Network dynamics of spiking neurons with adaptation

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Circuit level: population dynamics

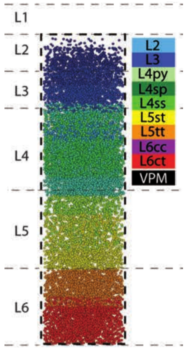


Fig.: Oberlaender (2012) *Cereb Cortex*

How to describe circuit level cortical dynamics?

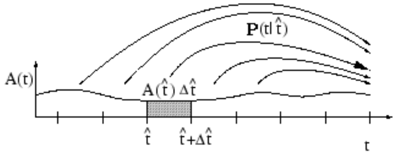
- Cortex is layered.
- Few neuron types per layer.
- Each type in a layer forms a population.
- Populations have finite size.

Here we derive a theory for such interacting populations, exploiting that neurons can be treated as quasi-renewal point processes, even in presence of spike-frequency adaptation.

Stochastic population integral equation

For a population of $\mathbf{N}$ neurons with

- spike trains $\mathbf{s_i(t)}$
- population activity $\mathbf{A(t) = \frac{1}{N}\sum_i^N s_i(t)}$
- inter-spike-interval (ISI) probability density $\mathbf{P(t|\hat{t})}$



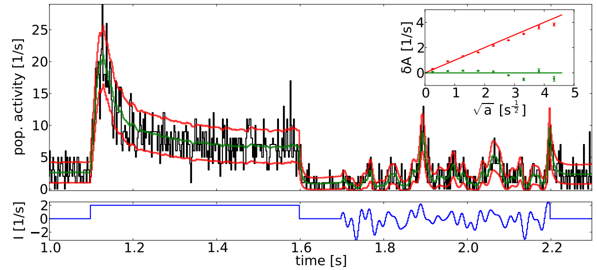
we have (cf. [4]) $$\mathbf{A(t) = \underbrace{\int_{-\infty}^{t} P(t|\hat{t})A(\hat{t})d\hat{t}}_{a(t)} + \delta A(t)}$$

with fluctuations $\mathbf{\delta A(t)}$ that obey (for $\tau \geq \mathbf{0}$, conditioned on $\mathbf{A(\hat{t} < t)}$)

$$\mathbf{\langle \delta A(t+\tau)\delta A(t)\rangle = N^{-1}a(t)\,\delta(\tau) - N^{-1}\int_{-\infty}^{t} P(t+\tau|\hat{t})P(t|\hat{t})A(\hat{t})d\hat{t}}$$

Because of synaptic coupling, external inputs and adaptation, $\mathbf{P(t|\hat{t})}$ depends on the past activity and on time (inhomogeneous renewal point process).

Network response to current injection

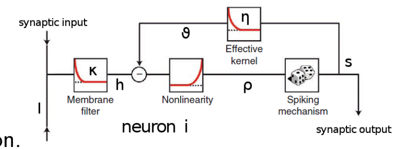


Population of 500 randomly connected excitatory neurons with adaptation.

From neuronal dynamics to populations

Generalized linear model neuron:

$$\boldsymbol{\rho_i(t) = c\, e^{h_i(t)-\vartheta_i(t)}}$$



Quasi-renewal approximation.

mean-field [4]	$\mathbf{h_i(t, \{t_j^f\}_{j,f}) \approx h(t)}$	$\mathbf{= (\kappa * [J\,A + I])(t)}$
adaptation [3]	$\mathbf{\vartheta_i(t, \{t_i^f\}_f) \approx \vartheta(t\vert\hat{t})}$	$\mathbf{= \eta(t-\hat{t}) + (\gamma * A)(t)}$
intensity	$\mathbf{\rho_i(t, \{t_j^f\}_{j,f}) \approx \rho(t\vert\hat{t})}$	$\mathbf{= c\, e^{h(t)-\vartheta(t\vert\hat{t})}}$

Then follows
survivor function $\mathbf{S(t|\hat{t}) = \exp(-\int_{\hat{t}}^{t}\rho(t'|\hat{t})dt')}$
interval density $\mathbf{P(t|\hat{t}) = \rho(t|\hat{t})\,S(t|\hat{t})}$

Linearized dynamics in the AI state

Assuming $\mathbf{A(t) = A_0 + \Delta A(t)}$, with small $\mathbf{\Delta A}$, we linearize the population integral equation:

$$\mathbf{a(t) = A_0 + (P_0 * \Delta A)(t) + A_0\frac{d}{dt}\{\mathcal{L} * [(\kappa J + \gamma) * \Delta A + \kappa * \Delta I]\}(t)}$$

Then $\mathbf{A(t) = a(t) + \sqrt{\frac{A_0}{N}}\xi(t)}$, with colored noise $\mathbf{\xi(t)}$.
$\mathbf{\langle \xi(t+\tau)\xi(t)\rangle = \delta(\tau) - \int_0^\infty P_0(s+\tau)P_0(s)ds}$

Spectral density of population activity

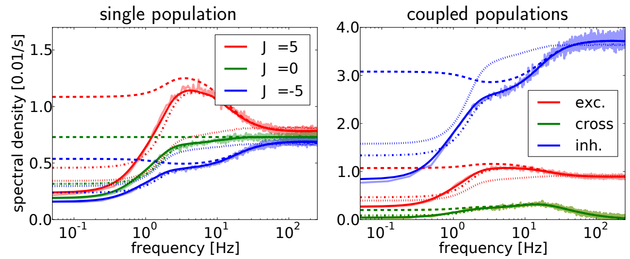
For renewal processes with ISI density $\mathbf{P_0}$ and rate $\mathbf{A_0}$, the power spectrum is

$$\mathbf{\tilde{C}_0(\omega) = A_0(1-\tilde{P}_0)^{-1}(1-\tilde{P}_0\tilde{P}_0^*)(1-\tilde{P}_0^*)^{-1}.}$$

Using the linear response function $\mathbf{R(t)}$ of the neurons to input currents, the spectral density of the population activity $\mathbf{A(t)}$ is

$$\mathbf{\tilde{C}(\omega) = \tilde{B}N^{-1}(\tilde{C}_0 + \tilde{R}\tilde{C}_I\tilde{R}^*)\tilde{B}^{T*}, \quad \tilde{B} = \left[1 - \tilde{R}(J + \tilde{\kappa}^{-1}\tilde{\gamma})\right]^{-1}}$$

Here $\mathbf{\tilde{C}_I}$ is the input spectrum, and $\mathbf{\tilde{R}(\omega) = (1-\tilde{P}_0)^{-1} i\omega A_0 \tilde{\mathcal{L}}\tilde{\kappa}}$.



Special cases:

$\mathbf{J = \tilde{\gamma} = \tilde{C}_I = 0}$:	renewal process, $\mathbf{\tilde{C} = \tilde{C}_0}$	dotted
$\mathbf{\tilde{\gamma} = 0}$:	linear response theory for LIF [2]	dash-dotted
$\mathbf{P_0 \sim \exp}$ and $\mathbf{\tilde{\gamma} = 0}$:	Hawkes process (Hawkes, 1971)	dashed

Conclusions

New theory explains effects of adaptation at the circuit level:

- Finite-size fluctuations in coupled, randomly connected populations.
- Noise-shaping through adaptation: reduction in low-frequencies.

References

1. Deger, Schwalger, Naud, Gerstner (in review) *arXiv:1311.4206*
2. Lindner, Doiron, Longtin (2005) *Phys Rev E*
   Trousdale, Hu, Shea-Brown, Josic (2012) *PLoS Comput Biol*
3. Naud & Gerstner (2012) *PLoS Comput Biol*
4. Gerstner (2000) *Neural Comput*