1}^{l}P_j\left(1-\sum_{i=j}^{l}(P_i\tau)\right)^{n-1}\qquad(5)$$

Obviously, the relation of τ versus p remains the same as in (1) for the case of equal transmission powers, i.e., when l=1. The normalized network throughput is also the same as (3).

At this time, we try to optimize the probabilities P_i's in order to maximize S. Rearranging (3) we can write:

$$S=\frac{E(L)}{T_s-T_c+\sigma\dfrac{T_c^*-(T_c^*-1)(1-\tau)^n}{n\tau(1-p)}},\qquad(6)$$

where $T_c^*=\dfrac{T_c}{\sigma}$. So, maximizing S is equivalent to maximizing the following equation:

$$S^*=\frac{n\tau(1-p)}{T_c^*-(T_c^*-1)(1-\tau)^n}.\qquad(7)$$

Now we show that S^* (and consequently S) is a decreasing function of p, when p is between 0 and 1. To this end, if we write (1) as in the following:

$$\tau=\frac{2}{w+1+pw(1+2p+(2p)^2+\cdots+(2p)^{m-1})},\qquad(8)$$

it is obvious that τ is a strictly decreasing function of p. Moreover, it can be shown that dS^*/dp is also negative. To this end, deriving S^* related to p, results in:

$$\frac{dS^*}{dp}=$$
$$\frac{d}{d\tau}\left(\frac{n\tau}{T_c^*-(T_c^*-1)(1-\tau)^n}\right)\frac{d\tau}{dp}(1-p)-\frac{n\tau}{T_c^*-(T_c^*-1)(1-\tau)^n}.\qquad(9)$$

So it's enough to show that $\dfrac{d}{d\tau}\left(\dfrac{n\tau}{T_c^*-(T_c^*-1)(1-\tau)^n}\right)$ is positive for τ between 0 and 1. It is equivalent to show that $(1-\dfrac{1}{T_c^*})(1-\tau)^{n-1}(1-\tau+n\tau)$ is smaller than 1. Since this expression is smaller than 1 for τ=0, and has a negative derivation related to τ for τ between 0 and 1, our goal is acquired.

Then, in order to maximize S, we need to minimize p. On the other hand, we have the following equation:

$$P_l=1-\sum_{i=1}^{l-1}P_i\ ,\qquad(10)$$

Now, we define the new functions as in the following:

$$h_1(P_1,P_2,\cdots,P_{l-1},\tau)=1-\sum_{j=1}^{l}P_j\left(1-\sum_{i=j}^{l}(P_i\tau)\right)^{n-1},\qquad(11)$$

$$g_2(p)=\frac{2(1-2p)}{(1-2p)(w+1)+pw(1-(2p)^m)},\qquad(12)$$

$$h_2(\tau)=g_2^{-1}(\tau)\ ,\qquad(13)$$

$$h_3(P_1,P_2,\cdots,P_{l-1},\tau)=h_1(P_1,P_2,\cdots,P_{l-1},\tau)-h_2(\tau),\qquad(14)$$

By defining $H(.)$ in (15) and using Lagrange multipliers we maximize h_1 subject to the condition h_3=0, as in the following:

$$H(P_1,P_2,\cdots,P_{l-1},\tau)=$$
$$h_1(P_1,P_2,\cdots,P_{l-1},\tau)-\lambda h_3(P_1,P_2,\cdots,P_{l-1},\tau),\qquad(15)$$
$$\frac{\partial H}{\partial P_k}=0\ \Rightarrow\ \frac{\partial h_1}{\partial P_k}-\lambda\left(\frac{\partial h_1}{\partial P_k}-\frac{\partial h_2}{\partial P_k}\right)=0\Rightarrow$$

$$(1-\lambda)\frac{\partial h_1}{\partial P_k}=0 \;\Rightarrow\; \frac{\partial h_1}{\partial P_k}=0;\, k=1,\cdots,l-1 \quad , \qquad (16)$$

$$\frac{\partial H}{\partial \tau}=0 \;\Rightarrow\; \frac{\partial h_1}{\partial \tau}-\lambda\left(\frac{\partial h_1}{\partial \tau}-\frac{\partial h_2}{\partial \tau}\right)=0 \;. \qquad (17)$$

The last equation does not affect the values of optimum P_i's or τ and may only be used to find λ. From (10), (11), (16) we will have:

$$\left(1-\sum_{i=k}^{l}(\tau P_i)\right)^{n-1}-(1-\tau P_l)^{n-1}+(n-1)\tau\sum_{j=k+1}^{l}P_j\left(1-\sum_{i=j}^{l}(P_i\tau)\right)^{n-2}=0. \qquad (18)$$

If we define $q_i=P_i\tau;\, i=1,2,\cdots,l$, then we have:

$$\left(1-\sum_{i=k}^{l}q_i\right)^{n-1}-(1-q_l)^{n-1}+(n-1)\sum_{j=k+1}^{l}q_j\left(1-\sum_{i=j}^{l}q_i\right)^{n-2}=0. \qquad (19)$$

Therefore,

$$q_k=1-\sum_{i=k+1}^{l}q_i-\left[(1-q_l)^{n-1}-(n-1)\sum_{j=k+1}^{l}q_j\left(1-\sum_{i=j}^{l}q_i\right)^{n-2}\right]^{\frac{1}{n-1}}, \qquad (20)$$

so, if we vary k from l-1 to 1, we obtain every q_i as a function of q_l . In addition we have the following equations:

$$\tau=\sum_{i=1}^{l}q_i\;,\quad P_i=\frac{q_i}{\tau}\;. \qquad (21)$$

Therefore, regarding (5) p can be written as a function of q_l. Now we can put the equivalents of τ and p as functions of q_l in the equation $\tau=g_2(p)$ and solve the one-variable equation obtained, using numerical methods. After obtaining q_l, we obtain the probability distribution corresponding to each transmission power level. In the next section, we will observe the results for our optimization problem for different parameters.

IV. Numerical Results

We have considered a network comprised of n wireless nodes. The typical values for the parameters in our analyses have been presented in Table I.

In the first set of analyses, we have considered 10 transmission power levels and changed the number of backoff stages. Then, we have plotted the maximized throughput. As we observe from Fig. 1, we can select the number of backoff stages such that beyond its value, the maximized throughput does not improve significantly. In Fig. 2, we have plotted optimum probabilities for different power levels for three numbers of nodes. As we observe, lower power levels should be selected with higher probabilities. Moreover, when the number of nodes decreases the optimum probability distribution approaches to a uniform distribution. Figs. 3-4, illustrate the maximized throughput for different number of power levels. As we observe, increasing the number of power levels can increase the maximized throughput. If we compare the results we observe that power randomization with 20 power levels is able to enhance the saturation throughput about 40%, 22% for n=50, and about 17%, 6% for n=10, corresponding to W_0=32, W_0=128, respectively. In other words, it is better to apply power randomization when the number of nodes is large because in this case collision probability is high.

TABLE I
TYPICAL VALUES IN OUR ANALYSES

Parameter	Value
σ	50 µs
Channel Bit Rate	1Mbit/s
E(L)	8184 bits
T_s	8982 µs
T_c	8713 µs

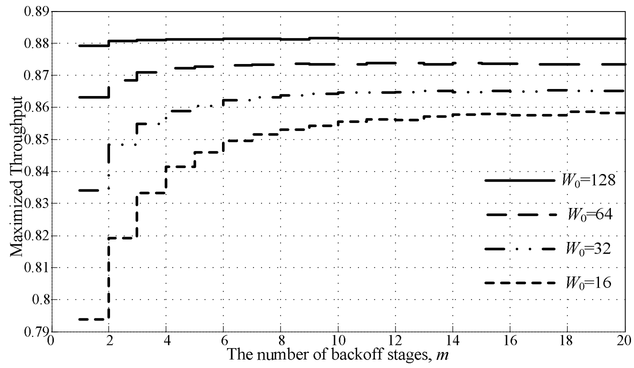

Fig. 1. The maximized throughput for different number of backoff stages and minimum contention window size, n=50, l=10

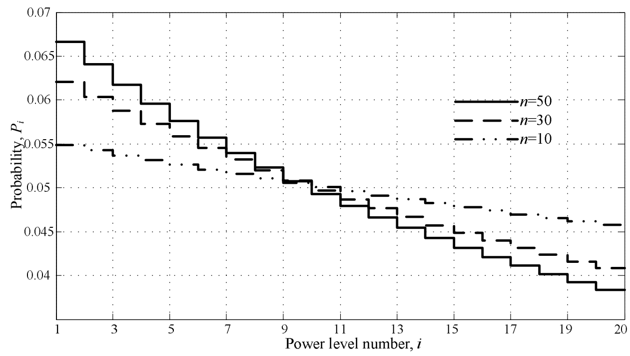

Fig. 2. The optimum probability distribution corresponding to power levels, W_0=32, l=20, m=5

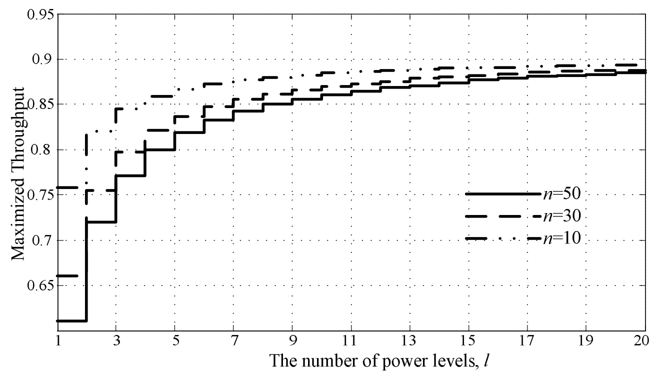

Fig. 3. The maximized throughput for different number of power levels, W_0=32, m=5

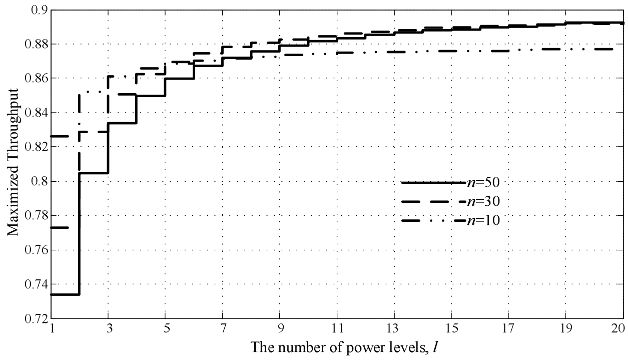

Fig. 4. The maximized throughput for different number of power levels, W_0=128, m=5

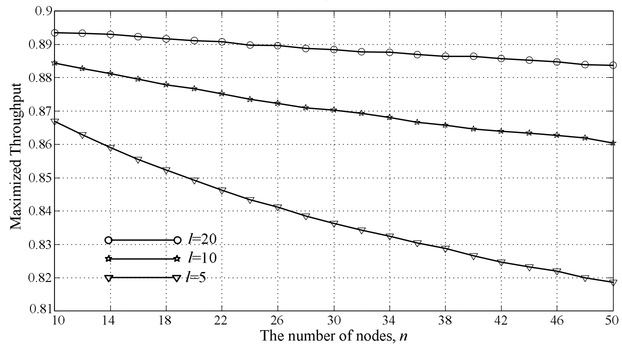

Fig. 5. The maximized throughput versus number of nodes, W_0=32, m=5

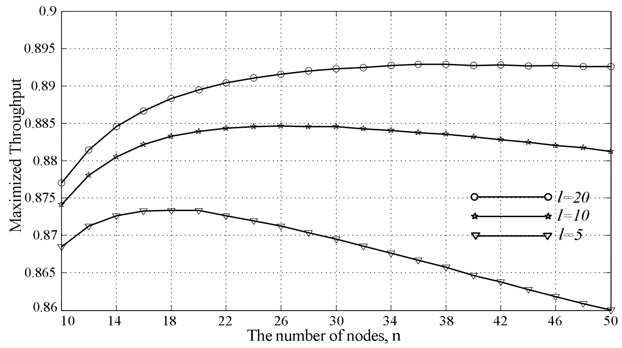

Fig. 6. The maximized throughput versus number of nodes, W_0=128, m=5.

Figs. 5-6 show the maximized throughput versus the number of nodes for different number of power levels. When W_0=32, increasing the number of nodes decreases the optimum throughput. However, for W_0=128 and regarding the number of power levels, increasing the number of nodes increases the maximized throughput up to an optimum point, afterwards the maximized throughput decreases. When the number of power levels increases the optimum value occurs for larger number of nodes. In fact, W_0=128 is not suitable for low number of nodes because in this case nodes are obliged to wait longer and many slots are wasted. By increasing the number of nodes, time slots are exploited more efficiently, leading to higher throughput. However, collision increases also, leading to lower throughput. Applying power randomization delays the optimum point due to tradeoff between these two contradicting factors, hence, the optimum throughput occurs for larger number of nodes. In addition, larger number of power levels smoothes the degrading effect of the second factor, i.e., collision, after the optimum point.

V. CONCLUSIONS

One of the important MAC protocols in different types of wireless networks is IEEE 802.11 DCF. In this paper, we evaluated the effect of power randomization in order to exploit some of the collided slots. To this end, we considered perfect capture model such that the strongest received packet is able to capture the receiver. Then, we modified the related equations for the analysis of IEEE 802.11 DCF without any capture effect. And we set up an optimization problem in order to compute the optimum distribution for transmission power selection. Finally, we showed the throughput enhancement in different set of parameters. According to our results, we observed that the throughput can be enhanced significantly depending upon the other parameters.

Considering the effect of power randomization in the case of non-perfect capture model as well as non-ideal propagation environments (e.g., Rayleigh fading) is one of our future works.

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