© 3rd IAHR Europe Congress, Book of Proceedings, 2014, Porto - Portugal. ISBN 978-989-96479-2-3

STEEL-LINED PRESSURE TUNNELS AND SHAFTS IN ANISOTROPIC ROCK

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Abstract

For the design of steel-lined pressure tunnels and shafts in anisotropic rock, it is common practice to assume for the computation that the system stands in isotropic medium with the lowest stiffness measured in situ. In this paper, stresses and displacements in the steel liner of a particular steel-lined pressure tunnel are numerically assessed taking into account the anisotropic behavior of the rock mass, for the pseudo-static case. The results are compared with the corresponding isotropic case.

Keywords: Pressure tunnels; Pressure shafts; Steel liner; Rock; Anisotropy.

1. Introduction

The basic criteria that are considered for the design of steel-lined pressure tunnels and shafts are (i) the working stress and deformation of the steel liner, and (ii) the load-bearing capacity of the surrounding rock mass (Schleiss, 1988). Criterion (i) includes the resistance of the liner to buckling (*i.e.* the stability of the steel liner under outer pressure), the limiting working stresses in the liner, and the crack bridging (*i.e.* the limitation of local deformation in the liner). Criterion (ii) aims at avoiding rock mass failure and therefore assuring the assumption of rock mass participation in the design. In principle, the limit of the load sharing capacity is reached when the tensile stresses transmitted to the rock due to inner pressure are larger than the in situ stress field.

When designing steel-lined pressure tunnels or shafts drilled in rock, designers usually make the assumption that the multilayer structure is in isotropic media. The stiffness considered for the rock is taken as the lowest modulus of elasticity measured in situ, and the stresses and displacements are computed using analytical methods based on compatibility condition of deformation of each layer of the radial symmetric system (Hachem and Schleiss, 2009). In Europe, according to the C.E.C.T. (1980) recommendations, calculation for primary stresses must be performed in the elastic range, and the allowable equivalent stresses (according to von Mises theory in triaxial state of stresses) must be below ratios compared to tensile or yield strength.

The influence of anisotropic rock behavior on the deformations and stresses in liners of pressure tunnels or shafts has been studied by several authors. Eristov (1967a) developed an analytical method for the lining behavior in elastic orthotropic media by partitioning the liner into beam elements. This method is similar to the Finite Element Method (FEM) approach presented in USACE (1997). Eristov (1967b) also studied experimentally the action of pressure tunnels linings in anisotropic media.

Postol'skaya (1986) performed a series of parametric investigations of the stress state of the crack-resistant lining, using the FEM, in different anisotropic media. More recently, Bobet (2011) has developed closed-form solutions for stresses and displacements for lined-tunnels in transversely isotropic rock. The multilayer system of steel-lined pressure tunnels or shafts in anisotropic rock has not been studied so far.

This study focuses on the influence of anisotropic rock on stresses and deformations in the steel liners of steel-lined pressure tunnels or shafts. A particular geometry with boundary conditions was chosen. A finite element model of the multilayer system was implemented and validated in isotropic rock compared to the analytical solution. Finally transversely isotropic rock behavior was considered and results were compared with the isotropic case.

2. Steel-lined pressure tunnels or shafts in isotropic rock

2.1 Axisymmetrical multilayer approach

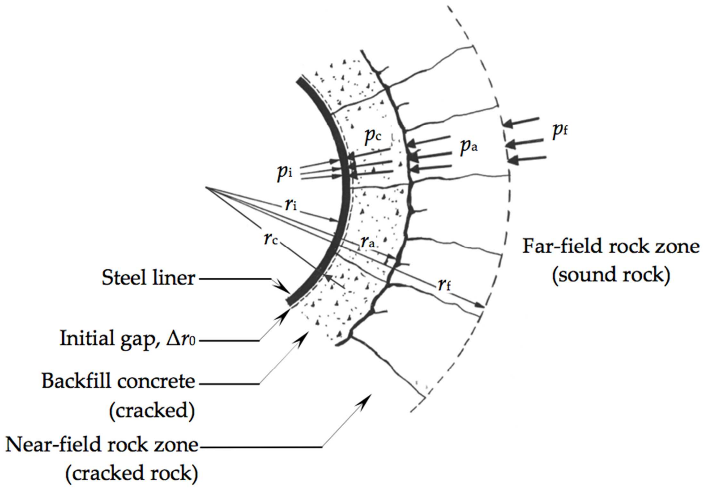

Figure 1. Definition sketch of the radial symmetrical multilayer system for steel lined pressure tunnels with axisymmetrical behavior (Hachem and Schleiss, 2011).

An axisymmetrical multilayer approach is considered (Figure 1), where five zones are distinguished (Hachem and Schleiss, 2011):

1. The steel liner: its inner surface is in contact with pressurized flow and is impervious.
2. An initial gap between the steel liner and backfill concrete due to steel shrinking as a consequence of the contact with cold water.
3. Backfill concrete: zone between the steel liner and the rock. The concrete usually cracks when subjected to inner pressure and thus it is assumed that it cannot transmit tensile stresses.
4. Near-field rock zone: disturbed part of the rock mass as a result of the excavation method and the change in the in situ stress field around the tunnel. It is also assumed that this zone cannot transmit tensile stresses.
5. Far-field rock zone: non-disturbed zone assumed as a homogeneous, isotropic and elastic material.

2.2 Analytical solution

The deformation of the system (Figure 1) is derived for each layer from the compatibility conditions at the interfaces (Hachem and Schleiss, 2011). It is expressed by Eq. [1] to Eq. [3]:

$$u_r^s(r = r_c) - \Delta r_0 = u_r^c(r = r_c) \quad [1]$$

with

$$\Delta r_0 \geq 0\,;$$

$$u_r^c(r = r_a) = u_r^{crm}(r = r_a)\,; \quad [2]$$

$$u_r^{crm}(r = r_f) = u_r^{rm}(r = r_f)\,; \quad [3]$$

where u is the displacement in the radial direction (indicated by subscript r), superscript s refers to steel, c to backfill concrete, crm to cracked rock mass (near-field zone) and rm to rock mass (far-field zone). Two assumptions are made: (i) uncracked layers are homogenous, elastic with axisymmetrical behavior, modeled according to the thick-walled cylinder theory (Timoshenko and Goodier, 1970) and (ii) cracked layers cannot transfer tensile stresses. It yields an analytical development to compute the radial displacements of the different layers, and therefore the stresses.

3. Constitutive model of anisotropic rock

The anisotropy which is considered for the rock mass in this study is the transverse isotropy, *i.e.* a medium with a plane of isotropy, particular case of orthotropy. Assuming that the y-axis is perpendicular to the plane of isotropy and that the z-axis is the longitudinal direction (*i.e.* out of the plane in Figure 1), the generalized Hooke's law is simplified and expressed by Eq. [4] to Eq. [9] (Lekhnitskii, 1963):

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_z) - \frac{\nu'}{E'}\sigma_y\,; \quad [4]$$

$$\varepsilon_y = -\frac{\nu'}{E'}(\sigma_x + \sigma_z) + \frac{1}{E'}\sigma_y\,; \quad [5]$$

$$\varepsilon_z = \frac{1}{E}(\sigma_z - \nu\sigma_x) - \frac{\nu'}{E'}\sigma_y\,; \quad [6]$$

$$\gamma_{yz} = \frac{1}{G'}\tau_{yz}\,; \quad [7]$$

$$\gamma_{xz} = \frac{1}{G}\tau_{xz} = \frac{2(1+\nu)}{E}\tau_{xz}\,; \quad [8]$$

$$\gamma_{xy} = \frac{1}{G'}\tau_{xy}\,. \quad [9]$$

It remains five independent constants to characterize a transversely isotropic material. E and E' are the Young's moduli in the plane of isotropy and perpendicular to it respectively. The Poisson's coefficients which characterize the reduction in the plane of isotropy for the tension in the same plane and the tension in a direction normal to it are written ν and ν' respectively. The shear moduli for the planes parallel and normal to the plane of isotropy are G and G' respectively. G' can be approximated by the empirical relation according to Eq. [10] (Amadei, 1996):

$$G' = \frac{E'}{1 + E'/E + 2\nu'} \,. \qquad [10]$$

4. Case study

The studied case considers a steel-lined pressure shaft, with parameters in typical application ranges. Table 1 presents the geometrical and loading properties and Table 2 the material properties (for the rock, only in the isotropic case).

Table 1. Geometrical and loading parameters of the case study.

PARAMETERS	r_i	t	Δr_0	r_c	r_a	r_f	p_i
UNITS	(m)	(m)	(m)	(m)	(m)	(m)	(bar)
VALUES	2.000	0.030	-	2.030	2.530	3.030	120

Table 2. Material properties of the case study (in isotropic rock).

PARAMETERS	E_s	ν_s	E_c	ν_c	E_{crm}	ν_{crm}	E_{rm}	ν_{rm}
UNITS	(GPa)	(-)	(GPa)	(-)	(GPa)	(-)	(GPa)	(-)
VALUES	210	0.29	20	0.2	10	0.2	10	0.2

5. Numerical modeling

The study is performed with the commercial finite element (FE) software Mechanical APDL (ANSYS) 14.0 (ANSYS®, 2011).

5.1 Finite element model

The implemented FE model assumes: (i) 2D plane strain conditions; (ii) the tunnel has a circular cross section; (iii) the tunnel stands in deep rock, *i.e.* the dimensions of the whole system are large enough so that the surrounding media is considered as infinite; (iv) initial gap is vanished; (v) contact between every layer is tied; (vi) materials are linear elastic; (vii) only the pseudo-static inner pressure due to water is taken into account; and (viii) only one quarter of the tunnel is implemented due to the symmetry in the transversely isotropic case.

To model the cracked layers (*i.e.* that cannot transmit tensile stress) using linear elastic materials, the concerned layers have been radially divided into homogenous parts to rapidly prohibit the circumferential tensile stresses after the layer interface, so that it behaves only in compression. The assumed opening of the cracks is 0.05 deg and the opening of the solid parts is 2.95 deg.

As the influence of discrete cracks was not the goal of the present study, a first layer of uncracked concrete (of thickness equal to 0.5t = 15 mm) was introduced arbitrarily in order to absorb the effects of discontinuities outside of the steel liner. The sketch of the FE model is shown in Figure 2.

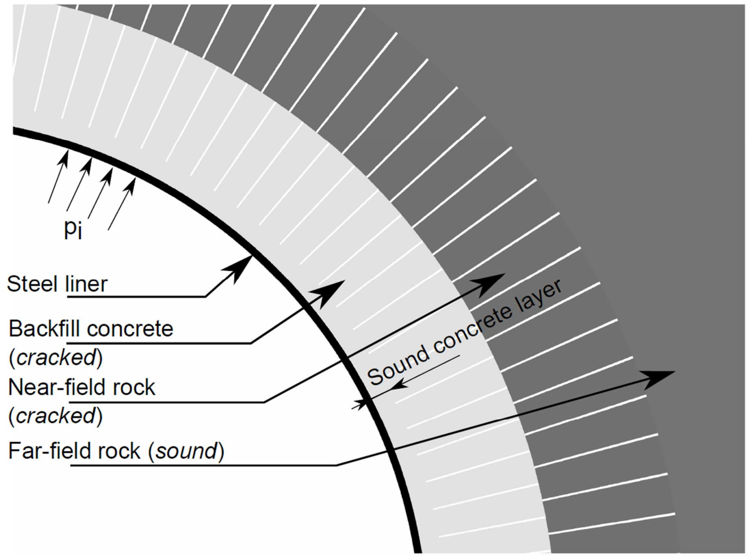

Figure 2. Sketch of the finite element model.

The mesh contains a total of 72’835 elements of type *PLANE183 8-node quadrilateral* (ANSYS®, 2011). The liner is discretized by 8 elements along its thickness and 570 elements along its quarter-circumference, for a total of 4’560 elements. The large number of elements is due to element-shape requirements induced by the small openings of the vacuums of the cracked layers. The convergence of the mesh was verified.

5.2 Validation in isotropic rock

In the case with isotropic rock behavior, the analytical development obtained by the compatibility conditions (Eq. [1] to Eq. [3]) indicates that radial displacements as well as circumferential stresses (hoop stresses) are constant for a given radius in the steel liner. For the studied case considering isotropic rock with an elastic modulus of 10 GPa, the analytical solution gives $2.38 \cdot 10^{-3}$ m for the maximum radial displacement and 268 MPa for the maximum circumferential stress. The results obtained with the finite element model are $2.31 \cdot 10^{-3}$ m and 272 MPa respectively. The error compared to the analytical approach is therefore +1.5% for the maximum stresses and -2.9% for the maximum radial displacements.

The finite element model has therefore a satisfying behavior in isotropic rock. It may be noted that the accuracy of the displacements and stresses in the liner also depends on the thickness of the first layer of sound concrete that has been introduced. Without considering the latter, the FE model is more conservative and gives larger stresses compared to the analytical solution, due to the influence of the radial cracks in backfill concrete.

With the same geometry but with isotropic rock with an elastic modulus of 20 GPa, the error compared to the analytical solution becomes +2.4% for the maximum stresses and -1.3% for the maximum radial displacements. This is the same order of magnitude than with an elastic modulus of 10 GPa, and thus the behavior of the model does not strongly depend on the rock stiffness.

Despite the use of the thin sound concrete layer, the stresses and displacements have a larger variation across the thickness of the liner as well as across the circumference in between the discontinuities in the backfill concrete. The system in the FE model is slightly stiffer (it gives larger radial displacements) but the maximum stresses are slightly larger due to the remaining influence of the discontinuities.

6. Systemic analysis in anisotropic rock

A parametric study is performed on the rock mass properties. Based on the corresponding isotropic case as a reference where the modulus of elasticity is equal to 10 GPa, five transversely isotropic rock masses are simulated, and compared to the results in the isotropic case. The properties of these rock materials are shown in Table 3.

Table 3. Properties of the tested rock masses.

	E/E'	E	E'	ν	ν'	G	G'
	(-)	(GPa)	(GPa)	(-)	(-)	(GPa)	(GPa)
ROCK 1	1.00	10	10	0.2	0.2	4.17	4.17
ROCK 2	1.20	12	10	0.2	0.15	5.00	4.69
ROCK 3	1.40	14	10	0.2	0.15	5.83	4.96
ROCK 4	1.60	16	10	0.2	0.15	6.67	5.19
ROCK 5	1.80	18	10	0.2	0.15	7.50	5.39
ROCK 6	2.00	20	10	0.2	0.15	8.33	5.56

These properties satisfy the thermodynamic constraints for a material that is transversely isotropic (Amadei, 1987).

7. Results

Simulations were performed for all types of rock masses introduced in Table 3. In the following, dimensionless results are presented in terms of simulated radial displacements and first principal stresses, representative of the tangential stresses (hoop stresses) in cylindrical coordinates.

7.1 Stresses

First principal stresses were obtained at the inner fiber of the steel liner. As in common practice the lowest stiffness of the rock is considered in an assumed isotropic case, the results in transversely isotropic rock masses are presented under dimensionless form, *i.e.* divided by the corresponding results in isotropic case. Dimensionless first principal stresses are plotted in Figure 3 over the angle of location in cylindrical coordinates. Maximum stresses occur at 0 deg, *i.e.* in the plane of isotropy of the rock medium, while minimum stresses occur at 90 deg, *i.e.* in the plane perpendicular to the plane of isotropy. For a ratio of anisotropy of 2, which is commonly found in practical cases, the maximum stresses in the liner are reduced by 22% compared to the corresponding isotropic case.

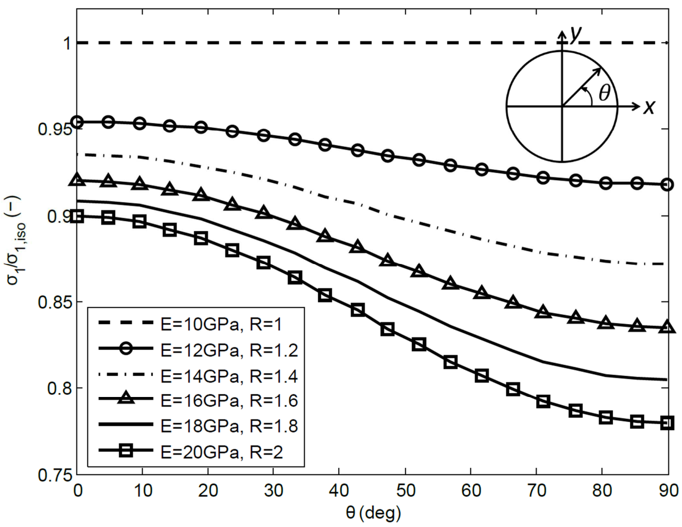

Figure 3. Dimensionless first principal stresses at the inner fiber of the steel liner obtained by finite element analysis as a function of anisotropy.

7.2 Displacements

Radial displacements were obtained at the inner fiber of the liner, as for stresses. Similarly, they are presented under dimensionless form, *i.e.* divided by the corresponding results in isotropic case. Dimensionless radial displacements are plotted in Figure 4 over the angle of location in cylindrical coordinates. Maximum displacements occur at 90 deg, *i.e.* in the plane perpendicular to the plane of isotropy, while minimum displacements occur at 0 deg, *i.e.* in the plane of isotropy of the rock medium. Compared to the corresponding isotropic case and for a ratio of anisotropy of 2, the maximum radial displacements are increased by 4%, while the minimum displacements are decreased by 34%.

8. Discussion

The use of a thin sound backfill concrete layer is required to diminish the stress concentrations in the liner due to the discontinuities in the cracked backfill concrete which are also not considered in the analytical solution. In isotropic rock with the appropriate thickness, the FE model gives satisfying results. When considering anisotropic rock behavior where radial displacements are no longer constant for a given radius, the sound concrete layer may transmit some tensile stresses that do not correspond to the theoretical assumption that cracked layers cannot do it. Nevertheless, it seems reasonable to neglect this influence due to the low stiffness of concrete compared to steel, as well as the assumed small thickness of this sound layer (in this case study 3% of the backfill concrete thickness). Finally, it has to be noted that the FE approach presented herein has not the purpose to model cracks behavior in the fissured layers. This model, however, uses a linear elastic approach (and thus ease in interpretation) to model steel-lined pressure tunnels or shafts multilayer system which behaves realistically in terms of stresses and displacements in the liner subjected to pseudo-static loads.

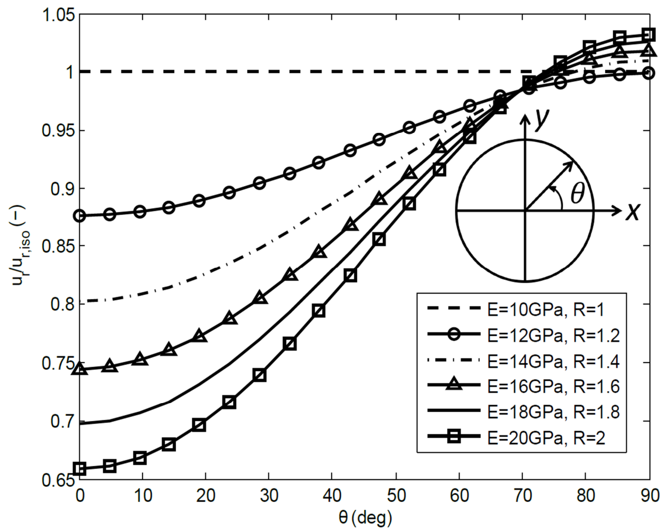

Figure 4. Dimensionless radial displacements at the inner fiber of the steel liner obtained by finite element analysis as a function of anisotropy.

9. Conclusions

A linear elastic FE approach was used to study the influence of transversely isotropic rock mass behavior on stresses and deformations in the steel liner of steel-lined pressure tunnels or shafts. A particular geometry of this hydraulic structure was studied, subjected only to inner water pressure, and a sensitivity analysis was performed on the rock mass properties. For a ratio of anisotropy of 2, first principal stresses are decreased by more than 20% in the steel liner compared to the corresponding isotropic case, and radial displacements are no longer constant for a given radius in the liner.

For the time being the study focused on pseudo-static loading, method commonly used by designers of such pressurized waterways. However, this FE approach provides outlooks for further studies considering transient loadings such water hammer, by modeling either sound or cracked layers in the system while still being in the linear elastic domain.

Acknowledgments

This study is part of the consortium *HydroNet 2: Modern methodologies for design, manufacturing and operation of hydropower plants*, a research project funded by the Swiss Competence Center Energy and Mobility (CCEM-CH).

References

Amadei, B., Savage, W.Z., and Swolfs, H.S. 1987. Gravitational stresses in anisotropic rock masses. *International Journal of Rock Mechanics and Mining Sciences*, 24(1):5-14.

Amadei, B. 1996. Importance of anisotropy when estimating and measuring in situ stresses in rock. *International Journal of Rock Mechanics and Mining Sciences*, 33(3):293-325.

ANSYS® 2011. Documentation, Release 14.0. ANSYS, Inc.

Bobet, A. 2011. Lined circular tunnels in elastic transversely anisotropic rock at depth. *Rock Mechanics and Rock Engineering*, 44(2):149-167.

C.E.C.T. 1980. Recommendations for the design, manufacture and erection of steel penstocks of welded construction for hydroelectric installations. European Committee of Boiler, Vessel and Pipe Work Manufacturers.

Eristov, V.S. 1967a. Computation of pressure tunnel linings in anisotropic rocks. *Hydrotechnical Construction*, 1(5):436-442.

Eristov, V.S. 1967b. Experimental studies of pressure-tunnel linings in anisotropic formations. Hydrotechnical Construction, 1(12):1054-1057.

Hachem, F.E., Schleiss, A.J. 2009. The design of steel-lined pressure tunnels and shafts, *International Journal on Hydropower and Dams*, 16(3):142-151.

Hachem, F.E., Schleiss, A.J. 2011. A review of wave celerity in frictionless and axisymmetrical steel-lined pressure tunnels, *Journal of Fluids and Structures*, 27(2):311-328.

Lekhnitskii, S.G. 1963. Theory of elasticity of an anisotropic elastic body. Holden-Day, Inc., San Franciso.

Postol'skaya, O.K. 1986. Effect of the structure of a rock mass and properties of rocks in the stress state of hydraulic pressure tunnels. *Hydrotechnical Construction*, 20(1):25-30.

Schleiss, A.J. 1988. Design criteria applied for the lower pressure tunnel of the North Fork Stanislaus River hydroelectric project in California, *Rock mechanics and Rock Engineering*, 21(3):161-181.

Timoshenko, S.P., Goodier, J.N. 1970. Theory of elasticity. McGraw-Hill Book Co., New York, N.Y.