Segmenting Planar Superpixel Adjacency Graphs w.r.t. Non-planar Superpixel Affinity Graphs – Supplement –

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1 Derivation of the Dual of the Decomposition

We now derive the dual of the Lagrangian decomposition. We begin by writing the decomposition below.

$$\begin{aligned}
\max_{\theta^{pmc},\psi} \quad & 1^T(\theta^{\mathrm{P}} - \theta^{pmc} - Y\psi) + 1^T(\psi + \theta^{\mathrm{NP}}) && (1)\\
\text{subject to} \quad & Z\theta^{pmc} \geq 0 && (2)\\
& -W\psi \geq \theta^{\mathrm{NP}} && (3)\\
& \theta^{\mathrm{P}} - \theta^{pmc} - Y\psi \leq 0 && (4)\\
& \min([0, -\theta^{\mathrm{P}}]) \leq -\theta^{pcm} - Y\psi && (5)\\
& \psi \geq 0 \ . && (6)
\end{aligned}$$

The dual of this decomposition for the planar multicut problem is derived in [1]. In order to be closer to the form of [1], we make the following adjustments to the notation. We define $\phi = \min([0, -\theta^{\mathrm{P}}])$ and $\hat{\lambda} = -\theta^{pmc} - \phi$. Notice that $\theta^{pmc} = -\hat{\lambda} - \phi$. We now rewrite the decomposition using $\hat{\lambda}$ and ϕ.

$$\begin{aligned}
\max_{\hat{\lambda},\psi} \quad & 1^T(\theta^{\mathrm{P}} + \hat{\lambda} + \phi - Y\psi) + 1^T(\psi + \theta^{\mathrm{NP}}) && (7)\\
\text{subject to} \quad & Z(-\hat{\lambda} - \phi) \geq 0 && (8)\\
& -W\psi \geq \theta^{\mathrm{NP}} && (9)\\
& \theta^{\mathrm{P}} + \hat{\lambda} + \phi - Y\psi \leq 0 && (10)\\
& \phi \leq \hat{\lambda} + \phi - Y\psi && (11)\\
& \psi \geq 0 \ . && (12)
\end{aligned}$$

We now move terms to different sides of the relevant inequalities. This facilitates the writing of the LP as a Lagrangian.

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$$\max_{\hat{\lambda},\psi} \quad 1^T(\theta^{\mathrm{P}} + \hat{\lambda} + \phi - Y\psi) + 1^T(\psi + \theta^{\mathrm{NP}}) \tag{13}$$

$$\text{subject to} \quad 0 \le -Z\hat{\lambda} - Z\phi \tag{14}$$

$$0 \le -\theta^{\mathrm{NP}} - W\psi \tag{15}$$

$$0 \le -\theta^{\mathrm{P}} - \phi + Y\psi - \hat{\lambda} \tag{16}$$

$$0 \le \hat{\lambda} - Y\psi \tag{17}$$

$$\psi \ge 0 \ . \tag{18}$$

We now write the above LP as a Lagrangian.

$$\min_{\gamma,\omega,\beta,\delta\ge 0} \max_{\hat{\lambda},\psi\ge 0} \quad 1^T(\theta^{\mathrm{P}} + \hat{\lambda} + \phi - Y\psi) + 1^T(\psi + \theta^{\mathrm{NP}}) \tag{19}$$

$$+ \gamma^T(-Z\hat{\lambda} - Z\phi) \tag{20}$$

$$+ \omega^T(-\theta^{\mathrm{NP}} - W\psi) \tag{21}$$

$$+ \beta^T(-\theta^{\mathrm{P}} - \phi + Y\psi - \hat{\lambda}) \tag{22}$$

$$+ \delta^T(\hat{\lambda} - Y\psi) \ . \tag{23}$$

Now we group the terms that have components of ψ and $\hat{\lambda}$ together. This facilitates the creation of new constraints.

$$\min_{\gamma,\omega,\beta,\delta\ge 0} \quad 1^T(\theta^{\mathrm{P}} + \phi) + 1^T\theta^{\mathrm{NP}} - \gamma^T Z\phi - \omega^T\theta^{\mathrm{NP}} + \beta^T(-\theta^{\mathrm{P}} - \phi^T) \tag{24}$$

$$+ (1^T - \gamma^T Z - \beta^T + \delta^T)\hat{\lambda} \tag{25}$$

$$+ (-1^T Y + 1^T - \omega^T W + \beta^T Y - \delta^T Y)\psi \ . \tag{26}$$

Since $\hat{\lambda}$ is unbounded and ψ is non-negative, the LP with the following constraints is equivalent to the Lagrangian above.

$$\min_{\gamma,\omega,\beta,\delta\ge 0} \quad 1^T(\theta^{\mathrm{P}} + \phi) + 1^T\theta^{\mathrm{NP}} - \gamma^T Z\phi - \omega^T\theta^{\mathrm{NP}} + \beta^T(-\theta^{\mathrm{P}} - \phi^T) \tag{27}$$

$$\text{subject to} \quad \gamma^T Z + \beta^T = 1^T + \delta^T \tag{28}$$

$$\omega^T W \ge \beta^T Y - 1^T Y + 1^T - \delta^T Y \ . \tag{29}$$

Except for transposes of some terms, this is the final form of the Lagrangian decomposition used in the main manuscript.

References

1. J. Yarkony. *MAP inference in Planar Markov Random Fields with Applications to Computer Vision.* PhD thesis, University of California, Irvine, 2012.