

Evaluation of Eurocode damage equivalent factor based on traffic simulation

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ABSTRACT: The Eurocodes provide a simplified fatigue design control through the use of Fatigue Load Model 3 (FLM3) and a damage equivalent factor, λ , which is now widely used. In applying the Eurocode method, the first problem arises when designers try to determine the corresponding fatigue span length, especially for bridge static systems not listed. A second problem is that in case of long bridges, the probability of having several heavy vehicles simultaneously over the bridge exists (in single lane traffic as well as in multi-lane traffic), which results in larger cycles and, accordingly, higher contributions to total damage. This paper first evaluates the damage equivalent factors and then provides an all-inclusive method for determining the fatigue critical length for any bridge static system and the corresponding damage equivalent factor. It proposes modifications for improving the fatigue design method using damage equivalent factors.

1 INTRODUCTION

Fatigue is an important consideration in the design of bridges especially those made of steel. Multiple passages of heavy vehicles can eventually lead to cracks and failure. The stress ranges (in number and amplitude) from the passage of a heavy vehicle over a bridge depend on many parameters; some are known (Nowak 1993, Laman & Nowak 1996, Mori et al. 2007) while others are not studied or poorly studied. Some of the known parameters are: bridge static system, detail location, span length, the vehicle geometry and weight. However, the effect of having several heavy vehicles on bridge at the same time has not studied in fatigue design of bridges. In addition, there is the probability of crossing or overtaking in the case of bridges with heavy vehicles on several lanes with bidirectional or unidirectional traffic which complicates the problem.

The concept of the damage equivalent factor for the fatigue load model is effective for expressing the traffic actions with equivalent stress range at two million cycles and to compare with the resistance of a detail. Development of this concept expands validity range of the damage equivalent factor, and improves its accuracy; study on the damage equivalent factor also improves our knowledge of multi-lane traffic effect, as demonstrated in this article.

2 CONCEPT OF DAMAGE EQUIVALENT FACTORS BASED ON EUROCODES

The concept of the damage equivalent factor was proposed to eliminate the tedious calculation procedure of damage accumulation. Thanks to this method which is presented in Figure 1, the computation of the usual cases is performed once during development of the code. The left side of Figure 1 illustrates different elements involved with fatigue verification using damage accumulation including:

- A simplified model of real traffic
- The corresponding stress history including dynamic effect
- The extracted stress cycles and calculation of equivalent stress range.

The right side of the diagram shows the application of the fatigue load model to obtain maximum stress, $\sigma_{FLM,max}$, and minimum stress $\sigma_{FLM,min}$, by placing the fatigue load model on the most severe positions. To obtain the same value as the equivalent stress range, $\Delta\sigma_{E2}$, which takes into account the damage accumulation, the engineers will then correct the value of $\Delta\sigma_{FLM}$ with damage equivalent factor λ . The fatigue assessment can then be carried out as follows (EN 1993-2, 2006):

$$\gamma_{Ff}(\Phi_2 \lambda \Delta\sigma_{FLM}) \leq \frac{\Delta\sigma_c}{\gamma_{Mf}} \quad (1)$$

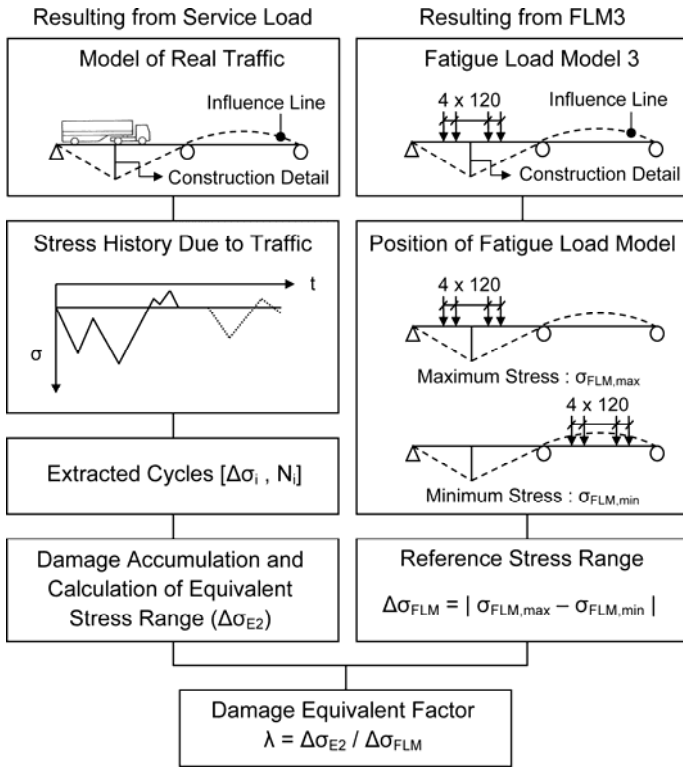


Figure 1. Procedure of determination of damage equivalent factor.

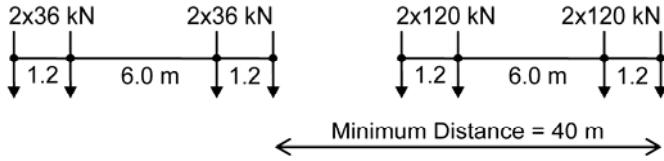


Figure 2. Geometry of Fatigue Load Model 3 (FLM3).

where γ_{FF} is the partial safety factor for fatigue loading, Φ_2 is the impact factor (which is included in the fatigue load model 3 for good pavement quality), $\Delta\sigma_{FLM}$ is stress range due to FLM3), $\Delta\sigma_c$ is the reference value of fatigue strength (at 2 million cycles) and γ_{MF} is the partial safety factor for fatigue strength.

The damage equivalent factor, λ , can be obtained from:

$$\lambda = \lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4 \leq \lambda_{max} \quad (2)$$

where λ_1 is the factor for damage effect of traffic depending on critical length, λ_2 is for modification of traffic volume, λ_3 is for modification of the bridge design life, λ_4 is the factor which adds up the effect of traffic on the other lanes to the first lane, and λ_{max} is the maximum value which takes into account the fatigue limit.

In EN 1993-2 (2006), the factor λ_1 is determined for various bridge types with span lengths range from 10 m to 80 m, by applying the FLM3. It is separately represented for midspan and support section as a function of the critical length.

The geometry of FLM3 based on EN 1991-2 (2002), which is in accordance with damage equivalent factors, is shown in Figure 2. The weight of each axle is 120 kN. Where relevant, a second set of axles in the same lane should be taken into account. The geometry of the second set is similar, but the weight of each axle is equal to 36 kN (instead of 120 kN). The minimum distance between two vehicles measured from center-to-center is at least 40 m. This model tries to take into account the effect of multiple heavy vehicles on the bridge.

It is important to mention the critical span length is defined in EN 1993-2 (2006) on a case-by-case basis; consequently, the critical span length is unknown for undefined cases. Some of the defined critical lengths are:

- for a single-span bridge, equal to span length,
 - in support sections of a continuous-span bridge, the mean of the two spans adjacent to that support,
 - for reaction of intermediate supports of a continuous-span bridge, the sum of two adjacent spans.
- In addition, the factor λ_2 adapts the traffic volume passing over a bridge, and it can be calculated as:

$$\lambda_2 = \frac{Q_{m1}}{Q_0} \left(\frac{N_{obs}}{N_0} \right)^{1/5} \quad (3)$$

where Q_{m1} is the average gross weight of the heavy vehicles in the slow lane, N_{obs} is the annual number of heavy vehicles in the slow lane, Q_0 and N_0 are the base values in determination of λ_1 .

The multi-lane traffic effect can be calculated using the factor λ_4 . This factor takes into account the effect of having more than one lane by using following formula:

$$\lambda_4 = \left[1 + \frac{N_2}{N_1} \left(\frac{\eta_2 Q_{m2}}{\eta_1 Q_{m1}} \right)^5 + \frac{N_3}{N_1} \left(\frac{\eta_3 Q_{m3}}{\eta_1 Q_{m1}} \right)^5 + \dots \right]^{1/5} \quad (4)$$

where N_i is the annual number of heavy vehicle in the corresponding lane, Q_{mi} is the average gross weight of heavy vehicles in the corresponding lane and η_i is the transverse distribution factor of the corresponding lane (always positive).

When for a particular case, all cycles due to the passage of the heavy vehicle traffic are lower than the constant amplitude fatigue limit (CAFL), fatigue life is unlimited. Accordingly, the factor λ_{max} is given to control the fatigue limit.

The aforementioned method for calculation of damage equivalent factors has shortcomings. For example, continuous flow of traffic on a bridge is not considered in calculation of the damage equivalent factors, and more than one heavy vehicle in each lane is neglected. In addition for multi-lane traffic, several heavy vehicles might be on bridge simultaneously, which indicates the occurrence of one big cycle instead of two smaller cycles.

3 METHODOLOGY

3.1 Hypotheses and traffic conditions

For modeling traffic over a bridge accurately and extracting the cycles from stress history, a program (WinQSIM) based on Microsoft C# has been developed by Meystre (2006). Statistical parameters of actual traffic are based on Weigh-In-Motion (WIM) measurements from stations of Gotthard (as an international station) and Mattstetten (as a national station) of Switzerland in 2009. The program randomly chooses heavy vehicle properties from a database which is in accordance with real traffic. In addition, another program (FDABridge) based on Microsoft C# is developed to calculate damage sum using the Miner linear damage sum. It is also able to determine damage equivalent factor using defined fatigue load model and S-N curve parameters.

The simulation program is able to model fluid traffic as well as congested traffic close to reality; however, in the current study with the aim of fatigue analysis, the traffic is assumed to be always fluid and the vehicles circulate with constant speed. Such a simulation allows having several heavy vehicles over bridge simultaneously, which have an important effect as pointed out in the initial studies by Maddah & Nussbaumer (2011).

In the current study, a real estimation of hourly variation of traffic is applied based on the measurements of Switzerland (Maddah et al., in prep.). Besides, it is decided to simulate traffic on weekdays only, since heavy vehicle traffic is negligible on weekends and holidays. The hourly variation of heavy vehicle traffic (as a percent of daily heavy vehicle traffic) on weekdays, for different WIM stations of Switzerland as well as the average of stations are shown in Figure 3a. Based on the average, the hourly variation of heavy vehicle traffic is proposed. Similarly, hourly variation of all vehicle traffic (Fig. 3b) and hourly variation of heavy vehicle proportion in traffic (Fig. 3c) are proposed. The maximum and average of annual proportion of heavy vehicle traffic on weekdays are respectively 25 percent and 10 percent in Switzerland. For comparison purpose, the simulations are done for both 10 and 25 percent.

In the current study, heavy vehicle is defined as any vehicle with total gross weight over 100 kN. The vehicles with weight lower than 100 kN assumed as cars whose weight are neglected. Moreover, based on the former studies results (Maddah & Nussbaumer 2010), the traffic simulations are performed for one year, and then number of resulted of cycles are multiplied by 100, as design life of bridges.

In the case of single lane traffic, two traffic conditions are simulated: highways and main roads. The annual number of simulated heavy vehicles is 2'000'000 for the highways, and it is 500'000 for

the main roads. The summary of different traffic conditions are shown in Table 1.

For double lane traffic, the same bridge static systems are chosen. The traffic simulation parameters as well as calculation of damage equivalent factors are similar to the single lane traffic. Five double lane traffic conditions are studied, as summarized in the Table 1 (two cases for the bidirectional traffic and three cases for the unidirectional traffic).

The bidirectional traffic conditions include main road with 500'000 annual heavy vehicles traffic and highway with 2'000'000 annual heavy vehicles in each direction.

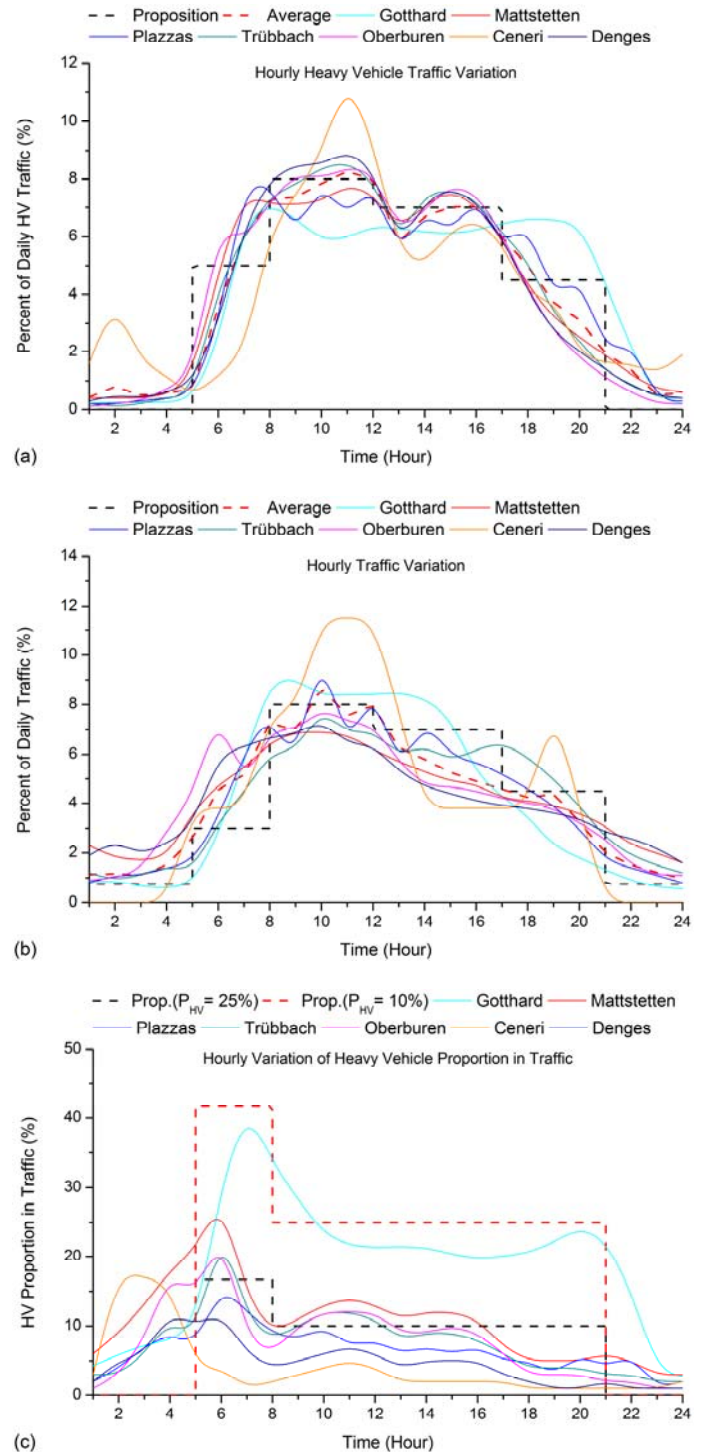


Figure 3. Hourly variation of traffic on weekday: (a) heavy vehicles, (b) all vehicles, (c) proportion of heavy vehicle.

Table 1. Parameters of different simulated traffic conditions

Single lane traffic condition					
Abbr.	Traffic	Type	N_{obs} ($\times 10^3$)	Q_m (kN)	P_{HV} (%)
G25HW	Gotthard	highway	2000	313	25
M25HW	Mattstetten	highway	2000	282	25
G25MR	Gotthard	main road	500	313	25
G10HW	Gotthard	highway	2000	313	10
Double lane traffic conditions (Gotthard, $P_{HV} = 25\%$)					
Abbr.	Direction	Type	N_2 ($\times 10^3$)	Section	
UD10020	unidirectional	highway	400	box	
UD10010	unidirectional	highway	200	box	
UD10020IG	unidirectional	highway	400	girder	
BD100100MR	bidirectional	main road	500	box	
BD100100	bidirectional	highway	2000	box	

In the case of unidirectional traffic, slow lane always has highway traffic with 2'000'000 annual heavy vehicles, and the fast lane traffic in one case is 10 percent and in another case is 20 percent of the slow lane traffic.

The main objective of the double lane simulation is to study the effect of heavy vehicles which are crossing (or overtaking) on the bridge. The determination of actual frequencies of these situations is not part of this study. The transverse distribution of load has the key role on influence rate of other lanes. In the case of box section, the most unfavorable case, there is no transverse distribution, and it is acceptable that the box section can be uniformly charged regardless of axle position. In order to study the cross section effect, simulations performed for a double I-Girder Bridge where the transverse distribution factor for slow lane traffic is 1 and for the fast lane is 0.4. For the latter case, the traffic condition is unidirectional with 2'000'000 heavy vehicles on the slow lane and 400'000 heavy vehicles on the fast lane.

Damage equivalent factors for the following static systems and detail locations are analyzed:

- single-span bridges, midspan moment (1SS-MM),
- two-span continuous bridges with equal spans length, negative moment of second support (2CS-2SM) and reaction (2CS-2SR), as well as midspan moment (2CS-MSM),
- three-span continuous bridges with equal spans length, midspan moment of second span (3CS-MM), second support moment (3CS-2SM), as well as first and second supports reaction (3CS-1SR and 3CS-2SR).

For all static systems, the bridges span length ranges from 1 m to 200 m. The fatigue resistance curve of steel is considered, as defined in EN 1993-1-9 (2005), with slope of 3 for cycles with stress ranges higher than constant amplitude fatigue limit (CAFL), and slope of 5 for cycles lower than CAFL, also cycles lower than cut-off limit are dismissed.

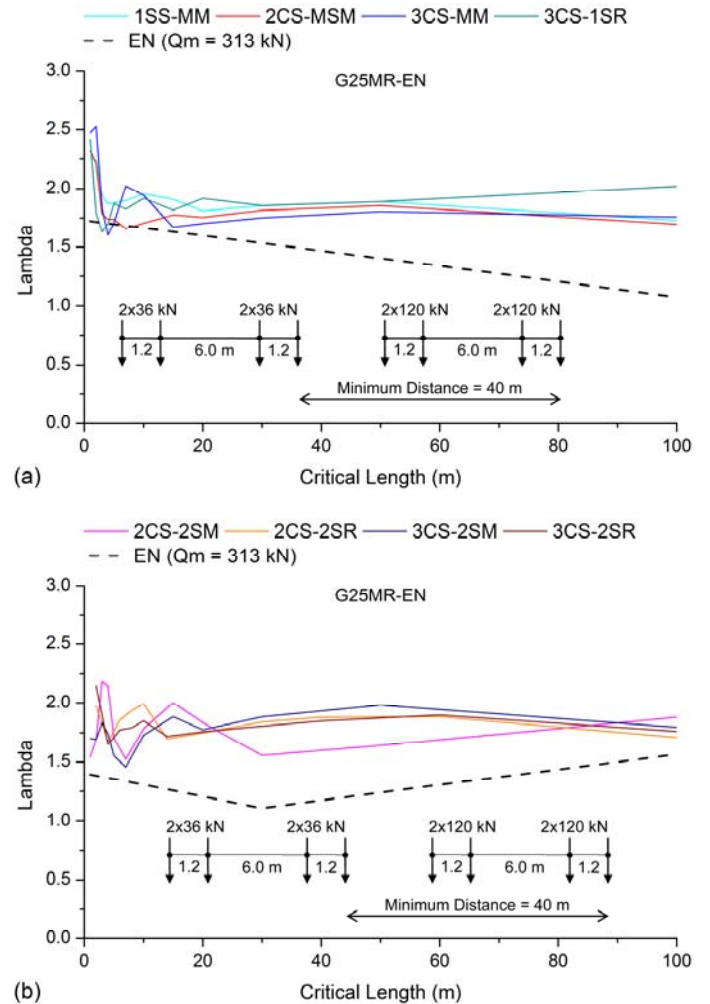


Figure 4. Factor λ for different bridge cases, Gotthard main road traffic ($P_{HV} = 25\%$): (a) at midspan (b) at support.

Furthermore, the dynamic amplification factor is considered based on the total weight of traffic on bridge at a given time (Ludescher & Brühwiler, 2004). When the total weight on the bridge is lower than 300 kN, the dynamic factor is 1.4. When the total weight of traffic on the bridge is more than 1500 kN the dynamic factor is 1.0. For the total weights in-between, the dynamic factor changes linearly between 1.0 and 1.4.

3.2 Damage Equivalent Factor for Single Lane Traffic

The first simulations are performed for the main road traffic with 25 percent annual proportion of heavy vehicle traffic (G25MR) to compare damage equivalent factors resulting from simulations with Eurocode. Figure 4 shows the results of the first simulations for different bridge static systems; the fatigue load model (FLM3) is also plotted for clarity. The corresponding damage equivalent factors of EN 1993-2 (2006) are also shown in Figure 4. Since the average gross weight of heavy vehicles on station Gotthard (2009) is 313 kN, partial equivalent factor, $\lambda_2 = 313 / 480$, is multiplied to calculate the damage

equivalent factor, λ , of the code. Figure 4 shows that the damage equivalent factor obtained for both mid-span and support sections are above the curve of the code, expressing the code is non-conservative. The safety margin of the Eurocode curves (midspan and support) depend on the span length and bridge static system, which is not desirable.

With the same simulations, the factor λ_{max} is determined for various bridge static systems under the same traffic condition (G25MR), as represented in Figure 5. The λ_{max} obtained for different cases are also greater than the curve of Eurocode, indicating the factor λ_{max} , same as λ_I , given by Eurocode is non-conservative.

3.3 Damage Equivalent Factor for Double Lane Traffic

The same bridge static systems are studied for double lane traffic. Traffic simulation parameters as well as calculation of damage equivalent factors are similar to single lane traffic. The properties of double lane traffic conditions are listed in Table 1.

Figure 6 illustrates the factor λ_4 obtained for different bridge cases with the bidirectional highway

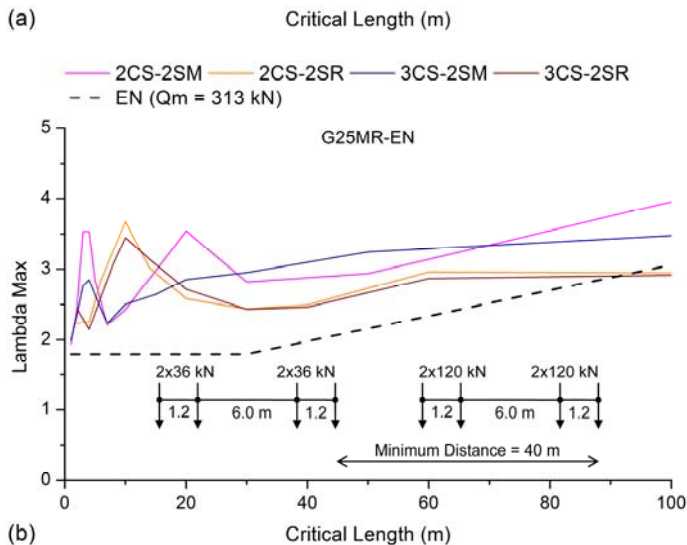
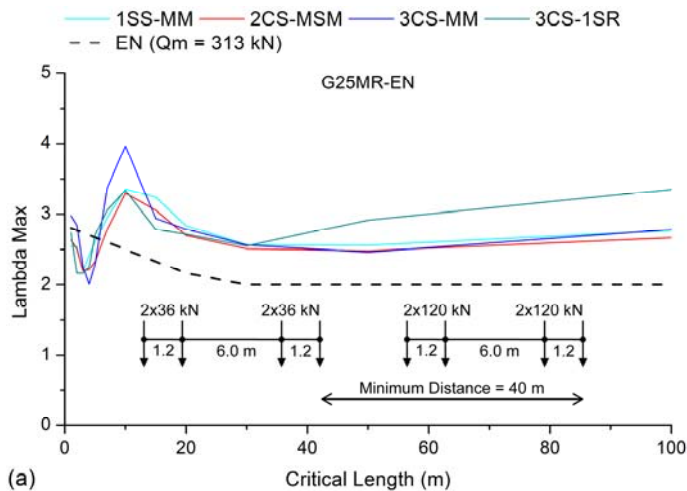


Figure 5. Factor λ_{max} for different bridge cases, Gotthard main road traffic ($P_{HV} = 25\%$): (a) at midspan (b) at support.

traffic condition (BD100100). For comparison, the factor λ_4 of Eurocode calculated with Equation 3 and it is also shown in Figure 6. For the most simulated bridge cases, the λ_4 obtained from simulations is higher than the value given by the code, indicating that the code is non-conservative. This variation between the simulation and the code is mostly due to the effect of vehicles crossing. Figure 6 justifies that this effect should be taken into account.

In addition, the factor λ_4 in the case of mid support reaction of two-span of continuous bridge, obtained for different double lane traffic conditions, is illustrated in Figure 7. For comparison, the factor λ_4 of Eurocode corresponding to each traffic condition is also calculated with Equation 3 and plotted. Figure 7 shows that the accuracy of λ_4 given in the Eurocode is limited to very short-span bridge with critical length lower than 10 m. For bridges with longer critical length, the probability of crossing or overtaking on the bridge increases, and the code underestimates the damage.

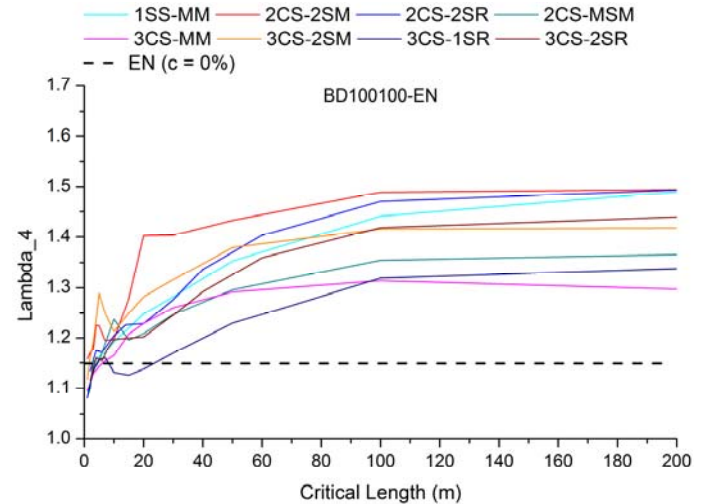


Figure 6. Factor λ_4 for different bridge cases, bidirectional highway traffic condition (BD100100).

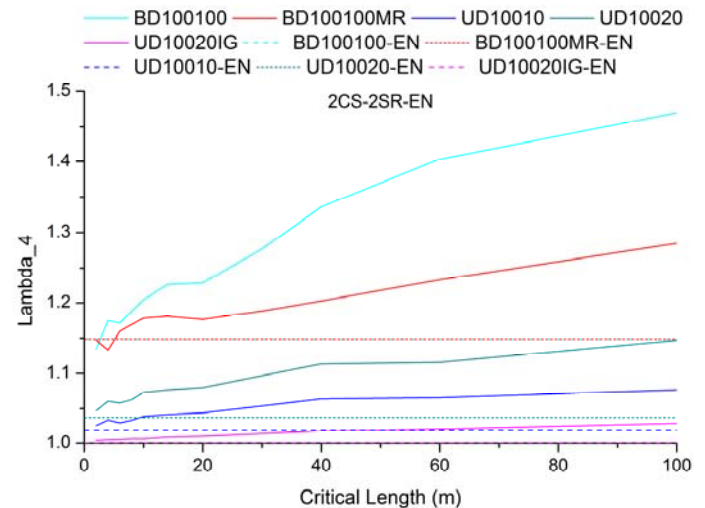


Figure 7. Factor λ_4 for different double lanes traffic conditions, mid support reaction of two-span continuous bridges.

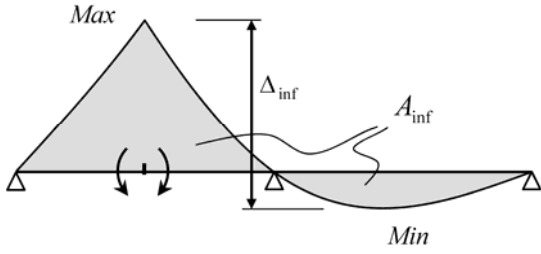


Figure 8. Schematic influence line to determine parameters of fatigue equivalent length.

In addition, the crossing or overtaking ratio depends on annual number of heavy vehicles on each lane. For instance, in two bidirectional traffic conditions, comparing the case of main road (BD100100MR) with highway (BD100100) in Figure 7, the factor λ_d given in the Eurocode is similar for both cases. The λ_d resulting from simulations for the case of highways is clearly higher than the case for main roads and this difference constantly grows by increasing critical length. This can be explained because the probability of crossing is higher in the case of highways in which the annual number of heavy vehicles in each lane is four times more. Also for longer bridges, it is more probable to have several heavy vehicles in the other lanes of the bridge.

The effect of crossing or overtaking on a bridge is more pronounced in the case of a box cross section, where the transverse distribution factor for the second lane is equal to one. Figure 7 shows that this effect can be neglected in the case of bridges with I-Girder cross section (UD10020IG) in which the fast lane transvers distribution factor is 0.4.

4 PROPOSED DAMAGE EQUIVALENT FACTOR

4.1 Single Lane Traffic

Several attempts have been made to define appropriate damage equivalent factor as well as the fatigue load model and influence line length. It has been seen that using the fatigue load model with one axle would lead to less scattering in the damage equivalent factors for various bridge static systems. Therefore, it is proposed to use a single load model with the weight of 480 kN being the same as the total weight of fatigue load model in EN 1991-2 (2002). In addition, the fatigue critical length is denominated by the fatigue equivalent length, L_λ , which can be determined as follows:

$$L_\lambda = \frac{A_{inf}}{\Delta_{inf}} \quad (5)$$

where A_{inf} is absolute sum of area under influence line, as shown in Figure 8, and Δ_{inf} is difference between maximum and minimum values of influence line.

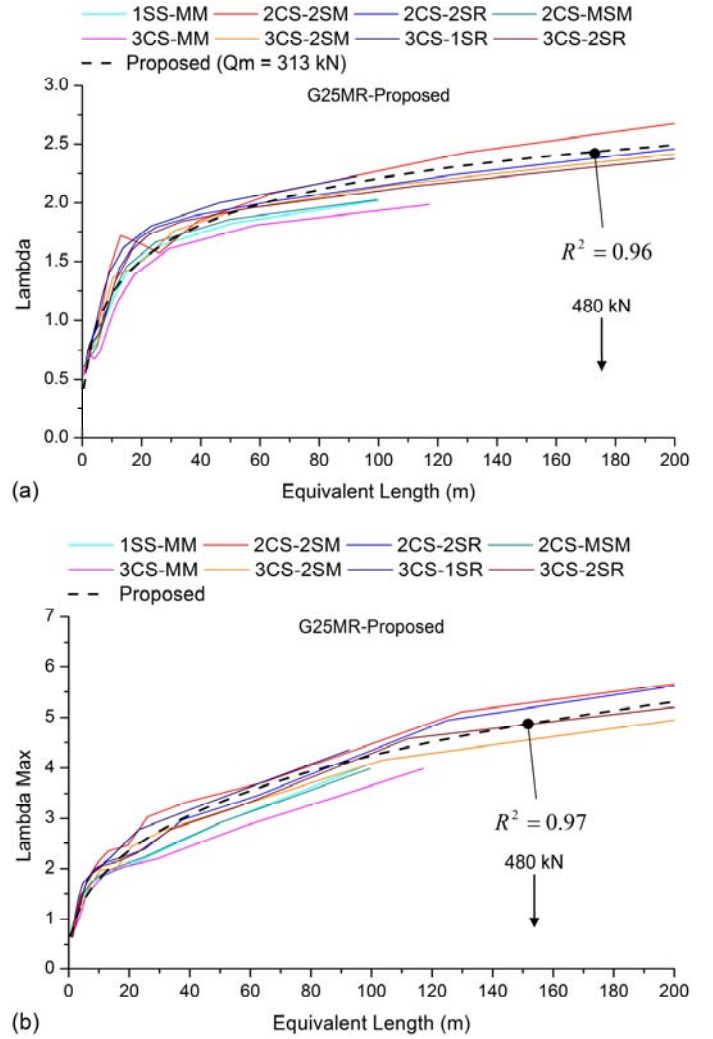


Figure 9. Damage equivalent factors for different bridge static systems, Gotthard main road traffic ($P_{HV} = 25\%$): (a) factor λ (b) factor λ_{max} .

The same simulations are performed, and the damage equivalent factor, λ , as well as λ_{max} , are determined by the new hypothesis for different bridge cases, as illustrated in Figure 9; the single axle fatigue load model is also plotted for clarity. The proposed curve is based on the 500'000 passages of heavy vehicle per slow lane with average gross weight of 313 kN and design life of 100 years.

The curves obtained for different static systems show a clear trend with a narrow dispersion band, well represented by the proposed curve. The range of equivalent length axis in Figure 9 is extended up to 200 m, since in some bridge cases the equivalent length is longer.

Dividing damage equivalent factor, λ , by the factor λ_2 (Eq. 3), the factor λ_1 for various traffic conditions of the single lane traffic are determined. As in EN 1993-2 (2006), the base average gross weight, Q_0 , is taken as 480 (kN) and the base value of annual heavy vehicle traffic, N_0 , is taken as 500'000 for calculation of λ_2 . Figure 10 demonstrates the λ_1 obtained for the different traffic conditions for second support reaction of two-span bridges. For comparison, the curve of λ (before dividing it by λ_2) is also

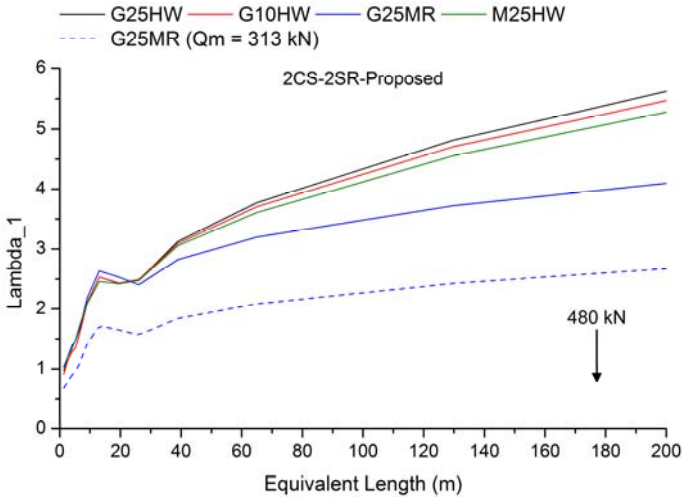


Figure 10. Factor λ_1 for different single lane traffic conditions, mid support reaction of two-span continuous bridges.

plotted on the same figure for the case of main road traffic (G25MR). In Figure 10, there is a slight difference between different traffic conditions, except in the case of main road traffic condition (G25MR). Such a difference shows the average gross weights can properly be adapted by λ_2 ; on the contrary, the annual number of heavy vehicle cannot be adapted by λ_2 . This is explained by the fact that the probability of having several heavy vehicle together on a bridge depends on the annual number of heavy vehicle traffic. This variation is more pronounced for bridges with longer equivalent length for the same reason.

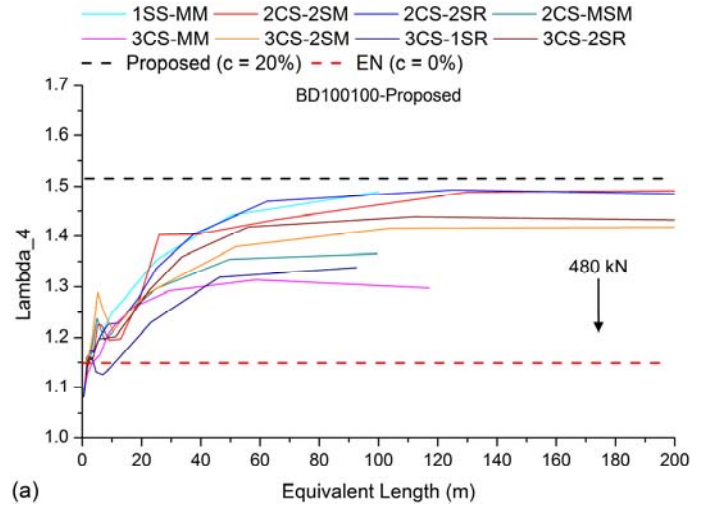
4.2 Double Lane Traffic

The factor λ_4 adds up the damage from other lanes with the first lane, thus it is always an increasing factor. In EN 1993-2 (2006), this factor is calculated in a simplified way accepting that the damage due to the other lanes can be accumulated with a constant slope fatigue resistance curve equal to 5. The current definition of λ_4 does not embrace the effect of having several heavy vehicles simultaneously (side-by-side) on a bridge. It is possible to redefine λ_4 to improve this shortcoming. Using the Miner summation rule and assuming the stress response spectrum due to passage of heavy vehicles simultaneously on both lanes is proportional to the stress response spectrum due to passage of traffic on slow lane, we can modify Equation 4 as:

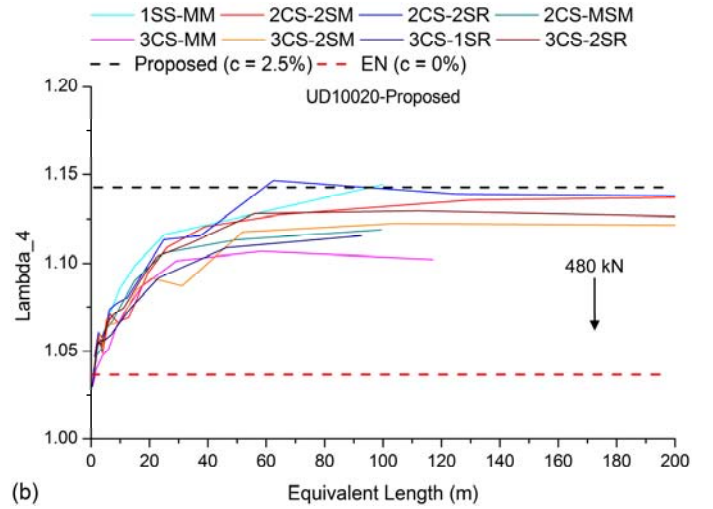
$$\lambda_4 = \left[(1-c) + \left(\frac{N_2}{N_1} - c \right) \cdot \left(\frac{\eta_2 Q_{m2}}{\eta_1 Q_{m1}} \right)^5 + c \cdot \left(1 + \frac{\eta_2 Q_{m2}}{\eta_1 Q_{m1}} \right)^5 \right]^{1/5} \quad (6)$$

where c is crossing (or overtaking) ratio.

The factor λ_4 for each bridge static system based on the hypothesis proposed for λ_1 , as described in the Section 4.1, is calculated and illustrated in Figure 11 for two traffic conditions of unidirectional highway traffic (UD10020) and bidirectional highway traffic



(a)



(b)

Figure 11. Factor λ_4 for different bridge static systems: (a) bidirectional traffic, BD100100, (b) unidirectional traffic, UD10020.

(BD100100). The corresponding λ_4 from EN 1993-2 (2006) as well as λ_4 obtained from Equation 6 are also plotted. The effect of crossing and overtaking in the case of unidirectional and bidirectional traffic, as shown in Figure 11, depends on the traffic condition and bridge static system, and it is more pronounced than the effect of traffic volume, which is considered in the Eurocode.

By trial and error, it is found that the simulation results can be conservatively represented assuming: $c = 20$ percent for bidirectional highway traffic condition (BD100100) and $c = 2.5$ percent for unidirectional highway traffic condition (UD10020). The proposed crossing and overtaking ratios are preliminary results and the study is still in progress. The principal parameters that have an influence are: the traffic volume on each lane, direction of traffic, and fatigue equivalent length. Future study will clarify the crossing and overtaking ratio based on the aforementioned parameters.

5 CONCLUSIONS

The goal of this paper is to propose modifications to improve the damage equivalent factors (λ) for fatigue design of road bridges according to the concept of Eurocodes. The factor λ can be decomposed into several partial factors, i.e. $\lambda = \lambda_1 \cdot \lambda_2 \cdot \lambda_3 \cdot \lambda_4$. This study especially focuses on λ_1 , λ_2 , λ_4 and λ_{max} .

Our study on damage equivalent factors was carried out with simulation of real continuous traffic, in a manner as realistic as possible, on different bridge static systems. The results are thus more accurate than the former simulations which have been done within writing of the Eurocode. Based on these simulations, a fatigue load model with a single axle is proposed, because it leads to a significant decrease in the dispersion of the values of λ_1 as well as λ_{max} obtained for different static systems. In addition, a new method for determining the fatigue equivalent length is proposed (see Eq. 5). This expression provides a simpler method for uniquely determining the fatigue length for all influence lines.

In addition, λ_2 is evaluated by performing simulations for different single lane traffic conditions. The results show that λ_2 adapts the average gross weight of heavy vehicles in different traffic conditions; however, the annual number of heavy vehicle traffic cannot properly be modified by λ_2 . It can be explained because λ_2 does not take into account the probability of having simultaneity on a bridge depends on the number of heavy vehicle.

For λ_4 , similar simulations have been done to study the effect of double lane traffic for different bidirectional and unidirectional traffics. The obtained results for box cross section, which has uniformed transverse distribution, show that the effect of overtaking or crossing cannot be neglected in λ_4 ; however, λ_4 in the Eurocode only considers the damage accumulation due to volume of traffic in the additional lanes. Consequently, a new λ_4 is proposed, Equation 6, which adds up damage due to crossing (or overtaking) as well as damage due to traffic volume on fast lane. In addition, based on the traffic simulations for highways, the maximum crossing ratio in bidirectional highway traffic condition is proposed 20 percent, and the maximum overtaking ratio in unidirectional traffic, assuming the annual heavy vehicle traffic of the fast lane is 20 percent of the slow lane, is proposed 2.5 percent.

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