# Analyzing Corona Breakdown with a Finite Element-based Electromagnetic Solver

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***Abstract*—This paper demonstrates how an already developed finite element code for solving electromagnetic problems can be manipulated and simply extended so as to investigate complicated corona breakdowns. A safe criterion based on the eigenvalue analysis is used in order to predict the maximum electric field that a structure can withstand without suffering from a corona breakdown. Comparison with other semi-analytical techniques developed by researchers solely focusing on high power phenomena verifies the results of the developed algorithm.**

***Index Terms*—finite element method; microwave corona breakdown; space applications**

## I. INTRODUCTION

The corona breakdown, also called gas discharge, is produced due to a locally rapid increase of the electron population because of the ionization of gas molecules surrounding the aperture of antennas or trapped within the gaps of waveguides onboard satellites. This type of discharge is produced when the RF field accelerates the environmental electrons and they impact against the gas neutrals with enough energy as to ionize them, releasing new electrons. Nevertheless, the electrons tend to move away from the zones of high density trying so to attain the chemical equilibrium leveling out the electron population density, process that is called diffusion. Moreover, under high pressure regimes a certain amount of free electrons are attached to the external shells of the ions due to their trend to become electrically neutral. Corona breakdown occurs above a certain value of the electric field for which the ionization is produced so quickly that the electrons are not able to escape fast enough from the zones where a high concentration of electrons is produced and the attachment rate is not high enough as to compensate the ionization. This loss of local equilibrium between ionization, diffusion and attachment, increases dramatically the electrical conductivity of the once-neutral air, triggering an electron avalanche with the subsequent glowing emission [1], [2].

Besides the increase of the return loss, the harmful effects of corona breakdown for filter-based waveguide technology and coaxial connectors include a dramatic temperature rise due to the ohmic losses produced in the plasma that is formed, provoking the partial or total destruction of the onboard components. Corona breakdown occurring at antennas onboard planes and spacecrafts results in a decrease of the transmitted signal intensity, a change in the input impedance and the radiation pattern of the antenna, and a modification of the pulse shape and noise modulation of the signal [3]–[5]. This can provoke the radio blackout between the spacecraft and the earth-station when some important manoeuvres take place, jeopardizing the complete mission; or it may provoke the malfunctioning of some devices on airplanes.

Despite such severe consequences, the numerical codes developed for corona breakdown predictions are very limited and sensitive to the device geometry. On the other hand, researchers of the electromagnetic (EM) community have developed plenty of debugged, optimized and parallelized Finite Element (FE) codes, either in-house or commercial ones. The aim of the current contribution is to take advantage of such mature codes and clarify the additional steps required so as to obtain a robust and accurate corona breakdown solver.

## II. MATHEMATICAL MODELING

The diffusion, attachment and ionization mechanisms outlined in Section I are connected through the local continuity equation of the electron population density, $n$:

$$\frac{\partial n}{\partial t} = (\nu_i - \nu_a)n + \nabla \cdot D\nabla n, \tag{1}$$

where $D$ is the diffusion coefficient, and $\nu_i$ and $\nu_a$ are the ionization and the attachment rates, respectively. Different formulas can be found for all three parameters in the literature. The diffusion coefficient and the attachment rate only depend on the pressure in general, whereas the ionization rate is strongly influenced both by the pressure and the magnitude of the local electric field [6]–[8].

The corona breakdown criterion is established by the equilibrium between the increase of the electrons population density due to the ionization rate and the decrease due to the diffusion and attachment processes:

$$0 = \frac{\nu_i - \nu_a}{D} n + \nabla^2 n, \tag{2}$$

where a location-independent diffusion coefficient, $D$, has been assumed. Dirichlet boundary condition is applied on the edge of the computational space for the electron population density ($n = 0, \forall n \in \partial V$).

Suitable approximations of the ionization rate, $\nu_i$, have been proposed in the literature in such a way that a semi-analytical

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solution can be found for (2) [9]–[11]. Such approximations result in simple or even closed-form expressions for the breakdown condition, but they are based on a-priori known electric field distributions and they are strictly case sensitive. Therefore, if the geometry of the structure or the mode of the propagating electric field changes, a new approximation has to be evaluated. Not to mention a plethora of antenna problems, where the electric field cannot be represented in a closed-form but only numerical evaluations are possible. Finally, discrepancies between different ionization rate models are not clearly identified any more due to the aforementioned enforced approximations. Consequently, a general method for predicting corona breakdowns in complicated structures is needed.

## III. FROM MICROWAVE ANALYSIS TO CORONA BREAKDOWN

In order to solve the corona breakdown problem for arbitrary geometries and fields, a numerical scheme needs to be implemented. The numerical approach followed hereby is based on the FE technique, which has extensively been applied to waveguide and antenna problems in the past. The intrinsic adaptivity of the FE method, able to handle a wide range of physical problems based on partial differential equations, is the link between the electromagnetic and the corona modeling.

To begin with, let us suppose that a FE code for solving the Helmholtz equation is at hand:

$$\nabla^2\phi + k^2\phi = 0 \tag{3}$$

with Dirichlet boundary conditions, while $\phi$ refers to any of the EM potentials. By expanding the unknown $\phi$ into a sum of weighted basis functions $\{\phi\}$, the weak formulation of the FE representation of (3) results in the following matrix equation:

$$[K]\{\phi\} - k^2[T]\{\phi\} = 0, \tag{4}$$

If a Galerkin testing is applied, the entries of $[K]$ and $[T]$ for the $l^{th}$ finite element, are respectively:

$$[K]^l_{pq} = \int_{V_l} \{\nabla\Lambda\}_p \{\nabla\Lambda\}_q dV \tag{5}$$

$$[T]^l_{pq} = \int_{V_l} \{\Lambda\}_p \{\Lambda\}_q dV, \tag{6}$$

where $\{\Lambda\}_p$ is the $p^{th}$ basis and $\{\Lambda\}_q$ the $q^{th}$ test function. As the eigenvalue $k^2$ does not generally depend on the position, (4) boils down to a generalized linear eigenvalue problem which can be solved in an efficient way using specialized algorithms. If a similar procedure is followed to solve (2), its weak formulation can be written as follows:

$$[K]\{n\} - [T']\{n\} = 0. \tag{7}$$

Compared to (4), there is only one different term to be computed, namely:

$$[T']^l_{pq} = \int_{V_l} \{\Lambda\}_p \frac{\nu_i - \nu_a}{D} \{\Lambda\}_q dV \approx \frac{\nu_i - \nu_a}{D}[T]^l_{pq}, \tag{8}$$

with the approximation being valid if the mesh is dense enough so that the variation of $\frac{\nu_i - \nu_a}{D}$ within the $l^{th}$ finite element is negligible. Comparing (4)-(6) with (7)-(8), it is evident that no additional computational tool is needed to assemble (7). In fact, the dependence of the term $\frac{\nu_i - \nu_a}{D}$ on the local electric field strength transforms (7) from a generalized eigenvalue problem to a general nonlinear one:

$$[M]\{n\} = 0, \tag{9}$$

where $[M] = [K] - [T']$. Consequently, a sweep of the input power (or of the electric field amplitude), which in turn affects the values of $\nu_i$, is necessary to find the breakdown power (or breakdown electric field) for which 9 has non-trivial solutions, i.e., $det([M]) = 0$. Alternatively, instead of a direct calculation of the determinant of $[M]$, the current analysis is based on the tracking of the corresponding eigenvalues of the matrix $[M]$. This procedure provides a more complete and accurate analysis of the phenomenon as it is discussed in Section IV.

## IV. CORONA BREAKDOWN ANALYSIS

In order to determine the corona breakdown power (or electric field) for a given pressure, the first eigenvalue of the matrix $[M]$ is tracked. The power limit for which this eigenvalue switches from a positive to a negative value defines the corona breakdown threshold. Moreover, the tracking of this eigenvalue provides a robust and fast convergent way of finding the required level of input power so that the determinant vanishes. It additionally guarantees that this zeroing corresponds to the lowest possible power at which a breakdown can occur.

In an attempt to demonstrate the logical steps of the algorithm in a better way, the analysis of the corona phenomenon inside a circular waveguide with a radius of 1 cm is presented. The waveguide is excited by its fundamental mode, i.e. TE11, and the operating frequency is 10 GHz, as shown in Fig. 1.

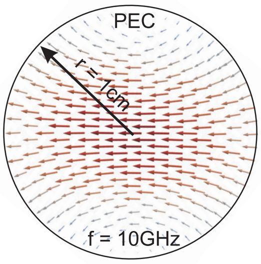

Fig. 1. A circular waveguide with a radius of 1cm is excited at 10 GHz with its fundamental mode, i.e. TE11.

Therefore, for a given pressure, the input power is swept and the first eigenvalue of the matrix $[M]$ is tracked. As far as two input powers resulting in a negative and a positive eigenvalue are found, the bisection method can be used. Thus, in only a few iterations the input power that renders the first eigenvalue of $[M]$ zero can be found. The same procedure is repeated for different values of pressure for which the device is expected to operate. The tracking of the eigenvalues versus the magnitude of the electric field is shown in Fig. 2 for different values of pressure.

Concerning the computation time of solving the eigenvalue problem (9), the calculation of the higher eigenvalues of

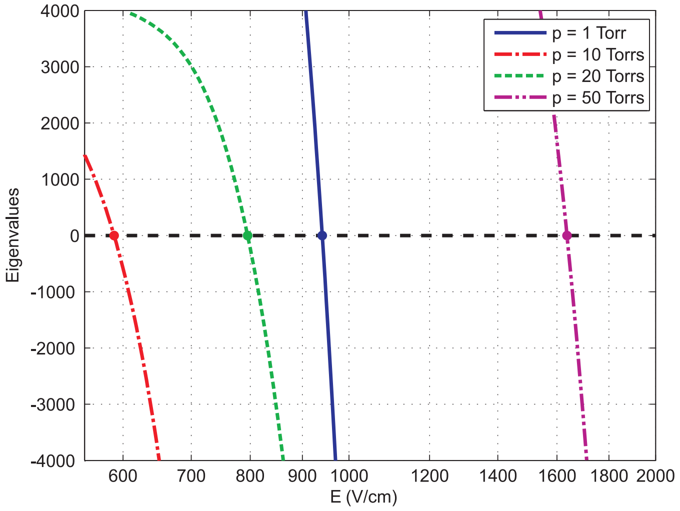

Fig. 2. Tracking of the first eigenvalue of $[M]$ for different levels of electric field and pressure. The breakdown for a certain pressure occurs at that level of electric field for which the corresponding eigenvalue changes its sign. In this example, the waveguide of Fig. 1 is meshed with 2176 triangles.

the matrix $[M]$ is redundant. Consequently, faster numerical algorithms that provide a converged value only for the first eigenvalue of a matrix can be used to accelerate the solution process.

A summarizing plot that is quite familiar in the corona breakdown community is the Paschen curve. For each pressure it indicates the maximum power (or electric field) that a device can withstand without suffering from a corona breakdown. For validating the current numerical method, the corona breakdown limit for the circular waveguide of Fig. 1 is simulated and compared with the values found in the literature [9]. These reference values were obtained by applying specific approximations to the ionization rate, since the electric field distribution of the TE11 mode is well-known.

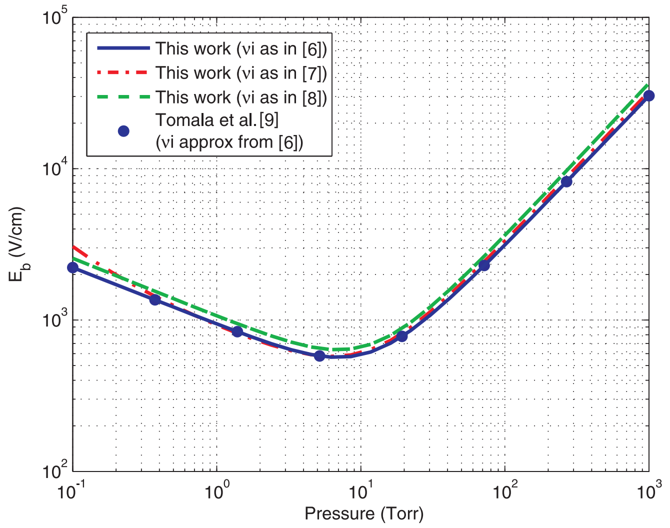

Fig. 3. Predicted levels of electric field breakdown $E_b$ using the proposed numerical method with three different ionization rate models [6]–[8] and the semi-analytical approach [9].

The agreement between the proposed fully numerical and the reference semi-analytical method is excellent, as shown in Fig. 3. Moreover, if other ionization rate models, proposed in the literature, are used, then the Paschen curve varies slightly. Such variations cannot be taken into account with semi-analytical methods and only a numerical approach can reveal them. Using the proposed FE scheme, a fast convergent method based on the tracking of the first eigenvalue is available and an accurate prediction of the corona breakdown is possible. Finally, the generality of the scheme has to be pointed out, as the corona breakdown analysis can be performed for arbitrary microwave components and excitation fields without any modification of the numerical algorithm.

## V. CONCLUSION

A fully numerical approach to investigate the corona breakdown phenomenon based on a FE method is discussed. The proposed method minimizes the additional development effort needed if a EM FE code is available and demonstrates the generality of such a tool in handling various microwave breakdown problems and corona parameter models.

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