Waste collection routing with time windows and intermediate disposal trips

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Overview

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  - volume capacities
  - weight capacities
  - fixed costs
  - unit distance running costs
  - hourly driver wage rates
  - speeds
  - site dependencies (accessibility constraints)

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- A set of containers placed at collection points with time windows
- A set of dumps (recycling plants) with time windows

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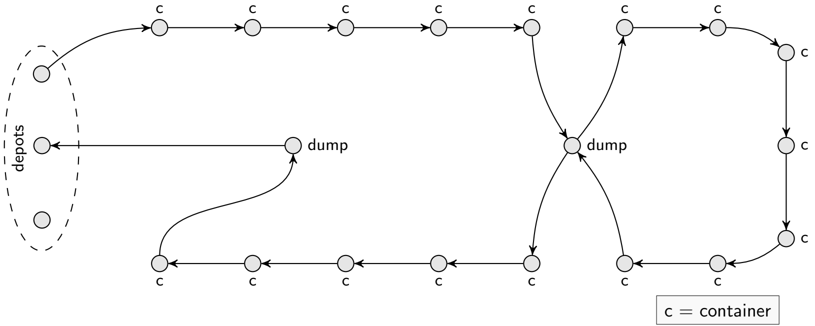
- A set of depots
- A set of containers placed at collection points with time windows
- A set of dumps (recycling plants) with time windows
- Maximum tour duration, interrupted by a break
- A tour is a sequence of collections and disposals at the available dumps, with a mandatory disposal before the tour end
- A tour need not finish at the depot it started from

Figure 1: Tour illustration



Overview

1. Introduction
2. Literature
3. Formulation
4. Solution Approach
5. Case Study
6. Conclusion
7. References

- VRP with intermediate facilities:
  - VRP with satellite facilities (Bard et al., 1998)
    no time windows, no driver break, homogeneous fleet
  - Waste collection VRP (Kim et al., 2006)
    time windows, driver break, homogeneous fleet
  - MDVRP with inter-depot routes (Crevier et al., 2007)
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- Heterogeneous fixed fleet VRP:
  - Proposed by Taillard (1996)
  - Best exact solutions by Baldacci and Mingozzi (2009)
  - Best heuristic solutions by Subramanian et al. (2012) and Penna et al. (2013)

- Model formulations:
  - Bard et al. (1998) - no time windows, no driver break, homogeneous fleet
  - Sahoo et al. (2005) - time windows, driver break, homogeneous fleet
  - Crevier et al. (2007) - no time windows, no driver break, homogeneous fleet, depots and intermediate facilities coincide
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- Exact algorithms - Bard et al. (1998):
  - Branch-and-cut with subtour elimination constraints and lifted $\overrightarrow{D}_k$ and $\overleftarrow{D}_k$ inequalities
  - Algorithms for detecting the above
  - Results on test problems with 15, 18, 20 stops and 0, 1, 2 satellite facilities

- Heuristics - Kim et al. (2006):
  - Kim et al. (2006) - simulated annealing; propose 10 instances with up to 2092 stops and 19 intermediate disposal facilities
  - Ombuki-Berman et al. (2007) - genetic algorithm; distance improvement of 15%, fewer vehicles
  - Benjamin (2011) - variable neighborhood tabu search; distance improvement of 15%, fewer vehicles
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  - Crevier et al. (2007) - adaptive memory (Rochat and Taillard, 1995), tabu search, set covering; propose two sets of instances with 48 to 288 customers
  - Tarantilis et al. (2008) - hybrid guided local search; improvement of $< 1\%$
  - Hemmelmayr et al. (2013) - variable neighborhood search with dynamic programming for the insertion of the intermediate facilities; improvement of 1-3%

- Contribution:
  - Multiple depots
  - Multiple capacities
  - Realistic cost-based objective function
  - Simplification in the modeling of the dump visits
  - Non-time window constrained break
  - Incentive, rather than enforcement, to go back to the origin depot

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ϕ_k	$=$ fixed cost of vehicle k
β_k	$=$ unit distance running cost of vehicle k
θ_k	$=$ hourly wage rate of vehicle k

Decision Variables:

$$x_{ijk} = \begin{cases} 1 & \text{if vehicle } k \text{ traverses edge } (i,j) \\ 0 & \text{otherwise} \end{cases}$$

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S_{ik} = start-of-service time of vehicle k at point i

Q^v_{ik} = cumulative volume on vehicle k at point i

Q^w_{ik} = cumulative weight on vehicle k at point i

$$\text{Min} \quad f = \sum_{k \in K} \left(\phi_k y_k + \beta_k \sum_{i \in N} \sum_{j \in N} \pi_{ij} x_{ijk} + \theta_k \left(\sum_{j \in O''} S_{jk} - \sum_{i \in O'} S_{ik} \right) \right) \tag{1}$$

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$$x_{ijk} \leqslant \alpha_{ijk}, \qquad \forall k\in K, i\in N\setminus O'', j\in N\setminus O' \tag{8}$$

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& Q^w_{ik} + \rho^w_j \leqslant Q^w_{jk} + \left(1 - x_{ijk}\right) M, && \forall k \in K, i \in N \setminus O'', j \in P && (14)\\
& S_{ik} + \epsilon_i + \delta b_{ijk} + \tau_{ijk} \leqslant S_{jk} + \left(1 - x_{ijk}\right) M, && \forall k \in K, i \in N \setminus O'', j \in N \setminus O' && (15)\\
& \left(S_{ik} - \sum_{m \in O'} S_{mk} \right) + \epsilon_i - \eta \leqslant \left(1 - b_{ijk}\right) M, && \forall k \in K, i \in N \setminus O'', j \in N \setminus O' && (16)\\
& \eta - \left(S_{jk} - \sum_{m \in O'} S_{mk} \right) \leqslant \left(1 - b_{ijk}\right) M, && \forall k \in K, i \in N \setminus O'', j \in N \setminus O' && (17)\\
& b_{ijk} \leqslant x_{ijk}, && \forall k \in K, i, j \in N && (18)\\
& \left(\sum_{j \in O''} S_{jk} - \sum_{i \in O'} S_{ik} \right) - \eta \leqslant \left(\sum_{\substack{i \in N \setminus O'' \\ j \in N \setminus O'}} b_{ijk} \right) M, && \forall k \in K && (19)
\end{aligned}
$$

$$\text{s.t.} \quad \lambda_i \sum_{j \in N \setminus O'} x_{ijk} \leqslant S_{ik} \leqslant \mu_i \sum_{j \in N \setminus O'} x_{ijk}, \qquad \forall k \in K, i \in N \setminus O'' \qquad (20)$$

(23)

$$\text{s.t.} \quad \lambda_i \sum_{j \in N \setminus O'} x_{ijk} \leqslant S_{ik} \leqslant \mu_i \sum_{j \in N \setminus O'} x_{ijk}, \qquad \forall k \in K, i \in N \setminus O'' \tag{20}$$

$$\sum_{j \in O''} S_{jk} - \sum_{i \in O'} S_{ik} \leqslant \mathrm{H}, \qquad \forall k \in K \tag{21}$$

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$$x_{ijk}, y_k, b_{ijk} \in \{0, 1\}, \qquad \forall k \in K, i, j \in N \qquad (22)$$

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$$x_{ijk}, y_k, b_{ijk} \in \{0, 1\}, \qquad \forall k \in K, i, j \in N \qquad (22)$$

$$Q^v_{ik}, Q^w_{ik}, S_{ik} \geqslant 0, \qquad \forall k \in K, i \in N \qquad (23)$$

Extension:

$$z_{ijk} = \begin{cases} 1 & \text{if } i \text{ is the origin and } j \text{ the destination of vehicle } k \\ 0 & \text{otherwise} \end{cases}$$

Ψ = weight of relocation term

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$$\text{Min} \quad f = \text{Objective (1)} + \Psi \sum_{k \in K} \sum_{i \in O'} \sum_{j \in O''} \left(\beta_k \pi_{ji} + \theta_k \tau_{jik}\right) z_{ijk} \tag{24}$$

s.t. Constraints (2) to (23)

$$\sum_{m \in P} x_{imk} + \sum_{m \in D} x_{mjk} - 1 \leqslant z_{ijk}, \quad \forall k \in K, i \in O', j \in O'' \tag{25}$$

$$z_{ijk} = \{0, 1\}, \quad \forall k \in K, i \in O', j \in O'' \tag{26}$$

Overview

- For small instances, common solver for the MILP formulation enhanced by valid inequalities and elimination rules
- Valid inequalities and elimination rules include:
  - Impossible traversals
  - Time window infeasible traversals
  - Latest start/earliest finish
  - Minimum tour duration
  - Symmetry breaking for subsets of identical vehicles
  - Minimum/maximum number of dump visits

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- For realistic-size instances, a feasibility preserving local search heuristic
- A feasible tour satisfies three criteria:
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- The quality of the heuristic is assessed by benchmarking its results to the optimal ones obtained with the MILP model on small instances.

Figure 2: Temporal feasibility algorithm

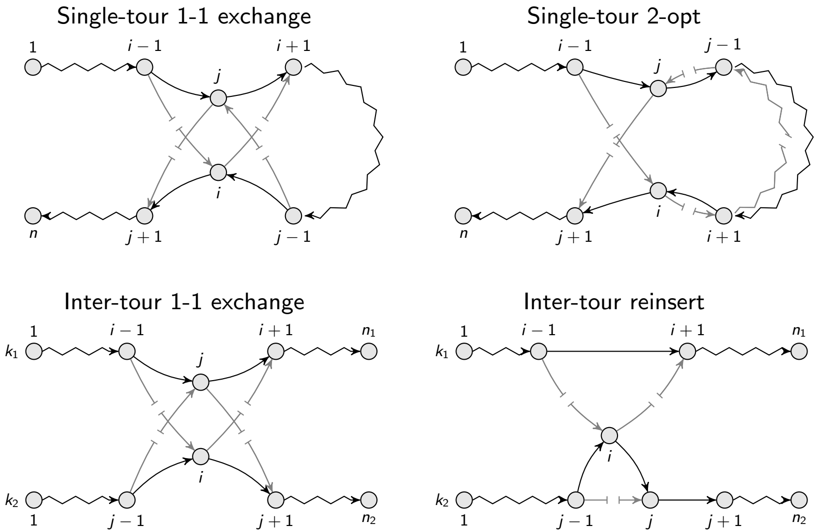
Data: tour k as a sequence of points $1, \ldots, n$ after a change
Result: start-of-service times, waiting times and temporal feasibility of tour k

set S_{1k} to earliest possible;
for $i = 2 \ldots n$ in tour k **do**
    // Calculate tentative start-of-service times
    $S_{ik} = S_{(i-1)k} + \epsilon_{i-1} + \tau_{(i-1)ik}$;
    // Insert break
    if $S_{(i-1)k} + \epsilon_{i-1} \leqslant S_{1k} + \eta$ **and** $S_{ik} + \epsilon_i > S_{1k} + \eta$ **then**
        $S_{ik} = S_{ik} + \delta$;
    end
    // Calculate waiting times
    if $S_{ik} < \lambda_i$ **then**
        $w_{ik} = \lambda_i - S_{ik}$;
        $S_{ik} = \lambda_i$;
    else
        $w_{ik} = 0$;
    end
end

Figure 1: Temporal feasibility algorithm, cont'd

```
// Check time window feasibility
if S_ik ⩽ μ_i, ∀i then
    // Forward time slack reduction
    for i = n...2 in tour k do
        S'_(i−1)k = S_(i−1)k;
        S_(i−1)k = min(S_(i−1)k + w_ik, μ_i−1);
        w_(i−1)k = w_(i−1)k + (S_(i−1)k − S'_(i−1)k);
        w_ik = w_ik − (S_(i−1)k − S'_(i−1)k);
    end
    w_1k = 0;
    // Check duration feasibility
    if S_nk − S_1k ⩽ H then
        tour k is temporally feasible;
    else
        discard tour k as duration infeasible;
    end
else
    discard tour k as time-window infeasible;
end
```

Figure 3: Neighborhood operators



- Tour construction:
  - Sequential feasibility preserving insertion heuristic
  - At every iteration an unassigned container is inserted at the point that yields the smallest increase in the objective value
  - When container insertions would violate capacity, a dump is inserted using the same logic
  - A dump insertion should allow for at least one subsequent temporally feasible container insertion

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- Tour improvement:
  - Alternation between inter-tour and single-tour improvement
  - The application of an inter-tour operator is followed by single-tour improvement of the affected tours
  - Every operator is applied for *maxOpIter* iterations and *maxOpNonImpIter* non-improving iterations, before changing to the next operator
  - Both single-tour and multi-tour improvement run for *maxIter* iterations and *maxNonImpIter* non-improving iterations
  - The resulting tour schedule is the best found during all iterations

Overview

- 5 instances extracted randomly from real underlying data
- 3 versions of each instance:
  - No time windows (iX)
  - Wide time windows (iX_tw) - randomly assigned
  - Narrow time windows (iX_ntw) - randomly assigned
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- 1 depot, 1 dump, 2 identical vehicles
- Tests on 2.60 GHz Intel Core i7, 8GB of RAM
  - Local search heuristic coded in Java
  - Model solved on Gurobi 5.6.2 warm-started with the solutions from the local search heuristic
  - Solver time limit set to 1000 sec

Table 1: Comparison between heuristic and solver on random instances
$maxOpIter = 100$, $maxOpNonImpIter = 13$, $maxIter = 100$, $maxNonImpIter = 1$

	Heuristic		Solver				
Instance	Objective	Runtime (sec.)	Objective	L Bound	MIP gap (%)	Runtime (sec.)	Opt gap (%)
i1	214.849	0.170	214.849	214.837	0.006	375.562	0.000
i1_tw	284.016	0.070	252.825	252.825	0.000	4.038	12.337
i1_ntw	428.539	1.093	394.817	394.817	0.000	0.922	8.541
i2	249.317	0.042	249.317	249.317	0.000	400.032	0.000
i2_tw	257.583	0.050	257.582	257.582	0.000	2.306	0.000
i2_ntw	460.635	0.756	439.769	439.769	0.000	2.420	4.745
i3	240.133	0.051	240.133	76.004	68.349	1000.000	0.000
i3_tw	245.457	0.070	245.457	245.457	0.000	2.894	0.000
i3_ntw	444.589	0.641	444.589	444.589	0.000	2.446	0.000
i4	138.643	0.077	138.643	138.643	0.000	521.509	0.000
i4_tw	140.204	0.030	140.204	140.204	0.000	7.660	0.000
i4_ntw	179.537	0.043	179.537	179.537	0.000	2.849	0.000
i5	220.770	0.070	220.770	129.834	41.190	1000.000	0.000
i5_tw	233.211	0.050	233.211	233.211	0.000	3.501	0.000
i5_ntw	405.622	0.848	405.622	405.622	0.000	3.051	0.000

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Table 2: Comparison between heuristic and solver on selected random instances
maxOpIter = 350, *maxOpNonImpIter* = 37, *maxIter* = 100, *maxNonImpIter* = 1

	Heuristic		Solver				
Instance	Objective	Runtime (sec.)	Objective	L Bound	MIP gap (%)	Runtime (sec.)	Opt gap (%)
i1_tw	252.825	0.410	252.825	252.825	0.000	3.487	0.000
i1_ntw	394.817	3.399	394.817	394.817	0.000	0.916	0.000
i2_ntw	439.769	3.080	439.769	439.769	0.000	2.309	0.000

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  - Mathematical model
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  - Mathematical model
  - Local search heuristic
  - The heuristic performs favorably with an average optimality gap of less than 2% in short computation times (less than 1 sec)
- Future work
  - Mathematical model improvement to solve larger instances
  - Extension of the heuristic to include all features of the mathematical model
  - Development of efficient inter-tour operators to respond to the challenge posed by the heterogeneous fleet
  - Sensitivity analysis of the parameters
  - Benchmarking for realistic-size instances against current state of practice

Thank you for your attention!
Questions?

Overview

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