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T0001 – MilliNewton force sensor

Frequency bandwidth

This document gives an approximate calculation of the frequency bandwidth of the MilliNewton – and serves as a starting point for calculations on other cantilever-type sensors. The mechanical resonance frequencies of MilliNewton are high, due to the very small cantilever size. The response is in fact limited by the electrical circuit at ca. 1 kHz (MilliNewton-A) or ca. 200 Hz (MilliNewton-B).

Thomas Maeder, 14.7.2014[1]

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[1] Replaces version 2004-04-19.

1. Mechanical resonance

1.1. Simple cantilever beam – analytical solution

According to Whitney[1] et Strässler[2], the resonance frequencies of a constant cross section cantilever are given by:

(1) $$f_i = \frac{1}{2\pi} \cdot \sqrt{\frac{E^* \cdot I}{\rho \cdot S}} \cdot \left(\frac{z_i}{L}\right)^2$$

f_i	frequency of the i^{th} mode
E^*	effective elastic modulus
I	flexure inertial moment of the cantilever
ρ	cantilever specific mass
S	cross section area
z_i	constant for the i^{th} mode
L	cantilever free length

The approximate values of z_i are given in table 1 below.

i	z_i	***i***	z_i
1	1.8751	4	10.9955
2	4.6941	5	14.1372
3	7.8546	6	17.2786

Table 1. Constants z_i of the cantilever resonance modes.

For a beam of rectangular cross section b x h, with $b >> h$, we have:

(2) $$E^* = \frac{E}{1-\nu^2}$$

(3) $$I = \frac{b \cdot h^3}{12}$$

(4) $$S = b \cdot h$$

E	Young's modulus
ν	Poisson's coefficient
b	cantilever beam width
h	cantilever beam thickness

Combining (1)–(4), we obtain finally:

(5) $$f_i = \frac{1}{4 \cdot \sqrt{3} \cdot \pi} \cdot \sqrt{\frac{E}{(1-\nu^2) \cdot \rho}} \cdot h \cdot \left(\frac{z_i}{L}\right)^2$$

In general, the first mode is taken in consideration only, because it limits the working frequency of the sensor in practice. In this case:

(6) $$f_1 \cong 0.16154 \cdot \sqrt{\frac{E}{(1-\nu^2) \cdot \rho}} \cdot \frac{h}{L^2}$$

1.2. Simple cantilever – mass-spring solution

One seeks to express the above solution in terms of a mass-spring system, in order to be able to easily introduce an extra mass (the force centring ball). For a mass m supporter by a mass-less spring of stiffness k, the resonance frequency is given by:

(7) $$f = \frac{1}{2\pi} \cdot \sqrt{\frac{k}{m^*}}$$

f	resonance frequency
k	spring elastic constant
$m*$	"effective" mass (see below)

For a cantilever, where the force is applied at a point $x \le L$:

(8) $$m_c = \rho \cdot S \cdot L$$

(9) $$k = 3\frac{E^* \cdot I}{x^3}$$

m_c	cantilever mass
x	force application point

For a cantilever beam of rectangular cross section b x h with $b >> h$ – see (2)-(4):

(10) $$m_c = \rho \cdot b \cdot h \cdot L$$

(11) $$k = \frac{E \cdot b \cdot h^3}{4x^3}$$

(12) $$m^* = u \cdot m_c$$

u	coefficient real mass – equivalent mass

If one writes frequency according to (6) :

(13) $$f = \frac{1}{2\pi}\sqrt{\frac{E}{\left(1-\nu^2\right)\cdot\rho}} \cdot \frac{h}{\sqrt{u \cdot x^3 \cdot L}} \cong 0.16154 \cdot \sqrt{\frac{E}{\left(1-\nu^2\right)\cdot\rho}} \cdot \frac{h}{L^2}$$

(14) $$u \cong 0.24267 \cdot \frac{L^3}{x^3}$$

1.3. Cantilever – addition of a mass on the beam

If one adds a mass m' at point x of the beam, one gets:

(15) $$f' \cong \frac{1}{2\pi} \cdot \sqrt{\frac{k}{u \cdot m_c + m'}}$$

(16) $$f' \cong 0.16154 \cdot \sqrt{\frac{E}{(1-\nu^2) \cdot \rho}} \cdot \frac{h}{L^2} \cdot \frac{1}{\sqrt{1 + \frac{m' \cdot x^3}{0.24267 \cdot \rho \cdot b \cdot h \cdot L^4}}}$$

1.4. Application to MilliNewton

The parameters for MilliNewton are given in table 2 below.

Symbol	Description	400 mN	1'000 mN	2'000 mN	unit
b	width	3.0	3.0	3.0	mm
h	thickness	0.25	0.40	0.635	mm
x	ball position	8.0	8.0	8.0	mm
L	free length	9.3	9.3	9.3	mm
ρ	density of Al_2O_3	3.9	3.9	3.9	mg/mm^3
m'	masse (ball)	4.1	4.1	4.1	mg
k	stiffness	7.6	30.9	124	N/mm
f	resonance (w/o ball)	4.30	6.87	10.91	kHz
f'	resonance (with ball)	3.64	6.16	10.16	kHz

Table 2. Calculated mechanical resonance frequencies.

In reality, the clamping of the cantilever is not perfect. Therefore, the real mechanical resonance frequencies will be slightly lower.

2. Electrical bandwidth

2.1. Measurement bridge & amplifier feedback

The electrical bandwidth of the measurement bridge is limited by:

(17) $$f_j \cong \frac{1}{2\pi \cdot k_j \cdot R_j \cdot C_j}$$

f_j	limiting frequency of the measurement bridge
k_j	capacitor factor, 1 (outputs) or ½ (ground)
R_j	measuring bridge resistance
C_j	measurement bridge filtering capacitor

For MilliNewton-A, we have: $C_j = 10$ nF, $k_j = 1$ (one capacitor across bridge output) and $R_j \cong 8$ kΩ (somewhat variable), therefore $f_j \cong 2$ kHz.
For MilliNewton-B, we roughly have the same values, except $k_j = 1/2$ (two capacitors, one between each bridge output and ground), which yields $f_j \cong 5$ kHz. However, there is an additional feedback capacitor C_f in parallel with an effective feedback resistor R_f (corresponding in fact to a feedback network), yielding a much lower frequency limit:

(18) $$f_f \cong \frac{1}{2\pi \cdot R_f \cdot C_f}$$

where $R_f \cong \frac{1}{2} R_j \cdot z$

f_f	limiting frequency of amplifier feedback
R_f	effective feedback resistor
C_f	amplifier feedback filtering capacitor
z	amplifier gain

With $C_f = 220$ pF and $z \cong 300$ (conservative, normally lower), we get a lower limiting frequency of $f_f \cong 700$ Hz, which dominates the electric response of MilliNewton-B.

2.2. Amplifier – gain bandwidth product

The amplification bandwidth is limited by:

(19) $$f_A \cong \frac{f_1}{z}$$

f_A	amplification limiting frequency
f_1	gain bandwidth product
z	amplification (gain)

For MilliNewton-A, we have: $f_1 \cong 1$ MHz[3] and $z \cong 300$, therefore $f_A \cong 3.3$ kHz.
For MilliNewton-B, we have: $f_1 \cong 0.8$ MHz[4] and $z \cong 300$, therefore $f_A \cong 2.7$ kHz.

2.3. Amplifier – slew rate

At large output swings, the working frequency may also be limited by the amplifier slew rate. In this case:

(20) $$f_S \cong \frac{U'_{max}}{\pi \cdot U_S \cdot S}$$

f_S	limiting frequency due to slew rate
U'_{max}	slew rate
U_S	supply voltage
S	full scale output span (ratiometric)

For MilliNewton-A[3], the minimal value of U'_{max} is 0.1 V/μs. For the nominal values $U_S = 5$ V and $S = 0.6$, one obtains $f_S \cong 10$ kHz. For MilliNewton-B[4], the higher slew rate of the amplifier ($U'_{max} = 1$ V/μs) yields a much higher limiting frequency of $f_S \cong 100$ kHz.

3. Conclusion

For MilliNewton-A, the working frequency is essentially limited by two electrical characteristics: the RC network formed by the measurement bridge and its filtering capacitor, and the gain bandwidth product of the used amplifier (LM 358). These two frequency are of the order of a few kHz. Therefore, on expects a notable alteration of the output signal beyond 1 kHz.

For MilliNewton-B, the limiting frequency is lower (~700 Hz), which arises almost exclusively from the amplifier feedback circuit. This is more in line with the specified < 10 ms response time, and would alter the signal beyond ca. 200 Hz.

The mechanical resonance frequencies vary with the cantilever thickness, and lie in general above the electrical limits, starting with > 3 kHz for the lowest force range.

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1 Whitney-S, "Vibrations of cantilever beams: deflection, frequency and research uses", University of Nebraska - Lincoln (UNL), 1999.

2 Strässler-S Ryser-P, "Project VIVIGLUS (mechanical)", EPFL-LPM, 2003.

3 National Semiconductor, "LM158/LM258/LM358/LM2904 Low Power Dual Operational Amplifiers", datasheet, 1999.

4 Maxim, "MAX4400-MAX4403 Single/dual/quad, low-cost, single-supply, rail-to-rail Op amps with shutdown", datasheet 19-1599, rev. 3, 2001.