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Natural Dynamics Modification for Energy Efficiency: A Data-driven Parallel Compliance Design Method

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***Abstract*— We present a data-driven method for designing parallel compliance. Designing such compliance helps the system to improve energy efficiency, mainly by reducing negative work. The core idea is to design a controller first and then find springs working in parallel with each actuator such that force-displacement graph is lined up around displacement axis. By doing so, we simply shape the natural dynamics for performing the task efficiently. Maximum torque reduction for actuators is a byproduct of this design method. The method can be used in different cyclic robotic application, especially in legged locomotion systems. In this paper, we design a spinal compliance for a bounding quadruped robot in Webots. The results show that the power consumption and the maximum torque are reduced significantly.**

I. INTRODUCTION

From control point of view, stability, energy efficiency, and high velocity are the most desired characteristics and ultimate goals in robotic systems. Robustness and adaptability are penultimate; especially when robots deal with uncertainties and work in unstructured environments. From design point of view, compliance is one of the most important and easy to use objects in our toolbox, and its potential role to attain mentioned characteristics is indisputable. Here are some instances: in a linear system, spring constant can relocate poles of the system and subsequently affect stability. Shaping the compliance to store energy and release it in a desired manner can lead to energy efficiency [1]. In SLIP model, average forward velocity can be controlled by leg compliance [2]. However, optimal compliance to reach each of these characteristics are either unknown or intractable to find.

Compliance is a passive element, thus it directly affects the passive/natural dynamics. Consistency of natural dynamics with the task can, at least, lead to energy efficiency ([3] and [4]). Therefore, changing dynamics toward the desired one is a reasonable methodology. We call this methodology "dynamics shaping". Dynamics shaping toward task-consistency, like morphological evolution in animals, leads to an intelligent design (see [5] and [6]). Simplicity in control system is one of the most important properties in intelligent design. Likewise, parallel compliance as a passive element in a robotic system does not impose any control complexity. Adaptability and robustness are byproducts of such simplicity; see [7] and [8]. Studying the natural dynamics of robotic system is a vital step in this methodology; see [9] for natural dynamics in walking.

Actuators along with external forces are energy consumption sources in a robotic system. Using compliance for each actuator in order to reduce the total energy consumption is a systematic approach. However, configuration of compliance in connection with actuator should be specified. Parallel and series compliance are two major design configurations. More complex configurations such as antagonistic mechanisms can be imagined as well (see 2.5.1 in [10]). Having compliance in series with actuator needs a mass in between. This extra mass adds complexity to the system dynamics ([11]). This problem does not exist in parallel compliance. Moreover, additive property of compliance and actuator forces make this configuration much easier to optimize. Assuming cyclic tasks, here we focus on fully passive and fixed parallel compliance to avoid complexities involved in using adaptable compliant actuators for shaping dynamics [10].

Compliance is specified by its force-displacement profile. Satisfying passivity, this profile can have any shape; linear or nonlinear. Realization limitation may impose other constraints like monotony on compliance profile ([12]). In [1], we show that nonlinearity in compliance profile gives us a flexibility in energy store-release shape of compliance, and subsequently improves energy efficiency of the system. In this work, our design methodology, based on the natural dynamics, results in a nonlinear parallel compliance.

In classical robotics, because of the tracking precision importance, stiffness is preferred over compliance. On the other hand, in legged locomotion, compliance is highly preferable. In both bipedal and quadrupedal systems, leg compliance can absorb ground impacts and help to use the energy for stable locomotion ([13]). In bipedal robots, adding parallel compliance to knee joints can result in energy efficiency ([14]). Recent researches show that compliance in spinal joint of a quadruped can lead to better behavior ([1], [15], and [16]). It is worth mentioning that in soft robotics, softness is preferred over rigidity and compliance. Soft robotics, next to rigid and compliant robotics, are considerable eras in robotics.

Task is the center point in robotics system. Body is designed based on the task, and control tries to fulfill the task. In legged locomotion, performing a gait, such as walking or running, can be considered as the task. Main property of tasks, in legged locomotion, is periodicity. Repetitive motion of joints simplify all the analysis to one cycle. We will use this property to construct our method.

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In this research, we focus on parallel compliance design that leads to improving energy efficiency. In [10], for variable compliance system, a strategy is proposed to tune compliance based on trajectory and torque as follows:

$$K(t) = \frac{dT(t)}{d\theta(t)} \tag{1}$$

This strategy cannot be applied to a non-variable compliance. It is also speculated that "...A good stiffness seemed to be the slope of the first order linear regression line of the torque-angle curve.". We will show that linear regression can be a good starting point for design; nevertheless, it does not result in optimum compliance. We will present a data-driven optimal compliance profile design for cyclic tasks based on force-displacement behavior.

We will use the proposed method for a simple mass-spring system and a 2-DOF manipulator as two basic systems. And finally, we try to design parallel compliance for a spinal joint of a quadruped in Webots environment.

II. PROPOSED METHOD

In this section, we discuss how parallel compliance affects control effort and how this can help us to design proper compliance.

A. Control Effort and Parallel Compliance

Consider a joint in a manipulator performing a periodic task. This task is defined by a set of trajectories for joints. Independent-joint control is depicted in Fig. 1. In this figure, $G(s)$ is a model for robotic joint, mapping from force to position. Controller and actuator are represented by $C(s)$. x_d is the reference trajectory, and effects of other joints are represented by $D(t)$ which is assumed to be periodic as well.

Having a perfect controller, we can ensure perfect tracking ($x = x_d$). In the first case (Fig. 1.a) applied force (F_{g1}) to plant ($G(s)$) is equal to actuator force and force applied by other joints ($F_{c1} + D(t)$). Now consider the second case, where a parallel compliance is added to the joint (Fig. 1.b). As long as it is a passive subsystem, $K(x)$ can have any profile for force-displacement. Again, with the assumption of perfect tracking ($x = x_d$). Applied force in second case (F_{g2}) is:

$$F_{g2}(t) = F_{c2}(t) + D(t) - F_{spr}(x(t)) \tag{2}$$

Perfect trajectory tracking implies that applied forces are equal ($F_{g1} = F_{g2}$). Threfore

$$F_{c2}(t) = F_{c1}(t) + F_{spr}(x(t)) \tag{3}$$

Eq. 3 says that behavior of the system (F_{c2}) with a particular parallel compliance ($K(x)$), can be simply calculated using F_{c1}. Having Eq. 3 and a cost function, we are able to develop several numerical methods to optimize $K(x)$ for a given task.

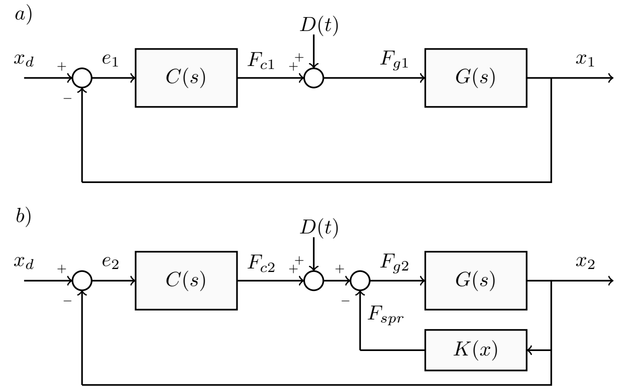

Fig. 1: A general independent-joint control system in two situations, (a) without and (b) with parallel compliance. $K(x)$ represents parallel stiffness. Effects of other joints are represented by $D(t)$ assuming to be periodic and equal in both cases.

B. Absolute Work as a Cost Function

Springs are passive elements that can store and release energy. Tuning their compliance, they can reduce actuator effort and improve energy efficiency. However, this is really important how to define the cost function based on the actuator effort. First we consider actuator work. Given the fact that most of robotic tasks are periodic, work, in one cycle, can be defined as:

$$W = \oint F dx \tag{4}$$

Considering force-displacement graph, this integral is equal to enclosed area. Defining work in this way means that actuators can absorb negative work (when Fdx is negative) and reuse the absorbed energy later. Most of robotic actuators, such as electrical motors, cannot recycle negative work, and decelerating is costly as accelerating. Interestingly, parallel compliance does not have any effect on this index. Therefore, a more meaningful way to define the cost function in our study is to punish negative work. Thus, we consider absolute work as:

$$W' = \oint |F dx| \tag{5}$$

We will use this index as a cost function throughout this paper. The same cost function is used in [14] to optimize the knee compliance of bipedal robot. Other cost functions, such as torque square ([17]), are also used for compliance design. Nevertheless, unlike them, minimization of our cost function results in both negative and positive work reduction.

C. Numerical optimization

In order to setup a numerical optimization, we need to parameterize spring profile. Before that, we should choose a class for this profile. First class, with minimum parameters, is the linear spring (spring constant and rest length as parameters). Increasing complexity, we encounter polynomials and piecewise linear cases. Note that passivity condition should be hold for all these cases.

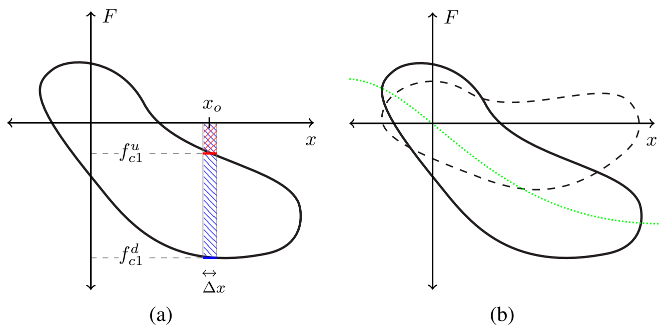

Fig. 2: a: Calculating work from force-displacement graph.
b: Effect of adding parallel compliance. Green-dotted graph shows the negative profile of the proper compliance. Black-solid and black-dashed graphs show force-displacement behavior before and after adding such compliance.

Using Eq. 5 as the cost function, and spring parameters as inputs for optimization, and taking advantage of Eq. 3, without running the system, we can use myriad of numerical and general optimization methods (such as genetic algorithm) to find the fittest parallel compliance. However, by delving into basic concept of parallel compliance and pondering upon force-displacement graphs, we can do better than just numerical optimization. We call it direct method.

D. Direct Method

Considering Fig. 2a as a typical example for force-displacement graphs for system depicted in Fig. 1.a. Calculating work is possible by calculating and integrating partial work on an element Δx around x_0. Work for this element is

$$\Delta W_1^{'} = (|f_{c1}^u| + |f_{c1}^d|)|\Delta x| \quad (6)$$

Using Eq. 3, we can calculate this partial work for system with parallel compliance (Fig. 1.b) as follows:

$$\Delta W_2^{'} = (|f_{c1}^u + F_{spr}(x_0)| + |f_{c1}^d + F_{spr}(x_0)|)|\Delta x| \quad (7)$$

Our goal is to minimize this partial work using $F_{spr}(x_0)$. It is easy to show that any choice that satisfies the following inequalities is an answer to this partial optimization.

$$-f_{c1}^u < F_{spr}(x_0) < -f_{c1}^d \quad (8)$$

While there are infinite optimum points, a good choice can be

$$F_{spr}(x_0) = -\frac{f_{c1}^u + f_{c1}^d}{2} \quad (9)$$

Having maximum distances from boundaries, and subsequently robustness is the first reason for this choice. Reducing maximum forces is the other. This flexibility in optimum point can be used to satisfy other design constraints such as passivity and monotony.

It is interesting to check what happens to force-displacement graph after adding parallel compliance. Such graphs, before and after adding parallel compliance are depicted in Fig. 2b. It can be seen that graph is lined up around horizontal axis ($F = 0$). Upper boundary of graph ($\dot{x} > 0$) now has positive force and lower boundary of graph ($\dot{x} < 0$) has negative force. This mean that force and velocity have the same signs, thus negative work is minimized.

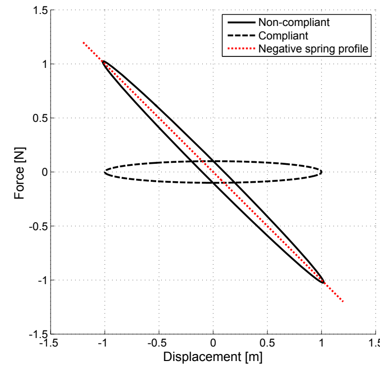

Fig. 3: Using proposed method to design a proper parallel compliance for a mass-spring-damper system. In this System, $m = 1kg$, $b = 0.1Nms^{-1}$. For PID controller, $k_p = 50$, $k_i = 1$, and $k_d = 5$.

III. SIMPLE EXAMPLE

Consider a simple mass-spring-damper system. The defined task is to move the mass on a sinusoidal trajectory with amplitude of $1m$ and frequency of $1rad/s$. In Fig. 3, we see the force-displacement graph when no parallel compliance is present. Using the proposed method, it can be easily concluded that a linear spring with $1N/m$ as constant (with rest length of zero) is the optimal spring. Note that this result is consistent with formula for natural frequency.

After adding parallel compliance, we see that force-displacement graph is lined up around x-axis, and maximum forces are reduced dramatically. Moreover, the area inside the graphs indicate damper energy dissipation; the bigger the damper constant, the larger the enclosed area.

It is interesting to note that, in this simple example, natural dynamics of system has changed in a way to match the task; we call this "dynamics shaping". Other side of the coin is to define the task in a way to exploit such natural dynamics. In order to reach a new level of optimality, these two approaches can be used to design body and task simultaneously.

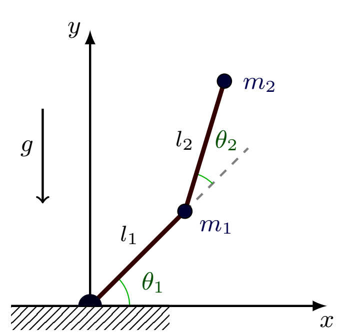

Fig. 4: 2-DOF planar manipulator ($l_1 = l_2 = 1m$, $m_1 = m_2 = 1kg$, and $g = 9.81$).

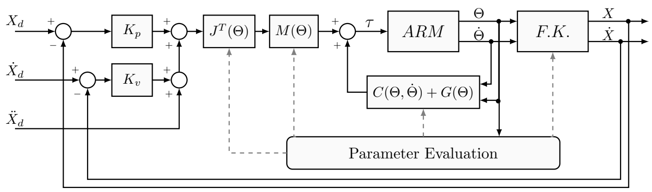

Fig. 5: Operational space control system. For gains, we have $k_p = 1000$ and $k_v = 500$. Applied torques are saturated to $[-20N\ 20N]$.

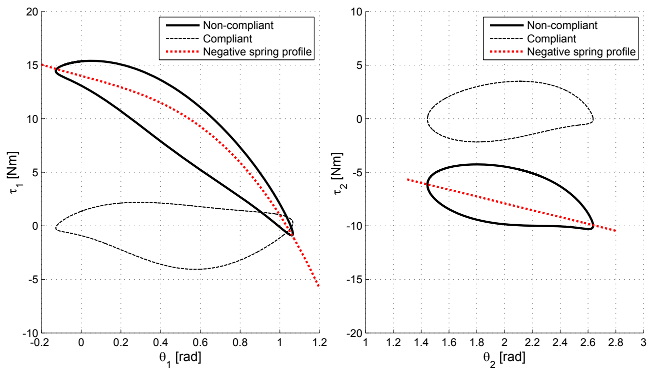

Fig. 6: Effect of adding proper parallel compliance to torque-angle behavior of joints. For the first and second, we designed the parallel compliance as $\tau_1^s = 8\theta_1^3 + 5\theta_1 - 14$ and $\tau_2^s = 3.2\theta_2 + 1.5$.

IV. SIMULATIONS

In this section, we will use the proposed method to design parallel compliance for a 2-DOF manipulator (in MATLAB/Simulink [18]) and a quadruped robot with flexible spine (in Webots [19]).

A. 2-DOF Planar Manipulator

In this section, we design compliance for a 2-DOF planar manipulator; see Fig. 4. Drawing a circle in the operational space using end-effector is the task. For controlling the end-effector on the desired trajectory (operational space control), we used inverse dynamics method and force-based control (Jacobian transpose) as depicted in Fig. 5. For more detail on operational control see [20] and [21]. In control schematic, J is the Jacobian matrix, and M,C, and G matrices are coming from dynamical equations of motion as follow:

$$M(\Theta)\ddot{\Theta} + C(\Theta, \dot{\Theta})\dot{\Theta} + G(\Theta) = \tau \qquad (10)$$

Simulation results for the non-compliant case is illustrated in Fig. 6. Energy consumption for this case can be found in TABLE I. Using the proposed method for this robot, springs' profile are designed accordingly as shown in Fig. 6. Power consumption for the compliant case can also be found in TABLE I. Comparing rigid and compliant cases, it can be seen that 75% improvement is achieved by adding compliance. In Fig. 7, the end-effector's trajectory for non-compliant and compliant cases are illustrated against the desired trajectory. In both cases the desired trajectory is followed very well. However, a discrepancy in transition behavior is visible. This is not expected since we use inverse dynamics method. It means that we expect the same solution

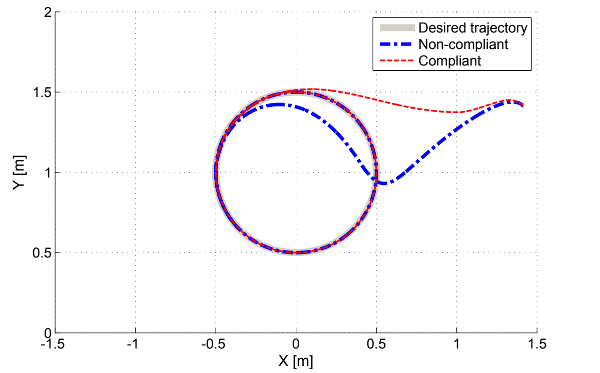

Fig. 7: Trajectory of the end-effector for non-compliant and compliant cases vs. desired trajectory.

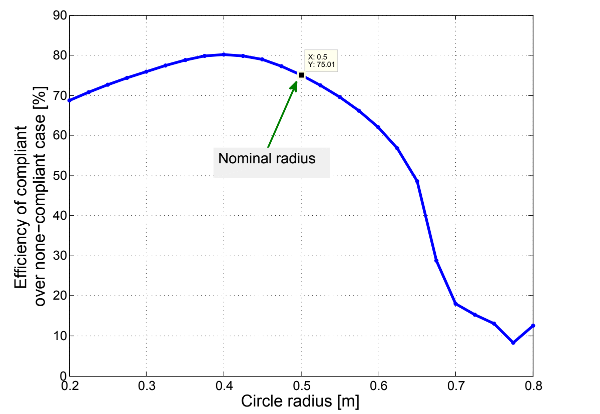

Fig. 8: Performance improvement of compliant over non-compliant case for different circle radius drawn by end-effector.

for the non-compliant and the compliant cases; both in steady state and in transition phases. The answer for this discrepancy lies in the motor torques saturation. Having a saturation for applied torque allows us to observe some of own system dynamics (natural dynamics). Here, better dynamics of the compliant system leads to smaller motor torques and a smoother transition behavior.

It is also interesting to check the robustness of designed compliance to the circle radius drawn by the end-effector. Parallel compliance is designed for radius of $0.5m$. Efficiency of the compliant robot over the non-compliant one (in percentage) is plotted against circle radius in Fig. 8. The parallel compliance designed for $0.5m$ can still drastically improve performance of a wide range of radii. Nonetheless, for better performance in each radius, parallel compliance should be designed accordingly. It seems that parallel spring plays two important roles: recycling negative work, and compensating the gravity. Latter is more effective on this robustness. Furthermore, we repeated the procedure in absence of gravity. Parallel compliance reduced power consumption by 42%; compare this to 75% in presence of gravity.

TABLE I: Power consumption for rigid and compliant cases

Case	Without Compliance	With compliance
Work per cycle	$39.4J$	$9.84J$
Average power	$6.27W$	$1.56W$

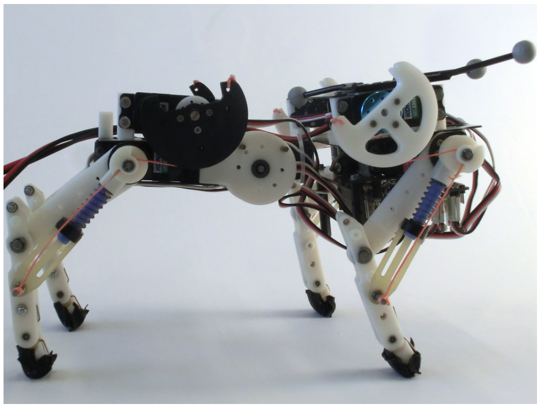

Fig. 9: Bobcat, a quadruped robot with flexible spine. Robot is actuated by 9 RC-servo motors; four hip joints, four knee joints, and spinal joint.

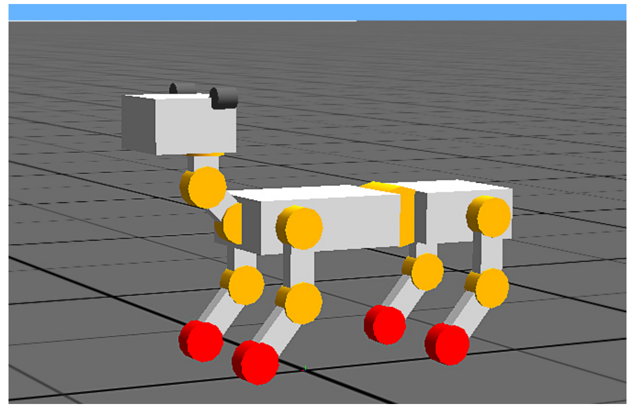

Fig. 10: Ghostdog robot with flexible spine in Webots simulation environment.

B. Quadruped with Flexible Spine

A quadruped robot with flexible spine is an interesting case study. It is long known that spinal joint helps to increase forward velocity of quadrupedal mammals such as cheetah and horse ([22] and [23]). Moreover, role of compliance in animal locomotion is investigated in [24]. Inspired by biology, recent robots are built to benefit from this fact; see MIT and Boston dynamics cheetah as paragons ([25] and [26]). In [15], we present the Bobcat, a cat-sized quadruped robot with flexible spine. Despite its contribution to forward velocity, adding an additional actuator, the spinal joint, will drastically increase the power consumption. Adding a parallel compliance and shaping the natural dynamics toward the task, which is considered to be the bounding gait, can result in lower power consumption and better cost of transportation. In this work, we check this hypothesis in simulation.

The goal is to design a proper parallel compliance for the spinal joint. Ghostdog robot with flexible spine is shown in Fig. 10. As in cheetah, Ghostodog's spine can bend upward and downward; enabling the robot to bound faster. However, adding an extra joint, the spinal joint, increases power consumption dramatically. Hopefully, by adding proper parallel compliance, cost of transportation can be improved significantly. Front and hind hip along with spinal joint are controlled on sinusoidal trajectories specified in TABLE II. To control active joints on their desired trajectory, a PID controller is used ($k_p = 100$, $k_i = 5$, and $k_d = 1$). Knee joints are passive and throughout this experiment gait parameters are fixed.

Torque-angle graph for 20 seconds of bounding, when no parallel compliance is assumed, is illustrated in Fig. 11. Using the method, a parallel compliance is designed to reduce spinal joint power consumption; see Fig. 11. Power consumption for non-compliant and compliant cases are shown in TABLE. III. Power consumption on the spinal joint in compliant case (second row) is reduced by 27.3% over the non-compliant case.

Legged robots are more sophisticated than manipulators. The higher complexity is the result of highly nonlinear dynamics and leg-ground impacts. Therefore, unlike manipulators, it is not straightforward in practice to use feedback linearization to attain a simple dynamics to control. Therefore, here we used just PID controllers at the Ghostdog joints to track the desired trajectory. Because of not canceling the nonlinearities and using a simple controller, adding the parallel compliance changes the robot's behavior (output). Any changes in the output behavior reduces the fitness of the parallel compliance. Nonetheless, we can overcome this issue by iterating the design procedure. Stop condition, however, is determined by the designer. In each iteration, new force-angle data will be saved and new proper parallel compliance will be designed accordingly. Third row of TABLE III shows the results for another iteration. The applied force and the trajectory of spine in the second row are used to redesign the compliance. It can be seen that the power consumption of spine is improved from $18.2Nm/s$ to 17.0. On the other hand, velocity and total power consumption of the robot are worsened; compare the second and the third rows in TABLE III. In this case, however, cost of transportation can be a better index for choosing compliance since it codes overall behavior of the robot; see the last column of TABLE III. Cost of transportation (COT) is defined as $COT = P/mgv$. where P and v are the robot's average consumed power and average velocity. Mass of the robot is shown by m, and g is the gravitational constant. This index is widely used in literature ([27]).

TABLE II: Bounding gait parameters for Ghostdog robot.

		Front hip	Hind hip	Spine
Frequency	$[Hz]$	2.2	2.2	2.2
Amplitude	$[rad]$	0.7	0.7	0.3
Offset	$[rad]$	0.1	-0.1	0.1
Phase lag	$[rad]$	0	2.2	0.2

TABLE III: Performance of Ghostdog robot.

	Spine average power	Robot average power	Average speed	Robot COT
Non-compliant	$25.0Nm/s$	$90.1Nm/s$	$1.66m/s$	0.64
Compliant (1st)	$18.2Nm/s$	$83.3Nm/s$	$1.65m/s$	0.59
Compliant (2nd)	$17.0Nm/s$	$84.9Nm/s$	$1.62m/s$	0.62

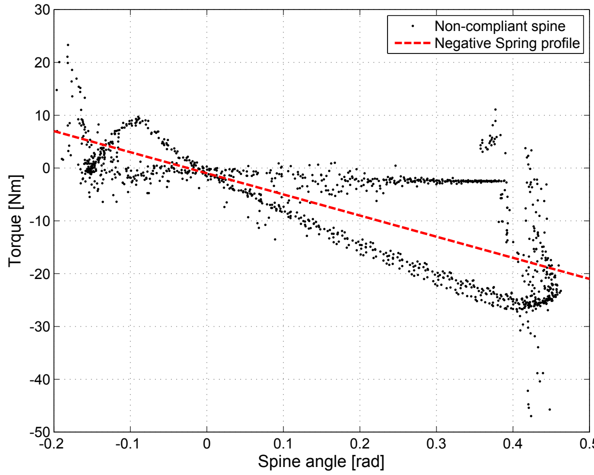

Fig. 11: Torque-angle graph for quadruped spine and the proper parallel compliance. Sampling Time is $4ms$. Spring profile is $\tau = 40\theta + 1$.

V. CONCLUSIONS

In this work, we presented a method for designing parallel compliance to reach lower power consumption. The designed compliance is task dependent. Additive force of parallel compliance and periodicity of motions helped us to construct this methodology. In the proposed method, compliance profile is directly extracted form force-displacement graph. In this method, dynamics of the system are considered implicitly since they are encoded in force-displacement graph. Compliance profile can take any form as long as it satisfies passivity conditions and realization limitation. We showed that for a simple mass-spring-damper, spring constant is tuned to match the natural frequency of the system with frequency of the task. This results in reducing the consumed energy. Another way to attain higher energy efficiency is natural dynamics exploitation where we modify the task to match robot's natural dynamics. Natural dynamics modification (shaping) and exploitation might be used alternatively.

We designed compliance for a 2-DOF manipulator resulting in 75% improvement in energy consumption. It was also shown that designed compliance is robust to reasonable changes in the task. 2-DOF manipulator was an interesting case due to its similarity to 2-segmented leg; consider first/second joint as hip/knee.

Finally, we tried to design parallel compliance for the spinal joint of a quadruped. Dynamics discontinuity due to ground impacts and subsequently imperfection in the tracking deteriorates the design performance. Nonetheless, energy consumption of spine was reduced by 27.3%.

Simulation results showed the effectiveness of the proposed method for designing parallel compliance. In future, we will try to use this methodology in real world experiments; designing spinal parallel compliance for Bobcat robot. We are also interested in combination of natural dynamics modification, as in this work, with natural dynamics exploitation, as in adaptive oscillators, in order to reach energy efficiency. This approach will be useful for redundant robots and cases where task is not fully defined.

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