ANGULAR DEPENDENT SOLAR GAIN FOR MULTIPLE GLAZING FROM OPTICAL AND THERMAL DATA

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ABSTRACT

The angle dissolving determination of the solar gain factor $g(\varphi) = \tau_e(\varphi) + q_i(\varphi)$ of an entire insulation glass (IG) – as described in [1] for a two pane glass – can be generalized for IGs with a pane number n>2. And this lacking the possibility to refer to the thermal reaction of the inner panes (ΔT_1 , … , ΔT_n) on an external signal S (solar source).

The elevated number of inner degrees of freedom of the entire system, corresponding to the heat transfer coefficients (Λ_{12} , … , $\Lambda_{n-1,\,n}$) and the individual absorptances (α_1 , … , α_n) can – regarding the response – be sufficiently represented by means of an equivalent two pane glass with absorptances (A_1, A_n) and Λ as global intrinsic quantities of the glazing. Whose ratio A_1/A_n is again fixed by means of one single thermal measurement alone. Under knowledge of the global absorptance $\alpha(\varphi)$ from an optical measurement their magnitudes (A_1, A_n) follow.

As an intrinsic invariant describing optics, the ratio A_1/A_n provides the possibility of conversion into the response under an arbitrary other boundary condition BC (h_e, h_i). The space these quantities (A_1, A_n) leave open behind concerning an identical response onto a signal S will be specified. This space shows the open margin left designing a glazing.

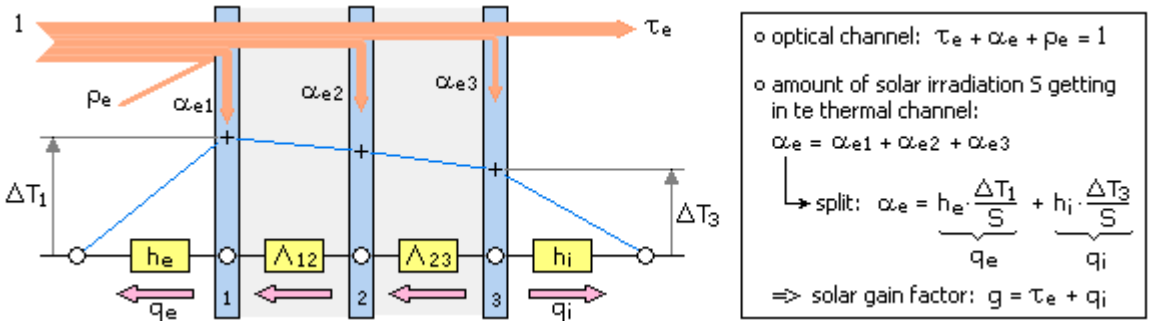

Figure 1: Node-model for a multiple glazing. The absorbed part of irradiation α_e appears in each pane as source, these represented by nodes. The theoretical considerations done with this model show that the information of the outer temperature elevations ΔT_1 and ΔT_3 under known experimental boundary conditions (h_e , h_i) determine the branching ratio $q_e(\varphi) / q_i(\varphi)$ under all other boundary conditions.

INTRODUCTION: MODEL ASSUMPTIONS – THERMAL NETWORK WITH SOURCES

The derivations given concerning the energy flows of the IGs are based on a model of a thermal network according to Fig. 1, in which the panes are represented by nodes:

pc1 The nodes are characterized by their *temperature elevation* (ΔT_1 , … , ΔT_n) towards the environment – e.g. the laboratory.

pc2 *Thermal energy exchange* only takes place between directly neighboured nodes and is described by *conductances* (h_e , Λ_{12} , … , $\Lambda_{n-1,\,n}$, h_i). Those having the property that energy flow *through them* is proportional to the temperature difference *over them*.

pc3 In place of the pane-nodes energy is coupled in by an *external source* S in extent of the *absorptances* (α_1 , … , α_n), which specify the part of S locally reaching the thermal channel. With these *absorptances* α_i are meant those, which are achieved resulting in the given compound of panes in the IG – see EN 410 [3].

pc4 Considered is the *stationary flow equilibrium* tuned in under the source S. Therefore terms in the equations, which describe the inner energy of the panes, can be omitted.

(**pc** = preconditions).

ESTABLISHING THE EQUATIONS FOR A STATIONARY FLOW EQUILIBRIUM

In a system defined as above the entries $S \cdot \alpha_j$ in place of the nodes each can be superposed in their effect, resulting in a linear response ΔT and these scaling with S. The according linear system of equations in an abbreviated matrix notation is:

(1) $\Delta T_i / S = M_{ij} \cdot \alpha_j$, $i = (1 , \dots , n)$, $j = (1 , \dots , n)$, $n =$ number of panes .

The matrix elements M_{ij} contain the thermal quantities Λ_{ij} und (h_e, h_i) by virtue of those the system gets rid of the irradiated energy. Linearity of ΔT in S involves that, with a single measurement under a given irradiation S alone, one acquires knowledge about the system under each other. Vice versa with a further stationary measurement under a changed irradiation S nothing new is emerged. Regardless the fact that *relative errors* of *measured variables* which are scaling with S enter inversely proportional to S.

Without evaluating the system of equations (1) above, for any desired n the following equation is valid at the *system boundary*, describing how this open system exchanges energy in equilibrium and therein gets rid of the irradiated energy:

(2) $\alpha = h_e \cdot \Delta T_e / S + h_i \cdot \Delta T_i / S$, *global absorptance* $\alpha = \alpha_1 + \dots + \alpha_n$,

the two terms on the right side describing the *secondary heat gain* related to irradiation S:

(3) $q_e = h_e \cdot \Delta T_e / S$, $q_i = h_i \cdot \Delta T_i / S$, *secondary heat gain coefficients* .

The useful equation (2) is not an additional equation to the equations (1), while it can be derived from those – in perfect consistence. With the definitions (3) equation (2) has the shape:

(4) $\alpha = q_i + q_e$.

In case the optical value $\alpha(\varphi)$ is already fixed, with (4) one knows already the sum of the two variables: The minimal information missing would be their *branching ratio* q_e/q_i.

GENERAL IMPLICATIONS DERIVED FROM THE EQUATIONS REGARDING THE DETERMINATION OF VALUES

cc1 So far *global absorptance* $\alpha(\varphi)$ is predetermined through an independent experiment – in our case through an optical measurement [2] – with (2) one already knows the irradiation S, that must have been present in the experiment, likewise one knows already the sum (4) of the two secondary heat gain coefficients (q_e, q_i). The value S here appears implicitly in the entire evaluation.

cc2 Nevertheless one could always take the way measuring the value S independently during the thermal experiment at the level of entrance of irradiation into the glazing. In this scenario the value α would follow with (2) and cannot be predetermined additionally. Deciding for one of the two ways is a matter of measuring accuracy. As we have $\alpha(\varphi)$ from a precise optical measurement [2], we decided for a procedure pursuant to (**cc1**), see [1].

cc3 Taking both ways, would contain a certain possibility to validate the reliability of the measurements, also the *boundary conductances* (h_e, h_i). These are modelled in their amount [1] and appear lately as referencing conductances in the entire evaluation procedure of the experiment. This seems to be a subtle point concerning the measuring error. The formal shape of the present discussion will show a certain diminished criticality.

cc4 A priory knowledge of both values (α_1, α_2) in a two pane glass under known boundaries (h_e, h_i) would open the possibility to achieve knowledge of the conductance Λ. This from an independent measurement of S. Measuring one temperature elevation $\Delta T_1/S$ or $\Delta T_2/S$ would be sufficient. The according equation given below (10) resp. (11) determines the conductance Λ. According to (2) the second temperature is not independent. So, measuring the second temperature would here not be from a substantial usability.

(**cc** = conclusions).

OUTER RESPONSE OF A MULTIPLE GLAZING UNDER ARBITRARY BC

Back to the problem originated from a lacking knowledge of the individual absorptances α_j. The proposition whose validity will be shown here reads like:

***Proposition**: A single measurement of the response (ΔT_1 , ΔT_n) of a thermal network defined as above under a signal S and under known boundary conductances (h_e , h_i) and known global conductance Λ determines the response under all other boundary conductances (h_e^N, h_i^N). This is valid for systems of an arbitrary number of panes n.*

Deriving this proposition is not free of charge comparable with direct conclusions getting from the fact that responses are scaling with the signal S: The boundary conductances (h_e, h_i) do not enter as a simple scaling factor. The proposition was already shown in the case n=2 [1]. In the case n>2 now inner degrees of freedom are added.

Anyway, regarding the response again only the outer responses (ΔT_1, ΔT_n) are measured. It has to be shown additionally that the left open degrees of freedom (n>2) can be sufficiently represented by an *equivalent two pane glass* concerning the outer behaving – this under specification the procedure within the parameter of this representing glass can be found.

APPROACH FOR THE SOLUTION: FIND AN EQUIVALENT TWO PANE GLASS

In the case n=2 the proposition follows directly from (1): Two equations determine all absorptances (α_1, α_2). All temperatures are already acquired with the outer pane temperatures (ΔT_1, ΔT_2). An independent measurement of $\alpha(\varphi) \equiv \alpha_1 + \alpha_2$ determines further the irradiation S.

The additional minimal information α_1/α_2 then would be sufficient, to determine both (α_1, α_2). Indeed starting from (1) an isomorphism between this *minimal information* α_1/α_2 and the *temperature detuning* $\Delta T_1/\Delta T_2$ – this interceded by (h_e , h_i , Λ) – can be derived. Consequently complete knowledge over the system is achieved in the case n=2:

$$(5)\quad \frac{\Delta T_1}{\Delta T_2} \xleftrightarrow{(h_e,h_i,\Lambda)} \frac{\alpha_1}{\alpha_2} \qquad , \qquad \alpha = \alpha_1 + \alpha_2 \qquad .$$

The ratio α_1/α_2 herein has the status of a sufficient determining information because it is an intrinsic value of the glass – itself not depending on the interceding values (h_e, h_i, Λ).

Now backwards – from the once determined α_1/α_2 – the *temperature detuning* ($\Delta T_1^N/\Delta T_2^N$) can be calculated for each other BC (h_e^N, h_i^N). Finally the *branching ratio* q_e^N/q_i^N follows with (6) and by means of (4) their amounts (q_e^N, q_i^N):

$$(6)\quad \frac{q_e^N}{q_i^N} = \left(\frac{\Delta T_1^N}{\Delta T_2^N}\right) \cdot \frac{h_e^N}{h_i^N} \qquad , \qquad \alpha = q_e^N + q_i^N \qquad .$$

Now a glass n=2 is the simplest configuration, which is just able at all – this over α_1/α_2 – to tune the *branching ratio* q_e/q_i – particularly capable of an asymmetric behaviour regarding the outflows (q_e, q_i) over intrinsic quantities alone. So, if for all cases n>2 an equivalent glass n=2 could be found, the proposition above would be proved. In this *equivalent two pane glass* the absorptances (α_1 , ... , α_n) will have to be represented by the *representing absorptances* (A_1, A_n), whose direct sum equals again α. So, an isomorphism analogue to (5) has to be found:

$$(7)\quad \frac{\Delta T_1}{\Delta T_n} \xleftrightarrow{(h_e,h_i,\Lambda)} \frac{A_1}{A_n} \qquad , \qquad \alpha = A_1 + A_n \qquad .$$

Such an isomorphism (7) found, the procedure to determine the branching ratio q_e/q_i would follow in a fully analogue way, finally pursuant to (6).

In the cases n>2 however the intrinsic values (α_1 , … , α_n) and (Λ_{12} , … , $\Lambda_{n-1,\,n}$) strike up an alliance regarding the outflow of energy on both sides – resulting in a *branching ratio* (6). From both types of information only their global quantities are known:

$$(8)\quad \alpha = \alpha_1 + \ldots + \alpha_n \qquad , \qquad \textit{global absorptance} \qquad ,$$

$$(9)\quad \Lambda = (1/\Lambda_{12} + \ldots + 1/\Lambda_{n-1,\,n})^{-1} \qquad , \qquad \textit{global heat transfer coefficient} \qquad .$$

The temperatures of the inner panes hidden, the inner degrees of freedom are left undetermined within (8) and (9), of which one only knows the global amounts α and Λ. Nevertheless the parameter A_1/A_n in (7) would – according to case n=2 – represent an intrinsic invariant of the system, whose amount together with $\alpha = A_1 + A_n$ would have a representing meaning concerning the response, but would now leave open a field of equivalent constellations. Therein a manufacturer of glazing has an open field to design a glazing with given properties.

CONCRETE IMPLEMENTATION – DERIVATION OF THE PROPOSITION

To derive equations (5) and (7) and to obtain the concrete shape of (A_1, A_n) establishing the equations (1) is indispensable. Starting from case n=2 subsequently the case n=3 will be treated. The shape of the representing absorptances (A_1, A_3) will in substance be deduced by comparing the coefficients within the two cases n=2 and n=3. The generalization to arbitrary cases n follows by virtue of inductive conclusions:

- In the case n=2 the equations (1) are given by:

(10) $$\frac{\Delta T_1}{S} \cdot \frac{\alpha_1 \cdot (\Lambda + h_i) + \alpha_2 \cdot \Lambda}{\Lambda \cdot (h_i + h_e) + h_i \cdot h_e} \quad ,$$

(11) $$\frac{\Delta T_2}{S} = \frac{\alpha_2 \cdot (\Lambda + h_e) + \alpha_1 \cdot \Lambda}{\Lambda \cdot (h_i + h_e) + h_i \cdot h_e} \quad .$$

Expression (11) corresponds to formula (13) in EN 410 [3] for q_i. This under usage of (3) $q_i = \Delta T_2 / S \cdot h_i$ .The quotient of (10) and (11) delivers the searched shape of (5):

(12) $$\frac{\Delta T_1}{\Delta T_2} = \frac{\Lambda \cdot (1 + \alpha_1/\alpha_2) + h_i \cdot \alpha_1/\alpha_2}{\Lambda \cdot (1 + \alpha_1/\alpha_2) + h_e} \quad \Leftrightarrow \quad \frac{\alpha_1}{\alpha_2} = \frac{h_e \cdot \Delta T_1/\Delta T_2 + \Lambda \cdot (\Delta T_1/\Delta T_2 - 1)}{h_i - \Lambda \cdot (\Delta T_1/\Delta T_2 - 1)} \quad .$$

- In the case n=3 (see Fig. 1) the equations (1) are given by:

(13) $$\frac{\Delta T_1}{S} = \frac{\Lambda \cdot (\alpha_1 + \alpha_2 + \alpha_3) + h_i \cdot [\alpha_1 + \alpha_2 \cdot \Lambda_{12}/(\Lambda_{12} + \Lambda_{23})]}{\Lambda \cdot (h_i + h_e) + h_i \cdot h_e} \quad .$$

(14) $$\frac{\Delta T_2}{S} = \frac{\Lambda \cdot (\alpha_1 + \alpha_2 + \alpha_3) + [h_i \cdot \Lambda_{12}(\alpha_1 + \alpha_2) + h_e \cdot \Lambda_{23} \cdot (\alpha_2 + \alpha_3) + h_e \cdot h_i \cdot \alpha_2]/(\Lambda_{12} + \Lambda_{23})}{\Lambda \cdot (h_i + h_e) + h_i \cdot h_e} \quad .$$

(15) $$\frac{\Delta T_3}{S} = \frac{\Lambda \cdot (\alpha_1 + \alpha_2 + \alpha_3) + h_e \cdot [\alpha_3 + \alpha_2 \cdot \Lambda_{23}/(\Lambda_{12} + \Lambda_{23})]}{\Lambda \cdot (h_i + h_e) + h_i \cdot h_e} \quad .$$

The quotient of (13) and (15) for the two outer panes delivers the searched shape of (7):

(16) $$\frac{\Delta T_1}{\Delta T_3} = \frac{\Lambda \cdot (1 + A_1/A_3) + h_i \cdot A_1/A_3}{\Lambda \cdot (1 + A_1/A_3) + h_e} \quad \Leftrightarrow \quad \frac{A_1}{A_3} = \frac{h_e \cdot \Delta T_1/\Delta T_3 + \Lambda \cdot (\Delta T_1/\Delta T_3 - 1)}{h_i - \Lambda \cdot (\Delta T_1/\Delta T_3 - 1)} \quad .$$

Whereas formal identity between (12) and (16) is reached under absorptances (A_1, A_3), those approaching the searched *two pane glass* with an equivalent behaviour to the outside like this:

(17) $A_1 = \alpha_1 + \alpha_2 \cdot \Lambda_{12} / (\Lambda_{12} + \Lambda_{23}) = \alpha_1 + \alpha_2 \cdot (1 - \Theta)$,

(18) $A_3 = \alpha_3 + \alpha_2 \cdot \Lambda_{23} / (\Lambda_{12} + \Lambda_{23}) = \alpha_3 + \alpha_2 \cdot \Theta$.

The resulting values (A_1, A_3) are achieved by a prorated admixture of the inner absorptance α_2 to the outer absorptances α_1 und α_3. The *admixture ratio* Θ representing the inner degree of freedom ($0 < \Theta < 1$) is weighting this contribution in proportion to $\Lambda_{12}/\Lambda_{23}$.

- First the direct sum of the quantities (A_{e1}, A_{e3}) equals the global absorptance α:

(19) $A_1 + A_3 = \alpha_1 + \alpha_2 \cdot \Lambda_{12} / (\Lambda_{12} + \Lambda_{23}) + \alpha_3 + \alpha_2 \cdot \Lambda_{23} / (\Lambda_{12} + \Lambda_{23}) = \alpha_1 + \alpha_2 + \alpha_3 = \alpha$.

- The quantities (A_1, A_3) and therefore their ratio A_1/A_3 are not sensitive to Λ. The ratio characterizing the system is found to be detached from the amount of the global value Λ:

(20) $$A_1/A_3 = \frac{\alpha_1 + \alpha_2 \cdot \Lambda_{12}/(\Lambda_{12} + \Lambda_{23})}{\alpha_3 + \alpha_2 \cdot \Lambda_{23}/(\Lambda_{12} + \Lambda_{23})} \quad .$$

Independence from Λ still was a property of the invariant in the case n=2 where with α_1/α_2 no thermal property at all is entering. This was given by the simple structure. In the case n=3 now the ratio A_1/A_3 has the meaning to describe the *temperature detuning* $\Delta T_1/\Delta T_3$ outside and this again detached from the global thermal property Λ. In this sense this ratio has again an optical intrinsic meaning, furthermore again not depending from the BC (h_e, h_i).

Starting from the values (A_1, A_3), these once determined under a single BC, backwards by means of (16) the *temperature detuning* under each other BC (h_e^N, h_i^N) is determined. Therewith over (6) the new *branching ratio* q_e^N/q_i^N is determined – interceded by the quantities (h_e^N, h_i^N, Λ). The derivation yields this property of invariance given in A_1/A_3 and this by prescribing in (17) and (18) in which manner the invariance is kept within the constellations (8) and (9). The pane $\alpha_1 + \alpha_2 + \alpha_3 = \alpha$ contains a *solution space* within a glazing exposed to a signal S has an equivalent behaviour to the outside in the sense of indistinguishable responses.

VALUE RANGE DISCUSSION IN THE SIMPLE BUT IMPORTANT CASE N=3

The question concerning the shape of the value range of the absorptances α_j that could have been underlaid for a glazing under an outer measurement leading to (A_1, A_3) can be discussed in an illustrative way for the case n=3. The experimentalist or the manufacturer determines (A_1, A_3) together with the global absorptance ($\alpha = A_1 + A_3$). The possible constellations of values (α_1, α_2, α_3) are in first instance restricted to a pane within the positive octant given by:

(21) $\alpha_1/\alpha + \alpha_2/\alpha + \alpha_3/\alpha = 1$, intercept form of the pane .

Choosing the absorptance α_2 as control variable in equations (17) and (18) under a fixed *admixture ratio* Θ this implies moving on straight lines, whose parametric form is given by:

(22) $\alpha_1 = A_1 - \alpha_2 \cdot (1 - \Theta)$
(23) $\alpha_3 = A_3 - \alpha_2 \cdot \Theta$ } $(0 < \Theta < 1)$.

therein the *admixture ratio*: $\Theta = \Lambda_{23}/(\Lambda_{12} + \Lambda_{23})$ resp. $1 - \Theta = \Lambda_{12} / (\Lambda_{12} + \Lambda_{23})$.

In Fig. 2 the pane (21) is presented containing some straight lines in case (A_1 , A_3) = (0.4 , 0.2):

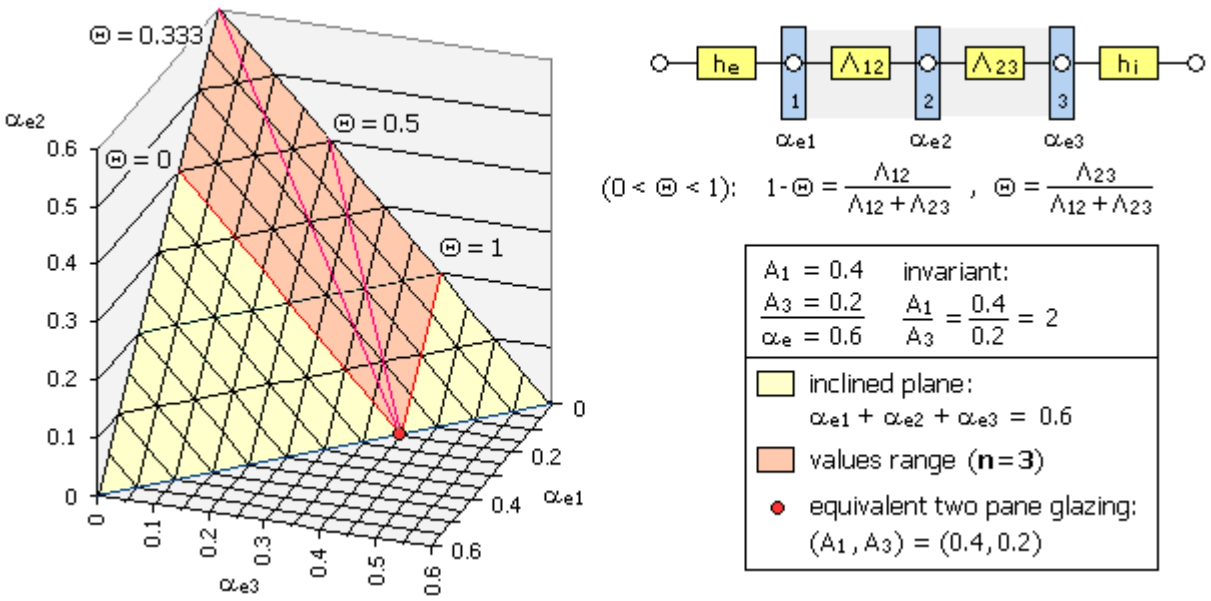

Figure 2: For a given triple glazing (Λ) given boundary condition (h_e , h_i) all these cases give the same answer (ΔT_1 , ΔT_3) to a signal S. They are indistinguishable so long as the temperature of the inner pane ΔT_2 – depending on the ratio Θ – is not accessible.

Under admixture within (0 < Θ < 1) the associated straight lines subtend the permissible value range, in which the system purveys identical responses: Therein the *admixture ratio* Θ is representing the degree of freedom in case n=3, delivering the possibilities to tune in the given values (A_1, A_3) under which outer pane temperatures ΔT_2 (13) and ΔT_3 (15) do not change.

However the temperature ΔT_2 (14) of the inner pane changes, but it remains a *hidden parameter*. In a glazing n=3 with α_2=0 the searched glass n=2 with (A_1, A_3) corresponds directly to (α_1, α_3). In this point, where the admixture Θ has no repercussion, the according straight lines intersect.
If a designer follows the strategy to have no weak link in the chain regarding the conductances Λ_{ij}, he moves on the straight line Θ=0.5. If a designer pursues the extremal constellation of giving an absorptance exclusively to the inner pane – α_2=0.6 in the present example – in agreement with (23) he would choose $\Theta = A_3/\alpha_2 = 0.2/0.6 = 0.333$. With n=3 panes he is able now to obtain a desired asymmetric optical behaviour by tuning with the *admixture ratio* Θ.

EXTENDING THE RESULTS ON ARBITRARY NUMBER OF PANES N

The arbitrary case n is attained by concluding in an inductive way, starting from the case n=3 and equations (17) (18). In the case n=4 the admixtures to α_1 and α_4 have the shape:

$$\left.\begin{array}{ll} (24) & A_1 = \alpha_1 + \alpha_2 \cdot (1/\Lambda_{23} + 1/\Lambda_{34}) \cdot \Lambda + \alpha_3 \cdot (1/\Lambda_{34}) \cdot \Lambda \\ (25) & A_4 = \alpha_4 + \alpha_3 \cdot (1/\Lambda_{12} + 1/\Lambda_{23}) \cdot \Lambda + \alpha_2 \cdot (1/\Lambda_{12}) \cdot \Lambda \end{array}\right\} \quad \Lambda = (1/\Lambda_{12} + 1/\Lambda_{23} + 1/\Lambda_{34})^{-1} \quad .$$

The sum of both equivalent absorptances A_1 und A_4 results in the correct global absorptance:

$$(26) \quad A_1 + A_4 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 = \alpha \quad .$$

And in an analogue way the admixture of each pane j with her particular absorptance α_j takes place to both outer panes for the case of an arbitrary number of panes n.

ERROR DISCUSSION IN THE EVALUATION FOR ENTIRE MEASUREMENT PROCEDURE

The subtle point is given by modelling the boundary conductances (h_e, h_i) these appearing as referencing resistances. Anyhow possible systematic errors compensate each other particulately in the ratio (16) therewith the intrinsic optical invariant A_1/A_n gets fixed. The global absorptance $\alpha(\varphi) \equiv A_1+A_n$ originating from an independent optical measurement [2] opens only just the way to regress on the more harmless behaving ratio A_1/A_n.

CONCLUSIONS

Starting from the question, what can be found out from a glazing regarding its energetical behaviour, without disassembling it that means under an exclusively usage of global characteristic values and available only outer responses in a steady state equilibrium, the *ensemble acting* of panes in compound could be clarified on a principal level. This inversely allows the manufacturer to design the global behaviour in a targeted strategy: The free scope for designing in principle being available as well this in a conclusive formalism, now problems of practicability and feasibility can be treated in a precise way.

Regarding the *response to a signal* and *temperature detuning* of the two outer panes the procedure given here is based on the search for an invariant *intrinsic parameter* which can be attributed to an according *equivalent two pane glass*. This is – as pointed out – the simplest configuration showing all outer possibilities of responses, being capable of an asymmetric behaviour.

The system based was a serial network with nodes in which linear associations between entries and reactions are valid. The deduced propositions are valid in the frame of the model. Nevertheless characteristic values based on simple minimal configurations and the appropriate procedure to find the parameters for the system within the model, open as well opportunities to understand and design systems with a higher complexity. Here i.e. the insight according to which the absorptances are weighted by an *admixture ratio* which corresponds to the two conductances the absorbed energy has to cover on each side.

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