#### INTRODUCTION

Usual soundproofing materials are generally a robust and cost-effective solution to address noise issues, but not always effective or too bulky at low frequency [1]. Active noise control technology can be used alternatively, particularly when annoying sounds have strong low-frequency components. This approach generally requires sensors and control loudspeakers connected together by a control algorithm to cancel noise on the principle of destructive interferences [2]. Sound pressure reduction in closed spaces can also be achieved by controlling the dynamics of boundaries. Rather than targeting a total cancellation which requires significant input of additional acoustic energy, the goal is to prevent sound reflexion on the walls. In closed spaces, the use of loudspeaker as electroacoustic absorbers can assist to absorb the acoustic energy propagating near their diaphragm (or the mechanical structure attached to them). To help with this process, a control circuit (electrical load) is connected across the electrical terminals in order to modify the transducer internal dynamics. Specific applications of these impedance-based control strategies can be found in noise reduction within cavities such as rooms or ducts [3, 4, 5]. As reported in [6] for instance, an arrangement of electroacoustic absorbers disposed in the corners in a lightly damped room provides significant damping of the low-frequency resonances. This helps to actively equalize sound fields with low modal density.

This article uses the concept of electroacoustic absorbers previously introduced in [7] and discusses an alternative way to design the dedicated electrical load from the mathematical model of the loudspeaker. The main attraction of electroacoustic absorbers is to achieve broadband sound absorption with conventional loudspeakers without the need for any sensor. In the remaining of the paper, the comprehensive knowledge required for modeling an electrodynamic loudspeaker is presented. Then, the loudspeaker is turned into an electroacoustic resonator with broadband absorptivity through an impedance-based matching approach. Finally, some computed results are provided and concluding remarks will consider the benefits of the methodology over other alternatives. An overview of foreseen future developments will also be provided.

#### **CONE LOUDSPEAKER: DESCRIPTION AND MODELING**

### **Overview and Special Features**

Cone loudspeakers are reversible electrodynamic transducers, wherein a voice coil enables a cone-shaped diaphragm to produce sound power in response to an electrical audio signal input. It is basically a single-degree-of-freedom oscillator driven by the voice coil. The latter provides driving force to the diaphragm by the response of a magnetic field to the current flowing through it [8]. Alternating current will therefore move the cone back and forth. A common feature to reversible or bilateral electromechanical transducers, i.e. those which give rise to mechanical motion from electrical energy or the other way round, is their sensing and actuating capability. With this special feature, the loudspeaker has the ability to provide information about the sound field, and obviously the possibility to interact with it (while possibly interacting with it). The way the dual sensing and actuating functionality can be exploited to create a broadband electroacoustic resonator is the main motivation of this paper.

### **Governing Equations**

Referring to Fig. 1, the mechanical part can be represented by a mass-spring system below the first modal frequency of the diaphragm. The governing equation of the mechanical part follows from the Newton's second law and can be written using phasor representation, as

$$S\underline{p} = \left(j\omega M_{ms} + R_{ms} + \frac{1}{j\omega C_{ms}}\right)\underline{v} - Bl\underline{i}$$
(1)



**FIGURE 1:** Schematic representation of a cone loudspeaker with terms of electromechanical coupling included:  $\underline{F}_{mag} = Bl \underline{i}$  is the force of electric origin resulting from the magnetic field acting on a moving free charge (current), and  $\underline{\varepsilon} = -Bl \underline{v}$  is the back electromotive force (voltage) induced by the motion of the voice coil within the magnetic field.

where  $\underline{p}$  is the surrounding sound pressure acting on the diaphragm,  $\underline{v}$  is the diaphragm velocity,  $\underline{i}$  is the electrical current flowing through the voice coil, and  $\omega$  is the radial frequency. It is assumed that all forces acting on the transducer are small enough so that the displacements remain proportional to applied forces (small signal modeling). For the model parameters, S is the effective piston area, Bl is the force factor,  $M_{ms}$  and  $R_{ms}$  are the mass and mechanical resistance of the moving body, and  $C_{ms}$  is the equivalent mechanical compliance accounting for the flexible edge suspension and spider. When the loudspeaker is loaded by a rear volume  $V_b$ , the reaction of the fluid that acts on the rear face is commonly modeled as an acoustic compliance  $V_b/(\rho c^2)$ , where  $\rho$  is the density of the medium and c is the speed of sound in the medium [8].

The system electrical dynamics is commonly addressed by linear circuit theory generalized to include the effect of electromechanical coupling. The governing equation is based on Kirchhoff's laws and can be written as

$$\underline{e} = (j\omega L_e + R_e)\underline{i} + Bl\underline{v}$$
<sup>(2)</sup>

where  $\underline{e}$  is the input voltage applied across the electrical terminals,  $L_e$  and  $R_e$  are the self inductance and dc resistance of the coil. Equation (2) clearly shows that the voltage drop across the electrical terminals is the sum of a contribution proportional to the current applied and a contribution proportional to the velocity of the mechanical part.

Introducing the mechanical impedance  $\underline{Z}_m = j\omega M_{ms} + R_{ms} + 1/(j\omega C_{mc})$ , where  $C_{mc}^{-1} = C_{ms}^{-1} + \rho c^2 S^2 / V_b$ , and the blocked electrical impedance  $\underline{Z}_e = j\omega L_e + R_e$ , Eqs. (1-2) can be expressed more compactly as

$$\frac{S\underline{p} = \underline{Z}_m \underline{v} - Bl\underline{i}}{e = Z_e i + Blv}$$
(3)

and allows the basic analysis of an electrodynamic loudspeaker system. Notice that the radiation impedance is excluded of the following development with a view of providing general properties of the loudspeaker (apart from the radiation conditions of the diaphragm).

## **Input Impedance**

Input impedance  $\underline{Z}_{in}$  is the electrical impedance 'seen' by any equipment (such as an audio amplifier) connected to the input terminals of the loudspeaker. It can be shown that without driving sound pressure (Sp = 0), the input impedance can be derived after

$$\underline{Z}_{in} = \frac{\underline{e}}{\underline{i}} = \underline{Z}_e + \frac{(Bl)^2}{\underline{Z}_m} \tag{4}$$

which is the combined effect of all resistances and reactances of the loudspeaker system [8].

#### **Connecting an External Electrical Load**

Let us consider an electrical load of complex impedance  $\underline{Z}_L = R_L + jX_L$ , where the real part of impedance is the resistance  $R_L$  and the imaginary part is the reactance  $X_L$ . When the coil is in motion and the electrical load is connected across the transducer terminals, the current which is generated from the back emf  $\underline{\varepsilon} = -Bl \underline{v}$  can be expressed as

$$\underline{i} = \frac{1}{\underline{Z}_e + \underline{Z}_L} \underline{\varepsilon}$$
(5)



**FIGURE 2:** Block diagram of an electrodynamic loudspeaker connected to an electrical load  $\underline{Z}_L$ .

### **ELECTROACOUSTIC RESONATOR: CONCEPT AND FORMULATION**

## Acoustic Absorptivity of the Diaphragm

The transfer function describing the relationship between the input pressure force and the output velocity of the diaphragm is easily obtained from Eqs. (3-5). The closed form expression of the specific acoustic impedance (in  $N \text{ sm}^{-3}$ ) that results can be written as

$$\underline{Z} = \frac{p}{\underline{v}} = \frac{\underline{Z}_m}{S} + \frac{(Bl)^2}{S(\underline{Z}_e + \underline{Z}_L)}$$
(6)

Below the first modal frequency of the diaphragm, the coil inductance is commonly neglected [8] and the specific acoustic admittance that results can be written in terms of damping ratio  $\zeta$ , undamped angular frequency  $\omega_0$  and system gain K, as

$$\underline{Y} = \frac{\underline{v}}{\underline{p}} = K \frac{j\omega}{(j\omega)^2 + j\omega 2\zeta \omega_0 + \omega_0^2}$$
(7)

where the parameters of the oscillator are

$$\omega_{0} = \frac{1}{\sqrt{M_{ms}C_{mc}}} \qquad \zeta = \frac{R_{ms} + \frac{(Bl)^{2}}{R_{e} + R_{L}}}{2M_{ms}\omega_{0}} \qquad K = \frac{S}{M_{ms}}$$
(8)

The reflection coefficient which defines the ratio of the reflected and incident sound pressures can be derived after

$$\underline{r} = \frac{\underline{Z} - \rho c}{\underline{Z} + \rho c} \tag{9}$$

The magnitude of  $\underline{r}$  is less than 1 if and only if the real part of the specific acoustic impedance is positive. Any surface having this property absorbs acoustic energy. The sound absorption coefficient at normal incidence is commonly used for assessing the acoustic performance of materials. It is derived after extracting the magnitude  $|\underline{r}|$  of the reflection coefficient as

$$\alpha = 1 - |\underline{r}|^2 \tag{10}$$

and is valid for the steady-state response of the system to a harmonic excitation.

#### Assigning a Target Acoustic Impedance

The general objective is to achieve broadband sound absorption by matching the diaphragm impedance to the characteristic impedance  $\rho c$  of the medium. To meet the acoustic requirements, the transducer dynamics must be taken into account, i.e. impedance matching should be achieved over a bandwidth that is consistent with the transducer capability. Developing Eq. (6) clearly shows that the loudspeaker has a dynamic response of the bandpass type which must be observed. The specific acoustic impedance (in N s m<sup>-3</sup>) that is to be assigned should be represented as a complex quantity  $\underline{Z}_0 = (R_0 + jX_0)/S$ , where the real part of impedance is the resistance  $R_0/S$  and the imaginary part is the reactance  $X_0/S$ . The latter expression can also be expanded in terms of capacitive reactance and inductive reactance, as

$$S\underline{Z}_{0} = j\omega M_{0} + R_{0} + \frac{1}{j\omega C_{0}}$$
<sup>(11)</sup>

where  $C_0$  is a mechanical compliance,  $R_0$  is a mechanical resistance and  $M_0$  is a mass. For convenience in the design process, Eq. (11) can also be rewritten in terms of undamped angular frequency  $\omega_0$ , damping ratio  $\zeta$  and system gain K, as

$$\underline{Y}_{0} = \frac{1}{\underline{Z}_{0}} = K \frac{j\omega}{(j\omega)^{2} + j\omega 2\zeta \omega_{0} + \omega_{0}^{2}}$$
(12)

where  $\underline{Y}_0$  defines the diaphragm velocity response when subject to externally applied forces, and the tuning parameters can be expressed as

$$\omega_0 = \frac{1}{\sqrt{M_0 C_0}} \qquad \zeta = \frac{R_0}{2} \sqrt{\frac{C_0}{M_0}} \qquad K = \frac{S}{M_0}$$
(13)

# **Expressing the Synthetic Load Impedance**

As described in Fig. 2, the complex impedance  $\underline{Z}_L$  is involved in the functional relationship between the voltage  $\underline{\varepsilon}$  induced in the voice coil and the current  $\underline{i}$  required so that the feedback force  $\underline{F}_{mag}$  opposes the pressure force  $\underline{Sp}$  accordingly. Equating Eq. (6) and Eq. (11), leads to formally identifying  $\underline{Z}_L$  (in  $\Omega$ ), as

$$\underline{Z}_L = -\underline{Z}_e + \frac{(Bl)^2}{S\underline{Z}_0 - \underline{Z}_m} \tag{14}$$

As clearly seen in Eq. (14),  $\underline{Z}_L$  can be split off into a negative series resistance-inductance, the role of which is to neutralize the blocked electrical impedance  $\underline{Z}_e$  of the voice coil, and a complex electrical filter which depends on both the mechanical impedance  $\underline{Z}_m$  and the desired specific acoustic impedance  $\underline{Z}_0$ .

#### **Stability criterion**

The complex impedance given by Eq. (14), however, is not feasible due to causality issue. For a practical implementation, it should take the opposite. The synthetic load required to meet acoustic specifications should take the form of an electrical admittance (in  $\Omega^{-1}$ ), and can be written as

$$\underline{Y}_{L} = \frac{1}{\underline{Z}_{L}} = -\frac{(j\omega)^{2}a_{2} + j\omega a_{1} + a_{0}}{(j\omega)^{3}b_{3} + (j\omega)^{2}b_{2} + j\omega b_{1} + b_{0}}$$
(15)

where

$$a_{2} = M_{ms} - M_{0} \qquad b_{3} = a_{2}L_{e}$$

$$a_{1} = R_{ms} - R_{0} \qquad b_{2} = a_{1}L_{e} + a_{2}R_{e}$$

$$a_{0} = 1/C_{mc} - 1/C_{0} \qquad b_{1} = a_{0}L_{e} + a_{1}R_{e} + (Bl)^{2}$$

$$b_{0} = a_{0}R_{e}$$
(16)

Information about absolute stability of the synthetic load can be obtained directly from the coefficients of Eq. (15) using Routh's stability criterion [9]. If any of the coefficients in the denominator are zero or negative in the presence of at least one positive coefficient, there is a root (or pole) that is imaginary or has positive real part. In such a case the system is not stable. This implies that  $M_{ms} > M_0$ ,  $C_{mc} < C_0$  and

$$R_{ms} > R_0 - \frac{(Bl)^2}{R_e} - (M_{ms} - M_0) \frac{R_e}{L_e}$$
(17)

The stability condition also emphasizes that the loudspeaker selection is essential in order to achieve the broadband sound absorption through the synthesis of dedicated electrical loads.

## **COMPUTED RESULTS AND DISCUSSION**



**FIGURE 3:** Bode plot of computed input impedance (left) and the synthetic electrical admittance to achieve the target acoustic impedance  $\underline{Z}_0$  (right).

Figure 3(a) illustrates the frequency response of the input impedance (see Eq. (4)) when the loudspeaker is loaded by a rear sealed enclosure, the volume of which is 1.3 L. As shown in Fig. 3(a), the input impedance that is 'seen' by the synthetic electrical load is essentially resistive at low and middle frequencies, except an area where the behavior is dominated by the mechanical resonance. As the frequency increases, the inductance of the coil becomes large, as does the modulus of input impedance which also becomes reactive. Figure 3 (b) is the frequency response of the synthetic load admittance required to achieve various acoustic impedances at the diaphragm. When analyzing the frequency responses in Fig. 3, a major issue is likely to arise due to the inductive behavior of the moving-coil transducer which will interact with the synthetic load dynamics. As in any active systems, the gain must not be greater than unity (zero dB) when the phase crosses  $-\pi$ . A key point in the design stage is therefore to match the synthetic load and the loudspeaker input impedance so as the whole operates in a stable manner. This is a challenging task since both have their own dynamics that may interact with each other and affect performance in closed loop.



**FIGURE 4:** Real part (—) and imaginary part (--) of the computed specific acoustic impedance (left) and corresponding sound absorption coefficient (right).

Figure 4(a) illustrates the frequency response of the specific acoustic impedance computed when the loudspeaker input terminals are left open circuit (case A) and that corresponding to a perfect (case C) and intermediate (case B) impedance matching. In order to improve the acoustic absorptivity of the diaphragm, it is shown that some additional resistance must be provided to the system while reducing both the inductive and capacitive reactances. The expected outcome in terms of sound absorption coefficient is given in Fig. 4(b). It can be seen that the effect of such additional electrical load enables to significantly improve the absorptivity of the diaphragm. Table 1 summarizes the computed control results in terms of resonant frequency  $f_0 = \omega_0/(2\pi)$ , damping ratio  $\zeta = R_0/(2M_0\omega_0)$ , system gain  $K = S/M_0$  and bandwidth  $B = 2\zeta f_0$ , compared to the open circuit case. The technical data of the Monacor SPX-30M, the 3" full range loudspeaker used for running the simulation, can be found in Tab. 2.

TABLE 1:	Setting cases	and	corresponding	control	results
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Case	<b>Resonance frequency</b>	Damping ratio	Gain	Bandwidth
	fo	ζ	K	B
A (open circuit)	$152\mathrm{Hz}$	0.09	1.5	26 Hz
В	$158\mathrm{Hz}$	0.46	4.2	$147~\mathrm{Hz}$
С	$158\mathrm{Hz}$	0.96	5.1	$303~\mathrm{Hz}$

As given in Tab. 1, the expected bandwidth, which is the width of the range of frequencies for which the energy is at least half its peak value, is clearly increased with the help of the synthetic load. Expected acoustic performance with an off-the-shelf loudspeaker is quite attractive compared to state of the art soundproofing solutions. To treat low frequencies particularly, it is an appropriate option in terms of efficiency, size and integration. In addition, some versatility can

Parameter	Notation	Value	Unit
dc resistance	$R_{e}$	6.4	Ω
Voice coil inductance	$L_e$	0.18	mH
Force factor	Bl	3.05	$N.A^{-1}$
Moving mass	$M_{ms}$	2.1	g
Mechanical resistance	$R_{ms}$	0.38	$ m N.m^{-1}.s$
Suspension compliance	$C_{ms}$	1.23	$mm.N^{-1}$
Effective area	$\boldsymbol{S}$	32	$\mathrm{cm}^2$
Equivalent volume	$V_{as}$	1.78	$\mathbf{L}$
<b>Resonant frequency</b>	$f_s$	100	Hz
Mechanical Q factor	$Q_{ms}$	3.45	
Electrical Q factor	$oldsymbol{Q}_{es}$	0.89	

TABLE 2: Technical data of the Monacor® SPX-30M loudspeaker

be provided since the sound absorptivity of the diaphragm can be readily varied in a controlled fashion (see cases B and C in Fig. 4(b)).

Even if the practical realization of the synthetic load is out of the scope of the paper, it can be noted that a voltage-current converter should necessary be involved in the loop so that the synthetic load may comply with an electrical admittance, as discussed in Sec. 3.4. As depicted in Fig. 2, the signal input  $\varepsilon$  is picked up across the transducer terminals and goes through the electrical load. The output current that results is used to drive the voice coil through  $F_{mag}$  and hence the diaphragm attached, appropriately. In this way, the closed-box loudspeaker is turned into a built-in electroacoustic resonator controlled by a current source.

#### CONCLUSION

This paper discussed a methodology to design a built-in electroacoustic resonator for active noise reduction purposes. It is shown that assigning a desired acoustic impedance at the transducer diaphragm is equivalent to the implementation of a functional relationship (transfer function) between the electrical current and voltage across the transducer terminals, and vice versa. The main attraction of such approach is to achieve active sound absorption without the need for any sensor. By operating without microphone, the risk of acoustic feedback is alleviated, whereas it is the weakness of most of active noise control techniques. The desired dynamic response of the loudspeaker for any sound disturbance is actually incorporated within a synthetic electrical load admittance, the role of which is to adjust the loss and offset some of the reactive parts of the transducer. The acoustic impedance of the diaphragm (or the mechanical structure attached) can thus modified to partially or totally absorb incident sound waves. Although improvements are still needed, this sensorless control technique offers a promising direction for practical applications of active noise control devices by taking advantage of the natural absorptivity of the loudspeaker diaphragm.

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