Improving Human-Compiler Interaction Through Customizable Type Feedback

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Abstract

Type errors reported by compilers can sometimes be cryptic, or difficult to understand. In this paper, we propose a type debugging framework that exposes a high-level representation of the typechecking decision-making process that users normally do not have access to in state-of-the-art compilers. This representation makes it easier for non-experts to analyze complex type errors. Our system is implemented by instrumenting the existing Scala typechecker, but without modifying it. We also provide generic search algorithms that can be used as basic building blocks to build a number of systems, from visual type debugging tools to customized error reporting for normal Scala users as well as to users of domain-specific languages. Using our framework, it is possible to overcome well-known limitations of local type inference by providing precise type error messages to mainline Scala users or Scala DSL users alike. In addition, the framework provides better user feedback for non-trivial type errors from existing Scala libraries and DSLs.

Keywords type system, type errors, type debugging

1 Introduction

Writing software using statically-typed languages inherently involves fixing type errors. The subject of what constitutes a good error message, and therefore properly directs the user, has been well studied [14, 32]. Yet, in practice, it is hard to achieve both generality (*i.e.,* typechecking rules should apply to as many conflicts as possible) and precision (*i.e.,* type errors convey enough information for users to be able to easily navigate to and fix the error) without the full type derivation tree, and without affecting the performance of regular compiler runs. Since language architects and compiler writers must provide enough information to assist both novice,

```
val xs: List[Int] = List(1, 2, 3)
xs.foldRight(Nil)((x, ys) => (x + 1) :: ys)
// type mismatch;
//  found   : List[Int]
//  required: Nil
//              (x + 1) :: ys)
//                      ^
```

Listing 1. Incrementing a list of integers gone wrong

as well as expert users (such as embedded DSL authors in Scala), achieving this trade-off is even more difficult.

An example of an innocuous code snippet which leads to a puzzling type error is shown in Listing 1. In this example, we simply have a list of integers that we want to increment using the `foldRight` method from the Scala standard library. The call to `foldRight` traverses the elements of a `List` from right to left, starting with the first argument as the first element, in this case `Nil` (an empty `List`), and applies the second argument, a function, to each element, which in this case appends the incremented elements.

In this example, the error comes from a limitation of type inference for local type parameters – type inference flows from left to right and only from parameter list to parameter list. We argue that cases like these, or in DSLs, or at intersections of advanced type system features, it should be possible to *adapt* type error feedback rather than placing the burden of adapting to cryptic error messages on the programmer.

The idea of improving type errors, however, is not new. One approach involves providing a separate, post-error, typing phase [1] which recreates a partial type derivation tree representing the error in question. Improved type error feedback is usually achieved by constraint solving [12, 30] and reporting conflicting source locations[1], which may involve standard techniques of program slicing [10] or providing error messages with more decision context. Unfortunately, this leads to a partial duplication of the typing phase, which has obvious software engineering implications – changes to the original typechecker are not propagated to the duplicated typechecker, leading to inconsistent error-reporting behavior. Furthermore, the duplicated typechecker can often only

[1] Source file, line and/or character position.

support a subset of language features [13], and, as a result, rarely end up being adopted in industrial languages. Such a constraint-oriented approach additionally suffers from scenarios where so many constraints are generated, that solving for them becomes intractable [20, 24].

An orthogonal approach to improving error reporting is aimed at developing tools that attempt to automatically provide suggestions which resolve type errors. These automatic techniques rely on the existence of heuristics defined by language architects [16] to modify code according to well-known coding errors. For example, such a technique might encounter a type mismatch in the argument list of a function which it might attempt to resolve by simply permuting the arguments until the type error is resolved. Such tools typically do not take into account the decision process of the typechecker [9] – they simply generate a large number of potential solutions from the generated permutations, and perform a ranking analysis that provides the most likely solutions to the user. In that sense they perform in a similar way to code completion tools. The disadvantage is that such techniques can report false positives [15], are computationally intensive, and are often ineffective for languages that support some sort of polymorphism [8].

This paper takes a step towards more principled human-compiler interaction by (a) introducing a novel approach to type error customization and debugging based on type derivation trees from the actual type checking process, that (b) we show can be used as the foundation for many systems to interact with and customize type error feedback, from graphical type debugging tools, to domain-specific type error customizations. Crucially, our approach is based upon the construction of a high-level representation of typing decisions that is obtained directly from derivation trees built from the result of the compiler's type checking, all without modifying the logic of the existing typechecker or reducing its features. To achieve this representation, we map low-level, context-free typechecking information, extracted from instrumented compiler runs, into what essentially becomes instantiations of typing rules. Built on top of this high-level representations of typing decisions, we provide a set of generic routines for analyzing the type derivation tree, and for adding custom type errors. This information is accessible to users via an API for traversing, analyzing, and building on the derivation tree with custom error feedback.

We achieve a sweet spot between generality and precision, such as in the case of the cryptic error message from the type inference example above in Listing 1. In this case, the framework is able to suggest the following helpful type ascription:

```
// Inferred expected type using the following location(s):
// xs.foldRight(Nil)((x: Int, ys: List[Int]) => (x + 1) :: ys)
//              ~~~
// You may try to annotate your code like: 'Nil:List[Int]'
```

Beyond such type ascriptions, helpful user feedback can include program slicing, descriptions of the type derivation tree, suggestions on how to better guide type inference, or even graphical tools for exploring the type derivation tree.

Finally, our framework is not limited to users of the mainline Scala distribution. Domain-specific library (DSL) authors with shallow and deeply embedded DSLs are also able to take advantage of customized type error analysis and reporting since the framework doesn't require or include a duplicate typechecking pass, it uses the normal typechecker.

1.1 Selected Related Work

Chameleon [30] translates Haskell's global type inference into a constraint solving problem. During typechecking, the Haskell compiler will generate a large number of constraints, each of which carries source location information for debugging purposes. Debugging essentially involves finding a minimal set of constraints that can explain a typing error using a fixed set of constraint reduction rules. Our framework is focused on object-oriented languages, where it has been shown [27] that local type inference is more suitable in terms of providing better localized error messages and that handling subtyping constraints is still tractable [24].

The work of El Boustani and Hage [1] is the only work, to the best of our knowledge, which focuses on modern object-oriented programming languages that support parametric and subtype polymorphism. They present a number of heuristics that improve error messages for Java with Generics by implementing a typechecker which collects subtyping constraints for very localized problems, specifically only method invocations that have local type parameters and variance. Our approach crosses the boundaries of function application, and therefore performing a more global type error analysis. In fact, our approach can also be used for explaining typing decisions related to a much richer type inference mechanism, and other type system features such as overloading, higher-kinded types and implicit search. In our evaluation, we show that heuristics used in [1] can be implemented in our framework.

Tsushima *et al.* [33] attempt to build type derivation trees by treating an existing typechecker as an oracle that recursively typechecks subtrees of the original program. The reconstruction of this type derivation tree is performed by interacting with users. While the main idea of our framework is to reconstruct similar typing rules, we avoid any user interaction which would be infeasible when attempting to visualize and debug advanced type system features in an straightforward way [28].

1.2 Contributions

This paper makes the following contributions:

- (1) A novel approach to type error customization and debugging framework that is based on type derivation trees from the actual type checking process (achieved by instrumenting the Scala compiler in a lightweight way), and (2) a programmer-accessible high-level representation of typechecking decisions. Thus, our high-level typechecking representation reflects the behavior of the real world Scala typechecker, while still supporting the full set of Scala language features.

- A set of programmer-facing generic routines that make it possible to traverse and analyze the high-level typechecking representation and to suggest and verify code modification for type errors. Such routines can be used to build a number of useful system for interacting with and customizing type error feedback, from graphical type debugging tools to type analysis plugins for IDEs.
- A plug-in architecture that can be used by the authors of libraries and embedded DSLs. This makes it possible for developers of such DSLs to provide additional domain-specific error analysis and type errors to provide their users with comprehensive and accurate domain-specific feedback, rather than relying on the basic error-reporting infrastructure of the host language.
- An extensive real world validation of these techniques; we integrate our framework into the full-fledged Scala compiler, and perform a detailed analysis of several advanced Scala libraries and frameworks which make advance use of the type system.

2 Type Derivation Trees: A Basis

Typechecking a program can be thought of as instantiating inference rules, which simply means to substitute concrete terms and types into the general inference rule, and then constructing a *type derivation tree* based on those inference rule instantiations. Such type derivation trees are central to our framework.

Basing type debugging and customization on type derivation trees is an advantageous choice for numerous reasons. Because such trees are based on the actual instrumented Scala typechecker,

- all Scala language features are supported by default,
- changes to the typechecker will generally never result in the type debugging and customization framework becoming out-of-sync,
- the framework can deal with OO concepts such as subtyping, and
- the framework works even for embedded DSLs.

In this section, we'll see how type derivation trees can provide a high-level view into where errors appear in programs. We'll achieve this by first constructing a type derivation tree for the innocuous code snippet shown in Listing 1, and then by analyzing it, to locate the typing error.

Section 2.1 introduces an established formalism [25] that we will use in section 2.2 to construct the type derivation tree which we will use to illustrate how such trees can aid in type debugging.

2.1 Formalization

Since the cryptic type error shown in Listing 1 is caused by a known limitation of Scala's local type inference, we

Definition 1. Core language syntax, from [25, pg 3].

$$\begin{array}{llcl}
\textbf{Terms} & E, F & = & x \mid E.x \\
 & & \mid & \mathrm{fun}[\overline{a}](x:\ T)E \mid \mathrm{fun}(x)E \\
 & & \mid & F[\overline{T}](E) \mid F(E) \\
 & & \mid & E.x \\
 & & \mid & \{x_1 = E_1, ..., x_n = E_n\} \\
\textbf{Types} & T, S, R & = & a \mid \top \mid \bot \\
 & & \mid & T \xrightarrow{a} S \\
 & & \mid & \{x_1 : T_1, ..., x_n : T_n\} \\
\textbf{Environments} & \Gamma & = & x : T \mid \epsilon \mid a \mid \Gamma, \Gamma'
\end{array}$$

adopt the formalism and type system described in Odersky *et al.*'s treatment of Scala's colored local type inference [25] to build our type derivation tree.

For the sake of brevity and clarity, we present a fragment of the typing rules of local type inference, as defined in [25].

2.1.1 Grammar

Definition 1 shows our core language syntax, which itself is based on $F_{\leq}$ extended with records, as per [25, 27].

The grammar gives terms, types and environments of the language. A term can be a variable x, a record constructor $\{x_1 = E_1, ..., x_n = E_n\}$ or record selection $E.x$. It also has two versions of function application and abstraction: ones with explicit type parameters and type arguments ($F[\overline{T}](E)$ and $\mathtt{fun}[\overline{a}](x\ :\ T)E$) and those that elide them, if possible, by conveniently inferring them from the context ($F(E)$ and $\mathtt{fun}(x)E$), respectively. The overbar in $\overline{a}$ signifies a finite sequence of local type parameters, and is equivalent to $a_1, ..., a_n$ for some n. A type is a either a type variable a, a top or bottom type in the type hierarchy ($\top$ and $\bot$ respectively), a potentially polymorphic function type $T \xrightarrow{a} S$ (with type variables written over the arrow), or a record type.

2.1.2 Typing Rules

A fragment of the local type inference type system is presented in Figure 1.

The inference judgment, $P, \Gamma \vdash^{w} E : T$ consists of the term to be typed, E, that eventually is assigned type T in a type environment Γ, and a *prototype* P representing parts of the type of E that are inherited from the context. P can be treated as a normal type, potentially having some type constants ? in place of the missing parts. For instance, given a judgment of $Int\ \to ?, \epsilon \vdash^{w} \mathtt{fun}(x)x : Int \to Int$ we know that the type of the only parameter x is Int and is inherited, whereas the return type is synthesized from the body of the function.

Since we are only interested in constructing a type derivation tree for the `foldRight` example shown in Listing 1, we only discuss rule `(app)` for function application with elided type arguments. The first premise of `(app)` expresses typechecking of function F without any *prototype*, and it requires the inferred type to be of a function type $S \xrightarrow{a} T$. Hence, in the second premise, the argument can be typed with an expected type coming from the just resolved function, with all the unknown type parameters substituted by constant type ?.

Matching between the expected *prototype* and synthesized type is expressed through operator $\nearrow$. $T \nearrow P$ means

$$(\textsc{abs})\ \frac{P, \Gamma, \overline{a}, x : T \vdash^w E : S}{T \xrightarrow{a} P, \Gamma \vdash^w \textbf{fun}(x)E : T \xrightarrow{a} S} \qquad (\textsc{abs}_{tp,?})\ \frac{?, \Gamma, \overline{a}, x : T \vdash^w E : S}{?, \Gamma \vdash^w \textbf{fun}\,[\overline{a}]\,(x : T)E : T \xrightarrow{a} S}$$

$$(\textsc{app})\ \frac{?, \Gamma, \vdash^w F : S \xrightarrow{a} T \quad [?/\overline{a}]\, S, \Gamma \vdash^w E : S' \\ \vdash_a S' <: S \Rightarrow C_1 \qquad \vdash_a T <: \top \searrow P \Rightarrow C_2}{P, \Gamma \vdash^w F(E) : \sigma_{C_1 \cup C_2, T} T \nearrow P} \qquad (\textsc{app}_{tp})\ \frac{?, \Gamma, \vdash^w F : S \xrightarrow{a} T \quad \left[\overline{R}/\overline{a}\right] S, \Gamma \vdash^w E : \left[\overline{R}/\overline{a}\right] S}{P, \Gamma \vdash^w F\left[\overline{R}\right](E) : \left[\overline{R}/\overline{a}\right] T \nearrow P}$$

$$(\textsc{var})\ P, \Gamma \vdash^w x : \Gamma(x) \nearrow P$$

Figure 1. Fragment of $P, \Gamma \vdash^w E : T$ judgment (as presented in [25, pg. 11])

that either T is structurally equal to P, with $?$ filled by some arbitrary types, or we can find the smallest supertype of T which is structurally equal to P. The operation $T \searrow P$ is the dual of $T \nearrow P$, where the greatest subtype of T is structurally equal to P.

2.2 Constructing the Type Derivation Tree

Consider the following definition of the `foldRight` function mentioned earlier:

```
foldRight = fun[a](elems: List[a]) fun[b](acc: b)
  fun(f: (a, b) -> b)
    elems.match {
      caseNil()        = acc
      caseCons(x, ys) = f(x, foldRight(ys)(acc)(f))
    }
```

This definition uses a straight-forward encoding of lists (summarized in Appendix B).

`foldRight` simply traverses the initial list of elements, `elems`, until an end is reached, `caseNil`, where it returns the initial accumulator. Then it will recursively backtrack, and apply function `f` to the intermediate result of applying `foldRight` to the result of the suffix of the list. Using the typing rules from [25, pg. 6], function `foldRight` has type, $List[a] \xrightarrow{a} b \xrightarrow{b} ((a, b) \rightarrow b) \rightarrow b$, which allows us to express the erroneous application from Listing 1 as: $\texttt{foldRight}(\texttt{Cons}(1, \texttt{Nil}())(\texttt{Nil}())((x, y) \rightarrow \texttt{Cons}(x{+}1, y))$

By applying the typing rules from Figure 1 to this definition of `foldRight`, we can construct the type derivation tree shown in Figure 2.

2.3 Using Type Derivation Trees to Debug `foldRight`

The typing error from Listing 1 is highlighted in the derivation tree in Figure 2 as **(type-mismatch)**. It results from the structural inequality between the inferred type, $List\,[Int]$, of the body of the function which actually performs the list manipulation, and an inherited expected type, $List\,[\bot]$. $List\,[Int]$ is inferred because lists are covariant in their type parameter, and the least substitution of Int `<:` a and $\bot$ `<:` a for a is Int, which means that the conflicting type originates in a different location in the type derivation tree.

In order to debug this, one has to figure out where the *prototype* $List\,[\bot]$ comes from. Type derivation trees allow us to backtrack through the type inference process, and to see how individual premises of the typing rules were arrived at. For the sake of clarity, in the figure, we highlight the types which led to the type mismatch in gray boxes, and use numerical superscripts to illustrate the order of our explanation.

Function f is the argument of the application of `foldRight(xs)(Nil())`, which inherits an expected type $(Int, List\,[\bot]) \rightarrow List\,[\bot]$ due to the `(abs)` rule from Figure 1. This type comes from the inference of the type of the function $\texttt{foldRight}(xs)(\texttt{Nil}())$ (superscript 4). There, we can see that the type responsible for the type mismatch was instantiated from the type parameter b (superscript 5), which in turn resulted from the application of the partially applied function $\texttt{foldRight}(xs)$ to `Nil()`. In this case, the type of the argument, `Nil()`, is simply $List[a]$. As previously alluded to, since variable a is in a covariant position, and carries no other constraints, the most appropriate solution coming from $\searrow$ results in the substitution of a to $\bot$. This, in effect, imposes the only constraint on type variable b, which is $List\,[\bot]\ <:\ b$. This constraint results in the highlighted substitution (superscript 10) that eventually leads to a type mismatch.

Given such a simple code snippet, one might be surprised at the complexity of the fragment of the type derivation tree shown in Figure 2. While presenting raw type derivation trees to users is certainly impractical, we nonetheless argue that type debugging via these type derivation trees is *powerful*. Traversing and analyzing such trees can provide essentially any information about the typechecking process, from a detailed picture of how language features interact, to the revelation of limitations of local type inference. Furthermore, as one might be able to infer from the grayed-out types in Figure 2, type derivation trees can be traversed and pruned so as to simplify and speed up the localization of types of interest, as we will show in section 4.1. We will also show in later sections how this approach serves as a foundation for a number of different systems to customize or debug type errors, from interactive visualization tools for type debugging [28], to custom domain-specific type errors for embedded DSLs.

3 From Typechecker to Type Derivation Tree

A central novelty of our approach is that a high-level representation of typing decisions is obtained directly from the result of an existing compiler's type checking process, all without modifying the logic of the existing typechecker or reducing its features. Throughout the remainder of the paper, in the context of the type debugging framework, we refer to typing rules as *goals*, which can have some *premises* that, by definition, must be fulfilled for the rule (or goal) to be applied.

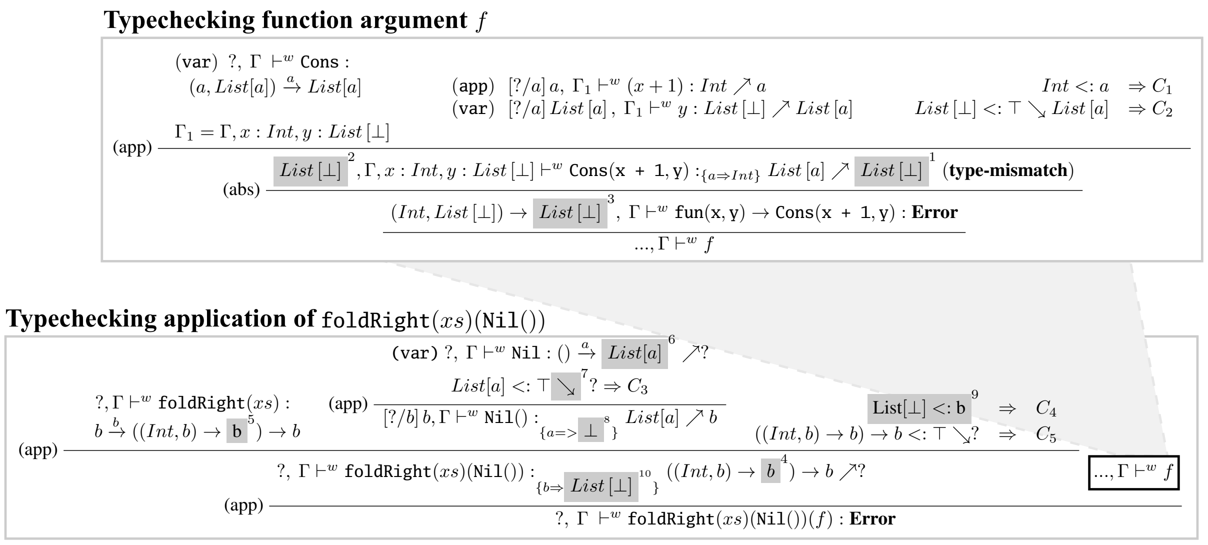
Typechecking function argument f
(var) ?, $\Gamma \vdash^w$ Cons : $(a, List[a]) \xrightarrow{a} List[a]$
(app) $[?/a]\, a,\ \Gamma_1 \vdash^w (x+1) : Int \nearrow a$
(var) $[?/a]\, List[a],\ \Gamma_1 \vdash^w y : List[\bot] \nearrow List[a]$
$Int <: a \Rightarrow C_1$
$List[\bot] <: \top \searrow List[a] \Rightarrow C_2$
$\Gamma_1 = \Gamma, x : Int, y : List[\bot]$
(app)
$List[\bot]^2, \Gamma, x : Int, y : List[\bot] \vdash^w$ Cons(x + 1, y) $:_{\{a \Rightarrow Int\}} List[a] \nearrow List[\bot]^1$ (type-mismatch)
(abs)
$(Int, List[\bot]) \to List[\bot]^3,\ \Gamma \vdash^w$ fun(x, y) → Cons(x + 1, y) : Error
$..., \Gamma \vdash^w f$
Typechecking application of foldRight(xs)(Nil())
(var) ?, $\Gamma \vdash^w$ Nil : $() \xrightarrow{a} List[a]^6 \nearrow ?$
$List[a] <: \top \searrow^7 ? \Rightarrow C_3$
?, $\Gamma \vdash^w$ foldRight(xs) : $b \xrightarrow{b} ((Int, b) \to b^5) \to b$
(app)
$[?/b]\, b, \Gamma \vdash^w$ Nil() $:_{\{a => \bot^8\}} List[a] \nearrow b$
$List[\bot] <: b^9 \Rightarrow C_4$
$((Int, b) \to b) \to b <: \top \searrow ? \Rightarrow C_5$
(app)
?, $\Gamma \vdash^w$ foldRight(xs)(Nil()) $:_{\{b \Rightarrow List[\bot]^{10}\}} ((Int, b) \to b^4) \to b \nearrow ?$
$..., \Gamma \vdash^w f$
(app)
?, $\Gamma \vdash^w$ foldRight(xs)(Nil())(f) : Error

Figure 2. Fragment of type derivation tree for application foldRight(xs)(Nil())(f), where xs = Cons(1, Nil()) and f = fun(x, y) → Cons(x + 1, y)

Our type customization and debugging framework builds on top of the existing implementation of Scala [23], a hybrid object-oriented and functional language. Apart from having Java-like features and running on the JVM, Scala has an advanced type system which includes features such as higher-kinded types [18], path-dependent types [21] and implicits [26], all of which are supported by our framework.

In this section, we will step through the construction of this high-level representation, beginning in section 3.1 with details of our compiler instrumentation API. Section 3.2 discusses how *goals* are defined in our high-level representation. Finally, section 3.3 describes how we can transform low-level data obtained from instrumenting the Scala compiler to high-level *premises* and *goals* introduced in section 3.2.

3.1 Instrumenting the Compiler

The type debugging framework collects low-level typechecking information by instrumenting[2] the existing Scala compiler using a minimal API, a set of low-level instrumentation classes, and an infrastructure for debugging. A small example is shown in Listing 2, where we instrument the body of a method that typechecks some AST (`tree`) given a *prototype* (`tpe`).

In this example 2, an `EV` object collects all instrumentation notifications. `TyperTypecheck`, `AstTyped`, and `TyperTypecheckFinished` of type `Event` are concrete instrumentation classes that carry raw typechecking information, such as type or symbol references.

[2] We chose to manually instrument the Scala compiler since the alternative is to modify bytecode (*e.g.*, `http://eclipse.org/aspectj/`), which is too coarse-grained. That is, bytecode modification only provides means to extract compiler data at the entry and exit point of method calls; obtaining typechecking information from within the body of methods is not possible to recover without reconstructing the logic of the mainline typechecker.

```
def typecheckAst(ast: Tree, tpe: Type): Tree = {
  EV <<< TyperTypecheck(ast, expectedType)
  ... // instrumented typing of ast
  EV << AstTyped(...)
  ...
  EV >>> TyperTypecheckFinished(...)
  ...
}
```

Listing 2. A brief look at the Instrumentation API.

Importantly, we introduce the notion of an *instrumentation block*, which makes it possible for *structural* information to be collected during instrumentation, which results in instrumentation data that more closely reflects the premises-conclusion relations in the typing rules, as opposed typical "flat" instrumentation data. These instrumentation blocks are delimited by the <<< and >>> operators, and typically also contain other (potentially nested) instrumentation blocks, delimited using the same operators, as well as single instrumentation points (defined using the << operator). As a result, the framework understands that direct instrumentation points between the <<< and >>> method calls can be considered as typechecking dependencies, without having direct references to them in the source code.

While stuctured, the type of information that instrumentation collects is quite low-level. *Raw* compiler data, such as ASTs that are being typechecked, expected types inherited from the context, or type parameters to be instantiated, are among the bits of low-level data collected. Indeed, this sort of data is a far cry from the high-level type derivation trees constructed from that we're after, as it carries no knowledge of related premises that need to be satisfied, nor of enclosing typechecking context information.

Finally, while instrumentation does incur non-negligible compilation time performance penalties while collecting data, our framework manages to sidestep noticeable slowdowns by selectively enabling instrumentation only when typechecking regions of the source code that directly affect the detected errors. We additionally save performance by making use of the fact that definitions in the Scala compiler are initialized in a lazy manner *i.e.,* their type signatures are verified or inferred once per compilation, at the first point when they are referenced during typechecker's run.

3.2 High-Level Typechecking Representation

In this section, we provide a brief overview of our high-level typechecking representation, based on type derivation trees. In section 3.3, we delve into the details of our high-level representation and how it maps to our low-level instrumentation data.

```
abstract class Goal {          | trait Typecheck extends Goal {
 def allPremises: List[Goal] |   def typeg:   TypeGoal
 def parent:      Goal        |   def adaptg:  AdaptGoal
}                              | }
```

Listing 3. Base class of high-level representation and goal `Typecheck` that represents typechecking of an AST

Listing 3 provides a class for a `Goal` (or typing rule) that has a reference to all its premises, and a conclusion, `parent`, which is also of type `Goal`. Subclasses of `Goal` and their members provide concrete requirements for each of the typechecking decision points. Nodes of a type derivation tree can be constructed from subclasses of `Goal`s which represent numerous concepts such as typing of an abstraction or typing of function application.

`Typecheck` is an example of such a subclass. In order to satisfy this goal, for example, its members require that it first be typed and then adapted. Note that the process of typing and adaptation in this example are represented via the types of `Typecheck`'s members, which are also subclasses of `Goal`.

3.3 Mapping From Low-Level to High-Level

To understand how we map from low-level instrumentation data to high-level type derivation trees, we provide an explanation from two different angles. We first begin with a side-by-side example of instrumented code, and high-level goals in Figures 4 and 5, respectively, which gives an intuition for how our implementation achieves this mapping. We then provide a formalization of this mapping, in an effort to provide a complete and general depiction of how our framework translates from low-level instrumentation events to high-level goals and premises.

3.3.1 Instrumention to Goals & Premises: An Example

Listing 4 presents a simplification of the actual Scala compiler's implementation, and instrumentation, of typing function application. In this example, the method `typeApplication` takes an argument, `ast`, which is an `Apply` AST node (an AST node that represents function application). This AST has two parameters, the AST for the function, and the AST for the function's arguments, both of which are extracted via a pattern match on line 2.

The first instrumentation block is delimited by instrumentation classes `TypeApplication` and `TypeAppDone`. This creates a logical block of instrumentation which essentially specifies that any instrumentation executed in between is part of the logic responsible for typing an function application. The function itself is typechecked via method `typecheckAst`, which is itself also instrumented (not shown).

The rest of Listing 4, beginning on line 4, represents the logic for actually typing function application. Lines 5, 11, and 14 illustrate different possible paths through this typechecking method; a function type having type parameters (`PolymorphicType`, line 14), a monomorphic function type (`MethodType`, line 11), and an erroneous function type (`ErrorType`, line 5). Importantly, each different path through this `typeApplication` method result in the instantiation of different sorts of instrumentation classes.

Now that we've seen an example of a typechecking method, we now look at the high-level representation that we'll create from the low-level instrumentation events.

Listing 5 gives a high-level representation for typing function application. We use inheritance to abstract over different typechecker executions (one can think of `traits` in Scala as a simple Java `interface` in this context). These interfaces give users of the high- level representation a handle on the possible decisions the typechecker can make. As we will see in section 4, this also enables users to pattern match [7] on these decisions.

In Listing 5, a `TypeApp` trait states that any typing of function application has to always first typecheck the function (member `typecheckFun`) and directly corresponds to the first premise of typing rule `app` in Figure 1. Traits `TypeAppFallback` and `TypeAppCorrect` correspond to different typechecker executions involving erroneous and error-free results of typechecking a function. Trait `TypeApplicationMain`, whose type is listed as the type of a premise of `TypeAppCorrect`, represents the typing of an application given a (valid) function type. Its subtraits `TypeAppMonomorphic` and `TypeAppPolymorphic` correspond to executions involving typing of monomorphic and polymorphic applications, respectively. This distinction between subtraits of `TypeApplicationMain` directly reflects the presence, or lack thereof, of type variables in the inference rule (`app`). Member `targsFromExpectedType` corresponds to the potential inference of type arguments from the result type of the method and the expected type (recall `InferTypeArguments` in Listing 4), and member `inferMethInstance` corresponds to the act of inferring type arguments from the typechecked arguments and formal parameters.

In order to relate these low-level instrumentation events to this high-level representation, we return to the low-level in-

```
def typeApplication(ast: Apply, expected: Type): Apply = {
 EV <<< TypeApplication(ast)
 val Apply(funAst, argsAsts) = ast
 val fun1 = typecheckAst (funAst, ...)
 val typedApp: Apply = if (fun1.tpe == ErrorType) {
   EV <<< InvalidFunApp(...)
   ...
  } else {
   EV <<< TypeApp1(...)
   fun1.tpe match {
    case MethodType(parameters, _) =>
     val args1 = argsAsts map (arg => typecheckAst (arg, ...))
     if (hasError(args1)) ... else ...
    case PolymorphicType(tParams,MethodType(params,resultType)) =>
     EV <<< InferTypeArguments(expected, tparams)
     val argsTypes = expectedTypeForArgs(typeParams,
                        resultTpe, expected)
     EV >>> Inferred(argsPt)
     val args1 = (argsAsts zipWith argsType) map (
            (arg, argExpectedType) => typecheckAst (arg, ...))
     if (hasError(args1)) { ... } else {
      EV <<< InferInstance(argsTypes, params, args1)
      val inferredMethod = ...
      EV >>> InferredMeth(inferredMethod)
      if (hasError(inferredMethod)) ...
      else typeApplication(..., expectedType)
     }
   }
   EV >>> TypeAppFinished(...)
 }
 EV >>> TypeAppDone(typedApp)
 typedApp
}
```

Listing 4. Example of instrumented typing of function application

strumentation data. These low-level instrumentation events collected by instances of EV are essentially a sequence of type checking events, which carry low-level information like symbols and expected types. Importantly, these events are delimited by methods <<< and >>>, and instances of instrumentation classes. These delimiters enable us to reconstruct the nesting structure characteristic of type derivation trees.

In order to determine which high-level goal a given instrumentation event corresponds to, we pattern match on these sequences. If we succeed in matching, we create a corresponding high-level representation. In the following section, we'll see how this matching works. These high-level representations refer back to the low-level events that they were created from. Subtle differences between low-level instrumentation, especially in the presence of type errors, result in different high-level type derivation trees.

3.3.2 Formalization

In this section, we formalize a mapping function which maps instances of low-level instrumentation into its high-

```
trait TypeApp              extends TypeGoal {
  def typecheckFun:                 TypeGoal
}
trait TypeAppFallback      extends TypeApp { ... }
trait TypeAppCorrect       extends TypeApp {
  def typeApp:                      TypeApplicationMain
}
trait TypeApplicationMain extends Goal
trait TypeAppMonomorphic  extends TypeApplicationMain {
  def typecheckArguments:           List[TypeGoal]
}
trait TypeAppPolymorphic  extends TypeApplicationMain {
  def targsFromExpectedType:        InferPotentialTypeArguments
  def typecheckArgs:                List[TypeGoal]
  def inferMethInstance:            InferMethodInstance
  def typeInferredApp:              Option[TypeApplicationMain]
}
```

Listing 5. A fragment of the high-level representation corresponding to typechecker decisions necessary to type a function application

level counterparts, in a one-to-many relation. The algorithm transforms low-level instrumentation recursively in a depth-first postfix manner. In other words, for any block of instrumentation data, we first map all low-level instrumentation contained within it, and then use that information as a context for the selection of a single high-level representation, based on the types of members of potential high-level goals.

Definition 2. High-Level Representation & Dependencies

$$
\begin{array}{lll}
\mathtt{F} & ::= & (f,\ \mathtt{S}) \qquad\qquad \text{(premise)} \\
\mathtt{S} & ::= & \mathtt{List[T]} \mid \mathtt{T} \quad \text{(type of premise)}
\end{array}
$$

Any high-level representation dictates its requirements on typechecking decisions through premises. Definition 2 presents each premise, F, in the form of a tuple consisting of a name, *f*, and its type, S, similarly to how we define members for Goals.

Definition 3. Mapping: Type Signatures

$$
\begin{array}{rl}
\mathtt{premises}: & \mathtt{T} \Rightarrow \overline{\mathtt{F}} \\
\mathtt{linearlization}: & \mathtt{T} \Rightarrow \overline{\mathtt{T}} \\
\mathtt{reverse}: & \overline{\mathtt{T}} \Rightarrow \overline{\mathtt{T}} \\
\mathtt{spec}: & T \Rightarrow \overline{\mathtt{S}} \\
\mathtt{targetT}: & \mathtt{Event} \Rightarrow \overline{\mathtt{T}} \\
\mathtt{compare}: & \mathtt{Event} \Rightarrow \mathtt{MatchR} \Rightarrow \mathtt{MatchR} \Rightarrow \mathtt{Int} \\
\mathtt{sort}: & \overline{\mathtt{MatchR}} \Rightarrow (\mathtt{MatchR} \Rightarrow \mathtt{MatchR} \Rightarrow \mathtt{Int}) \Rightarrow \overline{\mathtt{MatchR}} \\
\mathtt{matchF}: & \overline{\mathtt{T}} \Rightarrow \overline{\mathtt{S}} \Rightarrow \mathtt{T} \Rightarrow \mathtt{MatchR} \\
\mathtt{head}: & \overline{\mathtt{MatchR}} \Rightarrow \mathtt{T} \\
\mathtt{map}: & \mathtt{Event} \Rightarrow \overline{\mathtt{T}} \Rightarrow \mathtt{T} \\
\\
\mathtt{MatchR} ::= & \mathtt{Success}\ \overline{\mathtt{S}}\ T \mid \mathtt{Partial}\ \overline{\mathtt{S}}\ T
\end{array}
$$

Definition 3 gives type signatures of functions used for defining the mapping of low-level instrumentation (subtypes of base type Event), to high-level representation, T (subtypes of Goal). We use the same notational conventions as in section 2.1.1.

Definition 4. Mapping: Function Definitions

$$
\begin{array}{lcl}
\mathtt{premises}(t) & = & \begin{cases} \epsilon & \textbf{if}\ \mathtt{t}\ <:\ \mathtt{Goal} \wedge \mathtt{Goal}\ <:\ \mathtt{t} \\ \overline{\mathtt{F}} & \textbf{if}\ \mathtt{t}\ <:\ \mathtt{Goal} \wedge \neg(\mathtt{Goal}\ <:\ \mathtt{t}) \end{cases} \\
\mathtt{spec}(t) & = & [S_f \mid (\mathtt{f}, S_f) \leftarrow \mathtt{premises}(T_n), \\
 & & \quad T_n \leftarrow \mathtt{reverse(linearization(t))}] \\
\mathtt{head}(matches) & = & \begin{cases} t & \textbf{if}\ \mathtt{matches}\ =\ \mathtt{(Success\ s1\ t),...,}\ m_n \\ t & \textbf{if}\ \mathtt{matches}\ =\ \mathtt{(Partial\ s1\ t),...,}\ m_n \end{cases} \\
\mathtt{map}(ev)(ps) & = & \mathtt{head(sort(}\ [\ \mathtt{(matchF(}ps\mathtt{)(spec(T}_G\mathtt{))(T}_G\mathtt{)}\ \mid \\
 & & \qquad \mathtt{T}_G \leftarrow \mathtt{targetT}(ev)\ ]\mathtt{)(compare(}ev\mathtt{)))}
\end{array}
$$

The *premises* function returns a sequence of all *required* premises for a given static type T, respecting the order of declarations (T <: Goal means that Goal is among super types of T and ϵ represents an empty sequence) *e.g.,* in the case of TypeAppCorrect it would be only type TypeApplicationMain. The *linearization* function returns the chain of super types up to type Goal. Function *spec* follows the *linearization* chain to retrieve all (declared and inherited) types of premises. *targetT* function declares possible high-level representations for every low-level instrumentation (it is defined by compiler experts). In order to express the accuracy of mapping a low-level instrumentation to some concrete high-level representation, we use an internal data structure of MatchR. MatchR represents the result of matching *actual* premises to *required* ones *i.e.,* how already mapped low-level instructions within the block (recall the postfix order of mapping), fit with members of high-level Goal. It can be either Success, meaning that the mapping function has found real dependencies ($\bar{\mathtt{S}}$) for each of the *required* premises for the Goal of type T, or Partial, meaning that only some ($\bar{\mathtt{S}}$) have been found.

The function matchF(ps)(ts)(g) attempts to map each *required* premise $t \in ts$ to an *actual* premise $p \in ps$. This mapping respects the order of the *actual* and *required* premises. To express the properties of matchF more formally, we use a function *match* of type $(\mathtt{MatchR}, \mathtt{Int}) => \bar{\mathtt{T}}$. It extracts a sequence of *actual* premises, $\overline{Y}$, that have been associated with a *required* premise at a provided index. Using *match*, we can express the order-preserving mapping of matchF as follows:

1. $\forall i \forall j, j > i \wedge \mathtt{match}(m, i) = \overline{Y} \wedge \mathtt{match}(m, j) = \overline{Y'} \Rightarrow (\forall k, l Y_k \in \overline{Y} \wedge Y'_l \in \overline{Y'} \Rightarrow l > k)$
2. $\forall i, \mathtt{match}(m, i) = \overline{Y} \Rightarrow (\forall a \forall b, a < b \wedge Y_k^a \in \overline{Y} \wedge Y_l^b \in \overline{Y} \Rightarrow k < l)$

The subscript j in Y_j^k represents an index of the premise Y within an initial sequence of ps, while superscript k represents an index of an element within sequence $\overline{Y}$. The first statement ensures that *actual* premises respect the order in which they were mapped to *required* premises, while the second ensures that the order within a sequence associated with a member is also preserved.

Sorting Matching Results The role of function matchF is only to deterministically associate *required* with *actual* premises for a single high-level representation. It does not guarantee that it will always return a single Success object if we apply it to different high-level Goals coming from targetT function. In fact, since we do not restrict in any way how high-level representations are defined, or what mapping is provided by compiler experts, we can have multiple Success results for a one-to-many mapping between the representations or none at all. Thus, in order to disambiguate, we sort the matching results using *compare*.

The first parameter of *compare* expresses the fact that the comparison is bound with a particular instance of low-level representation. The *compare* function returns a negative value, zero or a positive value as the second argument is less specific than, equal to, or more specific than the third one. The sorting function *sort* takes a sequence of (possibly) ambiguous results of matching the premises, and sorts them using this instance-bound *compare*.

The Final Mapping Step The mapping function *map* in Definition 4 extracts the first, most specific, matching result and returns the high-level representation associated with it using the *head* function. As a result, *map* is capable of reconstructing type derivation trees by mapping low-level instrumentation to a high-level representation. As (potentially) ambiguous mappings are detected during the *head* operation since it checks if the next matching result in sequence is strictly less specific.

4 Programmatically Exploring Typechecking

Our framework implements a number of generic search algorithms on top of a high-level representation that can answer common typechecking questions in the form of located Goals. In section 4.1, we describe a generic technique that uses the shape of the type derivation tree and the types of terms to precisely prune the search space. Section 4.2 gives an overview on how to combine this searching and pruning with the high-level representation to understand decisions made by typechecker. Section 4.3 gives an example on how we can analyze type constraints used in type inference, and finally, in section 4.4 we show how our framework can suggest and verify code modification for type errors.

4.1 Pruning the Type Derivation Tree Search Space

Brute force navigation through the high-level representation is impractical for even small programs, while using heuristics that perform undirected depth-first search over type derivation trees is imprecise. We present a simple mechanism that we call TypeFocus, which allows us to gradually build knowledge about the evolution of a type of interest as we step through typing rules and at the same time prune the search space.

TypeFocus is a function that takes a type and extracts some part of it based on its shape. Definition 5 provides examples for function and polymorphic function type (Error simply means that type extraction was invalid). We create instances of TypeFocus to exploit the fact that parts of *prototype* are used in a consistent way in the premises of typing rules *e.g.,* abstraction, (abs), uses only the result type of the *prototype*. More importantly, we can freely compose TypeFocus *i.e.,* the composition of TypeFocus, say f and g, applied to some type, $(f \circ g)(tp)$, is equivalent to $f(g(tp))$. This gives us an opportunity to create compositions of TypeFocus instances from multiple, potentially distant, typing rules and still extract specific portion of a type.

Applying TypeFocus to types of terms allows us decide whether a potential type inference, that is performed as part of a typing rule, should guide our type derivation tree exploration. During the first stage of type mismatch explo-

Definition 5. *Type focus.*

$$
\begin{array}{ll}
\texttt{Focus} : T \Rightarrow T & \\
\texttt{foc-fun-res}(tp) & = \left\{ \begin{array}{ll} S & \textbf{if } \texttt{tp} = T \rightarrow S \\ \texttt{Error} & \textbf{otherwise} \end{array} \right\} \\
\texttt{foc-fun-param}_n(n)(tp) & = \left\{ \begin{array}{ll} T_n & \textbf{if } \texttt{tp} = (T_1, ..., T_n, ..., T_m) \rightarrow S \\ \texttt{Error} & \textbf{otherwise} \end{array} \right\} \\
\texttt{foc-poly-res}(tp) & = \left\{ \begin{array}{ll} T \rightarrow S & \textbf{if } \texttt{tp} = T \xrightarrow{a} S \\ \texttt{Error} & \textbf{otherwise} \end{array} \right\} \\
\texttt{foc-all}(tp) & = tp
\end{array}
$$

ration, we want to understand how parts of the *prototype* have been used to typecheck the argument so that we can build an appropriate TypeFocus composition. Later, we can engage TypeFocus, by applying it to types of terms, in order to identify premises that perform type inference.

4.1.1 Building TypeFocus for Type Mismatch Errors

Definition 6 presents a recursive build function that backtracks from an actual error to function application. During this process, we construct an appropriate composition of TypeFocus instances based on the sort of typing rule used. Such construction stops at the function application typing rule because we know that its *prototype* does not affect the *prototype* used for typing the erroneous argument. The function parent-rule returns a reference to the instance of a parent typing rule it is part of in the derivation tree. Helper functions, *arg$_n$-error* and *result-type-error*, allow us to determine whether an error occurred in a premise that types an argument or simply because of the method result type mismatch.

Definition 6. Building TypeFocus from type mismatch:

$$
\begin{array}{l}
\texttt{build}\text{: } \textit{typing-rule} \Rightarrow \texttt{Focus} \Rightarrow \texttt{Focus} \\
\texttt{build}((\text{abs}))(f) = \texttt{build}(\textit{parent-rule}(abs))(f \circ \texttt{foc-fun-res}) \\
\texttt{build}((\text{app}))(f) = \left\{ \begin{array}{lll} f \circ \texttt{foc-fun-param}_n & \textbf{if} & \textit{arg}_n\textit{-error}(app) \\ f \circ \texttt{foc-fun-res} & \textbf{if} & \textit{result-type-error}(app) \end{array} \right\} \\
\texttt{build}((\text{var}))(f) = \texttt{build}(\textit{parent-rule}(var))(f)
\end{array}
$$

build for a derivation tree that typechecks an argument of type mismatch from Figure 2 will therefore construct composition foc-all ∘foc-fun-res ∘ foc-fun-param$_0$.

4.1.2 Analysis of Type Derivation Trees Based on TypeFocus

As each of the typing rules assigns a type to a term, we can use TypeFocus in order to understand if typing rules perform type inference that later leads to a type error. Figure 3 defines analysis for each of the typing rules present in the local type inference formalization. The analysis judgment, $\texttt{Fc}, P, \Gamma \vdash^w t, \overline{S}$, is similar to the type inference judgment, except for TypeFocus Fc, which guides exploration. Also, unlike type inference, the judgment infers a sequence of types, $\overline{S}$, that affect the inference of a type extracted by TypeFocus.

In the application rule, (APP-FOC), we use the information extracted from the function type to determine if type inference should affect our derivation tree exploration. This is because a particular function type may contain type variables that are of no importance to us. If that is the case, we continue exploration of the function with an updated TypeFocus.

In the case that our derivation tree for foldRight(xs)(Nil()) represents function application, the composition of TypeFocus would be foc-all ∘ foc-fun-res ∘ foc-fun-param$_0$ ∘ **foc-fun-res** ∘ **foc-poly-res**. Applying it to the type of the function, $b_1 \xrightarrow{b_2} ((Int, b_3) \rightarrow b_4) \rightarrow b_5$, results in a still uninstantiated type variable b_4 (indexes in type variable are only for presentational reasons). Since we have extracted type variables from the type using the TypeFocus, we know that the type inference that will be performed as part of that typing rule will also instantiate them, end eventually lead to type mismatch.

Rule (APP$_{tp}$-FOC), gives an example where applying TypeFocus identifies explicit type arguments that will lead to type errors.

The framework provides also a more refined TypeFocus that is built from a high-level representation of failed algorithmic subtyping operations. This allows for even more precise error reporting but also requires some refinements in existing judgment, hence we describe the details only in Appendix C.

4.2 Exploring Type Derivation Trees

In this section, we show that our high-level representation and TypeFocus do a good job of implementing exploration techniques in a generic way, *i.e.,* without making any assumptions about the source code or user input.

In order to express analysis rules our framework conveniently allows for pattern-matching on instances of Goals to extract their premises[3].

Listing 6 shows that we can conveniently enumerate through a list of typing rules by pattern matching on goal in line 2. While we match on Goals of typing conditionals, member selection, variables, and application, respectively, we can integrate analysis for further typing rules by simply adding new pattern matching cases. An example of such extension is given in method applicationContext, which essentially implements the analysis rule APP-FOC. Apart from the already-discussed function applications, we now also pattern match on TypeAppOverloaded, and as a result can support method overloading.

Listing 6 shows that we can conveniently enumerate through a list of typing rules by pattern matching on goal in line 2. While we match on Goals of typing conditionals, member selection, variables and application, respectively, we can integrate analysis for further typing rules by pattern matching on their high-level representation. An example of such extension is given in method applicationContext which essentially implements analysis rule APP-FOC. Apart from the already discussed polymorphic and monomorphic function applications, we now also pattern match on

[3] Scala allows for providing custom extractors [7] for pattern matching and the framework provides them for every Goal subtype

$$\text{(APP-FOC)}\ \frac{\begin{array}{l} ?, \Gamma, \vdash^{w} F : S \xrightarrow{a} T \\ \mathtt{Fc}_1 = \mathtt{Fc} \circ \mathtt{foc\text{-}fun\text{-}res} \circ \mathtt{foc\text{-}poly\text{-}res} \\ \mathtt{Fc}_1(S \xrightarrow{a} T) = X, \mathtt{tvars} = \textit{type-variables}(X) \\ \overline{R} = \left\{ \begin{array}{ll} \mathtt{tvars} & \textbf{if}\ (\mathtt{tvars} \neq \emptyset) \\ \overline{R_0} & \textbf{if}\ \mathtt{Fc}_1, ?, \Gamma \vdash^{w} F, \overline{R_0} \end{array} \right\} \end{array}}{\mathtt{Fc}, P, \Gamma \vdash^{w} F(E), \overline{R}}$$

$$\frac{\begin{array}{l} ?, \Gamma \vdash^{w} F : S \xrightarrow{a} T \\ \mathtt{Fc}_1 = \mathtt{Fc} \circ \mathtt{foc\text{-}fun\text{-}res} \circ \mathtt{foc\text{-}poly\text{-}res} \\ \mathtt{Fc}_1(S \xrightarrow{a} T) = X, \mathtt{tvars} = \textit{type-variables}(X) \\ \overline{U} = \left\{ \begin{array}{ll} [\, r \mid a \in \mathtt{tvars},\ (r,\ a)\ \leftarrow \textit{zip}\ \overline{R}\ \overline{a}] & \textbf{if}\ (\mathtt{tvars} \neq \emptyset) \\ \overline{U_0} & \textbf{if}\ \mathtt{Fc}_1, ?, \Gamma \vdash^{w} F, \overline{U_0} \end{array} \right\} \end{array}}{\mathtt{(APP}_{tp}\mathtt{\text{-}FOC)} \qquad \mathtt{Fc}, P, \Gamma \vdash^{w} F[\overline{R}](E), \overline{U}}$$

$$\text{(ABS-FOC)}\ \frac{\begin{array}{l} P, \Gamma, \overline{a}, x : T \vdash^{w} E : S \\ \overline{R} = \left\{ \begin{array}{ll} T & \textbf{if}\ (\mathtt{Fc}(T \xrightarrow{a} S) = T) \\ \overline{R_0} & \textbf{if}\ (\mathtt{Fc}(T \xrightarrow{a} S) = S\ \wedge \\ & \quad \mathtt{Fc} = \mathtt{Fc}_0 \circ \mathtt{foc\text{-}fun\text{-}res}\ \circ\ \mathtt{foc\text{-}fun\text{-}res}\ \wedge \\ & \quad\quad \mathtt{Fc}_0, P, \Gamma, \overline{a}, x : T \vdash^{w} E, \overline{R_0} \\ \emptyset & \textbf{else} \end{array} \right\} \end{array}}{\mathtt{Fc}, T \xrightarrow{a} P, \Gamma \vdash^{w} \mathsf{fun}(x)E, \overline{R}}$$

$$\text{(VAR-FOC)}\ \frac{\overline{R} = \left\{ \begin{array}{lll} \mathtt{Fc}(\Gamma(x)) & \textbf{if} & (\mathtt{Fc}(\Gamma(x)) \neq Error) \\ \emptyset & \textbf{else} & \end{array} \right\}}{\mathtt{Fc}, P, \Gamma \vdash^{w} x, \overline{R}}$$

Figure 3. Fragment of analysis of typing rules based on TypeFocus

TypeAppOverloaded, and as a result can support method overloading.

```
0  def dispatch(goal: TypeGoal, focus: TypeFocus): List[Goal] =
1   goal match {
2    case TypeIfElse(_, thenP, elseP, leastUpperBound) => ...
3    case TypeMemberSelection(qualifier, memberSel)    => ...
4    case TypeVariable()                               => ...
5    case TypeApp(_)          => applicationContext(goal, focus)
6   }
7  def applicationContext(app: TypeApp, focus: TypeFocus) =
8   app match {
9    case TypeAppCorrect(fun, app@TypeAppPolymorphic(_, infer, _)) =>
10    val focus1 = focus compose FocFunRes compose FocPolyRes
11    val tpePart = focus1(typeOfFun(fun))
12    if (tpePart == Error) app :: Nil
13    else if (hasTypeVariables(tpePart)) inferInstance(infer, focus1)
14    else app :: dispatch(fun, focus1)
15   case TypeAppCorrect(fun, app: TypeAppMonomorphic)   => ...
16   case TypeAppCorrect(fun, app: TypeAppOverloaded)    => ...
17  }
```

Listing 6. Fragment of a generic routine that finds the source of the type, based on TypeFocus

While the high-level representation is clearly necessary for navigating derivation trees, it is not sufficient. TypeFocus operates on low-level type information in order to guide type exploration. While in line 11, we extract low-level information using function typeOfFun, we therefore make our exploration implementation dependent.

4.3 Reconstructing Local Type Inference Decisions Using the High-Level Representation

An advantage of local type inference is that all inference decisions are local, in the sense of the proximity of Goals, and are based on type constraints that are also locally collected. In this section, we exploit this fact by describing how, in using only TypeFocus and a type inference Goal, we can recreate specific source code locations that affected type inference.

```
trait InferMethodInstance extends InferInstance {
 def subExpectedTpeWithResTpe: SubtypeCheck
 def subArgsTpesWithFormals:   List[SubtypeCheck]
 def inferInstantiations:      List[SolveTVar]
}
```

Listing 7. Instantiation of a polymorphic method

In Section 4.1, we have shown that by using TypeFocus we can precisely identify typing rules where inference will instantiate type variables from the extracted portion of a type. We therefore assume that we can identify InferMethodInstance Goals, presented in Listing 7, which are high-level representations of type inference decision process.

The interface of InferMethodInstance clearly states that before performing any instantiation of type variables in Goals of type SolveTVar, two typechecking decisions will have to be made, both of which involve a Goal of type SubtypeCheck. More importantly, SubtypeCheck Goals represent a high-level representation for subtype relation checks between the two types. Hence, the first two members, give a reference to derivation trees that perform subtyping checks between the function result type and the expected type, and types of arguments and formal parameters, respectively. In a type inference sense, derivations represented by the two members represent primary sources of type constraints.

The main concern is to identify only those type constraint Goals that affected the instantiation of particular type variables. Here, again, we can make use of TypeFocus to select only those SolveTVar Goals from the member of inferInstantiations that have been in the extracted type part of the polymorphic function type. As each type variable instantiation Goal, SolveTvar, also has a low-level reference of used type constraints, we can identify correct SubtypeCheck Goals by simply traversing derivation trees they represent in search for those constraints.

Knowing which subtyping relation checks led to type constraints is still far from the actual identification of source code locations used in type inference. This is because a corresponding low-level information of SubtypeCheck Goals only has information about types rather than terms. From the discussion in previous sections, we may recall that InferMethodInstance is itself a premise to a polymorphic function application goal, TypeAppPolymorphic. The latter has a member typecheckArgs of type List[TypeGoal], which represents the decision process for typechecking the arguments, and more importantly has low-level information about the expressions that were being typechecked at that point. Subtyping checks performed in the member subArgsTpesWithFormals of the type inference Goal, come directly from typechecking those arguments. As a result, we are able to identify expressions that affected type inference of selected type variables, without encoding that information directly in either the low-level or high-level representation.

4.4 Providing Type-Correct Code Modifications

The high-level representation provides a convenient environment for compiler experts and library authors to use their experience to suggest direct code modifications.

Since many of the errors related to limited local type inference, the framework exposes two of the typechecker's internal methods responsible for calculating *least upper bound* (*lub*) and *greatest lower bound* (glb) from a list of types (both take simply arguments of type `List[Type]`). This allows us to investigate a more suitable type instantiation for local type parameters that is not limited by the boundaries of parameters' list *i.e.,* in the motivating example of `foldRight` application we provide original type constraint `List[Nothing]` coming from `Nil` (discovered using technique from Section 4.3), as well as an inferred type `List[Int]` to *lub*, where typechecker returns better instantiation, namely `List[Int]`.

```
def typecheckAscription(exprPos: Position, lub: Type): Boolean
def typecheckTypeArguments(appPos: Position, targs: List[Type]): Boolean
def typecheckMethodSignature(appPos: Position, methodDef: Symbol,
                             newTypeSig: Type): Boolean
```

As a programmer, we still want to make sure that such change is precise and type correct, so the framework has to provide an API that can typecheck the modifications. The listing above presents a subset of API provided in the framework – `typecheckAscription` verifies an explicit type ascription of `lub` type for an expression at location `exprPos`, `typecheckTypeArguments` verifies an application of type arguments `targs` for a polymorphic function application at position `appPos`, and `typecheckMethodSignature` which modifies a type of a polymorphic method `methodDef` to a new type `newTypeSig` and typechecks its usage in function application at position `appPos`. This way programmers can experiment with different heuristics for code modifications both in generic and plugin approach and still remain conservative in terms of type feedback.

5 Domain-Specific Type Errors

Generic exploration algorithms and code modifications might still reveal internal details of the data structures in the debugging feedback, which is correct but not always desirable. This section describes a complementary feature of the framework infrastructure–type debugging plugins–that allows for further type error customization for eDSLs and general purpose libraries.

```
trait DebuggerPlugin {
  def definedFor: PartialFunction[Goal, Option[Fix]]
}
```

Any domain specific plugin has to implement a `DebuggerPlugin` trait presented in listing above. A single method defines a handler for the erroneous case by means of a partial function from `Goal` to an optional fix, `Fix`, that carries information necessary to generate an additional feedback (we leave out the definition of `Fix` for irrelevance).

Whenever an error `Goal` is encountered, the debugging framework will first check if there exists a plugin that can potentially handle it. If the custom plugin produces some user feedback, then it will be reported. On failure, the mechanism will find other potential plugins, or fallback to general-purpose type error debugging algorithms. Hence we allow for initial lightweight matching that is later verified through, potentially expensive, more broad type derivation tree analysis. Lightweight error matching ranges from simple regular expression matching on the error message to basic overview of the derivation subtree in the vicinity of the error.

```
  it should "check the status value" in {
      class Foo { def status: String = "OK" };  val id = new Foo
      id should ('status("OK"))
  }
// overloaded method value should with alternatives:
//  (notExist: org.scalatest.words.ResultOfNotExist)(implicit
//      existence: ...)Unit <and> ... <and>
//  (rightMatcherX1: org.scalatest.matchers.Matcher[Foo])Unit
// cannot be applied to
// (org.scalatest.matchers.HavePropertyMatcher[AnyRef,Any])
//    id should ('status("OK"))
//       ^
```

Listing 8. ScalaTest error for 'should' operator

Consider a fragment of a simple test specification written using a popular testing framework, ScalaTest[4], in Listing 8. Programmer has defined a correct expectation regarding an instance of class `Foo` in a human-readable style where `'status("OK")` represents an expectation on the member `status` to return a concrete string value. The error message, that spans over multiple lines, reveals intimidating internal details of the testing DSL.

```
0 def shouldOpError(realError: ErrorGoal) = realError.parent match {
1  case app@TypeAppOverloaded(_, typecheckArgs, inferAlts, _) =>
2   val TypeAppCorrect(fun, _) = app.parent
3   fun match {
4    case TypeMemberSelection(qual, TypeMemberInAdaptedQual(
5                                      adaptQual, typeAdapted)) =>
6     if (isCorrectShouldSymbol(typeAdapted)) {
7      val names = inferAlt map shouldMethodNames
8      ...
9 }}}
```

Listing 9. Fragment of ScalaTest plugin that handles overloaded 'should' operator

Listing 9 presents a fragment of a ScalaTest-specific plugin that identifies such operator overloading problem. Plugins do not enforce any rules on how particular errors should be identified and analyzed. The typical approach identifies the vicinity of the error location by pattern matching on type derivation trees and identifying used low-level symbols and types.

In line 1 we pattern-match on the parent of the error and expect high-level representation for method overloading, `TypeAppOverloaded`. Clearly reported error mentioned multiple alternatives in function application.

As the library author we also know that function is a member selection and *should* operator is provided through a DSL-specific implicit conversion. Our error handler ensures that this is the case in lines 4-6 by pattern-matching on

[4] http://www.scalatest.org

corresponding typing rule using high-level representation for member selection, `TypeMemberSelection`. The second argument, `TypeMemberInAdaptedQual`, specifies that typechecker had to adapt the qualifier to a particular member. In line 6 we use low-level symbol information from the adapted member selection, to make sure that the selected member is the `should` method provided in the DSL.

The DSL-author can provide specialized, human-readable information about each of the possible alternatives, which are expressed in premise `inferAlts` (line 2), in order to provide a more acceptable error message, such as:

```
'should' misses an operator to handle property 'status("OK").
Providing one of the operators like 'should not exist',
'should exist', 'should contain', 'should have', 'should be'
... or an implicit conversion to 'ShouldMatcher' is enough.
```

Since specialized type error debugging relies on identical high-level representation as regular debugging, plugins' programmers can reuse elements of the infrastructure to explore type derivations in a uniform way. This includes provided algorithms that identify the source of the expected type, locate constraints that affect type inference or even analysis of implicit search.

6 Real World Validation

In this section, we present several scenarios where we have applied our framework in practice, from the mainline Scala compiler, to a visual type debugging tool, to embedded DSLs. Additional validation is in the appendix; we apply our framework to the experimental validation found in [6], and show that our framework applies appropriate feedback in all cases (Appendix E), and we apply it to Scala Virtualized [19] to provide custom type errors (Appendix F).

6.1 Instrumenting the Mainline Scala Compiler

The framework supports two versions of the Scala compiler: 2.11.x trunk branch (`Scala 2.11.0-M2`) and 2.10.x stable branch (`Scala 2.10.2`). With 230 instrumentation classes and no logical modifications to the compiler, we covered most of the typechecker implementation that is typically exercised by libraries and DSL programmers. The framework supports different compilers with the same set of instrumentation (modulo code changes) as well as algorithms that analyze type derivation trees. This follows our intuition that while the compiler is actively developed, the logic of typechecker remains mostly intact, and therefore hardly ever alters our instrumentation.

6.2 Visualizing Typechecking

Our type error customization and debugging framework is also capable of visualizing type derivations trees in an incremental way, in the spirit of (and in many ways subsuming) [28]. We've found that such a tool is advantageous to those with only a cursory knowledge of type systems. All of the generic search algorithms described in Section 4 can be run and visualized, therefore enabling interactive exploration of the erroneous scenarios. A screenshot of our tool is shown in Figure 4 in Appendix D.

6.3 Type Debugging Existing eDSLs and Libraries

For the evaluation of the applicability of our framework in explaining non-trivial type errors we have looked at popular libraries and DSLs:

- Testing libraries[5] (Specs2, Scalatest, Scalacheck)
- Shapeless[6] – a type class and dependent type based generic programming library that uses advanced type system features (more elaborate discussion is available in Appendix K).
- Standard Scala library – an official collections framework that uses higher-order functions and higher-rank polymorphism.
- Lightweight Modular Staging (LMS) [29] – a framework for type-driven staging to allow for deep embedding. We tested LMS with OptiML, an eDSL for machine learning written in Scala and based on LMS.

Customization offered by library- and DSL-specific plugins mostly resolves to correct identification of the erroneous scenario and analysis of results provided by the generic algorithms in order to give even more specialized user feedback. In many cases we have relied purely on generic algorithms to analyze derivation tree and provide appropriate error messages.

Table 1 presents an overview of type-system features encountered in non-trivial problems in the given libraries, and for which our framework was able to identify the root of the problem and provide better feedback to users. Type errors were collected from real code examples posted on mailing lists, forums like StackOverflow, public repositories, as well as directly from authors of the libraries. As a result we collected 92 self-contained examples representing various nontrivial problems. Many of the examples exercise the typechecker's ability to infer correct types. This involves debugging type errors related to type inference

- Implementation limitations related to inference composed of only local type constraints, including for higher-kinded types.
- For type parameters having lower or upper type bounds (either concrete or parametrized) (we also compare related work in that are in Appendix E).
- That is directed by the parametrized result type of the method or where term is part of an assignment.
- For member selection where we deal with type parameters belonging to methods and class declarations.
- For path-dependent types and partially applied function applications.

[5] etorreborre.github.io/specs2, scalatest.org and scalacheck.org

[6] github.com/milessabin/shapeless

	Testing frameworks	Shapeless	Scala collections	LMS/OptiML
Type inference	(5)	(14)	(19)	(3)
Implicits	(6)	(13)	(12)	(3)
Dependent-method types	-	(4)	-	-
Subtyping & Variance	-	(15)	(27)	-
Overloading resolution	(8)	-	(2)	(8)
Parsing	(2)	-	-	(2)

Table 1. Level of testing type errors, expressed as the number of tests exercising a particular feature, in external libraries and DSLs, explained in more detail in Section 6.3. Note that some tests exercise multiple type system features at the same time.

Type inference for higher-kinded types in Scala is very limited[7] and often manifests itself in cryptic type errors involving bottom type `Nothing`. While this is a known limitation of Scala's implementation that can be often solved by modifying type signatures of methods, related questions regularly appear on Stackoverflow or mailing lists. We have implemented heuristics in our framework that identify such situations based solely on high-level representation exposed by our framework (examples are provided in Appendix I). Type Debugger's approach was tested on 14 real world examples, where it provided code modifications that correctly fix those type errors.

Shapeless also uses Scala macros [2] that allow for convenient compile-time metaprogramming. Since macro expansion is part of regular typechecking and performs typechecking as well, our framework was still able to expose type derivations trees to library-specific plugin and eventually produce better error feedback.

While the framework has proved to be precise when explaining local type inference limitations, it failed to provide comprehensive user feedback in situations when source of term's type has been distributed among different type inference locations *i.e.,* if a type depends on an instantiation of a type parameter that itself depends on an instantiation of another type parameter at some other location, the framework will not automatically identify such dependency between the two locations. This is not caused by any technical limitation but rather our aim to keep type error feedback succinct, as we could very easily overwhelm users with the amount of information.

Implicits [26] are used in the design of flexible eDSLs and libraries with convenient APIs. LMS and OptiML use implicit conversions to perform lifting of values to their staged equivalents, and operations on staged values through type-classes. Many of the methods in Shapeless have an implicit parameter that allows for *type-level computation*, whereas Standard Scala collections use them to avoid code duplication [22].

Our framework can deal with typical type errors related to implicit search *i.e.,* diverging implicits, ambiguous implicits coming from the same as well as different scopes or simply no implicits found. Library-specific plugins can analyze `Goals` representing implicit search information and present it in a user-friendly way.

[7] http://adriaanm.github.io/research/2010/10/06/new-in-scala-2.8-type-constructor-inference/

Our type debugging framework is the first one to analyze a popular design pattern used in all of the analyzed libraries where implicit definition takes implicit parameters themselves, essentially triggering a chain of implicit searches on application. While convenient to use, failures carry little or none useful information to debug the problem. Since our framework instruments implicit search implementation, we can provide better feedback (we provide a simple example in Appendix J).

Method overloading is commonly used in the tested libraries and LMS's DSLs. We provide more user-friendly error messages by analyzing type signatures of the available alternatives. We can also explain type inference limitations in the presence of method overloading, such as not inferring types of parameters for closures. While our framework certainly exposes enough information to explain why alternatives were not selected according to language specification or implementation, we do not perform it. Unless controlled by users, which we currently do not support, the debugging process would produce large amount of information that is hard to parse visually.

To summarize, mature libraries and DLSs used for evaluation of our type error customization and debugging framework present non-trivial type errors that are encountered in every day programming. Our results have shown that the framework exposes enough of typechecking information in a format suitable enough to handle advanced type features. Finally, thanks to plugins infrastructure we were able to specialize error feedback for libraries and DSLs, that otherwise would reveal internal data structures.

7 Other Related Work

Type inference algorithm modification. Research on type error analysis has mostly focused on improving global type inference limitations in languages like Haskell or ML. Many implementations use a variation of the W algorithm [4] which is known to produce biased type errors. Different variations have been proposed such as M^{SYM} [17] or *UAQ* and *IEI* [14] that help with error localization. Apart from the work done in [34], which mixes local and global type inference elements, none of the solutions improve the state of local type inference typically encountered in object-oriented languages with subtyping. While modifications to the type inference algorithm may improve the location information they do not explain the decision process that led to type error.

Explanation systems. Explanation systems[5, 11, 14] typically collect information during typechecking to provide additional feedback to the users regarding problematic error. The process involves modifying the existing typechecker implementation to provide debugging capabilities or collecting constraints in a separate phase that essentially implements a simplified version of the typechecker. Generating a good message based on the type of constraint is challenging. Our framework, does not modify or replicate the typechecker and provides type derivations that can explored in conveniently.

Program slicing. Typechecking systems that have been translated into constraint-based systems led to the development of program slicing. Instead of generating an error message, all locations that affected the inference of a type error [10, 30] are provided. Unfortunately program locations needs to come with an additional information in order to be useful. Our framework is capable of providing both. Furthermore, it is unclear how these systems would behave in the context of more advanced type features such as implicits or path-dependent types where an error can not always be explained with the use of a simple location.

Interactive debugging. Chameleon [31] offers a more interactive approach to error messages in a subset of Haskell. The type inference problem is translated into a constraint solving problem, where constraints keep source location information. Therefore, the tool is able to respond precisely to type inference questions from the user. Our framework provides a visualization of derivation trees, based on work done in [28], which provides similar but more more powerful capabilities, as it visualizes every decision.

Automatic repair systems. Existing automatic repair systems [3, 17] typically attack the problem of ill-typed expressions by generating a large amount of plausible solutions which could potentially provide a quick solution. In order to reduce the amount of inevitable false-positives results, one has to devise a ranking mechanism that allows to quickly judge how the repair relates to others [15]. [9] presents a technique that explores a number of structure modifications that would fix an ill-typed term. This is limited to problems based only on the simply typed lambda calculus. In [3, 15, 16] the typechecker is used as an oracle for verifying program modifications which reduces the amount of false-positives. None of the approaches support type-classes (encoded in Scala as implicits [26]), which involve expensive exploration of scopes.

8 Conclusions

We have presented a new, unified way of improving type error feedback for regular programmers as well as DSL authors. We provide a type debugging framework which instruments the Scala compiler in a lightweight way and exposes typechecker's decision process in a high-level representation. Relying on high-level representation renders an opportunity to easier navigate and reason about typechecking from the programmer's point of view. We have exercised our framework by generating useful corrective code suggestions in order to overcome well-known limitations of local type inference. A complementary plugin mechanism is useful for library and DSL designers to customize type errors to generate better user feedback.

In future work, we plan to explore the interactive aspect of our framework. Users should be able to query every typing of their code, a natural progression of work, since that information is available in the high-level representation already. Finally the high-level representation offers an interesting opportunity for testing for regressions, and to verify the stability of the compiler during regular development.

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A Instrumentation Output for `foldRight` Application

To give a better idea of the structure of the collected low-level instrumented data, we present a simplified output for typechecking application `xs.foldRight(Nil())` (from our motivating example) in Listing 10. For presentation reasons instrumentation blocks increase the level of indentation, and `...` represents omitted instrumentation output. Extracted information about a typechecked function, `xs.foldRight` having type $(z : B) \xrightarrow{B} (op : (Int, B) \to B) \to B$, is within lines 1 and 6, type arguments are potentially inferred in lines 7-9, `Nil()` term is typechecked in lines 10-12, inference of type instantiation for type parameter B is output in lines 13-15, and finally typechecking of an application (with the just inferred type argument $List[Nothing]$) gives output in lines 16-18. While this particular fragment of typechecking execution is type correct, the true complexity of understanding such low-level output is particularly problematic when an erroneous case occurs and one of, potentially many, fallback mechanisms in the compiler is executed. Since the primary aim of our tool is to help with with understanding the decisions of typechecker in the presence of cryptic type errors, the latter is a common situation.

```
0  >TypeApplication               // xs.foldRight(Nil())
1    >TypeMemberSelection         // xs.foldRight
2      ...
3      >TypedSelection
4    >TypeAppl
5      >InferTypeArguments          // B from type '(z: B) =>
6        ...                        // (op: (Int, B) => B) => B'
7        >Inferred
8      >TypeGoal                    // Nil()
9        ...
10       >TypeGoalFinished
11     >InferInstance         // For type '(z: B) =>
12       ...                        // (op: (Int, B) => B) => B'
13       >InferredMeth
14     >TypeApplication             // For (z: List[Nothing]) =>
15       ...                        //   (op: (Int, List[Nothing]) =>
16       >TypeAppDone               //   List[Nothing]) => List[Nothing]
17     >TypeAppFinished
18   >TypeAppDone
```

Listing 10. Simplified instrumentation output for `xs.foldRight(Nil())`

B Encoding Lists in the Formalism from [25]

Using the minimal language we can provide intuitive definitions for lists using records with a single, `match`, method that uses `visitor` pattern for inspection (in the same manner as in [25]).

```
type List[a] = {
 match[b](v: ListVis[a,b]): b
}
```

```
type ListVis[a,b] = {
 caseNil(): a,
 caseCons(x: a, xs: List[a]): b
}
```

Type `List` takes a single type parameter, `a`, that represents the elements of the list, while the local type parameter of match, `b`, represents the type of the result of matching on such a list with a list visitor. `ListVis` is a record type requiring two functions that are called either when an underlying list is empty (`caseNil`) or when it is not (`caseCons`). Instances of list constructors - for an empty, `Nil`, and a non-empty `Cons`, lists - can then be expressed in a following way:

```
Nil[a]():List[a]= {
 match v = v.caseNil()
}
```

```
Cons[a](x:a, xs:List[a]): List[a]= {
 match v = v.caseCons(x,xs)
}
```

C `TypeFocus` Built From Algorithmic Subtyping

`TypeFocus` that is created solely from typing rules is not able to provide enough precision for complex types, such as type constructors having multiple type parameters. For instance, let us assume that the type constructor `Map` has two type parameters `A` and `B`, and we encounter a type error caused by the type mismatch between `Map[X,Y]` and `Map[I,J]`, where the type argument `Y` is not a subtype of `J`. Using the provided analysis, we would only identify types that affect type `Map[A, B]`, rather than just the type parameter `B`.

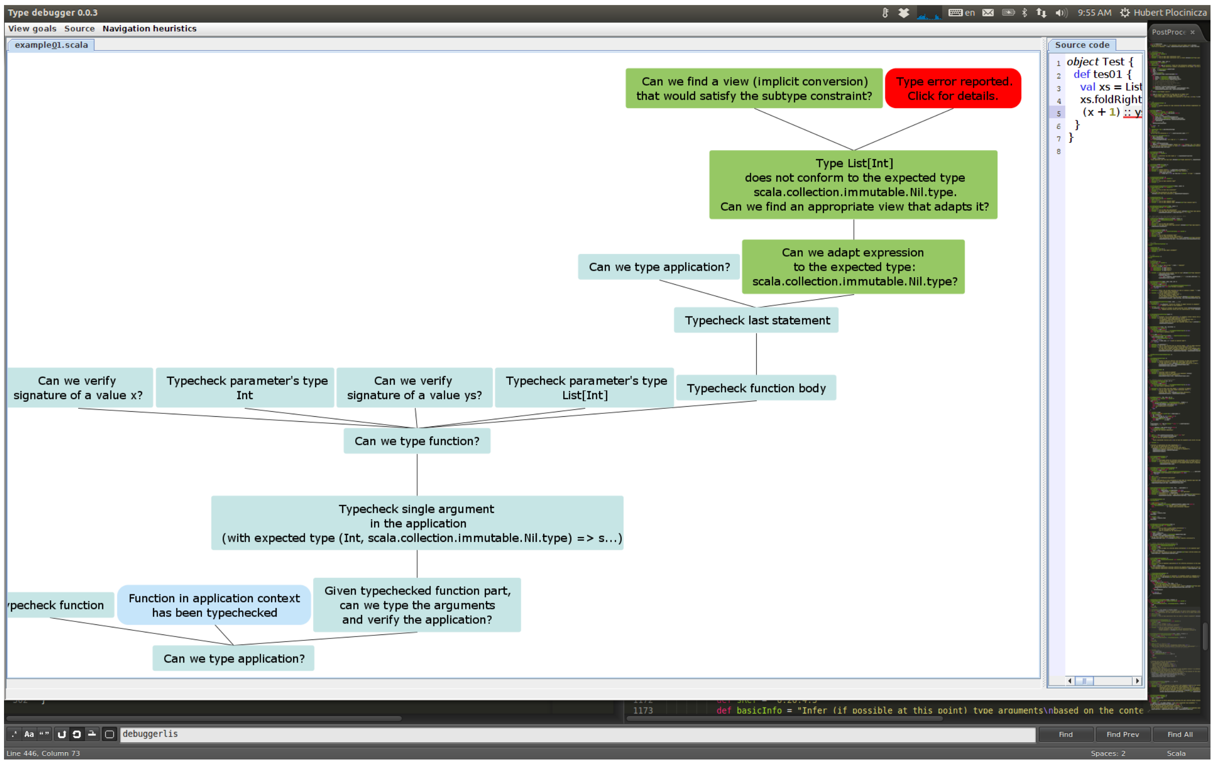
The framework provides an orthogonal `TypeFocus` that is built from the high-level representation of algorithmic subtyping. Such subtyping `TypeFocus` allows us to achieve a greater level of precision when identifying parts of types that lead to errors. At the same time, it requires minor refinements in the formalization of the analysis presented in Figure 3. To illustrate the problem, we consider `TypeFocus` that selects a first type argument in type application involving $List$ type constructor (*i.e.,* `foc-targ-0-list`$(List[a])$ = a for some type variable a) in
`foc-all` $\circ$ **foc-targ-0-list** $\circ$ `foc-fun-res` $\circ$
`foc-fun-param-0` $\circ$ `foc-fun-res` $\circ$ `foc-poly-res`
Applying full `TypeFocus` composition to the function type will be invalid, since b_4 is an uninstantiated type variable, rather than an expected type application. Therefore `TypeFocus` objects provide `partialFocus` method of type $T \Rightarrow (T, \texttt{Focus})$. It returns a tuple consisting of a type that has been extracted before an `Error`, and the remaining, *failed*, part of the `TypeFocus` composition. Analysis rules can be easily updated to reflect that change, and we use the remaining part of `TypeFocus` composition to analyze type constraints in type inference more precisely.

D Visual Type Debugger: Screen Shot

(See Figure 4)

E Debugging type errors related to Generics

The work of El Boustani and Hage [6] focuses on providing more informative error messages for problems related to Java Generics. In that sense they are the closest to our framework that authors are aware of, in terms of type debugging modern language that supports polymorphism and subtyping. We have translated all 22 examples that were used in the evaluation of the Java framework into Scala, 4 of which have been excluded as they were type correct in Scala.



Can we find a view (implicit conversion) that would satisfy the subtype constraint?

Type error reported. Click for details.

Type List[Int] does not conform to the expected type scala.collection.immutable.Nil.type. Can we find an appropriate view that adapts it?

Can we type application?

Can we adapt expression to the expected type: scala.collection.immutable.Nil.type?

Typecheck last statement

Can we verify signature of a value x?

Typecheck parameter's type Int

Can we verify signature of a value ys?

Typecheck parameter's type List[Int]

Typecheck function body

Can we type function?

Typecheck single argument in the application (with expected type (Int, scala.collection.immutable.Nil.type) => s...)

pecheck function

Function in application context has been typechecked

Given typechecked function part, can we type the arguments and verify the application?

Can we type application?

```
object Test {
  def tes01 {
    val xs = List
    xs.foldRight
     (x + 1) :: y
  }
}
```

Figure 4. Visual type debugging of `foldRight`

The examples handle method invocations that have, potentially many, local type parameters or type constructors with various type bounds, and arguments are either variables or functions which result type is a concrete type. Type Debugger for Scala provides comparable error messages for such type errors. The quality of feedback has been assessed in terms of localizing the source of error and highlighting type constraints that led to a type mismatch. Only examples involving wildcard types, that were translated to existential types in Scala, proved to be problematic, as they do not always follow the same instantiation semantics and locations as regular type parameters. More importantly our framework does not limit error reporting to just invocations of polymorphic methods.

Code modifications suggested by the Java framework only modify type arguments or types for already type annotated variables without actually verifying those code changes. We believe that if type annotation is provided by the user, it expresses a firm conviction that a term is of a particular type and there is a much smaller chance that a simple type argument change will be type correct for the definition of the variable. To illustrate the difference we present a small Java program taken from [6], Scala translation of the method and error produced at function application:

```
<T> List<T> foo(Map<T, ? super T> a){...}
Map<Number, Integer> m = ...;
List<Integer> ret = foo(m);
----
def foo[T](a: Map[T, _ >: T]): List[T]   // foo in Scala
```

Java framework generates type error:

```
Method <T>foo(Map<T, ? super T>) is not applicable for the argument
of type (Map<Number, Integer>), because:
[*] The type Integer in Map<Number, Integer> on 5:9(5:21) is not
 a supertype of the inferred type for T: Number. However,
 replacing Number on 5:13 with Integer may solve the type conflict.
```

while our Type Debugger would produce:

```
Type mismatch has been partially caused by the type of the
value ret that has an explicit type List[Integer].
Locations on the RHS of the assignment that directly led
to type mismatch:
    val m: Map[Number, Integer] = ???
               ~~~~~~
    val ret: List[Integer] = foo(m)
                             ~~~ ~
Part of the initialized value that inferred the conflicting type:
    val ret: List[Integer] = foo(m)
                             ~~~~~~
Type parameter T in method foo has been instantiated to Number
using the least upper bound of:
    val m: Map[Number, Integer] = ???
               ~~~~~~  ~~~~~~~
    val ret: List[Integer] = foo(m)
                             ~~~~~~
```

By following Java's suggestion and modifying the type annotation of `m` we will most likely lead to a type mismatch between the new type annotation of value `m` and its RHS. Hence, a more conservative approach in our framework.

F Virtualized Scala

Virtualized Scala [19] is an experimental branch of the Scala compiler that enables DSL authors to override standard language constructs (conditionals, variables, loops) by essentially transforming them into regular method calls. Methods prefixed with `infix_` are treated as infix operators and require even more modifications in typechecking function application in order to support them. Whenever function in function application is a member selection, such typechecker will always try to find an infix method in the scope for the member *i.e.,* `x.y(z)` will essentially trigger a search and typechecking of application `infix_y(x, z)`.

Our framework supports experimental compiler in order to improve the general DSL experience. Although Virtualized Scala modifies the existing Scala typechecker we had to add only 8 new instrumentation classes and 12 high-level `Goals` to regain the ability to correctly expose the same typechecking decision process as in a regular compiler. If typechecking of infix function fails, which would be most of the time, we fallback to regular typechecking of function application, for which a high-level representation already exists. For completeness we provide an example of high-level representation for virtualized function application in Appendix G.

Naturally, our exploration algorithms needed to be updated to reflect such change. Since analysis is expressed by pattern matching on high-level representation, we only had to add new pattern matching cases, rather than redesigning the whole framework. More importantly any changes are statically verified.

To evaluate our approach we have provided a custom debugger plugin specific to Scala Virtualized and libraries based on it. We were able to precisely identify common problems, involving type inference or operator overloading, and provide better error feedback.

G High-Level Representation for Virtualized Function Application

```
trait TypeVirtApply extends TypeGoal {
  def typeInfixMethod: TypeGoal
}
trait TypeVirtApplyFallback extends TypeVirtApply {
  def typeApply:       TypeApply
}
trait TypeVirtApplyCorrect extends TypeVirtApply {
  def typeQual:        TypeGoal
  def typeArg:         TypeGoal
}
```

Listing 11. High-level representation for virtualized function application

Goal `TypeVirtApply` expresses that in the typechecker of Virtualized Scala, we first attempt to type some method `infix_y` when encountering function application AST. Through member `typeApply` in `TypeVirtApplyFallback` we express the constraint that on failure, we fallback to the usual typing of function application, while on success our type debugger would return `TypeVirtApplyCorrect`, where premises mean that we need to typecheck (old) qualifier, `x`, and a single argument, `z`.

H Providing Customized Type Errors for Virtualized Scala

```
val x: Rep[Array[Int]] = ...
x foreach { x => x + "a" }
// found   : Rep[String]
// required: Rep[Unit]
//  x + "a"
    ^
```

Listing 12. An error in eDSL that uses LMS and Virtualized Scala

Using type debugging plugins described in Section 5, we can provide custom feedback to libraries that make use of Virtualized Scala. Lightweight Modular Staging (LMS) [29], being one example, often exhibits confusing type errors that reveal internals of the staging process. Currently there exists no technique to mitigate or customize such errors. Listing 12 presents a trivial case where a user attempted to apply some function to all elements of a staged `Array` value, `x`. Type constructor `Rep` tells LMS that the value is supposed to be staged. Both, regular and staged, types of `Array` provide method `foreach` through an implicit coercion, therefore DSL user could assume that the presented code is valid (without type `Rep` such code is a regular type-correct Scala code). Instead, an error is reported. In an ideal situation programmers would identify this problem by pattern matching on some function application where the function is `foreach` and it's a member of qualifier of type `Rep[Array[...]]`. Such scenario is therefore abstract enough to be expressed in terms of typing rules. Fortunately, our framework was perfectly able to do. LMS-specific plugin would pattern match on the type derivation tree using case

```
case TypeAppCorrect(TypecheckGoal(_,
      TypeMemberSelection(qual, mem)), _),
    typeAppMain) => ...
```

to identify erroneous function application. Without going into details, `TypeAppCorrect` represents an already encountered typing rule for function application, with `TypecheckGoal` representing typechecking of function, premise `TypeMemberSelection` refers to regular high-level representation of a member selection typing rule where `qual` premise refers to typechecking of a qualifier, and `mem` to typing of a member of that qualifier. More importantly `qual` has low-level data about the type of the qualifier, and `mem` about the member that is accessed. If LMS plugin successfully matches them with `Rep[Array[T]]` (for some `T`) and `foreach` on them, respectively, then it has identified that particular problem. We were able to exploit that knowledge to produce more useful message:

```
Unlike the regular Scala compiler, LMS does not insert
implicitly ‘()‘ value in the body of the ’foreach’ closure.
’foreach’ on ’Array’ differs from the one in Standard library.
```

I Limitations of Type Inference for Higher-Kinded Types in Scala

Our test-suite contains 14 real examples that specifically target compiler's inability to infer types for higher-kinded types. We illustrate the problem with a simple problem, where one of the type parameters, T, is used within the bounds of another, U, and is in higher-order position:

```
abstract class A;   abstract class B[T <: A]
class XA extends A; class XB extends B[XA]
def foo[U <: B[T], T <: A]( resolver: U ): Unit = ()
foo(new XB)
```

Example below presents an un-informative type error from the regular Scala compiler, and the Type Debugger feedback.

```
inferred type arguments [XB,Nothing] do not conform to
method foo's type parameter bounds [U <: B[T],T <: A]
foo(new XB)
^

Type Debugger feedback:
The current type signature of method foo
(of type [U <: B[T], T <: A](resolver: U): Unit)
limits the current's implementation ability to infer
an appropriate type argument for the type parameter T.
Inferred type argument for U, XB, is not within the upper
bound B[Nothing]. In order to track appropriately the constraints
for the type parameter T you can modify the type signature to:
def foo[U[_ <: A] <: B[_], T <: A](resolver: U[T]): Unit
```

J Errors related to implicit search

```
class Bar;                        class Foo;
implicit val b1: Bar = ...;      implicit val b2: Bar = ...;
implicit def foo(implicit x: Bar): Foo = ...
def test()(implicit x: Foo){ ... }
test()
//  ^
// Current Compiler error:
// could not find implicit value for parameter x: Foo

//Type debugging framework:
// could not find implicit value for parameter 'x: Foo'
// due to ambiguous implicits in the implicit search chain
// test()(foo([*ambiguous-implicit-values(b2,b1)*])
```

The example above defines three implicits in the scope - `b1`, `b2` and `foo`, and two classes. More importantly lack of an argument in function application `test()` triggers an implicit search. Type Debugger provides not only a reason for a type error, but also concrete arguments that led to it. Such feedback is crucial since Scala libraries typically define large number of potentially ambiguous implicits in them.

K Errors in Shapeless

Exotic combinations of advanced type system features, such implicits and higher-kinded types, that are present in Shapeless lead to type errors that are hard to parse for users. Nevertheless, we found Type Debugger to remain reasonably precise even without specialized debugger plugin.

```
def foo(x: Int :: String :: Nil) {};
foo(1 :: 2 :: Nil)
//  ^
// found   : shapeless.::[Int, shapeless.::[Int, shapeless.Nil]]
// required: shapeless.::[Int, shapeless.::[String, shapeless.Nil]]

// Type Debugger feedback:
// Conflicting expression and type:
// def foo(x: Int :: String :: Nil) = ...
//                   ~~~~~~
// foo(1 :: 2 :: Nil)
//          ~
```

A function application example above uses data structure for heterogeneous lists, HList. Type mismatch reported by the compiler blames full type for an error and reveals internal details of the implementation, whereas Type Debugger is able to identify exact conflicting locations using subtyping `TypeFocus`.