

INTRODUCTION

"Acoustic metamaterial" is a label that encompasses for acoustic structures that exhibit acoustic properties not readily available in nature. These properties can be a negative mass density, expressing the opposition of the acceleration of a particle to the application of pressure, or a negative bulk modulus, signifying the rarefaction of the particle in reaction to a compression (resp. a condensation in reaction to a depression). Recent publications [1, 2] report the achievement of simultaneous negative mass density and bulk modulus, based on the design of acoustic transmission lines periodically loaded with elastic membranes along the duct, and side holes ("stubs"). Looking at the unit-cell and deriving the equivalent lumped-element components of the transmission-line yields frequency-dependent acoustic equivalent (series mass and shunt compliance), taking successively negative and positive values as frequency increases. Thus, it can be observed a frequency region where both mass density and bulk modulus are simultaneously negative (yielding negative refraction index), and another where both are positive. Such acoustic meta-properties are also underlying in some other publications, employing 1D or 2D distribution of active loudspeakers, allowing distribution of controlled acoustic impedance [3], or with distributed active transverse Helmholtz resonators [4]. However, in all the preceding cases, the artificial behaviors result from a periodic arrangement of passive unit-cells, and not from individual "meta-properties" of each unit-cell.

The electroacoustic absorber (or more generally electroacoustic resonator) concept reported in Ref. [5] describes a feedback-based active impedance control concept, with the capability of achieving either total absorption through controlled acoustic impedance matching, but also potentially negative absorption (through the obtention of negative acoustic impedance). This specific feature reveals the capacity of a loudspeaker to reflect more sound energy than it receives from the field (negative resistance), but also to act as a negative acoustic reactance. This property reveals some similarities with the above-mentioned concept of meta-material with negative mass density or negative bulk modulus. But rather than achieving these meta-properties from a distribution of passive acoustic impedances, this active concept presents intrinsic meta-properties, allowing to present the individual active loudspeaker as a meta-material by itself.

It is also worth observing that such intrinsic metamaterial properties can be derived out of a passive electroacoustic resonator employing specific passive electric load as a shunt on a loudspeaker electric terminals, such as a simple series RLC resonator. This paper describes the metamaterial nature of such passive electroacoustic resonators through computational and experimental results.

PASSIVE ELECTROACOUSTIC RESONATOR

Description

Let's consider the closed-box electrodynamic loudspeaker of Fig. 1, in the air with mass density ρ and celerity of sound c . At the electric terminals, we denote the voltage drop e and the current intensity i , and at the diaphragm, we consider the total sound pressure p and diaphragm velocity v .

Expressed in the frequency domain (capitalized letters represent Fourier transforms of the time-domain quantities in minuscules), the equations coupling the electrical and mechanical parts of the loudspeaker are given as [6]:

$$\begin{aligned} SP(\omega) &= \left(j\omega M_{ms} + R_{ms} + \frac{1}{j\omega C_{ms}} + \frac{\rho c^2 S^2}{j\omega V_b} \right) V(\omega) - Bl I(\omega) \\ E(\omega) &= (j\omega L_e + R_e) I(\omega) + Bl V(\omega) \end{aligned} \quad (1)$$

where $\omega = 2\pi f$, f being the frequency of the harmonic disturbance in Hz, and the different

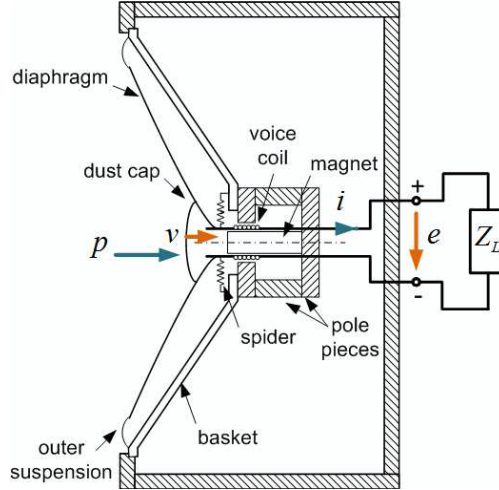


FIGURE 1: The shunt loudspeaker.

parameters being described in Table 1.

TABLE 1: List of the electroacoustic resonator parameters descriptions and values for the VISATON AL170 loudspeaker

Label	Description	Value	Unit
Bl	force factor	6.9	NA^{-1}
C_{ms}	mechanical compliance	$1.2 \cdot 10^{-3}$	m N^{-1}
L_e	electrical inductance of the coil	$0.9 \cdot 10^{-3}$	H
M_{ms}	moving mass	$13 \cdot 10^{-3}$	kg
R_e	electrical resistance	5.6	Ω
R_{ms}	mechanical resistance	0.8	N s m^{-1}
S	diaphragm area	$133 \cdot 10^{-4}$	m^2
V_b	enclosure volume	$10 \cdot 10^{-3}$	m^3

In the following, we will denote C_{mc} the total equivalent compliance of the enclosed loudspeaker, such as $\frac{1}{C_{mc}} = \frac{1}{C_{ms}} + \frac{\rho c^2 S^2}{V_b}$.

Behaving as a motor, the loudspeaker is fed by an electrical voltage E , and the electrical current I flowing in the voice-coil is responsible of a mechanical Laplace force $F = BLI$ which actuates the acoustic medium with a pressure P . In reaction, the medium exerts a pressure force SP on the diaphragm, leading the diaphragm to vibrate with the velocity V , depending on its mechanical impedance $Z_{mc}(\omega) = R_{ms} + j\omega M_{ms} + \frac{1}{j\omega C_{mc}}$. The voice-coil moving with velocity V in the air gap induces a voltage drop (electromotive force) $-BLV$ between the electric terminals, thus modulating the electrical current I circulating in the coil, depending on the electric load. From a control perspective, the problem is then to implement a functional relationship between the electrical variables at the loudspeaker's terminals, that is to say between E and I , in order to assign a certain vibrating velocity depending on the external sound pressure. To that purpose, different types of electrical networks can be used to implement the functional relationship that will shape the current after the electromotive force generated by the pressure force.

Effect of the Electric Load on the Acoustic Admittance of the Resonator

When the loudspeaker is loaded with an electric network of equivalent impedance Z_L , the second equation of Eq. 1 can be written as:

$$E(\omega) = -Z_L(\omega)I(\omega) = (j\omega L_e + R_e)I(\omega) + BlV(\omega) \quad (2)$$

Then, the normalized acoustic admittance presented by the loudspeaker diaphragm can be derived in a straightforward manner as:

$$Y(\omega) = \rho c \frac{V(\omega)}{P(\omega)} = \rho c S \frac{j\omega}{(j\omega)^2 M_{ms} + j\omega R_{ms} + \frac{1}{C_{mc}} + \frac{j\omega(Bl)^2}{j\omega L_e + R_e + Z_L(\omega)}} \quad (3)$$

It can be observed that an electric resistance R_L can provide additional damping to the acoustic resonator constituted by the loudspeaker diaphragm, which results in more or less sound absorption. And if the load Z_L has reactive components, it can also modify the total dynamic mass and compliance. In other words, the electric load is seen by the acoustic disturbance as additional mechanical impedance (combination of resistances, masses and compliances). The following will derive this result for the particular case of a series RLC resonator.

SIMULATIONS OF SHUNT ELECTRIC NETWORKS

In the following simulations (and in the following experimental measurement), we will consider different examples of shunt electric networks on a Visaton AL170 low-midrange loudspeaker (Thiele-Small parameters given in Table 1), with an enclosure of $V_b = 10\text{l}$. At 20°C in the air, $\rho = 1.18 \text{ kg}\cdot\text{m}^{-3}$ and $c = 343 \text{ m}\cdot\text{s}^{-1}$. The different series RLC shunt values are listed in Table 2.

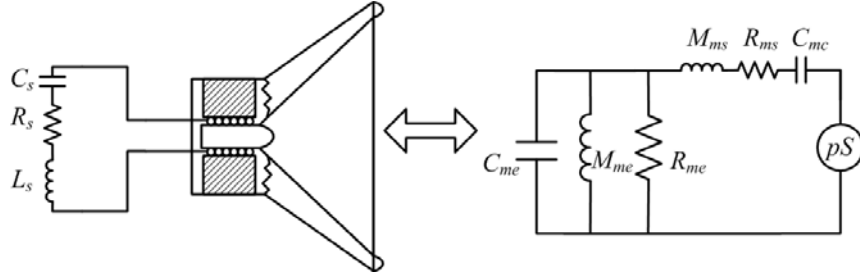


FIGURE 2: Topology of the RLC network and equivalent mechanical circuit (after [6]).

TABLE 2: Parameter settings for the simulations and experiments with series RLC electrical networks.

Topology	Case	R_s	L_s	C_s
Open circuit	A	∞	-	-
R shunt	F	4.7Ω	-	-
	I	1.5Ω	15 mH	$177 \mu\text{F}$
Series RLC	J	1.5Ω	8.3 mH	$406 \mu\text{F}$
	K	1.0Ω	5.5 mH	$550 \mu\text{F}$

Shunt Resistor

The simplest manner to turn the electroacoustic loudspeaker into a sound absorber is to plug a passive electrical resistance R_s at its terminals [5, 7], thus leading to the normalized acoustic impedance

$$Y(\omega) = \rho c S \frac{j\omega}{(j\omega)^2 M_{ms} + j\omega \left(R_{ms} + \frac{(Bl)^2}{R_e + R_s} \right) + \frac{1}{j\omega C_{mc}}} \quad (4)$$

It is then obvious that the electric resistance loading the loudspeaker terminals, affects the loudspeaker diaphragm dynamics as it is seen as an additional mechanical resistance $\frac{(Bl)^2}{R_e + R_s}$ damping even more the velocity of the diaphragm when an exogenous pressure force is exerted on its front face. If $R_{ms} < \rho c S$ and $R_e < \frac{(Bl)^2}{\rho c S}$ there even exists an optimal electric load R_{opt} , for which the total resistance of the diaphragm equals the value $\rho c S$ (characteristic impedance of the medium converted as a mechanical impedance for the diaphragm of surface S):

$$R_{opt} = \frac{(Bl)^2}{\rho c S - R_{ms}} - R_e \quad (5)$$

If the loudspeaker is loaded with such shunt resistor value, the diaphragm will be seen as an acoustic impedance surface matching the specific impedance of the air, thus leading to total absorption at the resonance of the loudspeaker defined by $f_s = \frac{1}{2\pi} \frac{1}{\sqrt{M_{ms} C_{mc}}}$.

Shunt Series RLC Network

In Ref. [8], it has been shown how active impedance control of a loudspeaker, namely a combined feedback control based on both sound pressure and diaphragm velocity sensing, could allow achieving negative acoustic mass and compliance. Such negative acoustic reactance can also be achieved by using a resonant series circuit as a shunt to the loudspeaker. The corresponding configuration is shown in Fig. 2.

The shunt electrical impedance of the series RLC network that is connected to the transducer terminals can be written as

$$Z_L(\omega) = R_s + j\omega L_s + \frac{1}{j\omega C_s} \quad (6)$$

Combining Eq. (6) with the characteristic equations (1) of the loudspeaker yields the closed-form expression for the specific acoustic admittance

$$Y(\omega) = \rho c S \frac{a_3(j\omega)^3 + a_2(j\omega)^2 + a_1(j\omega)}{b_4(j\omega)^4 + b_3(j\omega)^3 + b_2(j\omega)^2 + b_1(j\omega) + b_0} \quad (7)$$

where

$$\begin{aligned} a_3 &= L_e + L_s \\ a_2 &= R_e + R_s \\ a_1 &= C_s^{-1} \\ b_4 &= M_{ms} a_3 \\ b_3 &= M_{ms} a_2 + R_{ms} a_3 \\ b_2 &= M_{ms} a_1 + R_{ms} a_2 + C_{ms}^{-1} a_3 + (Bl)^2 \\ b_1 &= R_{ms} a_1 + C_{ms}^{-1} a_2 \\ b_0 &= C_{mc}^{-1} a_1 \end{aligned} \quad (8)$$

When combined to a series RLC network, the loudspeaker becomes a fourth-order system, as it can be observed on Fig. 4a with the values of Table 2. However, it is not obvious to simply quantify the effect of the RLC series network on the reactive component of the resonator impedance. With a view to further derive such equivalent components, we can consider the equivalent mechanical circuit corresponding to the loudspeaker connected with a series RLC resonator (see Fig. 2), corresponding to an equivalent mechanical impedance that can be derived as [6]:

$$Z_{me}(\omega) = \frac{(Bl)^2 j\omega}{(L_e + L_s)(j\omega)^2 + (R_e + R_s)j\omega + \frac{1}{C_s}} \quad (9)$$

The natural frequency of this resonant circuit of Eq. (9) is then $f'_s = \frac{1}{2\pi} \frac{1}{\sqrt{(L_e + L_s)C_s}}$.

From Eq. 9 and considering the equivalent mechanical circuit of Fig. 2, we can derive the asymptotic behaviors of the active mechanical impedance achieved with passive shunt series RLC resonator as:

$$\begin{aligned} Z_{me}(\omega) &\simeq -\frac{\omega^2(Bl)^2 C_s}{j\omega} & \text{and } \omega &\ll 2\pi f'_s \\ Z_{me}(\omega) &\simeq \frac{(Bl)^2}{R_e + R_s} & \text{and } \omega &\approx 2\pi f'_s \\ Z_{me}(\omega) &\simeq j\omega \left(-\frac{(Bl)^2}{\omega^2(L_e + L_s)} \right) & \text{and } \omega &\gg 2\pi f'_s \end{aligned} \quad (10)$$

It is now almost straightforward to identify from Eq. 10 the corresponding equivalent components obtained from this additional mechanical resonator, that is, the equivalent compliance C_{me} , resistance R_{me} and mass M_{me} as functions of the radial frequency ω :

$$\begin{aligned} C_{me}(\omega) &\simeq -\frac{1}{\omega^2(Bl)^2 C_s} \\ R_{me} &\simeq \frac{(Bl)^2}{R_e + R_s} \\ M_{me}(\omega) &\simeq -\frac{(Bl)^2}{\omega^2(L_e + L_s)} \end{aligned} \quad (11)$$

Depending on frequency, the equivalent components of the electroacoustic resonator takes successively the form of a constant positive resistance R_{me} , a negative mechanical compliance C_{me} and a negative mass M_{me} , the magnitudes of which decrease with frequency. This results illustrates a simple manner to achieve a mass and stiffness reduction of the electroacoustic absorber without requiring complex active impedance control system.

EXPERIMENTAL RESULTS

In order to assess experimentally the acoustic performance when using electrical matching networks, a closed-box Visaton AL-170 low-midrange loudspeaker is employed as an electroacoustic resonator. The specific acoustic admittance ratio and absorption coefficient are assessed after ISO 10534-2 standard [9], as depicted in Fig. 3.

In this setup, an impedance tube is specifically designed (length $L = 3.4$ m and internal diameter $\phi = 150$ mm), one termination of which is closed by an electroacoustic resonator, the other end being open with a horn-shape termination so as to exhibit anechoic conditions. A source loudspeaker is wall-mounted close to this termination. Two holes located at positions $x_1 = 0.46$ m and $x_2 = 0.35$ m from the electroacoustic resonator are the receptacles of 1/2" microphones (Norsonic Type 1225 cartridges mounted on Norsonic Type 1201 amplifier), sensing sound pressure $p_1 = p(x_1)$ and $p_2 = p(x_2)$. The transfer function $H_{12} = p_2/p_1$ is processed through a Pulse Bruel and Kjaer multichannel analyzer. In this paper, the practical realization of the series RLC network is not detailed.

We can observe that these experimental results (Fig. 4b) confirm the model (Fig. 4a). The connection to a dedicated series RLC network is capable of turning the electroacoustic absorber mass and compliance into negative values, at least on a limited frequency bandwidth around the resonance frequency f'_s . The main advantage here is that this negative acoustic properties are obtained without the need of external power source, the shunt series RLC network being a passive device.

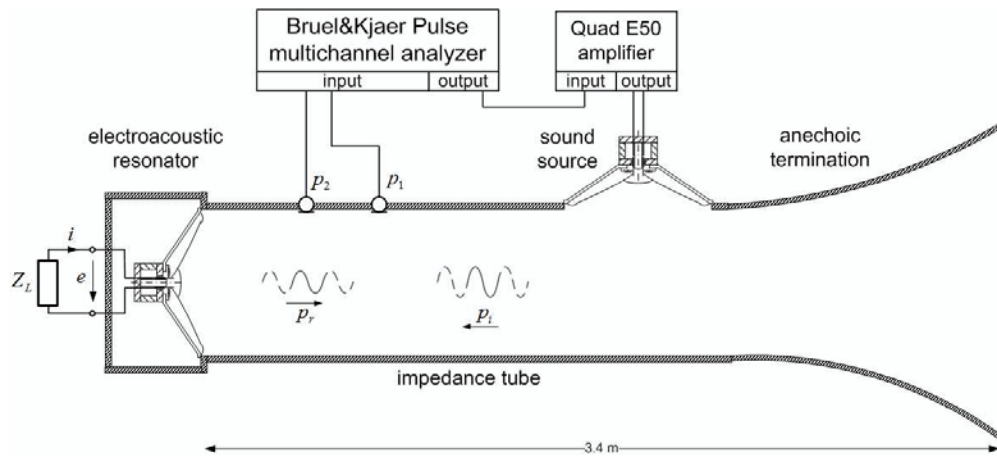


FIGURE 3: Picture of the experimental setup.

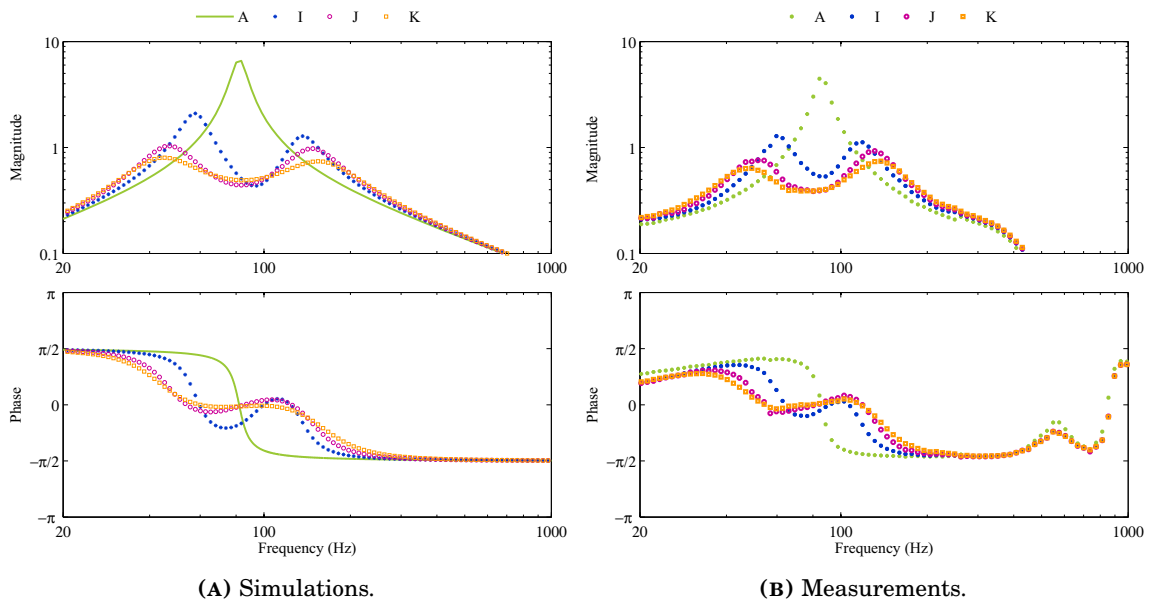


FIGURE 4: Normalized acoustic admittance achieved with the shunt series RLC resonators configurations (I,J,K), through simulations and measurements (up: magnitude in dB; bottom: phase in rad).

CONCLUSION

This study confirms that series RLC network used as shunt for electroacoustic absorbers are good candidates for achieving negative acoustic properties of the resonator without the need of active devices or periodic arrangements of passive acoustic resonators (membranes, side holes, Helmholtz resonators, etc.). Further studies are envisaged to see how such passive negative impedance components could advantageously be deployed within the acoustic transmission lines of Ref. [1], with a view to achieving extended negative acoustic properties.

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