From standing to breaking antidunes: a Hopf bifurcation?

J. Heyman[1,2], F. Mettra[1] and C. Ancey[1]

[1] Laboratory of Environmental Hydraulics, School of Architecture, Civil and Environmental Engineering, École Polytechnique Fédérale de Lausanne, Switzerland. [2] joris.heyman@epfl.ch

1. Introduction

Antidunes are bed morphologies often observed in steep slope mountain flows but also in small streams flowing on a sand beach. Linear stability analysis of the shallow water equations (SWE), when coupled to a sediment transport equation, predicts the growth of selected wavelength when the Froude number exceeds unity. If the bedform amplitude grows sufficiently (without being stabilized by nonlinear effects (Colombini and Stocchino (2008))), hydraulic jump can form on the lee side while a transcritical point, situated roughly on the crest of the dune, connects the sub and supercritical states.

This has been described in the literature as cyclic steps. Their stationary travelling wave solution has already been theoretically and experimentally investigated (Balmforth and Vakil (2012); Taki and Parker (2005); Sun and Parker (2005)).

In this talk, we provide numerical and theoretical evidence that a time dependent quasi-periodic solution of cyclic steps also exists in the SWE+Exner equations for a certain choice of the parameters. We compare this quasi-periodic phenomenon to breaking antidunes often observed in natural rivers (Fig. 1).

Figure 1: Antidunes in the rising stage.

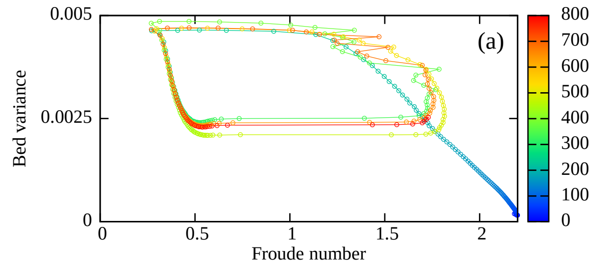
2. Numerical simulation

We solved numerically SWE+Exner equations via a robust finite volume scheme based on the Clawpack Fortran library (George (2008)). The boundary conditions were periodic. We used a generic bed erosion rate of the form:

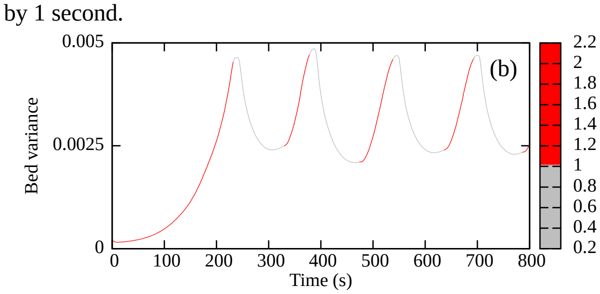
$$E = \alpha\,(\Theta - \Theta_c)^{\beta} \qquad (1)$$

where E is the erosion rate, Θ is the local Shields stress calculated from the friction law and Θ_c is the critical Shields stress. Deposition is not considered in this study.

For a certain range of parameters (initial Froude and Shields number, friction coefficient, α, β and Θ_c), the solution is found to oscillate between two state (Fig 2).



(a) Bed variance against Froude number. Simulation time (in seconds) is represented by the curve color and each points are separated by 1 second.



(b) Bed variance through time. Froude number is encoded by the curve color.

Figure 2: Numerical results

3. Conceptual model

To understand the dynamics involved, we built a conceptual model that captures the important features of the system. That is: (i) the bi-stability of the flow past an obstacle (Baines (1998)) and (ii) the bedform growth rate dependence on the Froude number. We show that under certain circumstances, the conceptual model undergoes an Hopf bifurcation, which leads to a stable periodic orbit of antidune growth and breaking.

References

Baines, P. G. (1998). *Topographic Effects in Stratified Flows.*

Balmforth, N. and Vakil, A. (2012). Cyclic steps and roll waves in shallow water flow over an erodible bed. *J. Fluid Mech.*, 695:35–62.

Colombini, M. and Stocchino, A. (2008). Finite-amplitude river dunes. *J. Fluid Mech.*, 611:283–306.

George, D. L. (2008). Augmented riemann solvers for the shallow water equations over variable topography with steady states and inundation. *J. Comput. Phys.*

Sun, T. and Parker, G. (2005). Transportational cyclic steps created by flow over an erodible bed. part 2. theory and numerical simulation. *J. Hydraul. Res.*, 43(5):502–514.

Taki, K. and Parker, G. (2005). Transportational cyclic steps created by flow over an erodible bed. part 1. experiments. *J. Hydraul. Res.*, 43(5):488–.