

Black-box optimization benchmarking of IPOP-s*^aACM-ES on the BBOB-2012 noisy testbed

Ilya Loshchilov
TAO, INRIA Saclay
U. Paris Sud, F-91405 Orsay

Marc Schoenauer
TAO, INRIA Saclay
U. Paris Sud, F-91405 Orsay
firstname.lastname@inria.fr

Michèle Sebag
CNRS, LRI UMR 8623
U. Paris Sud, F-91405 Orsay

ABSTRACT

In this paper, we study the performance of IPOP-s*^aACM-ES, recently proposed self-adaptive surrogate-assisted Covariance Matrix Adaptation Evolution Strategy. The algorithm was tested using restarts till a total number of function evaluations of $10^6 D$ was reached, where D is the dimension of the function search space. The experiments show that the surrogate model control allows IPOP-s*^aACM-ES to be as robust as the original IPOP-aCMA-ES and outperforms the latter by a factor from 2 to 3 on 6 benchmark problems with moderate noise. On 15 out of 30 benchmark problems in dimension 20, IPOP-s*^aACM-ES exceeds the records observed during BBOB-2009 and BBOB-2010.

Categories and Subject Descriptors

G.1.6 [Numerical Analysis]: Optimization—*global optimization, unconstrained optimization*; F.2.1 [Analysis of Algorithms and Problem Complexity]: Numerical Algorithms and Problems

General Terms

Algorithms

Keywords

Benchmarking, black-box optimization, evolution strategy, CMA-ES, self-adaptation, surrogate models, ranking support vector machine, surrogate-assisted optimization

1. INTRODUCTION

When dealing with expensive optimization objectives, the surrogate-assisted approaches proceed by learning a surrogate model of the objective, and using this surrogate to reduce the number of computations of the objective function in various ways.

Many surrogate modelling approaches have been used within Evolution Strategies (ESs) and Covariance Matrix Adaptation Evolution Strategy (CMA-ES): Radial Basis Functions

network [8], Gaussian Processes [17], Artificial Neural Network [3], Support Vector Regression [12], Local-Weighted Regression [11, 1], Ranking Support Vector Machine (Ranking SVM) [16, 13, 9]. In most cases, the surrogate model is used as a filter (to select λ_{Pre} promising pre-children) and/or to estimate the fitness of some individuals in the current population. An example of surrogate-assisted CMA-ES with filtering strategy can be found in [13].

A well-known drawback of surrogate-assisted optimization is a strong dependence of the results on hyper-parameters used to build the surrogate model. Some optimal settings of hyper-parameters for a specific set of problems can be found by offline tuning, however for a new problem they are unknown in the black-box scenario. Moreover, the optimal hyper-parameters may dynamically change during the optimization of the function.

Motivated by this open issues, new self-adapted surrogate-assisted s*^aACM-ES algorithm have been proposed combining surrogate-assisted optimization of the expensive function and online optimization of the surrogate model hyper-parameters [14].

2. THE ALGORITHMS

2.1 The $(\mu/\mu_w, \lambda)$ -CMA-ES

In each iteration t , $(\mu/\mu_w, \lambda)$ -CMA-ES [6] samples λ new solutions $x_i \in \mathbb{R}^D$, where $i = 1, \dots, \lambda$, and selects the best μ among them. These μ points update the distribution of parameters of the algorithm to increase the probability of successful steps in iteration $t + 1$. The sampling is defined by a multi-variate normal distribution, $\mathcal{N}(m^t, \sigma^{t2} C^t)$, with current mean of distribution m^t , $D \times D$ covariance matrix C^t and step-size σ^t .

The active version of the CMA-ES proposed in [7, 10] introduces a weighted negative update of the covariance matrix taking into account the information about $\lambda - \mu$ worst points as well as about μ best ones. The new version improves CMA-ES on 9 out of 12 tested unimodal functions by a factor up to 2, and the advantages are more pronounced in larger dimension. While the new update scheme does not guarantee the positive-definiteness of the covariance matrix, it can be numerically controlled [7]. Since in our study we do not observe any negative effects of this issue, we will use aCMA-ES, the active version of the CMA-ES, for comparison with the surrogate-assisted algorithms.

2.2 The s*^aACM-ES

The s*^aACM-ES [14] is the surrogate-assisted version of

the $(\mu/\mu_w, \lambda)$ -CMA-ES, where the surrogate model is used periodically instead of the expensive function for direct optimization. The use of Ranking SVM allows to preserve the property of CMA-ES of invariance with respect to rank-preserving transformation of the fitness function. The property of invariance with respect to the orthogonal transformation of the search space is preserved thanks to the definition of the kernel function by the covariance matrix, adapted during the search.

In s^* ACM-ES we perform the following surrogate-assisted optimization loop: we optimize the surrogate model \hat{f} for \hat{n} generations by the CMA-ES, then we continue and optimize the expensive function $f(x)$ for one generation. To adjust the number of generations \hat{n} for the next time, the model error can be computed as a fraction of incorrectly predicted comparison relations that we observe, when we compare the ranking of the last λ evaluated points according to $f(x)$ and \hat{f} . The s^* ACM-ES uses the generation of the CMA-ES as a black-box procedure, and it has been shown in [14], that the improvement of the CMA-ES from passive to active version (aCMA-ES) leads to a comparable improvement of its surrogate-assisted versions (s^* ACM-ES and s^* aACM-ES).

The main novelty of the s^* ACM-ES is the online optimization of the surrogate model hyper-parameters during the optimization of the fitness function. The algorithm performs the search in the space of model hyper-parameters, generating λ_{hyp} surrogate models in each iteration. The fitness of the model can be measured as a prediction error of the ranking on the last λ evaluated points. This allows the user to define only the range of hyper-parameters and let algorithm to find the most suitable values for the current iteration t .

The detailed description of s^* ACM-ES is given in [14].

2.3 The Benchmarked Algorithms

For benchmarking we consider s^* ACM-ES in IPOPOP restart scenario (IPOPOP- s^* aACM-ES) using default parameters and termination criteria as given in [7] and [14]. The only one parameter of the surrogate part of IPOPOP- s^* aACM-ES different from the default one is the index of generation g_{start} when we start to use the surrogate model. For noisy case we set $g_{start} = 5(i_{restart} + 1)$ instead of default $g_{start} = 10$. We found that sometimes it makes sense to postpone the surrogate-assisted search if several restarts ($i_{restart}$) were performed.

3. RESULTS

Results from experiments according to [4] on the benchmark functions given in [2, 5] are presented in Figures 3 and 4, and Figure 2. The **expected running time** (ERT), used in the figures and table, depends on a given target function value, $f_t = f_{opt} + \Delta f$, and is computed over all relevant trials (on the first 15 instances) as the number of function evaluations executed during each trial while the best function value did not reach f_t , summed over all trials and divided by the number of trials that actually reached f_t [4, 15]. **Statistical significance** is tested with the rank-sum test for a given target Δf_t using, for each trial, either the number of needed function evaluations to reach Δf_t (inverted and multiplied by -1), or, if the target was not reached, the best Δf -value achieved, measured only up to the smallest number of overall function evaluations for any unsuccessful trial under consideration if available. Tables 1 and 2 give the Expected Running Time (ERT) for targets

$10^1, -1, -3, -5, -7$ divided by the best ERT obtained during BBOB-2009 (given in the ERT_{best} row), respectively in 5-D and 20-D. Bold entries correspond to the best (or 3-best if there are more than 3 algorithms) values. The median number of conducted function evaluations is additionally given in *italics*, if $ERT(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{opt} + 10^{-8}$. Entries with the \downarrow symbol are statistically significantly better (according to the rank-sum test) compared to the best algorithm in BBOB-2009, with $p = 0.05$ or $p = 10^{-k}$ where $k > 1$ is the number following the \downarrow symbol, with Bonferroni correction of 30.

The IPOPOP- s^* aACM-ES outperforms IPOPOP- s^* aACM-ES usually by a factor from 2 to 3 on functions with moderate noisy. This is the case for Sphere ($f_{101}, f_{102}, f_{103}$) and Rosenbrock ($f_{104}, f_{105}, f_{106}$) functions with Gaussian, Uniform and Cauchy noise models. It seems that the moderate noise only slightly affects the quality of the surrogate model and allows to have a speedup comparable to one of the noise-less case.

On most functions with severe noise the surrogate model usually is not used (\hat{n} oscillates around zero), because it gives a very imprecise prediction of the expensive function. On 20-dimensional f_{124} Schaffer function with Cauchy noise IPOPOP- s^* aACM-ES requires about 7 times more functions evaluations to reach the optimum with $\Delta f_{opt} = 10^{-7}$ than IPOPOP-aCMA-ES (see Fig. 2). However, according to Table 2, the performance for $\Delta f_{opt} = 10^{-5}$ is exactly the same for both algorithms, therefore, we suppose that the loss of performance for $\Delta f_{opt} = 10^{-7}$ can be explained by some influence of surrogate-assisted search on restart conditions. We also found that if use default coefficient c_1 and c_μ for the covariance matrix update instead of noisy settings of $c_1/5$ and $c_\mu/5$, then IPOPOP- s^* aACM-ES performs as well as IPOPOP-aCMA-ES. However, the use of default coefficients worsen the results of IPOPOP-aCMA-ES and IPOPOP- s^* aACM-ES on other problems.

We also observe some loss in performance of f_{125} Griewank-Rosenbrock and f_{128} Gallagher functions for $d \leq 5$. For f_{115} we observe the speedup for $d = 5$ and loss for $d = 20$, probably because of the same reasons as for f_{124} .

The IPOPOP- s^* aACM-ES improves (sometimes insignificantly) the records in dimension 20 on $f_{101}, f_{102}, f_{103}, f_{104}, f_{105}, f_{106}, f_{107}, f_{109}, f_{112}, f_{114}, f_{117}, f_{118}, f_{120}, f_{122}, f_{123}, f_{127}, f_{129}$.

4. CPU TIMING EXPERIMENT

For the timing experiment the IPOPOP- s^* aACM-ES was run on noiseless f_1, f_8, f_{10} and f_{15} functions without self-adaptation of surrogate model hyper-parameters. The crucial hyper-parameter for CPU time measurements, the number of training points was set $N_{training} = \lfloor 40 + 4D^{1.7} \rfloor$ as a function of dimension D [14].

These experiments have been conducted on a single core with 2.4 GHz under Windows XP using Matlab R2006a.

On uni-modal functions the time complexity of surrogate model learning increases cubically in the search space dimension (see Fig. 1) and quadratically in the number of training points. For small dimensions ($D < 10$) the overall time complexity increases super-linearly in the dimension. The time complexity per function evaluation depends on the population size, because one model is used to estimate the ranking of all points of the population. This leads to a smaller computational complexity on multi-modal functions, e.g. f_{15} Rastrigin function, where the population becomes

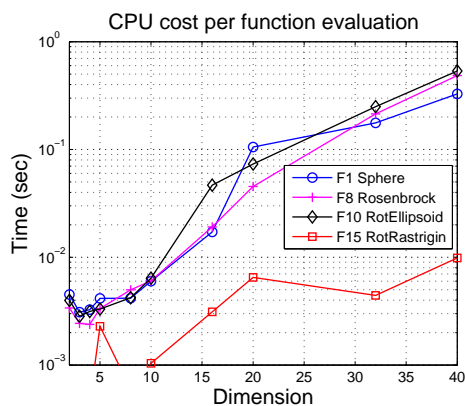


Figure 1: CPU cost per function evaluation of IPOP-aACM-ES with fixed hyper-parameters.

much larger after several restarts. The time complexity on noisy functions is more similar to the one of Rastrigin function, because in both cases the large populations are used.

The results presented here does not take into account the model hyper-parameters optimization, where λ_{hyp} surrogate models should be build at each iteration, which leads to an increase of CPU time per function evaluation by a factor of λ_{hyp} . For IPOP-^{s*}aACM-ES λ_{hyp} was set to 20.

5. CONCLUSION

In this paper, we have compared the recently proposed self-adaptive surrogate-assisted IPOP-^{s*}aACM-ES with the IPOP-aCMA-ES on noisy benchmark problems. The surrogate-assisted IPOP-^{s*}aACM-ES algorithm outperforms the original IPOP-aCMA-ES by a factor from 2 to 3 on the functions with moderate noise, and usually performs not worse on other functions. The IPOP-^{s*}aACM-ES algorithm improves the records on 15 out of 30 functions in dimension 20.

6. ACKNOWLEDGMENTS

The authors would like to acknowledge Anne Auger, Zyed Bouzarkouna, Nikolaus Hansen and Thomas P. Runarsson for their valuable discussions. This work was partially funded by FUI of System@tic Paris-Region ICT cluster through contract DGT 117 407 *Complex Systems Design Lab* (CSDL).

7. REFERENCES

- [1] Z. Bouzarkouna, A. Auger, and D. Ding. Investigating the local-meta-model CMA-ES for large population sizes. In C. Di Chio et al., editor, *Proc. EvoNUM'10*, pages 402–411. LNCS 6024, Springer, 2010.
- [2] S. Finck, N. Hansen, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2010: Presentation of the noisy functions. Technical Report 2009/21, Research Center PPE, 2010.
- [3] L. Graning, Y. Jin, and B. Sendhoff. Efficient evolutionary optimization using individual-based evolution control and neural networks: A comparative study. In *Proc. ESANN'2005*, pages 27–29, 2005.
- [4] N. Hansen, A. Auger, S. Finck, and R. Ros. Real-parameter black-box optimization benchmarking 2012: Experimental setup. Technical report, INRIA, 2012.
- [5] N. Hansen, S. Finck, R. Ros, and A. Auger. Real-parameter black-box optimization benchmarking 2009: Noisy functions definitions. Technical Report RR-6869, INRIA, 2009. Updated February 2010.
- [6] N. Hansen and A. Ostermeier. Completely derandomized self-adaptation in evolution strategies. *Evolutionary Computation*, 9(2):159–195, 2001.
- [7] N. Hansen and R. Ros. Benchmarking a weighted negative covariance matrix update on the BBOB-2010 noiseless testbed. In *GECCO '10: Proceedings of the 12th annual conference comp on Genetic and evolutionary computation*, pages 1673–1680, New York, NY, USA, 2010. ACM.
- [8] F. Hoffmann and S. Holemann. Controlled model assisted evolution strategy with adaptive preselection. In *International Symposium on Evolving Fuzzy Systems*, pages 182–187. IEEE, 2006.
- [9] H. Ingimundardottir and T. Runarsson. Sampling strategies in ordinal regression for surrogate assisted evolutionary optimization. In *Proc. ISDA'2011*, page To appear, 2011.
- [10] G. A. Jastrebski and D. V. Arnold. Improving evolution strategies through active covariance matrix adaptation. In *Proc. CEC'2006*, pages 2814–2821, 2006.
- [11] S. Kern, N. Hansen, and P. Koumoutsakos. Local meta-models for optimization using evolution strategies. In Th. Runarsson et al., editor, *PPSN IX*, pages 939–948. LNCS 4193, Springer, 2006.
- [12] O. Kramer. Covariance matrix self-adaptation and kernel regression - perspectives of evolutionary optimization in kernel machines. *Fundam. Inf.*, 98:87–106, 2010.
- [13] I. Loshchilov, M. Schoenauer, and M. Sebag. Comparison-Based Optimizers Need Comparison-Based Surrogates. In J. K. R. Schaefer, C. Cotta and G. Rudolph, editors, *Proc. PPSN XI*, pages 364–373. LNCS 6238, Springer, 2010.
- [14] I. Loshchilov, M. Schoenauer, and M. Sebag. Self-Adaptive Surrogate-Assisted Covariance Matrix Adaptation Evolution Strategy. In *GECCO '12: Proceedings of the 14th annual conference on Genetic and evolutionary computation*, page to appear, New York, NY, USA, 2012. ACM.
- [15] K. Price. Differential evolution vs. the functions of the second ICEO. In *Proceedings of the IEEE International Congress on Evolutionary Computation*, pages 153–157, 1997.
- [16] T. P. Runarsson. Ordinal regression in evolutionary computation. In Th. Runarsson et al., editor, *PPSN IX*, pages 1048–1057. LNCS 4193, Springer, 2006.
- [17] H. Ulmer, F. Streichert, and A. Zell. Evolution strategies assisted by gaussian processes with improved pre-selection criterion. In *Proc. CEC'2003*, pages 692–699, 2003.

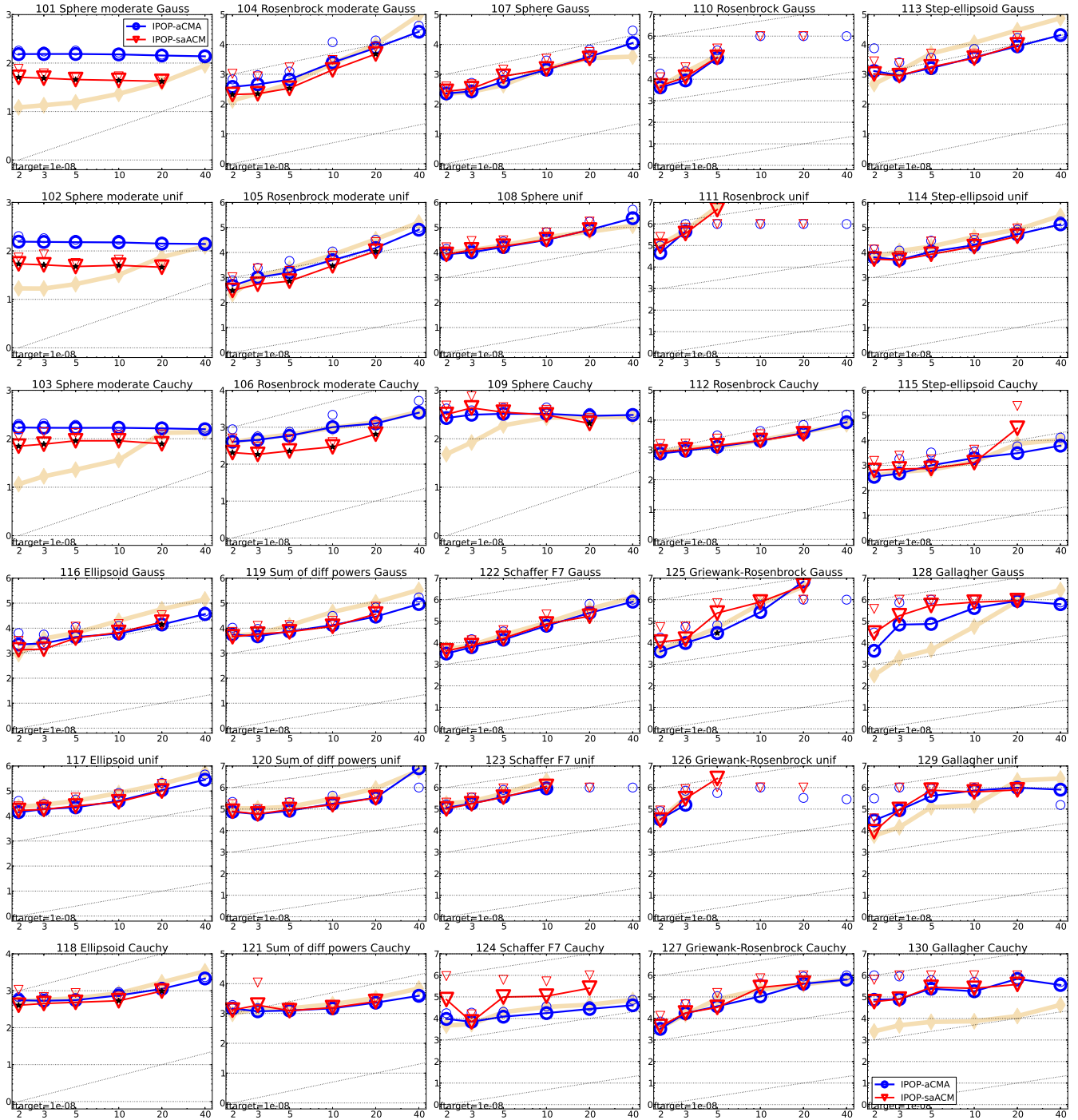


Figure 2: Expected running time (ERT) divided by dimension for target function value 10^{-8} as \log_{10} values versus dimension. Different symbols correspond to different algorithms given in legend of f_{101} and f_{130} . Light symbols give the maximum number of function evaluations from all trials divided by the dimension. Horizontal lines give linear scaling, the slanted dotted lines give quadratic scaling. Legend: \circ : IPOP-aCMA, ∇ : IPOP-saACM

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f101	11	37	44	62	69	75	15/15	f116	5730	14472	22311	26868	30329	31661	15/15
IPOP-aC	3.0(2)	3.0(1)	4.2(1)	5.8(2)	7.6(2)	9.3(2)	15/15	IPOP-aC	0.88(0.8)	0.62(0.6)	0.57(0.4)	0.73(0.5)	0.68(0.4)	0.68(0.4)	15/15
IPOP-sa	1.9(1)	2.1(0.5)*	2.2(0.5)*³	2.2(0.5)*⁴	2.5(0.3)*⁴	2.8(0.3)*⁴	15/15	IPOP-sa	1.0(1)	0.74(0.8)	0.56(0.5)	0.59(0.5)	0.56(0.4)	0.56(0.4)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f102	11	35	50	72	86	99	15/15	f117	26686	76052	1.1e5	1.4e5	1.7e5	1.9e5	15/15
IPOP-aC	3.5(3)	3.4(1)	3.8(0.9)	5.0(0.6)	6.1(0.8)	7.0(0.7)	15/15	IPOP-aC	0.65(0.5)	0.48(0.4)	0.44(0.2) ₂	0.47(0.2)	0.43(0.2)	0.46(0.2)	15/15
IPOP-sa	2.6(2)	2.1(0.7)	2.0(0.5)*⁴	1.9(0.4)*⁴	2.1(0.3)*⁴	2.2(0.3)*⁴	15/15	IPOP-sa	0.60(0.5)	0.43(0.3)	0.42(0.3)	0.54(0.3) ₂	0.59(0.3)	0.60(0.2) ₂	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f103	11	28	30	31	35	115	15/15	f118	429	1217	1555	1998	2430	2913	15/15
IPOP-aC	2.9(1)	4.1(1)	6.3(1)	11(2)	16(2)	6.5(0.6)	15/15	IPOP-aC	1.7(0.6)	0.88(0.3)	0.91(0.2)	0.91(0.1)	0.91(0.1)	0.90(0.1)	15/15
IPOP-sa	3.0(1)	2.8(0.7)*	3.4(0.9)*³	4.6(1)*⁴	7.1(3)*⁴	3.2(1)*⁴	15/15	IPOP-sa	0.49(0.1)	0.25(0.1)	0.33(0.1)	0.48(0.1)	0.63(0.1)	0.57(0.3)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f104	173	773	1287	1768	2040	2284	15/15	f119	12	657	1136	10372	35296	49747	15/15
IPOP-aC	1.5(0.5)	2.0(2)	1.7(1)	1.6(1.0)	1.5(0.9)	1.4(0.8)	15/15	IPOP-aC	1.5(1)	0.55(0.6)	0.58(0.5)	0.55(0.4)	0.49(0.2) ₂	0.61(0.4)	15/15
IPOP-sa	0.88(0.3)*	1.5(1)	1.1(1)	0.89(0.8)	0.80(0.7)*	0.73(0.7)*	15/15	IPOP-sa	4.9(10)	1.1(2)	0.99(0.9)	0.50(0.5)	0.47(0.2)	0.61(0.5)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f105	167	1436	5174	10388	10824	11202	15/15	f120	16	2900	18698	72438	3.3e5	5.5e5	15/15
IPOP-aC	2.8(0.5)	3.0(2)	1.3(0.8)	0.70(0.4)	0.70(0.4)	0.70(0.4)	15/15	IPOP-aC	18(39)	0.76(1)	0.65(0.5)	0.83(0.5)	0.43(0.3) ₂	0.66(0.5)	15/15
IPOP-sa	1.4(0.5)	1.5(2)	0.63(0.4)	0.33(0.2)	0.32(0.2)	0.32(0.2)	15/15	IPOP-sa	26(56)	1.1(1)	0.80(0.5)	0.52(0.3)	0.36(0.3)	0.64(0.5)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f106	92	529	1050	2666	2887	3087	15/15	f121	8.6	111	273	1583	3870	6195	15/15
IPOP-aC	2.3(0.7)	1.9(0.6)	1.6(0.3)	0.87(0.1)	0.90(0.1)	0.92(0.1)	15/15	IPOP-aC	3.0(3)	1.1(0.4)	1.1(0.3)	0.78(0.3)	0.84(0.1)	0.82(0.1)	15/15
IPOP-sa	1.7(0.9)	0.89(0.2)*²	0.60(0.2)*²	0.30(0.1)*³	0.32(0.1)*³	0.36(0.1)*³	15/15	IPOP-sa	3.3(4)	0.98(0.5)	0.97(0.5)	1.0(0.6)	0.95(0.4)	0.86(0.2)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f107	40	228	453	940	1376	1850	15/15	f122	10	1727	9190	30087	53743	1.1e5	15/15
IPOP-aC	1.9(2)	2.8(4)	1.9(2)	1.4(1.0)	1.3(0.7)	1.4(0.7)	15/15	IPOP-aC	2.9(3)	1.4(1)	0.71(0.6)	0.48(0.2)	0.66(0.3)	0.58(0.5)	15/15
IPOP-sa	1.4(2)	1.1(0.7)	1.4(1)	1.3(0.8)	1.4(1.0)	1.9(1)	15/15	IPOP-sa	2.6(3)	1.2(0.8)	0.75(0.5)	0.58(0.2) ₂	0.77(0.3)	0.66(0.3) ₂	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f108	87	5144	14469	30935	58628	80667	15/15	f123	11	16066	81505	3.4e5	6.7e5	2.2e6	15/15
IPOP-aC	11(17)	0.68(0.5)	0.64(0.2)	0.64(0.4)	0.66(0.3)	0.94(0.5)	15/15	IPOP-aC	28(60)	0.94(0.7)	0.66(0.4)	0.67(0.4)	0.56(0.2)	0.60(0.3)	15/15
IPOP-sa	7.8(9)	1.1(1)	0.60(0.4)	0.80(0.5)	0.74(0.4)	1.0(0.6)	15/15	IPOP-sa	23(52)	0.74(0.4)	0.70(0.5)	0.58(0.3)	0.80(0.7)	0.71(0.5)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f109	11	57	216	572	873	946	15/15	f124	10	202	1040	20478	45337	95200	15/15
IPOP-aC	3.1(2)	1.9(0.6)	1.1(0.3)	0.92(0.2)	1.2(0.2)	1.5(0.3)	15/15	IPOP-aC	2.6(3)	1.2(0.8)	2.1(3)	0.93(0.6)	0.94(0.5)	0.59(0.3)	15/15
IPOP-sa	2.9(2)	1.6(0.6)	0.83(0.3)	1.2(0.3)	1.2(0.3)	1.6(0.4)	15/15	IPOP-sa	2.6(2)	1.3(0.6)	3.1(4)	0.85(0.7)	0.97(0.8)	4.1(8)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f110	949	33625	1.2e5	5.9e5	6.0e5	6.1e5	15/15	f125	1	1	1	2.4e5	2.4e5	2.5e5	15/15
IPOP-aC	0.54(0.7)	4.3(5)	3.2(3)	0.79(0.8)	0.79(0.8)	0.79(0.8)	15/15	IPOP-aC	1(0)	26(22)	3142(3751)	0.53(0.4)	0.57(0.4)	0.57(0.4)	15/15
IPOP-sa	0.71(0.3)	5.6(7)	3.5(4)	0.78(0.8)	0.84(0.8)	0.84(0.8)	15/15	IPOP-sa	1.5(0)	18(18)	4308(4084)	0.56(0.6)	0.79(0.9)	4.8(6)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f111	6856	6.1e5	8.8e6	2.3e7	3.1e7	3.1e7	3/15	f126	1	1	1	∞	∞	∞	0
IPOP-aC	0.54(0.4)	7.3(8)	7.9(9)	∞	∞	∞	0/15	IPOP-aC	1(0)	62(98)	7882(5858)	∞	∞	∞	0/15
IPOP-sa	0.50(0.3)	7.4(8)	1.8(2)	1.00(1)	0.74(0.7)	0.74(0.6)	3/15	IPOP-sa	1(0)	165(444)	9361(8720)	1.3e7(1e7)	1.3e7(2e7)	1.3e7(1e7)	7/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f112	107	1684	3421	4502	5132	5596	15/15	f127	1	1	1	3.4e5	3.9e5	4.0e5	15/15
IPOP-aC	2.2(0.6)	1.2(0.6)	1.0(0.5)	1.1(0.3)	1.1(0.3)	1.1(0.3)	15/15	IPOP-aC	1.1(0)	30(24)	3085(2864)	0.50(0.5)	0.46(0.4)	0.46(0.4)	15/15
IPOP-sa	1.4(0.6)*²	1.5(2)	1.2(0.9)	1.2(0.6)	1.2(0.6)	1.2(0.6)	15/15	IPOP-sa	1(0)	20(22)	2432(3044)	0.30(0.3)	0.32(0.3)	0.33(0.3)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f113	133	1883	8081	24128	24128	24402	15/15	f128	111	4248	7808	12447	17217	21162	15/15
IPOP-aC	4.4(6)	0.78(1)	0.77(0.7)	0.32(0.2)	0.32(0.2)	0.32(0.3)	15/15	IPOP-aC	2.6(1)	1.8(2)	47(2)	30(3)	22(2)	18(2)	14/15
IPOP-sa	3.0(2)	0.82(0.8)	0.47(0.6)	0.35(0.4)	0.35(0.4)	0.35(0.4)	15/15	IPOP-sa	1.3(1.0)	16(21)	265(331)	203(342)	147(152)	124(212)	11/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f114	767	14720	56311	83272	83272	84949	15/15	f129	64	10710	59443	2.8e5	5.1e5	5.8e5	15/15
IPOP-aC	3.3(3)	0.61(0.5)	0.49(0.3)	0.60(0.2)	0.60(0.2)	0.61(0.2)	15/15	IPOP-aC	11(16)	10(5)	16(27)	7.0(8)	3.9(7)	3.5(6)	10/15
IPOP-sa	2.6(3)	0.73(0.4)	0.54(0.4)	0.44(0.2)	0.44(0.2)	0.44(0.3)	15/15	IPOP-sa	9.1(12)	10(21)	19(42)	13(18)	7.1(10)	6.3(9)	9/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f115	64	485	1829	2550	2550	2970	15/15	f130	55	812	3034	32823	33889	34528	10/15
IPOP-aC	1.8(1.0)	0.81(0.9)	1.1(1)	1.4(1)	1.4(1)	1.5(1)	15/15	IPOP-aC	1.4(0.8)	143(288)	391(625)	36(46)	35(45)	35(44)	11/15
IPOP-sa	1.6(0.6)	0.98(1)	1.2(1)	1.2(0.8)	1.2(0.8)	1.1(0.7)	15/15	IPOP-sa	1.4(0.9)	27(27)	126(131)	42(76)	41(74)	40(72)	13/15

Table 1: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 5. The central 80% range divided by two is given in braces. The median number of conducted function evaluations is additionally given in *italics*, if $\text{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\text{opt}} + 10^{-8}$. Best results are printed in bold.

Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f101	59	425	571	700	739	783	15/15	f116	5.0e5	6.9e5	8.9e5	1.0e6	1.1e6	1.1e6	15/15
IPOP-aC	6.1(1.0)	1.5(0.1)	1.6(0.1)	2.1(0.2)	2.7(0.1)	3.3(0.2)	15/15	IPOP-aC	0.36(0.1)	0.28(0.1)	0.23(0.0)	0.22(0.0)	0.23(0.0)	0.24(0.0)	15/15
IPOP-sa	4.7(1)*	0.86(0.2)*	0.73(0.1)*	0.78(0.1)*	0.88(0.1)*	0.99(0.1)*	15/15	IPOP-sa	0.46(0.1)	0.35(0.1)	0.30(0.1)	0.28(0.1)	0.29(0.1)	0.30(0.1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f102	231	399	579	921	1157	1407	15/15	f117	1.8e6	2.5e6	2.6e6	2.9e6	3.2e6	3.6e6	15/15
IPOP-aC	1.5(0.4)	1.6(0.2)	1.6(0.2)	1.6(0.1)	1.7(0.1)	1.8(0.1)	15/15	IPOP-aC	0.37(0.2)	0.35(0.2)	0.41(0.3)	0.45(0.2)	0.47(0.2)	0.56(0.3)	15/15
IPOP-sa	1.2(0.2)	0.93(0.2)*	0.75(0.1)*	0.61(0.1)*	0.60(0.0)*	0.60(0.0)*	15/15	IPOP-sa	0.29(0.1)	0.26(0.1)	0.29(0.1)	0.35(0.2)	0.42(0.2)	0.48(0.4)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f103	65	417	629	1313	1893	2464	14/15	f118	6908	11786	17514	26342	30062	32659	15/15
IPOP-aC	5.4(0.9)	1.5(0.1)	1.5(0.1)	1.2(0.1)	1.2(0.1)	1.2(0.1)	15/15	IPOP-aC	0.82(0.1)	0.78(0.2)	0.71(0.2)	0.63(0.1)	0.64(0.1)	0.66(0.1)	15/15
IPOP-sa	3.9(0.8)*	0.83(0.2)*	0.66(0.1)*	0.52(0.1)*	0.55(0.1)*	0.58(0.1)*	15/15	IPOP-sa	0.42(0.2)*	0.49(0.1)*	0.54(0.1)*	0.54(0.2)	0.56(0.1)	0.58(0.1)*	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f104	23690	85656	1.7e5	1.8e5	1.9e5	2.0e5	15/15	f119	2771	29365	35930	4.1e5	1.4e6	1.9e6	15/15
IPOP-aC	4.9(3)	1.6(1.0)	0.82(0.5)	0.80(0.5)	0.79(0.5)	0.77(0.4)	15/15	IPOP-aC	1.6(1)	0.62(0.4)	0.83(0.4)	0.42(0.3)	0.25(0.1)	0.28(0.1)	15/15
IPOP-sa	3.5(2)	1.1(0.6)	0.55(0.3)	0.52(0.3)	0.50(0.3)	0.49(0.3)	15/15	IPOP-sa	1.5(2)	0.74(0.3)	0.88(0.3)	0.33(0.1)	0.25(0.1)	0.37(0.3)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f105	1.9e5	6.1e5	6.3e5	6.5e5	6.6e5	6.7e5	15/15	f120	36040	1.8e5	2.8e5	1.6e6	6.7e6	1.4e7	13/15
IPOP-aC	1.2(0.3)	0.43(0.1)	0.43(0.1)	0.44(0.1)	0.44(0.1)	0.44(0.1)	15/15	IPOP-aC	0.62(0.3)	0.64(0.2)	0.79(0.4)	0.78(0.4)	0.44(0.2)	0.40(0.2)	15/15
IPOP-sa	1.0(0.4)	0.36(0.1)	0.35(0.1)	0.34(0.1)	0.34(0.1)	0.33(0.1)	15/15	IPOP-sa	0.65(0.5)	0.80(0.4)	0.80(0.4)	1.2(0.6)	0.87(0.5)	0.42(0.1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f106	11480	21668	23746	25470	26492	27360	15/15	f121	249	769	1426	9304	34434	57404	15/15
IPOP-aC	0.77(0.2)	0.76(0.2)	0.84(0.2)	0.89(0.1)	0.91(0.1)	0.91(0.1)	15/15	IPOP-aC	1.2(0.3)	1.0(0.2)	1.1(0.2)	0.77(0.1)	0.56(0.1)	0.64(0.1)	15/15
IPOP-sa	0.28(0.1)*	0.37(0.3)*	0.39(0.2)*	0.45(0.2)*	0.46(0.2)*	0.47(0.2)*	15/15	IPOP-sa	1.2(0.4)	0.85(0.2)	0.92(0.3)	0.69(0.1)	0.58(0.1)	0.71(0.1)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f107	8571	13582	16226	27357	52486	65052	15/15	f122	692	52008	1.4e5	7.9e5	2.0e6	5.8e6	15/15
IPOP-aC	0.81(0.4)	0.97(0.4)	1.3(0.6)	1.5(0.9)	1.3(0.7)	1.1(0.5)	15/15	IPOP-aC	2.3(3)	0.81(0.3)	0.97(0.6)	0.56(0.3)	0.66(0.1)	0.81(0.9)	15/15
IPOP-sa	0.83(0.5)	1.3(0.8)	1.4(0.7)	1.3(0.8)	1.0(0.5)	0.94(0.4)	15/15	IPOP-sa	2.5(3)	0.94(0.6)	0.81(0.3)	0.53(0.1)	0.51(0.2)	0.57(0.3)	15/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f108	58063	97228	2.0e5	4.5e5	6.3e5	9.0e5	15/15	f123	1063	5.3e5	1.5e6	5.3e6	2.7e7	1.6e8	0
IPOP-aC	0.74(0.3)	0.98(0.5)	1.0(0.6)	1.1(0.6)	1.5(0.4)	1.5(0.3)	15/15	IPOP-aC	6.4(4)	0.72(0.4)	0.88(0.4)	0.94(0.7)	0.50(0.1)	1.9(2)	0/15
IPOP-sa	0.72(0.3)	1.2(0.6)	1.1(0.6)	1.2(0.7)	1.6(0.3)	1.4(0.3)	15/15	IPOP-sa	7.6(8)	0.77(0.5)	0.81(0.5)	1.1(0.7)	0.63(0.4)	0.92(1)	0/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f109	333	632	1138	2287	3583	4952	15/15	f124	192	1959	40840	1.3e5	3.9e5	8.0e5	15/15
IPOP-aC	1.1(0.2)	1.2(0.1)	1.2(0.2)	1.2(0.1)	1.1(0.1)	1.1(0.1)	15/15	IPOP-aC	1.1(0.5)	3.9(11)	1.0(0.7)	0.91(0.6)	0.76(0.4)	0.62(0.3)	15/15
IPOP-sa	0.93(0.3)	0.78(0.2)*	0.78(0.2)*	0.89(0.3)*	0.79(0.2)*	0.75(0.1)*	15/15	IPOP-sa	0.86(0.4)	3.0(0.8)	2.2(2)	1.4(0.5)	0.76(0.3)	4.0(5)	14/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f110	∞	∞	∞	∞	∞	∞	0	f125	1	1	1	2.5e7	8.0e7	8.1e7	4/15
IPOP-aC	0/15	IPOP-aC	1(0)	827(912)	3.8e6(4e6)	1.1(1)	1.7(2)	1.7(2)	2/15
IPOP-sa	0/15	IPOP-sa	1(0)	1083(1766)	3.3e6(2e6)	0.61(0.5)	1.1(1)	1.1(1)	3/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f111	∞	∞	∞	∞	∞	∞	0	f126	1	1	1	∞	∞	∞	0
IPOP-aC	0/15	IPOP-aC	1(0)	6417(3982)	∞	∞	∞	∞	0/15
IPOP-sa	0/15	IPOP-sa	1(0)	7156(4580)	∞	∞	∞	∞	0/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f112	25552	64124	69621	73557	76137	78238	15/15	f127	1	1	1	4.4e6	7.3e6	7.4e6	15/15
IPOP-aC	0.84(0.2)	0.84(0.5)	0.89(0.5)	0.93(0.5)	0.94(0.5)	0.93(0.4)	15/15	IPOP-aC	1(0)	193(102)	2.8e5(4e5)	1.0(0.9)	1.1(1)	1.1(1.0)	15/15
IPOP-sa	0.89(0.3)	0.79(0.1)	0.84(0.1)	0.88(0.1)	0.88(0.1)	0.88(0.1)	15/15	IPOP-sa	1(0)	238(106)	3.3e5(4e5)	0.75(0.6)	0.93(0.5)	0.97(0.6)	14/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f113	50123	3.6e5	5.6e5	5.9e5	5.9e5	5.9e5	15/15	f128	1.4e5	1.3e7	1.7e7	1.7e7	1.7e7	1.7e7	9/15
IPOP-aC	0.60(0.4)	0.27(0.1)	0.27(0.1)	0.29(0.1)	0.29(0.1)	0.29(0.1)	15/15	IPOP-aC	0.51(0.6)	0.72(1)	1.0(2)	1.0(2)	1.0(1)	1.0(1)	7/15
IPOP-sa	0.54(0.2)	0.35(0.2)	0.31(0.1)	0.32(0.1)	0.32(0.1)	0.32(0.1)	15/15	IPOP-sa	1.3(2)	0.59(0.8)	1.1(1)	1.1(2)	1.1(1)	1.1(2)	8/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f114	2.1e5	1.1e6	1.4e6	1.6e6	1.6e6	1.6e6	15/15	f129	7.8e6	4.1e7	4.2e7	4.2e7	4.2e7	4.2e7	5/15
IPOP-aC	0.64(0.5)	0.35(0.1)	0.53(0.3)	0.62(0.3)	0.62(0.3)	0.63(0.3)	15/15	IPOP-aC	0.16(0.2)	0.27(0.3)	0.46(0.6)	0.46(0.6)	0.46(0.6)	0.46(0.6)	6/15
IPOP-sa	0.71(0.3)	0.39(0.3)	0.40(0.2)	0.50(0.3)	0.50(0.3)	0.55(0.3)	15/15	IPOP-sa	0.41(0.9)	0.28(0.5)	0.36(0.5)	0.36(0.5)	0.36(0.5)	0.36(0.5)	9/15
Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ	Δf_{opt}	1e1	1e0	1e-1	1e-3	1e-5	1e-7	#succ
f115	2405	30268	91749	1.3e5	1.3e5	1.3e5	15/15	f130	4904	93149	2.5e5	2.5e5	2.6e5	2.6e5	7/15
IPOP-aC	1.4(2)	1.2(0.6)	0.54(0.1)	0.45(0.0)	0.45(0.0)	0.45(0.1)	15/15	IPOP-aC	1.5(2)	66(99)	54(71)	54(71)	54(72)	53(70)	6/15
IPOP-sa	0.75(2)	0.67(0.3)	0.31(0.2)*	24.8(15)	4.8(15)	4.7(14)	15/15	IPOP-sa	0.55(1)	27(34)	30(43)	30(49)	30(39)	30(41)	12/15

Table 2: Expected running time (ERT in number of function evaluations) divided by the respective best ERT measured during BBOB-2009 (given in the respective first row) for different Δf values in dimension 20. The central 80% range divided by two is given in braces. The median number of conducted function evaluations is additionally given in *italics*, if $\text{ERT}(10^{-7}) = \infty$. #succ is the number of trials that reached the final target $f_{\text{opt}} + 10^{-8}$. Best results are printed in bold.

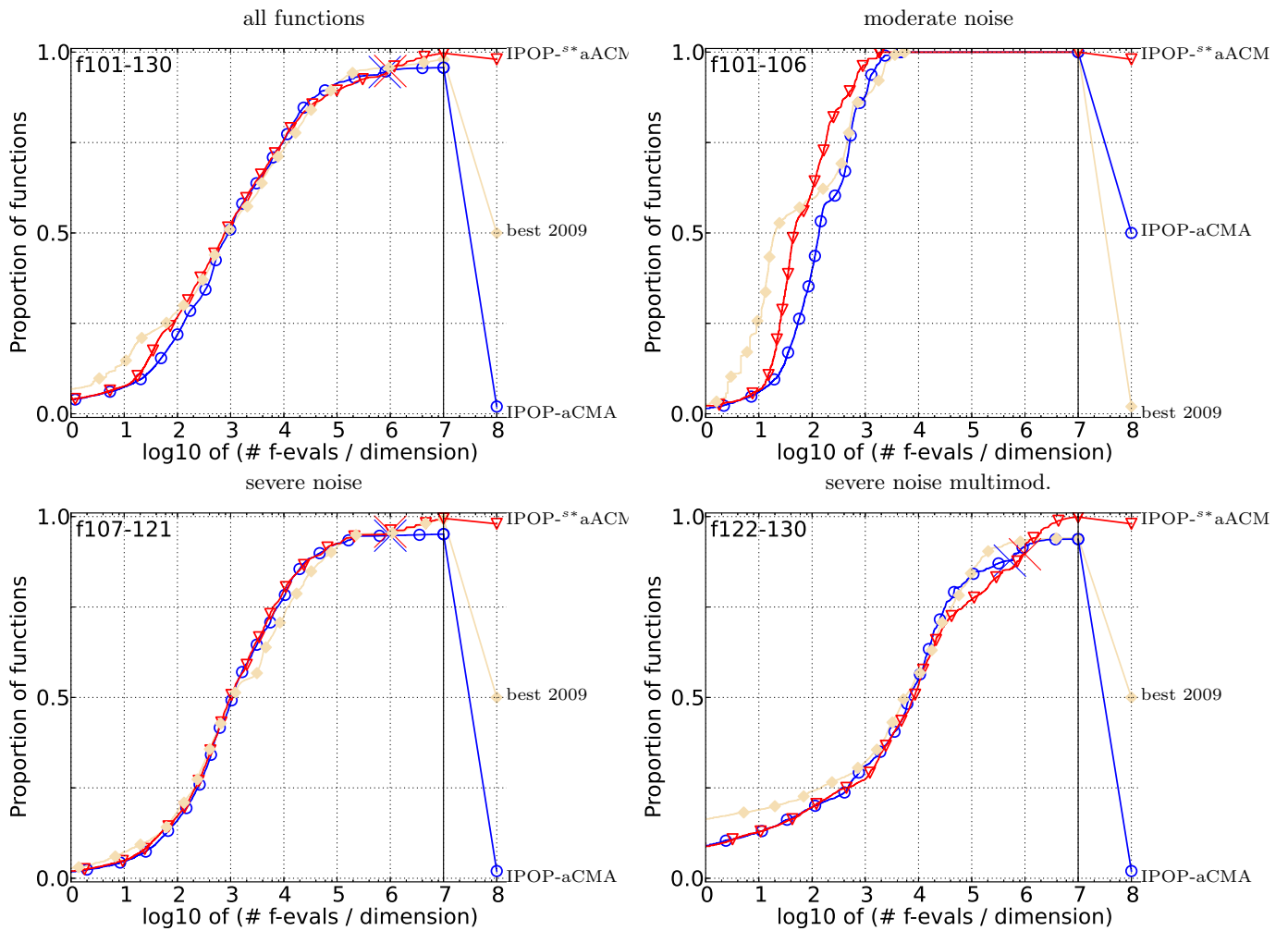


Figure 3: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 5-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.

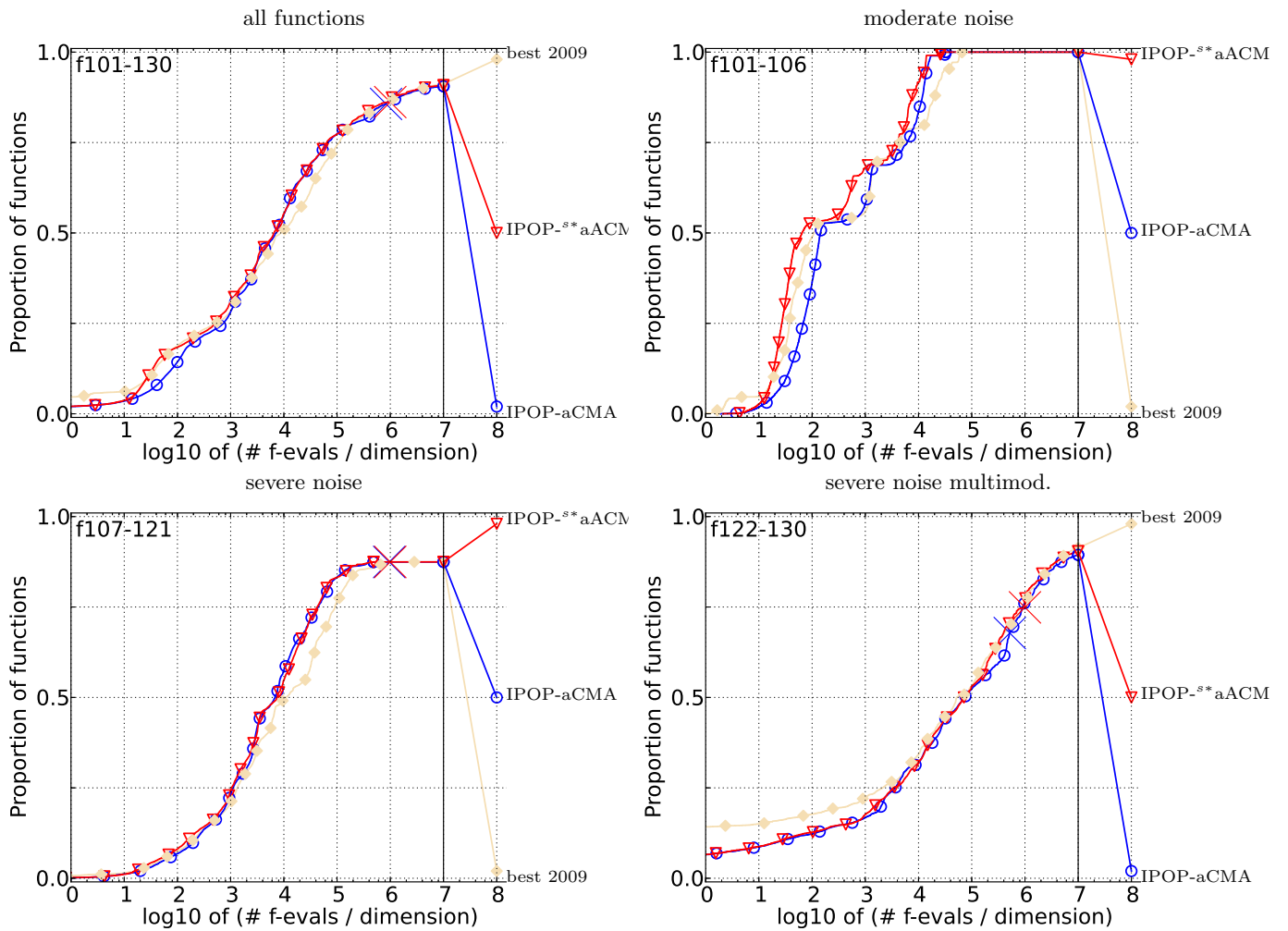


Figure 4: Bootstrapped empirical cumulative distribution of the number of objective function evaluations divided by dimension (FEvals/D) for 50 targets in $10^{[-8..2]}$ for all functions and subgroups in 20-D. The “best 2009” line corresponds to the best ERT observed during BBOB 2009 for each single target.