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Delayed multiattribute product differentiation ☆

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Abstract

We develop a two-stage model for versioning products with respect to both vertical and horizontal attributes. At first, a firm positions its top-quality "flagship" product in a market with an imperfectly known distribution of tastes and reservation prices. In the second stage, the firm learns these consumer characteristics and has the option of extending its product line by versioning the flagship product using pure horizontal differentiation, quality degrading, or both. The firm's nonconvex versioning problem is solved analytically for the two-product case. We find that *ex ante* extending the product line through vertical differentiation is optimal for low marginal cost of quality (development cost); otherwise pure horizontal differentiation is superior. Given quasilinear consumer preferences and a uniform distribution of consumer characteristics, versioning with respect to both horizontal and vertical attributes is never optimal. Under delayed differentiation the optimal policy is contingent on the observed demand realization and may lead to horizontal cannibalization and price dispersion for equal-quality products. The firm tends to increase its investment in product quality unless it adopts a state-contingent policy of horizontal versioning for high and vertical versioning for low demand realizations. Following a state-contingent policy, the optimal upfront development effort may be significantly lower than under full *ex-ante* commitment. The option value of delayed differentiation is generally nonmonotonic in the firm's development cost.
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Keywords: Multiproduct firms; Product versioning; Delayed differentiation

1. Introduction

Information goods such as computer software or electronic newspapers can be provided by firms at a low marginal cost, though in many cases large capital outlays are required to produce their first unit. The substantial setup cost is driven mainly by the cost of developing the top quality product. Having established this "flagship" product, a firm can degrade it or in other ways modify it, and in this way create a multitude of products with different attributes at a small "versioning" cost. Information goods are not the only products that enjoy such cost complementarities in development. Component standardization and design based on common platforms can be found in a wide range of industries [35]. In

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addition, information goods are often bundled in with physical goods, determining in part the functionality of these underlying physical products. Therefore, finding *optimal* versioning policies is becoming increasingly important, as often lower distribution costs and new-found customer intimacy render intricate second-degree price discrimination strategies feasible. We present a model of a firm that chooses a two-product portfolio sequentially, when products can differ with respect to both horizontal and vertical nonprice attributes. The firm determines its second, or *versioned product*, after it has completed the development of its *flagship product*, and uncertainty over the market acceptance of the latter has resolved.

It is well known that component sharing or, in a broader sense, cost complementarities between products can induce a firm to increase its product variety by keeping the number of components necessary for their assembly manageable [12]. On the other hand, over time the creation of product variety should be commensurate with the information available about demand, leading to the practice of delayed differentiation [1,21,33]. Cost complementarities and opportunities for delayed differentiation are especially large for information goods: provided a sufficiently modular product design, features can be easily disabled or rebundled, leading to vertically and horizontally differentiated versions of the initial flagship product [32]. Since most of the development effort goes into the design of the top quality product, the optimal initial research effort and therefore ultimately the choice of the flagship product's quality level depends on the firm's options for creating a versioned product once the market demand for the first product has been observed.[1] While this paper is not so much concerned with the timing of product introductions [24], we are interested in finding an optimal segmentation of a heterogeneous customer base of multiple characteristics both with and without delayed differentiation. We derive the option value for delaying the versioning decision and show that investment in product quality does not have to be higher when this option is available.

All consumers are initially endowed with a budget and personal taste. Each consumer of a certain (*taste,budget*)-type has a utility, which depends on price, product quality, and distance of the horizontal product attribute from one's own taste. Quality is a vertically differentiating instrument and can be thought of as performance or product breadth. If a product is characterized as a bundle of features [20], then a product including more features than another is of higher quality, whereas two products with different bundles of features, not distinguished in terms of performance or overall product breadth, can be seen as horizontally differentiated. Each of them appeals to consumer tastes "located" sufficiently close. Given a distribution of consumer types (here assumed to be uniform for simplicity), generated for instance by usage preferences or existing standards, the firm can choose product attributes so as to attract as many of the consumers with a sufficiently large budget (*i.e.*, potential buyers) as possible. More specifically, the goal of the risk-neutral firm is to create a certain number of product offerings of the form (*price, quality, horizontal product attribute*), such that its expected profits are maximized. Each product offering targets a subset or segment of the (*taste,budget*)-space, into which the consumers self-select by choosing the product that maximizes their respective utilities. However, at the outset the firm's ability to optimize the choice of its portfolio is limited by imperfect information about consumer characteristics. We assume that this subjective uncertainty resolves after the creation of its flagship product, which typically can be used to evaluate demand. This creates an option value of delaying differentiation to reduce the firm's partially irreversible commitment to a particular product portfolio.

The underlying problem of multiattribute product differentiation is highly nonconvex. We show that both pure horizontal and pure vertical versioning are *locally* optimal, whereas *mixed versioning*, *i.e.*, the simultaneous differentiation along both horizontal and vertical nonprice attributes, is *ex ante* never optimal in our quasi-linear homogeneous setting. Pure vertical versioning is *globally* optimal for relatively low development costs, whereas for high development costs pure horizontal versioning is superior. Under delayed differentiation the optimal policy is contingent on the demand realization ("state"). A consequence of this added flexibility may be that the firm's optimal *ex-ante* investment in product development may drastically drop, instead of increase as a result of diminished (strategic) irreversibility of the upfront sunk cost.

1.1. Literature review

Beginning with Hotelling's seminal paper on horizontal competition [15], numerous contributions have been made to product differentiation. The corresponding literature can be divided into locational models in the

[1] As an example, Microsoft's Office 2000 software was available in at least four different versions (Premium, Professional, Small Business, and Standard) essentially to segment the market and screen the heterogeneous consumer base. These versions contained both vertically (*i.e.*, more *vs.* fewer features) and horizontally (*i.e.*, disjoint features) differentiating attributes.

tradition of Hotelling, where each firm is given an "address" in product space, and so-called "non-address" models in the spirit of Chamberlin's monopolistic competition [8], where a representative consumer exhibits (probabilistic) preferences for different products.[2] An important distinction between the two groups of models is that in the latter group, *each* product is competing with all others, while in the former consumers are truly heterogeneous in their preferences, and some products may have no overlap, *i.e.*, may never be in direct competition.[3] In this paper we adopt the locational approach, which in our view better captures consumer heterogeneity and allows the explicit consideration of participation constraints that inevitably arise when dealing with a spatial distribution of endowed unobservable consumer characteristics. Our model is inspired by Salop's "circular city" [30], which we extend to a "cylinder" by adding a vertical product characteristic.

Mussa and Rosen [25], based on earlier work by Mirrlees [23], started a line of work on second-degree price discrimination of multiproduct firms with quality as the differentiating instrument. Cremer and Thisse [10] demonstrate that in many setups horizontal differentiation can be seen as a special case of vertical differentiation. However, as our model contains both horizontal and vertical features, it cannot be generally mapped into either a pure horizontal or a pure vertical one, as we shall see below. Jones and Mendelson [17] show that for quality-differentiated information goods and a uniform distribution of consumer types, no differentiation is optimal.[4] Our results do not confirm these findings. Interpreting one consumer characteristic as reservation price induces a type-dependent participation (or "feasibility") constraint: no consumer can pay more than her reservation price. This partitions the consumer base into segments buying different products.

In addition to allowing the multidimensional screening of a heterogeneous consumer base [19,29], product variety and broader product lines based on a modular product design[5] can help to delay differentiation along the supply chain [21,33]. The real-option value of delaying irreversible commitment to a full product line can be seen in analogy to financial decision making [22]: delaying irreversible investments can carry a significant value, sometimes comparable to the investment volume itself [28]. This option value is generally increasing in the magnitude of the uncertainty unless, as Ref. [16] shows, imperfectly correlated risks from different sources are pooled, potentially decreasing the value of managerial flexibility. We restrict ourselves to what [16] terms "market payoff variability," a single risk class, and thus circumvent this effect. Examining the real-option value of delaying the product versioning until – with release of the flagship product – overall demand has been observed, we find that it is generally non-monotonic in the cost of quality β. Naturally, the option value depends in an important manner on the performance of the default *ex-ante* versioning policy, and is therefore not necessarily monotonic in the amount of uncertainty over all β's. This points to resource flexibility as a driver of this option value [36]. Delaying differentiation does not generally allow one to reverse the upfront quality investment. Being able to make product-line-extension and pricing decisions *contingent* on the observed demand carries significant value depending on the firm's default policy. For instance for development costs, β, close to zero when the irreversibility of the investment in flagship quality is not very significant, the option to delay differentiation carries a high value. In fact we show that it is locally maximal at $\beta=0$. The effects of increased versioning flexibility on upfront investment in product development as well as on consumer surplus are ambiguous. We provide an example where delayed differentiation leads to a lower flagship quality as a result of a state-contingent policy where the mode of differentiation depends on the observed demand realization, which is generally the case if the dispersion in the firm's prior beliefs about demand is large. For very little demand uncertainty, the versioning mode is generally "locked in" *ex ante*, and the delay in the differentiation allows slight improvements on the previous pricing and positioning decisions. Consumer surplus depends both on the actual product development costs as well as on the chosen mode of product differentiation, and thus may vary in both directions when introducing an option to delay the versioning decision. To the best of our knowledge this paper, based on Ref. [37], provides a first complete analytical treatment of multiattribute versioning combining horizontal and vertical differentiation. We further extend this new approach to account for demand uncertainty by deriving optimal state-

[2] For a good bibliography see Refs. [2,4,34].

[3] A notable exception in this dichotomy is the model in Ref. [27] that combines characteristics from both locational and non-address models, driven by symmetry assumptions in the preferences of a representative consumer who is faced with localized products.

[4] These results have been generalized by Ref. [6], which provides sufficient conditions for goods with *nonzero* marginal costs and general utility functions under which a stable, incentive-compatible separation of the consumer base into segments can be reached.

[5] Ref. [18] provides an excellent survey of the recent literature on product development decisions.

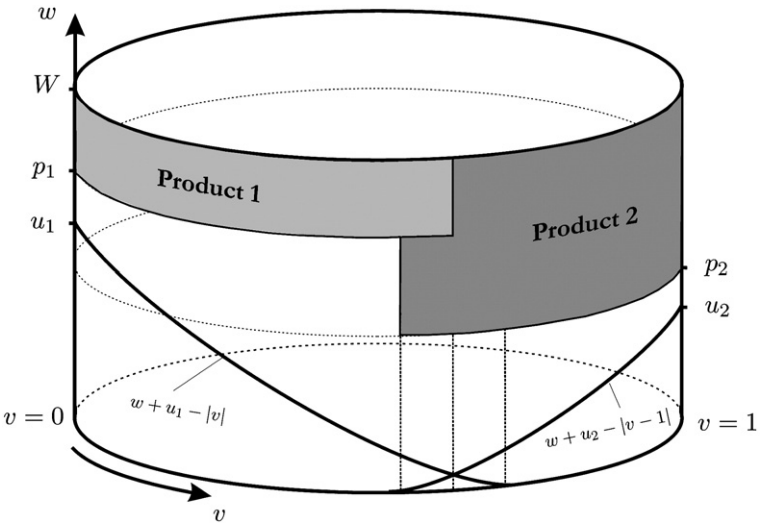

Fig. 1. Consumer utilities and resulting market segments ($q_1 > q_2$).

contingent policies under delayed differentiation. Information goods are chosen here for mere convenience. Everything in this paper equally applies to goods with positive marginal costs (*cf.* Section 4.4).

1.2. Outline

In the following section we introduce the basic model together with some structural simplifications that can be made without loss of any generality. In Section 3 we examine the *ex-ante* versioning case in which both flagship and versioned product are chosen simultaneously. We show that mixed versioning is never optimal and that investment in quality increases with estimated demand. Section 4 covers the case where the firm possesses the option to delay product versioning until after demand has realized. We determine the optimal versioning policy and compute the option value of delayed differentiation in this context. We also show that investment in product quality under delayed differentiation may either increase or decrease using a state-contingent horizontal–vertical versioning policy. Section 5 discusses the results and concludes with directions for further research.

2. The model

Let all consumers be distributed uniformly on $\mathcal{V} \times \mathcal{W}$, where $\mathcal{V} = \mathbb{R}/2\mathbb{Z}$ and $\mathcal{W} = [0, W]$ denote the sets of tastes and budget levels, $W > 0$ the maximum reservation price, and $\mathbb{R}$, $\mathbb{Z}$ the sets of reals and integers. The (quotient) space $\mathbb{R}/2\mathbb{Z}$ corresponds to an interval [0, 2] with the points 0 and 2 identified, so that $\mathcal{V} \times \mathcal{W}$ is a cylinder of radius $1/\pi$ and of height W (*cf.* Fig. 1).[6] The parameter W represents the maximum reservation price and is initially only imperfectly known, in terms of a bounded measurable probability density function f with compact support $\mathcal{S} \subset \mathbb{R}_{++}$. We assume that any information product offered can be uniquely described in terms of its attributes, price p, nonnegative quality q, and taste $z \in \mathcal{V}$. The *utility* of a consumer of type $(v, w) \in \mathcal{V} \times \mathcal{W}$ who buys a product of attributes (p, q, z) is assumed to be additively separable of the form $w - p + \kappa q - |v - z|$, and equal to w if she does not buy the product.[7] The positive constant κ defines the marginal valuation of quality relative to the other product attributes, price and taste.[8] This consumer can only afford the product if her reservation price (disposable budget) w is at least equal to the price p of the product,

$$p \le w. \tag{1}$$

[6] This model generalizes the common setup of "circular city" models in the spirit of Ref. [30], which are used to describe pure horizontal differentiation, typically to avoid the endpoint effects in Hotelling's "linear city." We have chosen the cylindrical setup for the mixed-differentiation model with precisely this in mind.

[7] General conditions for the additive separability of utility functions in budget have been provided by Ref. [14]. Instead of κq it is possible to incorporate any nonlinear strictly increasing quality measure $\phi(q)$ which simply corresponds to a rescaling (*cf.* also Eq. (4)).

[8] Without loss of generality the marginal utility of taste differences $|v - z|$ can be normalized to one, even when the cylinder circumference $2V$ (for some $V > 0$) is not precisely known. Varying V changes the relative marginal valuation of taste mismatch. By substituting $W' = W/V$ for W and using an appropriate prior probability density g of W' (instead of f) one obtains the same results as the ones derived for the unit cylinder $\mathcal{V} \times \mathcal{W}$, up to a constant factor. Thus, the results in this paper do also hold when both maximum reservation price *and* extent of the taste domain are *ex-ante* uncertain.

For any given maximum reservation price W the normalization constant for the consumer density is $1/(2W)$. The total budget in the set $\mathcal{V} \times \mathcal{W}$ is $W/2$, independent of the diameter of the cylinder.

2.1. Change of variables

To simplify some of the discussions that follow, we introduce the new variable $u = \kappa q - p$, so that consumer utility can be written in the form $w + u - |v - z|$. The new variable u represents the net utility gain at perfect horizontal fit. Using this simple change of variables, consumer utility becomes independent of price. The participation constraint (*i.e.*, for any consumer the utility obtained for the product has to weakly exceed utility for budget w) can be written as

$$|v - z| \le u. \tag{2}$$

Any optimization of the product portfolio at the firm level needs to account for the *feasibility* and *participation* constraints (1)–(2) as well as *incentive compatibility* in the sense that each consumer's choice is utility maximizing.

2.2. Cost structure

Throughout most of this paper we consider information goods with zero marginal cost, which simplifies the closed-form solutions. This assumption is not crucial and all results can be easily generalized to goods with positive marginal cost, as is shown in Section 4.4. For information goods, the costs of reproduction and distribution are indeed very small, so that this has become a standard assumption in much of the extant literature.[9] The cost βq of creating the flagship product depends linearly on the quality q, and can be written in the new variables as

$$C(p, u) = \frac{\beta}{\kappa}(p + u), \tag{3}$$

where β is a positive constant. Without loss of generality it is possible to set

$$\kappa = 1, \tag{4}$$

as the results for any arbitrary value of $\kappa > 0$ can be recovered by substituting everywhere $\beta' = \beta/\kappa$ for β. The weight κ in the utility function has the sole effect of scaling the firm's quality cost. In everything that follows we therefore assume that Eq. (4) holds.

[9] Information goods are chosen here merely to keep analytical complexity as low as possible. Of course, there are settings for which the small, but nonzero, nature of the marginal costs matters, such as in deriving asymptotic properties of large bundles of information goods [3,13].

2.3. Versioning problem

After having created the flagship product, the risk-neutral firm has the option to create a versioned product by varying product attributes in the following admissible ways:

(a) *horizontal version* (*i.e.*, same quality, different taste) at a cost $\alpha_H \ge 0$,
(b) *vertical version* (*i.e.*, lower quality, same taste) at a cost $\alpha_V \ge 0$,
(c) *mixed version* (*i.e.*, lower quality, different taste) at a cost $\alpha_M \in [\max\{\alpha_H, \alpha_V\}, \alpha_H + \alpha_V]$.[10]

This cost structure exhibits strong cost complementarities or economies of scope [26], as development costs are incurred only once to create the flagship product.[11] The firm maximizes profits subject to the consumers' choice and participation. Solutions to this profit-maximization problem exhibit *translation invariance*: profits will not change by just horizontally translating all products, as a direct consequence of the circular symmetry. One can therefore arbitrarily fix the horizontal location of the first product to zero. Any implied solution then naturally represents an entire equivalence class of solutions to the more general problem with arbitrary horizontal location of the first product.

2.4. Demand estimation

As has been pointed out above, the demand parameter W is initially only imperfectly known to the firm (*cf.* also footnote 8). For any product of price p the profit function depends on W through the marginal revenue density $p(W-p)/(2W)$. By replacing the random value W with a nonlinear estimator $\hat{W} = (E\ [1/W])^{-1}$ we obtain the *expected* marginal revenue density. The following technical lemma will be used throughout the paper without special mention.

Lemma 1. *(i)* $E\ [p(W-p)/(2W)] = p\ (\hat{W}-p)/(2\hat{W})$. *(ii)* $\hat{W} < \bar{W}$, *where* $\bar{W} = E(W)$.

The first part of this result is trivial but useful, as it allows replacing the imperfectly known W with the nonrandom $\hat{W}$ when computing expected profits. Part (ii) indicates that when maximizing expected profits, uncertainty in W prompts the firm to use as decision-

[10] This condition can also be written in terms of the r-norm of the row vector (α_H, α_V), *i.e.*, $\alpha_M = \|(\alpha_H, \alpha_V)\|_r = (\alpha_H^r + \alpha_V^r)^{1/r}$ for $r \in [1, \infty)$ and $\alpha_M = \|(\alpha_H, \alpha_V)\|_\infty = \max\{\alpha_H, \alpha_V\}$ for $r = \infty$.

[11] A multiproduct cost structure of this form has also been proposed by Ref. [31].

relevant "certainty equivalent" of W an estimator $\hat{W}$ that is always strictly (and sometimes substantially) lower than the expected maximum reservation price $\bar{W}$.[12] Our demand specification for the single-product firm corresponds to a linear demand curve

$$D(p,u) = (1-(p/W))\min\{1,u\}$$

with random intercept at $W>0$. The demand elasticity $p/(W-p)$ is independent of the consumers' net utility $u>0$. The demand specification considered here is somewhat nonstandard, since the consumer heterogeneity is with respect to their horizontal location and their income, while much of the extant literature has emphasized consumer heterogeneity with respect to horizontal location and marginal valuation of product quality.

3. Ex-ante versioning

3.1. Benchmark: Single-product firm

Before solving the multiproduct case we briefly discuss the single-product monopoly as a benchmark. Without loss of generality we can set $z_1=0$, in view of the translation invariance discussed above. With this the expected single-product monopoly profit $\bar{\Pi}_1(p_1, u_1)= E[\Pi_1(p_1, u_1; W)]$ as a function of (p_1, u_1) becomes

$$\bar{\Pi}_1(p_1,u_1) = \frac{p_1(\hat{W}-p_1)}{\hat{W}}\min\{u_1,1\} - \beta(p_1+u_1).$$

It is clear that $u_1^* \le 1$, since $\partial_{u_1}\bar{\Pi}(p_1, u_1)<0$ for any $(p_1, u_1)\in(0, \hat{W})\times(1, \infty)$. By virtue of the linearity of the profit function in $u_1^*\in[0, 1]$ maximum profits are achieved for $u_1^*\in\{0, 1\}$, so that

$$(p_1^*,u_1^*) = \begin{cases} (\hat{W}(1-\beta)/2,1), & \text{if } \beta \le 1, \\ (0,0), & \text{if } \beta > 1. \end{cases} \quad (5)$$

Thus, the single-product firm would like to achieve "full horizontal market coverage" (*i.e.*, $q_1^*=1+p_1^*$), whenever it decides to enter the market.[13]

Proposition 1. *(i) The optimal expected profit of the single-product firm is*

$$\bar{\Pi}_1^* = \frac{\hat{W}}{4}(1-\beta)^2 - \beta, \quad (6)$$

increasing in $\hat{W}$ and decreasing in β. (ii) The single-product firm enters the market if and only if

$$\hat{W} > \frac{4\beta}{(1-\beta)^2}. \quad (7)$$

Relation (7) is a *viability* condition that imposes a lower bound on the maximum-reservation-price estimator $\hat{W}$ as a function of β. In what follows we will use the optimal expected single-product monopoly profits as a benchmark for the performance of the various differentiation modes in a multiproduct setting.

3.2. Ex-ante product differentiation

Due to the symmetry of the problem it is possible, without any loss in generality, possible to set $z_1=0$ and to restrict z_2 to the interval [0, 1] (otherwise just switch products and translate). Any solution satisfying these constraints defines an equivalence class of solutions to the versioning problem. There is one further simplification, which directly results from the linearity of the problem in z_2. Indeed, the optimal value of z_2 must be at an extremity of [0, 1], *i.e.*, $z_1^*\in\{0, 1\}$. The induced two canonical cases of vertical and horizontal differentiation have to be distinguished in the solution to the versioning problem. Furthermore we assume that the quality of the second product does not exceed the quality of the first product ($q_1\ge q_2$), which after a linear variable transformation ($u=\kappa q-p$) can be equivalently written in the form

$$q_1 = p_1+u_1 \ge p_2+u_2 = q_2. \quad (8)$$

This assumption expresses the fact that in order for a "versioning" policy to be meaningful, the first (flagship) product has to be of higher quality than the derived product. Fig. 2 provides an overview of the various modes of differentiation that can arise: (1) no differentiation ($z_2=0$, $q_1=q_2$), which is *de facto* equivalent to the single-product monopoly; (2) pure vertical differentiation ($z_2=0$, $q_1>q_2$); (3) mixed differentiation ($z_2=1$, $q_1>q_2$); and, (4) pure horizontal differentiation ($z_2=1$, $q_1=q_2$). To simplify the exposition, we will first set versioning costs to zero, as their effect can be analyzed separately once all the main expressions for the zero-versioning cost base case have been established.

3.2.1. Pure vertical differentiation

The profit function $\bar{\Pi}_V$ is linear in u_1 and u_2, so that we can concentrate on the situation when $u_1=u_2=1$, since otherwise the firm would only enter with one

[12] The relative difference between $\bar{W}$ and $\hat{W}$ can become arbitrarily large, as can the relative difference in the corresponding profits (when using $\bar{W}$ instead of $\hat{W}$). As an example, consider a uniform distribution of W on an interval $[\varepsilon, 2-\varepsilon]$ with $0<\varepsilon<1$. Then $\bar{W}=1$ for all $\varepsilon\in(0, 1)$, while $\hat{W}$ goes to zero as $\varepsilon\to 0^+$.

[13] This result is driven by the linearity of the cost function. In the case where the cost function is strictly convex, say, of the form βq^2, there may exist interior solutions, making it impossible to derive closed-form solutions in the multiproduct case.

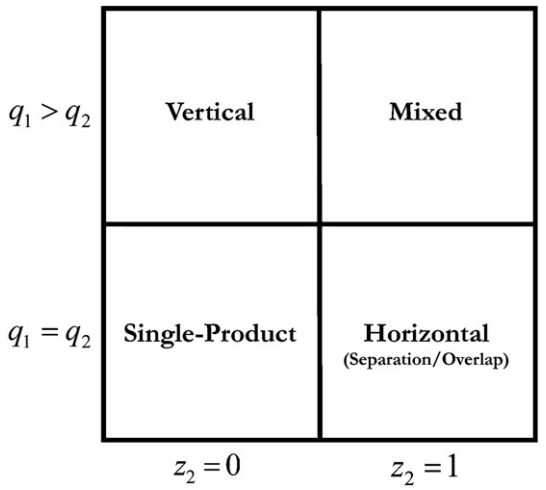

Fig. 2. Modes of differentiation.

product ($u_2=0$, *cf.* Section 3.1). The maximization problem can be written in the form

$$\max_{p_1,p_2} \bar{\Pi}_V(p_1,p_2) = \max_{p_1,p_2} \left\{ \frac{1}{\hat{W}} (p_1(\hat{W}-p_1)+p_2(p_1-p_2)) - \beta(p_1+1) \right\}, \tag{9}$$

the solution of which yields the unique interior maximizers, $p_1^*=2\hat{W}(1-\beta)/3$ and $p_1^*=\hat{W}(1-\beta)/3$, provided that $\beta<1$. Optimal expected profits are then

$$\bar{\Pi}_V^* = \frac{\hat{W}}{3}(1-\beta)^2 - \beta = \bar{\Pi}_1^* + \frac{\hat{W}}{12}(1-\beta)^2, \tag{10}$$

strictly greater than single-product monopoly profits $\bar{\Pi}_1^*$. Thus, pure vertical differentiation strictly dominates a single-product monopoly, given that versioning costs α_V are small enough. Pure vertical differentiation is *ex-ante* viable if and only if

$$\hat{W} > \frac{3\beta}{(1-\beta)^2}. \tag{11}$$

It is important to note that since $u_1^*=u_2^*$ at the optimum in pure vertical differentiation, a consumer of type (v, w) obtains equal surplus from both products. This follows from the latent assumption that *in equilibrium consumers buy the higher-price product if this yields exactly the same utility as the lower-price product* (*e.g.*, since resale value is higher also). If this were not the case, the firm could technically achieve stable separation by letting $u_1^*=u_1^*+\varepsilon$ for any (arbitrarily small) $\varepsilon>0$ without, in the limit, changing any of the results. The fundamental reason for the stable segmentation of the consumer population lies in the feasibility constraint (1): consumers never spend more than their budget w (reservation price) permits.

3.2.2. Mixed differentiation

In analogy to the single-product monopoly, profits $\bar{\Pi}_M$ only depend linearly on u_1 and u_2 (implying full horizontal coverage). Since the constraint (8) is by assumption not binding, it is $u_1=u_2=1$. The profit-maximization problem thus takes the form

$$\max_{p_1,p_2} \bar{\Pi}_M(p_1,p_2) = \max_{p_1,p_2} \left\{ \frac{1}{2\hat{W}} (p_1(\hat{W}-p_1)+p_2(\hat{W}-p_2) + p_2(p_1-p_2)) - \beta(p_1+1) \right\}. \tag{12}$$

Straightforward unconstrained maximization yields the optimal prices $p_1^*=\hat{W}(5-8\beta)/7$ and $p_2^*=\hat{W}(3-2\beta)/7$, which are well defined (*i.e.*, in $[0, \hat{W}]$ and such that $p_1^*>p_2^*$) for any positive $\beta<1/3$. Optimal expected profits are

$$\bar{\Pi}_M^* = \frac{\hat{W}}{7}\left(\frac{5}{4}-2\beta\right)^2 + \frac{\hat{W}}{16} - \beta = \bar{\Pi}_1^* + \frac{9\hat{W}}{28}\left(\frac{1}{3}-\beta\right)^2, \tag{13}$$

strictly increasing in $\hat{W}$ and strictly decreasing in β. In the mixed-differentiation mode, the multi-product monopolist enters if and only if

$$\hat{W} > \frac{\beta}{\frac{1}{7}\left(\frac{5}{4}-2\beta\right)^2+\frac{1}{16}}, \text{ and } \beta < \frac{1}{3}. \tag{14}$$

Given entry in mixed-differentiation mode, quality dispersion does in general occur, since by construction for $\beta<1/3$ that $q_1^*=\hat{W}(5-8\beta)/7+1 \geq \hat{W}(3-2\beta)/7+1=q_2^*$. However, vertical differentiation strictly dominates mixed differentiation. Indeed,

$$\bar{\Pi}_V^* - \bar{\Pi}_M^* = \hat{W}\left(\frac{(1-\beta)^2}{3} - \frac{1}{7}\left(\frac{5}{4}-2\beta\right)^2 - \frac{1}{16}\right) = \hat{W}\left(\frac{1}{20} - \frac{5}{21}\left(\beta-\frac{1}{10}\right)^2\right) > 0$$

for all $\beta \in (0, 1/3)$. Therefore, the mixed-differentiation mode does not play a role, which by itself is an interesting fact.

Proposition 2. *For any given viable market configuration $(\beta, \hat{W})$ either pure horizontal or pure vertical differentiation is optimal.*

Mixed versioning is dominated by pure vertical differentiation. As soon as goods are horizontally differentiated with $(z_2=1)$, it is best to reduce the quality of the flagship product so as to economize on providing costly horizontal coverage with each product. Hence, leaving slack in constraint (8) does not fully utilize the potential of the flagship product, whose quality will *ex ante* be reduced until $q_1=q_2$, *i.e.*, until pure horizontal differentiation is obtained.

3.3. Pure horizontal differentiation

In this mode constraint (8) is binding, so that $q_1=q_2$ or in other words

$$u_1 = u_2 + p_2 - p_1. \tag{15}$$

Expected profits are $\bar{\Pi}_H = \frac{1}{2\hat{W}}\big(p_1(\hat{W}-p_1)(1+u_1-u_2)+p_2(\hat{W}-p_2)(1-u_1+u_2)+\min\{p_1,p_2\}|p_2-p_1|(u_1+u_2-1)\big)-\beta(p_1+u_1)$ or, using Eq. (15),

$$\begin{aligned}\bar{\Pi}_H = \frac{1}{2\hat{W}}\big(&p_1(\hat{W}-p_1)(1-p_1+p_2)+p_2(\hat{W}-p_2)(1+p_1-p_2)\\&+\max\{p_1(p_2-p_1),p_2(p_1-p_2)\}(2u_2-1-p_1+p_2)\big)\\&-\beta(p_2+u_2),\end{aligned} \tag{16}$$

subject to $p_2 \in [p_1-1, p_1+1] \cap [0, \hat{W}]$ and $u_2 \in [(1+p_1-p_2)/2, 1]$. Thus, depending on the sign of $\partial_{u_2}\bar{\Pi}_H = \frac{1}{\hat{W}}\max\{p_1(p_2-p_1),\ p_2(p_1-p_2)\}-\beta$, either $u_2=1$ (for $\partial_{u_1}\bar{\Pi}_H>0$) or $u_2=(1+p_1-p_2)/2$ (for $\partial_{u_1}\bar{\Pi}_H<0$). In the latter case there is no overlap between the two products, which we term "separation" (*cf.* Fig. 3). In the case where $u_2=1$, there clearly is "overlap," and relation (15) entails that then $p_1 \geq p_2$. It is clear that overlap in practice means horizontal "product cannibalization," for some consumers are offered an individually rational choice (satisfying Eqs. (1)–(2)) between both products. It turns out that under *ex-ante* versioning horizontal overlap is never optimal, as the firm can do better by *either* eliminating cannibalization *or* purely vertically differentiating. This finding does not hold under delayed differentiation, where horizontal cannibalization may be optimal as the best possible "compromise" for certain intermediate demand realizations.

Proposition 3. *Let the parameters $0<\beta<1$ and $\hat{W}>0$ be given.*

(i) Horizontal differentiation with separation yields optimal expected profits of

$$\bar{\Pi}^*_{H_1} = \frac{\hat{W}}{4}(1-\beta)^2 - \frac{\beta}{2} = \bar{\Pi}^*_1 + \frac{\beta}{2}, \tag{17}$$

with symmetric product portfolio $(p_k^*, q_k^*) = (\hat{W}(1-\beta)/2,\ (1+\hat{W}(1-\beta))/2$, for $k=1, 2$.

(ii) There is no horizontal product cannibalization, i.e., optimal expected profits $\bar{\Pi}^*_{H_2}$ under horizontal differentiation with overlap are dominated,

$$\bar{\Pi}^*_{H_2} = <\max\left\{\bar{\Pi}^*_{H_1}, \bar{\Pi}^*_V\right\} = \bar{\Pi}^*. \tag{18}$$

When considering only horizontal differentiation, Proposition 3 implies that product cannibalization is not inherently ruled out, which may seem counterintuitive in our simple setting. In fact, for lower unit cost of quality β overlap (*i.e.*, product cannibalization) performs better than separation, which only yields a constant improvement of $\beta/2$ over single-product optimal profits $\bar{\Pi}^*_1$. Cannibalization implies an overinvestment in quality in the sense that more quality is provided than necessary for full horizontal coverage of the consumer base. Exploiting the consumers' feasibility constraint it is possible (for low β) to achieve a *vertical* separation of the consumer base offering one product at a lower price than the other. The high-price product is then solely preferred by high-w consumers of very good horizontal fit. This feature of the model explains the possibility of *price dispersion* in a purely horizontally differentiated market when the marginal cost of creating additional quality is sufficiently low. The following result summarizes the firm's entry decision into the market in the absence of an option to delay product differentiation (*cf.* Fig. 4).

Proposition 4. *Consider a firm that has the option to enter the market, given $(\beta, \hat{W}) \in (0, 1) \times \mathcal{S}$, with one or two information goods.*

(i) For any $(\beta, \hat{W})$ such that

$$2\beta/(1-\beta)^2 < \hat{W}, \tag{19}$$

the firm's entry is viable if versioning costs are small enough.

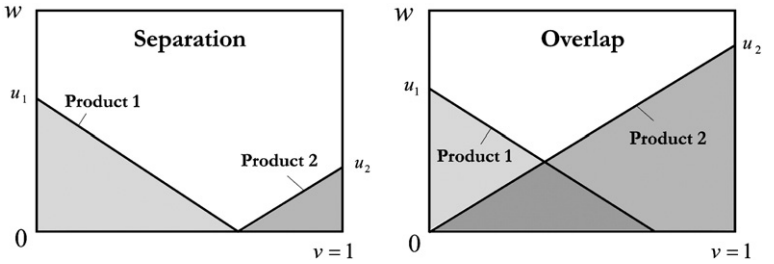

Fig. 3. Horizontal separation and overlap in terms of consumers' net utility.

(ii) In the case that Eq. (19) *does not hold, the firm's entry is not viable, even if versioning costs are zero.*

(iii) If entry is viable and $\hat{W} < 4\beta/(1-\beta)^2$*, then the firm needs to enter the market with two products.*

An interesting implication from the last proposition is that *in some markets for information goods that are unprofitable for any single product, a firm may still be able to enter profitably with a versioned portfolio of products.*

3.4. Effect of versioning costs

We have seen above that under perfect information mixed differentiation is strictly dominated by pure vertical differentiation. Thus, we can limit ourselves to adjusting and comparing expressions (6), (10), and (29). As a consequence of the earlier discussion, write $\bar{\Pi}_H^*$ (instead of $\bar{\Pi}_{H_1}^*$) for the optimal profits under pure horizontal differentiation (separation). Following Eq. (29), pure horizontal differentiation is dominated by single-product monopoly if and only if $\alpha_H > \beta/2$. On the other hand, it is dominated by pure vertical differentiation if and only if $\alpha_V < \alpha_H + \frac{\hat{W}}{12}(1-\beta)^2 - \frac{\beta}{2}$. The resulting partition of the (α_H, α_V)-plane is depicted in Fig. 5.

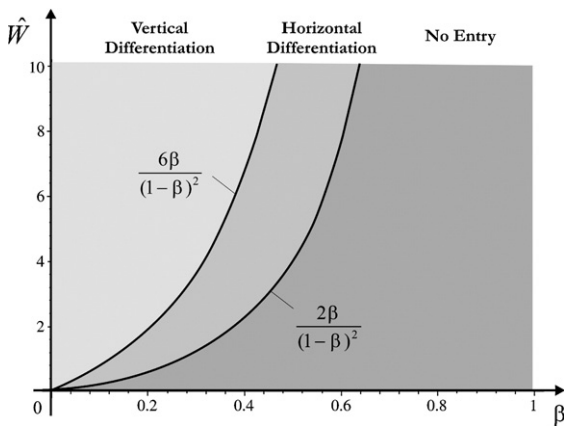

Fig. 4. Optimal *ex-ante* product versioning.

4. Delayed differentiation

Assume that the risk-neutral firm can delay the versioning decision until after the demand uncertainty has resolved through introduction of the flagship product. The exact timing proceeds over two stages ($t \in \{1, 2\}$): at $t=1$ the firm develops its flagship product of fixed[14] attributes (q_1, z_1) and presents it to the market without collecting revenues. In the second stage ($t=2$) firm learns the true value for the maximum reservation price W, decides about introducing a versioned product of nonprice attributes (q_2, z_2), and prices both products at p_1, p_2 respectively. As a result, the firm needs to choose its product portfolio such that its *ex-ante* expected profits achieve

$$\bar{\Pi}^{**} = \max_{q_1} \left\{ E\left[\max\{\Pi_V^*(q_1; W), \Pi_H^*(q_1; W)\} \right] \right\} \tag{20}$$

instead of the $\bar{\Pi}^* = \max\{\bar{\Pi}_V^*, \bar{\Pi}_H^*\}$ it obtains under *ex-ante* versioning according to Proposition 3. We solve this problem in stages, beginning with the second stage.

4.1. Optimal ex-post versioning

Assume the firm has observed the realization of W and has decided about the quality q_1 of its flagship product. As in Section 3, it is possible to differentiate the versioned product either vertically ($z_2=0$) or horizontally ($z_2=1$), provided that its quality does not exceed the

[14] Quality upgrades are not possible for $t>1$, since *e.g.*, the development team has been disbanded or upfront costs to restart significant quality improvements are prohibitive. The firm never has an incentive to change the horizontal nonprice attribute of the flagship product, $z_1=0$.

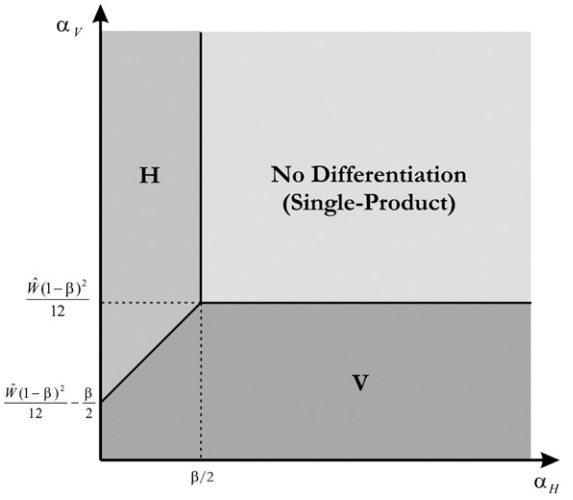

Fig. 5. Optimal modes of *ex-ante* differentiation for versioning cost tuples $(\alpha_H, \alpha_V) \geq 0$.

flagship product's, *i.e.*, $q_1 \geq q_2$ (*cf.* footnote 14). Alternately the firm has the option not to introduce a vertically versioned product at all.[15]

4.1.1. Pure vertical differentiation

Consider the case when $z_2=0$ and the resulting portfolio contains two products. Without loss of generality we can assume that $u_1^*=u_2^*=q_1-p_1^*$, since – as long as they are free – larger values for u weakly increase profits. The profit-maximization problem becomes

$$\max_{p_1,p_2} \Pi_V(p_1,p_2;q_1,W) = \max_{p_1,p_2}\left\{\frac{p_1(W-p_1)+p_2(p_1-p_2)}{W}\min\{1,q_1-p_1\}-\beta q_1\right\}$$

with solution equivalent to that of the *ex-ante* versioning problem, as long as q_1 is large enough so that full horizontal coverage holds. If on the other hand q_1 is small compared to the realization W, then horizontal coverage will not be full and product breadth is traded for the ability to charge high prices. In other words, "transportation costs" for consumers with significant misfit will be too high to buy any of the products. More specifically we obtain $(p_2^*, q_2^*)=(p_1^*/2, q_1-p_1^*/2)$, and

$$p_1^*=\begin{cases} 2W/3, & \text{if } q_1 \geq 1+2W/3, \\ q_1-1, & \text{if } q_1 \in [\bar{q}_V(W), 1+2W/3], \\ \left(4W+3q_1-\sqrt{(4W-3q_1)^2+12Wq_1}\right)/9, & \text{otherwise.} \end{cases} \tag{21}$$

where $\bar{q}_V(W)=\left(6+2W-\sqrt{9+4W^2}\right)/3 \in [1, \min\{2, 1+2W/3\}]$. From this one can derive an expression for the optimal profits as a function of q_1,

$$\Pi_V^*(q_1,W)=\begin{cases} W/3-\beta q_1, & \text{if } q_1 \geq 1+2W/3, \\ (q_1-1)\left(1-\frac{3(q_1-1)}{4W}\right)-\beta q_1, & \text{if } q_1 \in [\bar{q}_V(W), 1+2W/3], \\ p_1^*\left(1-\frac{3p_1^*}{4W}\right)(q_1-p_1^*)-\beta q_1. & \text{otherwise.} \end{cases} \tag{22}$$

Due to the cost complementarities a versioned product will always be introduced as long as the versioning cost α_V is small enough. The development cost for the flagship product is already sunk and a second product can then only increase profits (at least as long as α_V, α_H are small enough). It is straightforward to show that for any $W>0$ the profit function $\Pi_V^*(q_1; W)$ is continuously differentiable in $q_1>0$ with $\partial_{q1}\Pi_V^*(0)=\partial_{q1}\Pi_V^*(1+2W/3))=-\beta<\partial_{q1}\Pi_V^*(\bar{q}_V)=(\sqrt{4W^2+9}-3)/(2W)-\beta$. In addition, the state-contingent profit function Π_V^* has at most one maximum in $q_1>0$ but is generally not quasiconcave.[16]

4.1.2. Horizontal differentiation (including mixed)

When deciding to horizontally differentiate, the firm chooses $z_2=1$ and then needs to decide about p_1, p_2 and $q_2=u_2+p_2 \leq u_1+p_1=q_1$. As in the *ex-ante* versioning problem (*cf.* Section 3) there are two general cases, "separation" and "overlap." The details of the overlap case, which is synonymous with "horizontal product cannibalization," are discussed in the Appendix, where analytical solutions are provided. Under delayed differentiation it is indeed sometimes best to horizontally cannibalize products, whereas this is never optimal under *ex-ante* versioning (Proposition 5). Independent of the versioning costs, it is never optimal to version along both horizontal and vertical attributes at the same time.[17]

There are four possible configurations that all exhibit horizontal separation, depending on how large the

[15] The firm could also reprice the first product out of the market (at $p_1 \geq W$) and vertically version a second product, which is, however, strictly inferior when the versioning cost α_V is positive. If the firm does not introduce the second product at all, then it needs to solve $\max_{p_1' \in [0,W]} p_1'(W-p_1')\min\{1, q_1-p_1'\}$, with solution $p_1'^*$ that can be obtained by replacing W with $3W/4$ in Eq. (21). The firm does not incur a versioning cost, so that at least for large α_V not introducing a second product is optimal, given that vertical differentiation is considered. If on the other hand $\alpha_V=0$ (or $\alpha_V>0$, but small enough), then introducing the versioned product in the second round is a good idea, since it can always be accomplished by setting $p_2=0$.

[16] It is also locally maximal at $q_1=0$, as $\partial_{q_1}\bar{\Pi}_V^*|_{q_1=0}=-\beta<0$.

[17] This finding is somewhat driven by the quasi-linearity of the model in preferences and the assumption that the consumers are uniformly distributed. Including nonlinearities in the consumers' preferences, interior solutions favoring simultaneous differentiation with respect to more than one attribute cannot be ruled out.

quality q_1 is with respect to the demand realization W: (i) if $q_1 \geq 1+W/2$ (*i.e.*, $q_1 - p_1 \geq 1$), given separation there is just one product and $p_1^* = W/2$ analogous to the single-product firm. (ii) If $1/2 \leq q_1 - p_1 = 1-(q_2-p_2) < 1$, then

$$(p_1^*, p_2^*) = \arg\max_{p_1,p_2} \left\{ \frac{p_1(W-p_1)(q_1-p_1)}{2W} + \frac{p_2(W-p_2)(1-q_1+p_1)}{2W} - \beta q_1 \right\} = \left(\frac{W}{2}, \frac{W}{2}\right),$$

for $q_1 \in [(1+W)/2,\ 1+W/2]$. Note that in this case $q_2^* = 1+W-q_1$, resulting in *mixed* differentiation without price dispersion. (iii) If $q_1 \in [\bar{q}_H(W),\ (1+W)/2]$, then $p_1^* = p_2^* = q_1 - 1/2$, and $q_2^* = q_1$ (pure horizontal differentiation). Thereby $\bar{q}_H(W) = \left(2 + W - \sqrt{1+W^2}\right)/2 \in [1/2, \min\{1, (1+W)/2\}]$. (iv) If $q_1 - p_1 < 1-(q_2-p_2)$ (equivalent to $q_1 - p_1 < 1/2$), then $p_1 = p_2$ and $q_1 = q_2$, since the situation is symmetric (pure horizontal differentiation). We have

$$p_1^* = \arg\max_{p_1} \left\{ \frac{p_1(W-p_1)(q_1-p_1)}{W} - \beta q_1 \right\} = (W + q_1 - \sqrt{(W-q_1)^2 + Wq_1})/3, \quad (23)$$

for $q_1 < \bar{q}_H(W)$. Summarizing these results we obtain

$$\Pi_{H_1}^*(q_1; W) = \begin{cases} W/4 - \beta q_1, & \text{if } q_1 \geq (1+W)/2, \\ (q_1 - 1/2)(1-(q_1-1/2)/W) - \beta q_1, & \text{if } q_1 \in [\bar{q}_H, (1+W)/2], \\ 2p_1^*(1-p_1^*/W)(q_1-p_1^*) - \beta q_1, & \text{otherwise.} \end{cases} \quad (24)$$

In the case of realizations W such that $q_1 \in [(1+W)/2,\ 1+W/2]$ the firm would use mixed versioning; however, the following proposition shows that this is never optimal. Analogous to the pure-vertical-versioning case, it can be shown that the profit function Π_{H_1} is continuously differentiable and has at most one maximum in $q_1 > 0$.

Proposition 5. *(i) For any given realization of W in $\mathcal{S}$, either pure horizontal or pure vertical differentiation is optimal. (ii) Horizontal product cannibalization (overlap) may be optimal for certain "intermediate" realizations of W.*

Horizontal overlap in the product portfolio is most likely to be useful if the support $\mathcal{S}$ of demand realizations is such that for low states W vertical differentiation is optimal while for high states horizontal separation is best. Note that in the presence of horizontal product cannibalization, there generally is price dispersion (*cf.* the details on the overlap case in the Appendix). Figs. 6 and 7 show the optimal firm profits and modes of *ex-post* differentiation for $\mathcal{S} = [1, 2]$ and $\beta = 1/10$.

4.1.3. Effect of versioning costs

As a consequence of Proposition 5 mixed versioning cost, $\alpha_M \geq \alpha_H$, α_V, will never be incurred. With *perfect ex-ante* information about W, the effect of versioning cost is of course adequately displayed in Fig. 5 with $\hat{W}$ replaced by W. In the presence of uncertainty, the optimal policy depends on the magnitudes and the difference between α_V and α_H as these influence the attractiveness of one versioning mode over the other and of product-line extensions overall.

4.2. The product development decision

In the first stage of the versioning problem with delayed differentiation the firm decides about the appropriate development effort (*i.e.*, investment in product quality q_1^{**}) by solving problem (20). As pointed out in Section 1, irreversibilities in investments lead firms to value the option of delaying commitment [22,28,38]. Under delayed differentiation in the presence of demand uncertainty the marginal value of quality may be higher than under *ex-ante* versioning where an adaptation of the product portfolio to the realization of the uncertain demand is not possible.

Proposition 6. *If the optimal ex-post versioning policy is not state-contingent, i.e., the mode of differentiation does not depend on W, then the optimal ex-ante investment q_1^{**} is characterized by*[18]

$$q_1^{**} = 1 + \frac{2}{3} \frac{\int_{W_1(q_1^{**})}^{W_2(q_1^{**})} f(w)\mathrm{d}w - \beta + \int_{W_2(q_1^{**})}^{\bar{s}} p_1^*(w)\left(1 - \frac{3p_1^*(w)}{4w}\right) f(w)\mathrm{d}w}{\int_{W_1(q_1^{**})}^{W_2(q_1^{**})} \frac{f(w)}{w}\mathrm{d}w} \quad (25)$$

for pure vertical differentiation (with $\bar{s} = \max\{\mathcal{S}\}$, $W_1 = 3(q_1^{**}-1)/2$, $\bar{q}_V(W_2) = q_1^{**}$, and p_1^* as in Eq. (21) *for $w = W$), and*

$$q_1^{**} = \frac{1}{2} + \frac{\int_{W_1(q_1^{**})}^{W_2(q_1^{**})} f(w)\mathrm{d}w - \beta + 2\int_{W_2(q_1^{**})}^{\bar{s}} p_1^*(w)\left(1 - \frac{p_1^*(w)}{w}\right) f(w)\mathrm{d}w}{2\int_{W_1(q_1^{**})}^{W_2(q_1^{**})} \frac{f(w)}{w}\mathrm{d}w} \quad (26)$$

for horizontal differentiation without overlap (with $\bar{s}$ as above, $W_1 = 2q_1^{**} - 1$, $\bar{q}_{H_1}^{**}(W_2) = q_1^{**}$, and p_1^* as in Eq. (23) *for $w = W$).*

[18] It is clear that horizontal differentiation with overlap can never be the result of a non-state-contingent versioning policy.

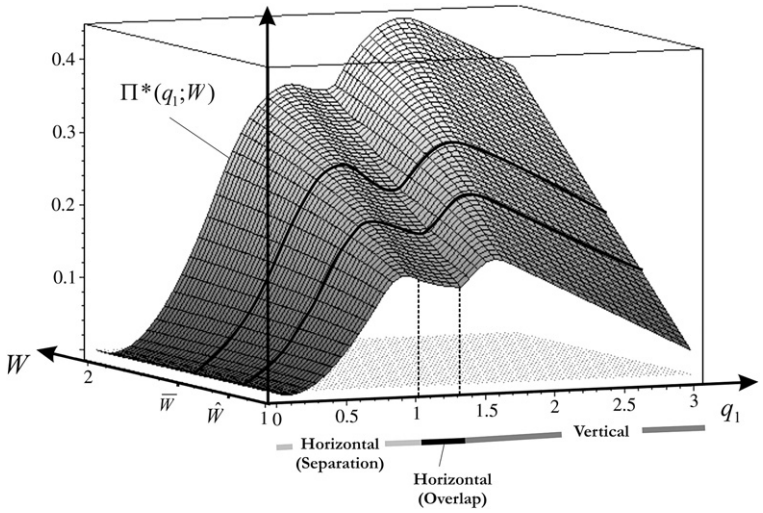

Fig. 6. Optimal cumulative profits as a function of q_1 and W (for $\beta=1/10$, $\mathcal{S}=[1, 2]$).

If the uncertainty about the demand characteristic W is low, then $[W_1, W_2]\cap\mathcal{S}=\mathcal{S}$, so that the expressions (25) and (26) are identical to the optimal *ex-ante* quality choice q_1^* under vertical and horizontal differentiation respectively (*cf.* Section 3.2). If the development cost β is zero, then clearly it is optimal to vertically version and choose $q_1^{**}=1+2\bar{s}/3$, a quality larger than the *ex-ante* optimal quality of $q_1^*=1+2\hat{W}/3$ for $\beta=0$, since uncertainty about demand implies uncertainty about the optimal pricing of the product portfolio. Hence, even if quality can be provided at an arbitrarily small positive cost, a firm lacking the flexibility to version and to reprice according to observed demand will generally limit its flagship product's quality.

4.2.1. Example

For *state-contingent* versioning policies a firm's investment in product development does not necessarily increase. Consider the case when the support $\mathcal{S}$ is finite containing only the positive elements $W_L=2$ and $W_H=4$, which correspond to a "low" and a "high" demand realization respectively. The probability of high demand is denoted by π_H and the development cost is $\beta=1/4$. Thus, the *ex-ante* demand estimator is $\hat{W}=4/(2-\pi_H)$, which satisfies the viability condition (19) for all $\pi_H\in[0, 1]$. For $\hat{W}$ less than $6\beta/(1-\beta)^2=8/3$ (*i.e.*, $\pi_H<1/2$) the firm decides *ex-ante* to horizontally version, whereas for $\pi_H>1/2$ vertical versioning is optimal. Under delayed differentiation the firm prefers to not commit to any particular mode of differentiation for a large range of intermediate π_H's. In the case of a "surprisingly" high realization of demand the *ex-ante* quality choice is relatively too small, so that horizontal versioning is optimal. The converse is true for a low demand realization, when it is better to vertically version in order to make use of the "unexpectedly" high flagship quality. Fig. 8 shows that the firm's development effort (proportional to q_1) may go both up and down with delayed differentiation as a result of actively using its added flexibility in the form of a state-contingent horizontal–vertical versioning policy.

4.2.2. Value of perfect information

The expected value of *perfect* information (EVPI) about the demand under *ex-ante* versioning is simply the difference between *ex-ante* optimal profit evaluated at W

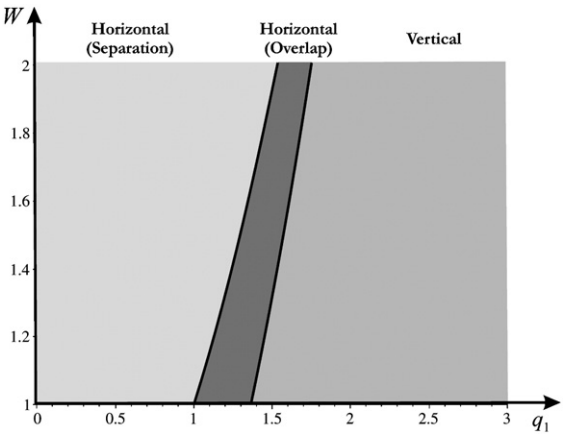

Fig. 7. Optimal modes of ex-post differentiation (for $\beta=1/10$, $\mathcal{S}=[1, 2]$).

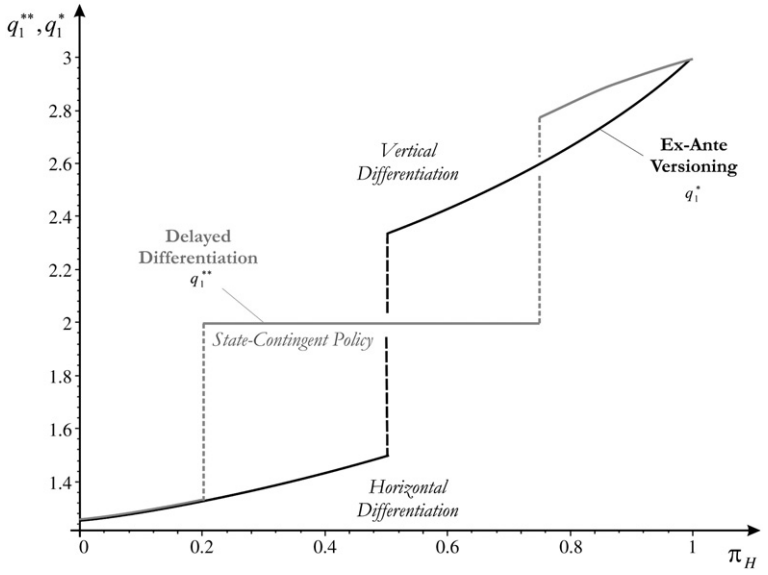

Fig. 8. Optimal choice of q_1 under *ex-ante* versioning and under delayed differentiation (for $\beta=1/4$, $\mathcal{S}=\{2, 4\}$).

(in expectation) and evaluated at $\hat{W}$ respectively: EVPI (*ex-ante* versioning)$=E\Pi^*(W)-\Pi^*(\hat{W})$, where

$$\Pi^*(W)=\begin{cases} W(1-\beta)^2/3-\beta, & \text{if } W\geq 6\beta/(1-\beta)^2,\\ W(1-\beta)^2/4-\beta/2, & \text{if } W\in[2\beta/(1-\beta)^2, 6\beta/(1-\beta)^2],\\ 0, & \text{otherwise.}\end{cases}$$

Clearly EVPI is an upper bound for the value of the option to delay differentiation [9]. It is generally *not* monotonic in the development cost β, as the way in which information alters the versioning decision depends on the firm's default no-information versioning policy.

4.2.3. Value of the option to delay differentiation

The option value of not having to commit to the product portfolio *ex ante* is given by $V_{\mathrm{d}}=\overline{\Pi}^{**}-\overline{\Pi}^{*}$. This value must be zero for large values of β, since both $\overline{\Pi}^{**}$ and $\overline{\Pi}^{*}$ are tightly bounded from below by zero and are strictly decreasing (and continuous) in β.

Proposition 7. *(i) The value of the option to delay differentiation, $V_d(\beta)$, is locally maximal at $\beta\in\{0, (3+\hat{W}-\sqrt{9+6\hat{W}})/\hat{W}\}$. (ii) Its slope with respect to the development cost β is given almost everywhere by the difference of the optimal flagship qualities with and without delayed differentiation,*

$$\partial_\beta V_{\mathrm{d}} \overset{a.e.}{=} q_1^* - q_1^{**}. \tag{27}$$

(iii) The option value is zero for large β's.

Development costs in which the firm is *ex ante* indifferent between vertical and horizontal versioning make the option to delay decisions about the composition of the product portfolio particularly valuable. A state-contingent policy specifying the mode of differentiation as a function of the realized demand clearly outperforms the noncontingent *ex-ante* decision. In particular, the value of flexibility is generally non-

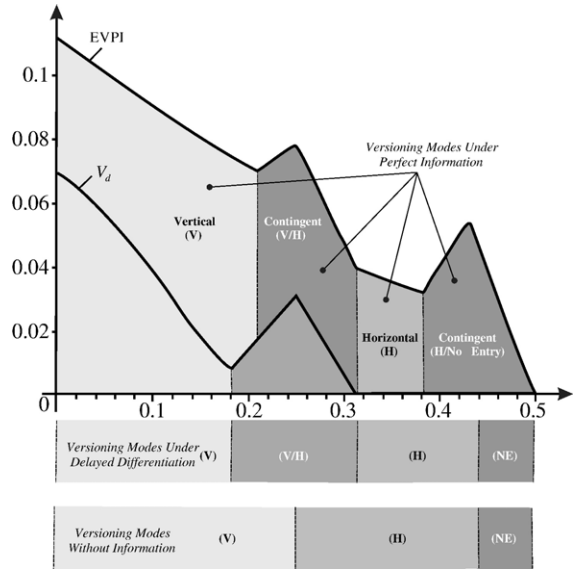

Fig. 9. Expected value of perfect information (EVPI) and value of the option to delay differentiation (V_{d}) as a function of the development cost β (for $\mathcal{S}=\{2, 4\}$, $\pi_{\mathrm{H}}=.5$).

monotonic in quality cost; it depends on the way in which a deviation from the firm's default *ex-ante* strategy influences expected profits. Varying the development cost, the option value varies proportional to the difference $q_1^* - q_1^{**}$, which can be positive and negative. Fig. 9 depicts the option value for our previous two-state example (with $\mathcal{S} = \{W_L, W_H\}$) together with the expected value of perfect information as a function of β.

4.3. Effects on consumer surplus

Consumer surplus depends directly on the firm's product portfolio and can be directly determined from the set $\Omega = \{(p_k, q_k, z_k)\}_{k=1,2}$.

Proposition 8. *(i) Given a demand realization W and the firm's product portfolio Ω, the consumer surplus is given in* Table 1 *(ii) Under ex-ante optimal versioning, the expected consumer surplus is given by*

$$\mathrm{CS} = \begin{cases} (2+\beta)/6, & \text{if } \beta \le \beta', \\ (1+\beta)/8, & \text{if } \hat{W} \in [\beta', \beta''], \\ 0, & \text{otherwise.} \end{cases} \tag{28}$$

where $\beta' = \left(3 + \hat{W} - \sqrt{9 + 6\hat{W}}\right)/\hat{W}$ and $\beta'' = \left(1 + \hat{W} - \sqrt{1 + 2\hat{W}}\right)/\hat{W}$.

The first part of Proposition 8 describes how consumer surplus can be determined from the results in Sections 4.1 and 4.2. The second part shows that under *ex-ante* differentiation consumer surplus is piecewise linearly increasing in β. At the points of discontinuity β' and β'', where the first mode of differentiation changes and then the firm decides not to enter the market, consumer surplus drastically decreases. For each given versioning mode, consumer surplus is increasing in β because the firm compensates for its decreasing quality investment by reducing price so as to ensure a large participation. In fact, expected market penetration[19] increases between two and four times faster than consumer surplus.

Table 1
Expected consumer surplus given a two-element product portfolio Ω

Mode of differentiation	Consumer surplus [$u_k = q_k - p_k$, $\delta = p_1 - p_2$]
Vertical	$\frac{W - p_2}{W}(u_1 - \min\{u_1, 1\}/2)\min\{u_1, 1\}$
Horizontal (separation)	$\frac{W - p_1}{W}\left(u_1 - \min\left\{u_1, \frac{1}{2}\right\}/2\right)\min\left\{u_1, \frac{1}{2}\right\}$
Horizontal (overlap)	$\frac{W - p_1}{W}\left(u_1 - \frac{1-\delta}{4}\right)\frac{1-\delta}{2} + \frac{W - p_2}{W}\left(u_2 - \frac{1+\delta}{4}\right)\frac{1+\delta}{2} + \frac{\delta}{W}\left(u_2 - \frac{\min\{u_2, 1\}}{2}\right)\min\{u_2, 1\}$

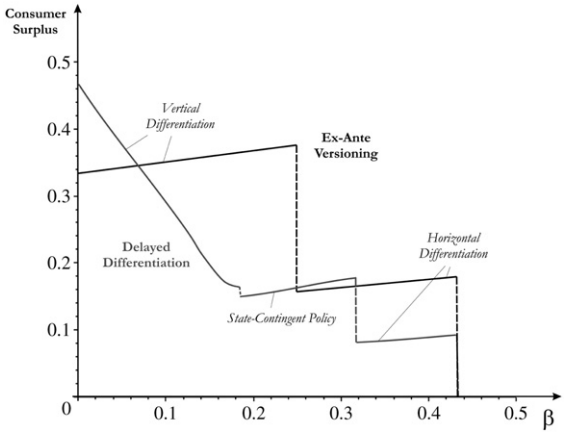

Fig. 10. Consumer surplus (CS) as a function of the development cost β (for $\mathcal{S} = \{2, 4\}$, $\pi_H = .5$).

As shown above, the firm generally values versioning flexibility, which may both increase or decrease the optimal *ex-ante* quality investment depending on the no-information default strategy. Interestingly, the increased profitability of the firm's investment does not always imply a decrease in consumer surplus. For low development cost, the benefits of the higher flagship quality may outweigh the firm's improved ability to screen the consumer base, leading for low β's to a higher consumer surplus under delayed differentiation than under *ex-ante* versioning (*cf.* Fig. 10).

4.4. Generalization to goods with positive marginal costs

A generalization of virtually all results in this paper to goods with positive marginal costs $c > 0$ can be achieved *via* a simple variable transformation. The only restrictive assumption is that the marginal costs for both goods be the same. This is realistic for many products, and especially those where the actual versioning decision can be taken far downstream in the supply chain [1], *e.g.*, by the retailer. Replacing the prices p_k in Section 3 by the markups $m_k = p_k - c$ (for $k = 1, 2$), together with using the constants $\hat{W}_c = \hat{W} - c$ and $\beta_c = \beta\hat{W}/(\hat{W} - c)$ is enough for

[19] Expected market penetration under *ex-ante* versioning is $(1+\beta)/2 \in [1/2, 1]$ for a horizontally and $(2+\beta)/3 \in [2/3, 1]$ for a vertically differentiated optimal product portfolio.

all profit-maximization problems under *ex-ante* versioning to carry over to the case with $c>0$. Under delayed differentiation the same change of variables can be used; the estimator $\hat{W}$ needs only to be replaced by the actual demand realization W. In addition, the firm needs to consider $q'_1=q_1-c$ for its second-stage versioning decision. Again all of the maximizers can be simply recovered by substituting the new constants. The overall effect of these transformations is that positive marginal costs are equivalent to increasing the development costs and also add an additional fixed cost of entry, changing the firm's viability requirements for market entry.

5. Conclusion and further research

Even when a company can extend its product line by varying horizontal *and* vertical attributes of a flagship product, it is – given our assumptions of quasilinear preferences and homogeneous consumer distribution – not optimal to do both at the same time.[20] This finding is independent of the versioning cost structure, as long as $\alpha_M \geq \alpha_V, \alpha_H$. Nevertheless, under delayed differentiation the firm's optimal versioning policy contingent on the demand realization generally incorporates both modes of differentiation. For low demand realizations it is best to differentiate vertically, adding a product of degraded quality to the then relatively high-performance flagship product to more adequately segment the consumer base by maintaining full horizontal market coverage. This can be accomplished by deliberately "damaging" the flagship product [11] to increase the product portfolio. On the other hand, for (unexpectedly) large demand realizations horizontal differentiation is generally superior, since the effective flagship product quality may not be high enough to guarantee full horizontal market coverage. For intermediate realizations of demand, horizontal product cannibalization may be the best compromise. The latter generally implies price dispersion for products of equal quality. Despite the quite general insight from option theory that added flexibility usually leads to an increase of a firm's *ex-ante* investment which has become "less irreversible," we find that for multiattribute versioning the upfront product development efforts may decrease if, due to the possibility of delayed differentiation, a policy contingent on the demand state becomes optimal. This result is driven by the nonconvexity of the problem, horizontal and vertical differentiation yielding locally optimal profits.[21]

Examining the option value of delayed differentiation further, we show that it contains a local maximum at the point of *ex-ante* indifference between the versioning modes, and that it naturally vanishes for very high product development costs. The effect of delayed differentiation on consumer surplus is mixed. Under *ex-ante* versioning consumer surplus heavily depends on the mode of differentiation used, and for each mode is monotonically increasing in the firm's development cost, as the firm tends to compensate an increase in quality cost β by lowering price and enlarging the consumer base. If differentiation is delayed, the versioning modes switch at different points, so that effects on consumer surplus are ambiguous. Overall it tends to decrease as the firm's ability to make effective pricing and product-line-extension decisions greatly increases.

Future research could proceed along the following three promising axes: (1) extend the approach to more than two products. One of the main challenges here is that the potentially multiple overlaps are hard to capture analytically; (2) admit more general cost functions and demand distributions; and (3) incorporate competitive dynamics into the multiattribute versioning decision.

Appendix A

Proof of Lemma 1. (i) This part is trivial. (ii) The probability density of W is by assumption given by the Lebesgue-measurable function $f\in L_1(\mathcal{S})$. Thus, we have using Fubini's theorem:

$$\begin{aligned}\frac{\bar{W}}{\hat{W}} &= \left(\int_{\mathcal{S}} x f(x)\mathrm{d}x\right)\left(\int_{\mathcal{S}} \frac{f(y)}{y}\mathrm{d}y\right)\\ &= \int_{\mathcal{S}\times\mathcal{S}} \frac{x}{y} f(x)f(y)\mathrm{d}(x,y)\\ &= \int_{\mathcal{S}\times\mathcal{S}} \frac{1}{2}\left(\frac{x}{y}+\frac{y}{x}\right) f(x)f(y)\mathrm{d}(x,y)\\ &> \int_{\mathcal{S}\times\mathcal{S}} f(x)f(y)\mathrm{d}(x,y) = 1,\end{aligned}$$

since $(x/y+y/x)/2>1$ for all $x, y>0$ with $x\neq y$. Note that the set of points $(x, y)\gg 0$ where $x=y$ is of measure zero in $\mathcal{S}\times\mathcal{S}$, so that the strict inequality is warranted. □

[20] This finding cannot be expected to hold for product portfolios with more than two products, the treatment of which in our framework poses significant analytical difficulties. It is also not necessarily what is observed in practice (*cf.* footnote 1).

[21] From a technical point of view, the maximizer (*i.e.*, the "optimal policy") is upper semicontinuous, essentially as a consequence of the continuity of the objective function. This is guaranteed by Berge's maximum theorem [5].

Proof of Proposition 3. (i) Since there is no overlap, the expected profits under horizontal differentiation (16) are under separation

$$\begin{aligned}\bar{\Pi}_{H_1}(p_1,p_2) = \frac{1}{2\hat{W}}&(p_1(\hat{W}-p_1)(1-p_1+p_2)\\&+p_2(\hat{W}-p_2)(1+p_1-p_2))\\&-\beta(1+p_1+p_2)/2.\end{aligned}$$

Note that $\bar{\Pi}_{H_1}(p_1,\ p_2)=\bar{\Pi}_{H_1}(p_2,\ p_1)$ and therefore, if there is an asymmetric maximizer, there must be at least two of them. Hence, without loss of generality, assume that $p_1 \geq p_2$. Let us now introduce the variable $\delta=p_1-p_2\in[0,\ 1]$, which leads to the problem of maximizing

$$\begin{aligned}\bar{\Pi}_{H_1}(p_2+\delta,p_2) = &-\frac{p_2^2}{\hat{W}}+\left(1-\beta-\frac{\delta(1-\delta)}{\hat{W}}\right)p_2\\&+\frac{1}{2}\left(\delta(1-\delta)\left(1-\frac{\delta}{\hat{W}}\right)-\beta(1+\delta)\right)\end{aligned}$$

with respect to $p_2\in[0,\ \hat{W}]$ and $\delta\in[0,\ 1]$, subject to the additional constraint $0\leq p_2+\delta\leq\hat{W}$. Neglecting the latter constraint, the first-order condition with respect to p_2 is $\partial_{p_2}\bar{\Pi}_{H_1}(p_2+\delta,p_2)=-\frac{2p}{\hat{W}}+1-\beta-\frac{\delta(1-\delta)}{\hat{W}}=0$, which yields $p_2^*(\delta)=\frac{\hat{W}}{2}(1-\beta)-\frac{\delta(1-\delta)}{2}\leq\frac{W}{2}(1-\beta)$. Thus,

$$\begin{aligned}\bar{\Pi}_{H_1}(p_2^*(\delta)+\delta,p_2^*(\delta)) &= \frac{\hat{W}}{4}(1-\beta)^2-\frac{\beta}{2}\\&\quad-\frac{\delta^2}{4\hat{W}}(1-\delta^2+2\beta\hat{W})\\&\leq\frac{\hat{W}}{4}(1-\beta)^2-\frac{\beta}{2}.\end{aligned}$$

The last inequality is binding if and only if $\delta=0$. The unique maximizers of the original maximization in the "separation" case, now satisfying all of the above constraints, are $p_1^*=p_2^*=\frac{\hat{W}}{2}(1-\beta)$, yielding optimal profits

$$\bar{\Pi}_{H_1}^*=\frac{\hat{W}}{4}(1-\beta)^2-\frac{\beta}{2}=\bar{\Pi}_1^*+\frac{\beta}{2}. \tag{29}$$

These profits are by the amount $\beta/2$ larger than profits for the single-product monopoly and are positive (*i.e.*, viable) if and only if

$$\hat{W}>\frac{2\beta}{(1-\beta)^2}. \tag{30}$$

In particular, a multiproduct monopolist firm can substitute horizontal differentiation for a quality increase, and save on cost (provided that $\alpha_H<\beta/2$).

(ii) When there is overlap, the expected profit function (16) becomes

$$\begin{aligned}\bar{\Pi}_{H_2}(p_1,p_2) = \frac{1}{2\hat{W}}&(p_1(\hat{W}-p_1)(1-p_1+p_2)\\&+p_2(\hat{W}-p_2)(1+p_1-p_2)\\&+\max\{p_1(p_2-p_1),p_2(p_1-p_2)\}\\&\times(1-p_1+p_2))-\beta(1+p_2).\end{aligned}$$

The single-product monopoly profits $\bar{\Pi}_1^*$ are attained for $p_1=p_2$ and $p_1=p_2+1$. As pointed out above, $p_1\geq p_2$. Let us again introduce $\delta=p_1-p_2\in[0,\ 1]$. Then

$$\begin{aligned}\bar{\Pi}_{H_2}(p_2+\delta,p_2) = &-\frac{p_2^2}{\hat{W}}+\left(1-\beta-\frac{\delta(1-\delta)}{2\hat{W}}\right)p_2\\&+\frac{\delta(1-\delta)}{2}\left(1-\frac{\delta}{\hat{W}}\right)-\beta.\end{aligned}$$

The first-order condition with respect to p_2, $\partial p_2\ \bar{\Pi}_{H_2}(p_2+\delta,\ p_2)=0$, yields $p_2^*(\delta)=\frac{\hat{W}}{2}(1-\beta)-\frac{\delta(1-\delta)}{4}$, and therefore

$$\begin{aligned}\bar{\Pi}_{H_2}(p_2^*(\delta)+\delta,p_2^*(\delta)) &= \frac{\hat{W}}{4}(1-\beta)^2-\beta\\&\quad-\frac{\delta(1-\delta)}{16\hat{W}}\left(\delta^2+7\delta-4\hat{W}(1+\beta)\right)\\&=\bar{\Pi}_1^*+\frac{\delta(1-\delta)}{16\hat{W}}(\delta-\delta_-)(\delta_+-\delta),\end{aligned} \tag{31}$$

where $\delta_\pm=\left(-7\pm\sqrt{49+16\hat{W}(1+\beta)}\right)/2$. It is $\delta_-<0<\delta_+$, and therefore $\bar{\Pi}_{H_2}^*$ in expression (31) is greater than $\bar{\Pi}_{H_2}^*$ if and only if $0<\delta<\min\{\delta_+,\ 1\}$. The following lemma helps establishing that the optimal profit $\bar{\Pi}_{H_2}^*$ is strictly inferior to $\max\{\bar{\Pi}_{H_1}^*,\bar{\Pi}_V^*\}$. In other words, if pure horizontal differentiation is superior to pure vertical differentiation, then separation will maximize profits.

Lemma 2. *Let the parameters $0<\beta<1$ and $\hat{W}$ be given. Then for any $\delta\in[0,\ 1]$:*

$$\delta(1-\delta)(\delta-\delta_-)(\delta_+-\delta)<\max\{8\beta\hat{W},(2\hat{W}(1-\beta))^2/3\}. \tag{32}$$

Proof. Let $\bar{\delta}=\min\{1,\delta_+\}$. As the RHS of Eq. (32) is always positive, we can limit ourselves to $\delta\in I=[0,\bar{\delta}]$ for which the LHS is nonnegative. Set $f(\delta)$ to be equal to the LHS of Eq. (32). Since f is a polynomial with a set of roots $\{\delta_-,\ 0,\ \delta_+,\ 1\}$, where $\delta_-<0<\bar{\delta}$, it is

$$\max_{\delta\in I}|f'(\delta)|=\max\{f'(0),-f'(\bar{\delta})\}.$$

More specifically, by the mean value theorem and the fact that $f''|_I<0$, for all $\delta\in I$:

$$f(\delta)\le\min\{f'(0)\cdot\delta,\,-f'(\bar\delta)(\bar\delta-\delta)\}\le\frac{f'(\bar\delta)f'(0)\cdot\bar\delta}{f'(0)-f'(\bar\delta)}. \tag{33}$$

Thereby $f'(0)=-\delta_-\delta_+$ and $-f'(\bar\delta)=\max\{\delta_+(\delta_+-\delta_-)(1-\delta_+),\,(1-\delta_-)(\delta_+-1)\}$. Let us first consider the case when $\bar\delta=1$, or equivalently

$$\hat W(1+\beta)\ge 2. \tag{34}$$

Then by Eq. (33)

$$\begin{aligned} f(\delta)&\le\frac{-\delta_-\delta_+(1-\delta_-)(\delta_+-1)}{-\delta_-\delta_++(1-\delta_-)(\delta_+-1)}\\ &=\frac{16\hat W(1+\beta)(\hat W(1+\beta)-2)}{4\hat W(1+\beta)+4(\hat W(1+\beta)-2)}\\ &=\frac{2\hat W(1+\beta)(\hat W(1+\beta)-2)}{\hat W(1+\beta)-1}. \end{aligned} \tag{35}$$

One can verify by straightforward manipulations that the RHS of Eq. (35) is less than $8\beta\hat W$ if and only if

$$\hat W(1-3\beta)<2\frac{1-\beta}{1+\beta},$$

which establishes inequality (32) for all

$$(\beta,\hat W)\in D_1=\left\{(\breve\beta,\breve W):0<\breve\beta<\frac{1}{3},\breve W<\frac{1-\breve\beta}{(1+\breve\beta)(1-3\breve\beta)}\right\}\cup([1/3,1)\times(0,\infty)).$$

On the other hand, one can verify that the RHS of Eq. (35) is less than $(2\hat W(1-\beta)^2/3$ if and only if

$$\frac{2}{1+\beta}+\frac{2}{3}\frac{(1-\beta)^2}{1+\beta}\hat W^2-\left(\frac{2}{3}\left(\frac{1-\beta}{1+\beta}\right)^2+1\right)\hat W>0. \tag{36}$$

Restricting β to the interval $(0,\,1/3]$, inequality (36) is satisfied if

$$\begin{aligned} &3+(1-\beta)^2\hat W^2-\left(\frac{(1-\beta)^2}{1+\beta}+\frac{3}{2}(1+\beta)\right)\hat W\\ &\ge 3+\left[\frac{2}{3}(1-\beta)\hat W-\left(\frac{(1-\beta)^2}{1+\beta}+\frac{3}{2}(1+\beta)\right)\right]\hat W>0. \end{aligned}$$

The expression in square brackets can be further minorized by setting $\beta=1/3$, based on the fact that the derivative of that expression with respect to β,

$$-\frac{4}{3}\frac{1}{1+\beta}-\frac{4}{3}\frac{1-\beta}{(1+\beta)^2}+2\frac{1-\beta}{1+\beta}+\frac{(1-\beta)^2}{(1+\beta)^2}-\frac{3}{2}$$

is negative on $(0,\,1/3]$. In particular,

$$\begin{aligned} &3+\hat W\left[\frac{2}{3}(1-\beta)\hat W-\left(\frac{(1-\beta)^2}{1+\beta}+\frac{3}{2}(1+\beta)\right)\right]_{\beta=1/3}\\ &\quad=\frac{1}{9}(\hat W-3)(4\hat W-9)>0 \end{aligned}$$

implies Eq. (32) for all $(\beta,\hat W)\in D_2=(0,\,1/3]\times((0,\,9/4)\cup(3,\,\infty))$. Alternately one can minorize the RHS of Eq. (36) by substituting

$$\hat W=\frac{2(1-\beta)^2+3(1+\beta)^2}{4(1-\beta)^2(1+\beta)},$$

its unique minimizer. Relation (36) is satisfied for a given $\beta\in(0,\,1/3)$ and all $\hat W>0$ if

$$23\beta^4-20\beta^3-150\beta^2-20\beta+23>0.$$

The last polynomial has the set of approximate roots $\{-2.0221,-.4945,.3269,3.0593\}$, so that Eq. (32) follows for $(\beta,\hat W)\in D_3=(0,\,3/10)\times(0,\,\infty)$. Since $D_1\cup D_2\cup D_3=(0,\,1)\times(0,\,\infty)$, we have indeed shown that Eq. (32) holds for all relevant[22] $(\beta,\hat W)$ whenever $\bar\delta=1$. Let us now examine the case when $\bar\delta=\delta_+<1$ or in other words

$$0<\hat W<\frac{2}{1+\beta}. \tag{37}$$

Using Eq. (33) write

$$\begin{aligned} f(\delta)&\le\frac{-\delta_-\delta_+^3(\delta_+-\delta_-)(1-\delta_+)}{-\delta_-\delta_++\delta_+(\delta_+-\delta_-)(1-\delta_+)}\\ &<\frac{\delta_+^2(\delta_+-\delta_-)(1-\delta_+)}{2-\delta_+}\\ &=\frac{1}{4}\frac{\sqrt{49+16\eta}(-9+\sqrt{49+16\eta})(-7+\sqrt{49+16\eta})^2}{-11+\sqrt{49+16\eta}}, \end{aligned} \tag{38}$$

where $\eta=\hat W(1+\beta)$ and by Eq. (37) it is $0<\eta<2$. Straightforward computations yield that expression (38) is less than $8\beta\hat W=8\eta(1-1/(1+\beta))$ if and only if

$$(\beta,\eta)\in E_1=\{(\breve\beta,\breve\eta):0<\breve\beta<R_1(\breve\eta),0<\breve\eta<2\}, \tag{39}$$

[22] $\bar\delta=1$ is equivalent to Eq. (34), which slightly restricts the relevant domain of $(\beta,\hat W)$ to a subset of $(0,\,1)\times(0,\,\infty)$.

where

$$R_1(\eta) = (2\eta^3 - 153\eta^2 - 537\eta - 294)^{-1}(2\eta^3 + 279\eta^2 + 306\eta - 1764 + \sqrt{49+16\eta}(15\eta + 12\eta^2 - 294) + [4032\eta^5 + 70,620\eta^4 - 91,110\eta^3 - 1,744,848\eta^2 - 444,528\eta + 7,260,624 + \sqrt{49+16\eta}(48\eta^5 + 6756\eta^4 + 14,538\eta^3 - 197,208\eta^2 - 232,848\eta + 1,037,232)]^{1/2}).$$

On the other hand, expression (38) is less than $(2\hat{W}(1+\beta))^2/3 = 4\eta^2(1-4/(1+\beta)+4/(1+\beta)^2/3$ if and only if

$$(\beta,\eta)\in E_2 = \{(\breve{\beta},\breve{\eta}) : R_2(\breve{\eta}) < \breve{\beta} < 1, 0 < \breve{\eta} < 2\}, \quad (40)$$

where

$$R_2(\eta) = \frac{\sqrt{49+16\eta}(23\eta+98) - 16\eta^2 - 273\eta - 686}{16\eta^2 + 295\eta + 686 - \sqrt{49+16\eta}(25\eta+98)}.$$

Since $R_1(\eta) > R_2(\eta)$ for all $0<\eta<2$, it is $E_1 \cup E_2 = (0, 1)\times(0, 2)$, and therefore Eq. (32) holds also for the case that $\overline{\delta} = \delta_+$, which concludes the proof of Lemma 2. □

As mentioned before, based on Lemma 2 as well as Eqs. (10), (29), and (31) pure horizontal differentiation in the overlap mode is strictly dominated by either pure vertical differentiation *or* pure horizontal differentiation in the separation mode. This concludes the proof of Proposition 3. □

Proof of Proposition 4. (i) For zero versioning costs, condition (19) is necessary and sufficient, since the viability domains of single-product monopoly, pure vertical differentiation, and mixed differentiation are subsets of the region of positive profits for pure horizontal differentiation in separation mode, given precisely by Eq. (19). Thus, if the latter condition is satisfied there exists $\varepsilon>0$ such that $\alpha_H \in [0, \varepsilon]$ is small enough for the resulting profits to be positive. (ii) This is an immediate consequence of Eq. (19) being necessary and sufficient for viability under zero versioning costs. (iii) From $\hat{W}\in(2\beta/(1-\beta)^2,\ 4\beta/(1-\beta)^2)$ it follows that a single-product monopoly is not viable according to Eq. (7), while at the same time Eq. (19) is satisfied. □

Details on the horizontal-overlap case under delayed differentiation. Since $q_1 \geq q_2$, we have that necessarily $p_1 \geq p_2$, which generally implies price dispersion for goods of equal quality. In the following it is useful to distinguish three cases. (i) If $q_1 - p_2 < 1$, then $q_1 - p_1 < 1$ and $q_2 - p_2 < 1$. We can conclude that $p_1, p_2 \in [q_1 - 1, q_1]$ or in other words that $p_1 - p_2 < 1$. Thus, $q_1 = q_2$ (pure horizontal differentiation). The profit-maximization problem is

$$\begin{aligned}(p_1,p_2) &= \arg\max_{p_1,p_2}\left\{\frac{p_1(W-p_1)(1-p_1+p_2)}{2W} + \frac{p_2(p_1-p_2)(q_1-p_2)}{W} + \frac{p_2(W-p_1)(1+p_1-p_2)}{2W}\right\} \\ &= \arg\max_{p_1,p_2}\left\{\frac{W-p_1}{2W}(p_1+p_2-(p_1-p_2)^2) + \frac{p_2(p_1-p_2)(q_1-p_2)}{W}\right\}. \quad (41)\end{aligned}$$

Setting $\delta = p_1 - p_2 \in [0, 1]$ it is possible to rewrite the profit function as

$$\Pi_{H_2}(p_2+\delta,\delta) = \frac{(W-p_2-\delta)(2p_2-\delta(1-\delta)) + 2p_2\delta(q_1-p_2)}{2W} - \beta q_1.$$

Any interior extremum satisfies the necessary optimality condition for p_2, from which we obtain

$$p_2^*(\delta) = \frac{2W + \delta(2(q_1-1) - (1-\delta))}{4(1+\delta)}.$$

By substituting this expression, the profit function Π_{H_2} can be expressed as a function of δ only, and the first-order condition with respect to δ is

$$a\delta^4 + b\delta^3 + c\delta^2 + d\delta + e = 0, \quad (42)$$

with $a = 27/16$, $b = (q_1+3)/2 - W$, $c = (q_1^2 - 5W)/4 - 17/16$, $d = (q_1^2 - 3q_1 + W)/2 + 1/8$, and $e = -W(W+1-2q_1)/4$. To analytically find the roots of this fourth-order polynomial we follow Ref. [7]. Substituting $y = \delta + b/(4a)$, Eq. (42) can be written equivalently in the reduced form

$$y^4 + Py^2 + Qy + R = 0, \quad (43)$$

where $P = (c/a) - 3b^2/8a^2$, $Q = d/a - cb/a^2 + b^3/8a^3$, and $R = e/a + (cb^2/16 - db/4)/a^2 - 3b^4/256a^4$. The solution behavior of Eq. (43) depends on the behavior of its resolvent cubic,

$$z^3 + 2Pz^2 + (P^2 - 4R)z - Q^2,$$

the roots of which are all real[23] and given by $z_k = 2\rho\cos(\varphi/3 + 2(k-1)\pi/3)$, for $k = 1, 2, 3$, with $\rho = \sqrt{4P^2 - 3Q}/3$ and $\varphi = \arccos((2P(P^2-4R)/3 + Q^2 - 16P^3/27)/(2\rho^3))$.

[23] This follows from the fact that the *discriminant* $\Delta = (P^2 - 4R - 4P^2/3)/3)^3 + (16P^3/27 - 2P(P^2-4R)/3 - Q^2)^2/4$ is negative in the relevant domain, which can be verified numerically.

From this, the roots of the reduced-form polynomial in Eq. (43) can be computed to

$$y_1 = (\sqrt{z_1} + \sqrt{z_2} - \sqrt{z_3})/2,$$
$$y_2 = (\sqrt{z_1} - \sqrt{z_2} + \sqrt{z_3})/2,$$
$$y_3 = (\sqrt{z_1} + \sqrt{z_2} + \sqrt{z_3})/2,$$
$$y_4 = (\sqrt{z_1} - \sqrt{z_2} - \sqrt{z_3})/2,$$

and a local maximizer of $\Pi_{H_2}(p_2^*(\delta)+\delta, p_2^*(\delta))$ is

$$\delta^* = y_1 - \frac{b}{4a}, \tag{44}$$

as the second-order condition is satisfied at that point.[24]

(ii) If $q_1 - p_2 \geq 1$, then the optimization problem is

$$\begin{aligned}(p_1,p_2) &= \arg\max_{p_1,p_2}\left\{\frac{p_1(W-p_1)(1-p_1+p_2)}{2W}\right.\\ &\quad\left.+\frac{p_2(p_1-p_2)}{W}+\frac{p_2(W-p_1)(1+p_1-p_2)}{2W}\right\}\\ &= \arg\max_{p_1,p_2}\left\{\frac{W-p_1}{2}(p_1+p_2-(p_1-p_2)^2)\right.\\ &\quad\left.+p_2(p_1-p_2)\right\}.\end{aligned} \tag{45}$$

The quality q_2 is actually indeterminate in $[1+p_2, q_1]$, but without loss of generality we set $q_2=q_1$ (which is true for even very small positive versioning cost). Note that $p_1-p_2 \leq 1$, otherwise there is only one product. Also, this case is equivalent to the corresponding *ex-ante* versioning situation for $\beta=0$. Introducing $\delta=p_1-p_2\in[0, 1]$, we obtain

$$\Pi_{H_2}(p_2+\delta,\delta) = \frac{(W-p_2-\delta)(2p_2+\delta-\delta^2)+2p_2\delta}{2W} - \beta q_1.$$

Assuming that there is an interior extremum the first-order condition with respect to p_2 is

$$2W - 4p_2 - \delta(1-\delta) = 0,$$

so that $p^*(\delta)=W/2-\delta(1-\delta)/4$. The resulting expression for the profits under horizontal versioning with overlap is

$$\Pi_{H_2}(\delta) = \frac{W}{4} - \beta q_1 + \frac{\delta(1-\delta)(\delta-\delta_-)(\delta_+-\delta)}{16W}, \tag{46}$$

where $\delta_\pm = (-7\pm\sqrt{16W+49})/2$. The first-order condition with respect to δ is then

$$\delta^3 + \frac{9}{2}\delta^2 - \left(2W+\frac{7}{2}\right)\delta + W = 0. \tag{47}$$

Using the cardanic formula to solve the cubic equation (47) it turns out that for any W there are three real roots, of which only one is associated with an interior maximum of the fourth-order polynomial at Eq. (46),

$$\delta^*(W) = \sqrt{\frac{8W+41}{3}}\sin\left(\frac{\pi}{6} - \frac{1}{3}\arccos\left(\frac{48(W+3)}{\sqrt{(8W+41)^3/3}}\right)\right) - \frac{3}{2}. \tag{48}$$

It is $\delta^*(W)\in[0, 1/2]$ strictly increasing in W with $\delta^*(0)=0$ and $\lim_{W\to\infty}\delta^*(W)=1/2$.

(iii) In the intermediary case with binding constraint, $q_1-p_2=1$, the optimization problem is identical to (45), subject to $p_2=q_1-1$,

$$\Pi_{H_2}(\delta) = \frac{(W-(q_1-1))(q_1-1)}{W} - \beta q_1 + \frac{\delta(1-\delta)(A-\delta)}{2W}$$

with the abbreviation $A=W-(q_1-1)$. The first-order necessary optimality condition is $3\delta^2-2(A+1)\delta+A=0$, and the maximizer is therefore

$$\delta^* = \frac{A+1-\sqrt{A^2-A+1}}{3}. \tag{49}$$

The second-order condition is satisfied here, since $\Pi''_{H2}(\delta^*) = -2\sqrt{A^2-A+1} < 0$. This concludes our H_2 analytical discussion of the horizontal-overlap case under delayed differentiation. □

Proof of Proposition 5. (i) It is sufficient to show that the profit function under mixed *ex-post* differentiation is dominated by pure horizontal differentiation with overlap. Let (p_1, q_1) be such that $1/2 \leq q_1-p_1<1$. Under horizontal separation (in which the local mixed-differentiation optimum occurs) this implies that $q_1-p_1=1-(q_2-p_2)<1$, and furthermore, as shown in Section 4.1, then $p_1^*=p_2^*=W/2$. Thus, also $q_1-p_2^*<1$ so that we can compare the profit function under horizontal separation to the maximand in Eq. (41) discussed earlier in this Appendix for the case that $q_1-p_2<1$. Subtracting the latter from the former yields $-p_1(W-p_1)\ (1-(q_1-p_2))/2-p_2(W-p_2)(q_1-p_2)-p_2(p_1-p_2)(q_1-p_2)(q_1-p_2)<0$, so that mixed differentiation is strictly dominated. (ii) The claim that horizontal cannibalization is sometimes optimal can be shown by example and thus Fig. 7 is sufficient as a proof. It is not optimal for large q_1 (compared to W), since it can be shown analogously to the computation in part (i) that pure vertical differentiation is globally better.

[24] An analytical test of the second-order condition is complicated, but it can be readily verified numerically.

For q_1 small compared to W horizontal overlap cannot be achieved. Thus, horizontal overlap can only be best for "intermediate" realizations of W. This concludes the Proof of Proposition 5. □

Proof of Proposition 6. Assume first that *ex post* pure vertical differentiation is optimal, independent of the demand realization W. Then differentiating the expression (22) for $\Pi_V^*(q_1; W)$ and then taking the expectation with respect to W yields[25]

$$-\beta+\int_{W_1}^{W_2}\left(1-\frac{3(q_1^{**}-1)}{2w}\right)f(w)\mathrm{d}w + \int_{W_2}^{\bar{s}} p_1^*\left(1-\frac{3p_1^*}{4w}\right)f(w)\mathrm{d}w=0,$$

where W_1, W_2, p_1^*, and $\bar{s}$ are as the corresponding part of Proposition 6. The last equation is equivalent to Eq. (25). Next assume that *ex post* pure vertical differentiation is optimal, independent of the demand realization W. As before we determine the first-order condition by first taking the derivative of $\Pi_{H_1}(q_1; W)$ in Eq. (24) with respect to q_1 and then taking the expectation with respect to W. This yields

$$-\beta+\int_{W_1}^{W_2}\left(1-\frac{2q_1^{**}-1}{w}\right)f(w)\mathrm{d}w + 2\int_{W_2}^{\bar{s}} p_1^*\left(1-\frac{p_1^*}{w}\right)f(w)\mathrm{d}w=0,$$

which is equivalent to Eq. (26) using the definitions of W_1, W_2, p_1^{**}, and $\bar{s}$ as in the corresponding part of Proposition 6. □

Proof of Proposition 7. (ii) By Berge's maximum theorem [5] the option value V_d is continuous in β as difference of two continuous functions $\overline{\Pi}^{**}$ and $\overline{\Pi}^{*}$. Also as a consequence of the maximum theorem both maximizers q_1^{**} and q_1^* are generally set-valued and upper semicontinuous in the parameter β. The maximizers are set-valued if and only if the firm is indifferent between several different q_1-values. Let $\hat{W}>0$ be given. Following our developments in Section 3, $q_1^*(\beta)$, $0\le\beta\le 1$, is given by

$$q_1^*(\beta)=\begin{cases}1+2\hat{W}(1-\beta)/3, & \text{if } \beta\ge\beta',\\ 1/2+\hat{W}(1-\beta)/2, & \text{if } \beta\in[\beta',\beta''],\\ 0, & \text{otherwise,}\end{cases}$$

where $\beta'=\left(3+\hat{W}-\sqrt{9+6\hat{W}}\right)/\hat{W}$ and $\beta''=\left(1+\hat{W}-\sqrt{1+2\hat{W}}\right)/\hat{W}$. Only for $\beta\in\{\beta', \beta''\}$ is q_1^* possibly set-valued; on $[0, \beta'')\setminus\{\beta'\}$ it is strictly monotonically decreasing. Similarly, the optimal flagship quality under delayed differentiation, q_1^{**}, can only be set-valued at $0<\beta_1,\ \beta_2,\ \beta_3<1$, where $\beta_1\le\beta'<\beta_2<\beta''\le\beta_3$, corresponding to the vertical-contingent (β_1), contingent-horizontal (β_2), and horizontal-no entry (β_3) mode transitions. We have noted in Section 4 that expressions (22) and (24) are continuously differentiable in q_1. The same is true for the corresponding expression in the overlap mode. In fact, these expressions are twice continuously differentiable ("smooth") almost everywhere, and thus the envelope theorem can be applied in the smooth portions, so that we obtain relation (27). (i) At $\beta=\beta'$ the firm is indifferent between horizontal and vertical versioning modes. But if the firm can observe demand $W=\hat{W}$ *before* taking the versioning decision,[26] then it generally will not be indifferent between horizontal and vertical differentiation, so that a state-contingent policy of intermediate *ex-ante* quality will at least weakly increase profits at $\beta=\beta'$. As a consequence, $q_1^{**}(\beta')\in(q_1^*(\beta'+), q_1^*(\beta'-))$ so that with Eq. (27) we obtain a local maximum of V_d at $\beta=\beta'$, in other words a state-contingent policy will generally improve profits around β'. Consider now $\beta=0$. Then $q_1^{**}(0)=1+2\bar{s}/3$ with $\bar{s}=\max\{\mathcal{S}\}$. On the other hand, it is following Section 3.2: $q_1^*(0)=1+2\hat{W}/3\le q_1^{**}(0)$ because generally $\bar{s}\ge\hat{W}$. (iii) This part is trivial: for $\beta\ge\beta_3$, β'', the firm does not enter the market and thus the option of being able to delay differentiation is not worth anything. □

Proof of Proposition 8. (i) Consider first the case of pure vertical differentiation. Consumer surplus CS_V is then

$$CS_V=\frac{1}{W}\int_0^{\min\{u_1,1\}}[(u_1-v)(w-p_1)+(u_1-v)]\mathrm{d}v = \frac{W-p_2}{W}\left[u_1v-\frac{v^2}{2}\right]_0^{\min\{u_1,1\}},$$

an expression equivalent to the first entry in Table 1. The consumer surplus for pure horizontal differentiation with separation follows in the same manner. We

[25] This means to switch differentiation and integration when determining the first-order conditions of the optimization problem (20). This naturally presupposes sufficient regularity of the probability density function f.

[26] If $W=\hat{W}$ (which can only happen with positive probability for certain "pathological" discrete distributions of W), then the firm might be indifferent between horizontal and vertical versioning modes, even under delayed differentiation.

now turn to the case to horizontal differentiation with overlap, for which the consumer surplus can be computed as follows:

$$CS_{H_2} = \frac{W - p_1}{W} + \frac{W - p_2}{W} \int_0^{\frac{1+p_1-p_2}{2}} (u_2 - v)\mathrm{d}v + \frac{p_1 - p_2}{W} \int_0^{\min\{u_1,1\}} (u_2 - v)\mathrm{d}v.$$

Using the abbreviation $\delta = p_1 - p_2$, we obtain the third expression in Table 1 by straightforward integration. (ii) Substituting the *ex-ante* optimal product portfolio determined in Section 3 into the first two expressions for consumer surplus in Table 1, one immediately obtains Eq. (28). □

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