Energy partition at the collapse of spherical cavitation bubbles - Supplemental Material -

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Validation of the estimation of the shock energy from the pressure sensor signal

In the paper, we developed a relationship between the shock energy E_{SW} and the pressure sensor signal $s(t)$:

$$E_{SW} \propto \int p_{max}^2 \, \tilde{p}(t)^2 \mathrm{d}t \propto p_{max}^2 \propto \left(\int s(t)\mathrm{d}t \right)^2 .$$

To obtain this relationship, we define the response of the sensor as $s(t) = h(t) * p(t)$, where $h(t)$ is the sensor's impulse response, $p(t)$ is the pressure and '$*$' denotes the convolution. We assume that the pressure $p(t)$ has a universal shape in the sense that $p(t) = p_{max}\, \tilde{p}(t)$, where $\tilde{p}(t)$ is the same function for all bubbles. The signal can then be expressed as $s(t) = h(t) * p_{max}\, \tilde{p}(t)$, and hence $\int s(t)\mathrm{d}t = p_{max} \int h(t) * \tilde{p}(t)\mathrm{d}t \propto p_{max}$. The validity of this reasoning is tested in the following paragraphs.

The assumption $p(t) = p_{max}\, \tilde{p}(t)$ implies that $s(t)/max(s(t))$ is the same function of time for all measurements because $h(t)$ and $\tilde{p}(t)$ are time-dependent only. Figure 1 shows the first 24 μs of the normalized signal $s(t)/max(s(t))$ for all measurement (dotted grey curves) and the mean normalized signal (solid black curve). The mean standard deviation is 0.09. The solid magenta curves on the figure represent the mean normalized signal $\pm$ the standard deviation. The standard deviation of the FWHM of the first peak on the normalized signal (i.e. the shock) is 0.63 μs (for a time resolution of 0.4 μs). Those results show that all the normalized signals are reasonably similar. Therefore, we conclude that the use of the assumption $p(t) = p_{max}\, \tilde{p}(t)$ is suitable for the estimation of E_{SW}. In turn, this implies that $\left(\int s(t)\mathrm{d}t\right)^2 \propto p_{max}^2$.

Alternatively, the validity of $p_{max}^2 \propto \left(\int s(t)\mathrm{d}t\right)^2$ is tested as follows. We have $\left(\int s(t)\mathrm{d}t\right)^2 = C_1 p_{max}^2$ where C_1 is a constant. We also have $\int s^2(t)\mathrm{d}t = C_2 p_{max}^2$ where C_2 is a constant. Dividing the former equation with the latter, we have

$$\left(\int s(t)\mathrm{d}t \right)^2 / \int s^2(t)\mathrm{d}t = C_1/C_2 = \text{const.}$$

Figure 2 shows this ratio of the integrals of the signal as a function of the potential energy E_0 for all bubbles. The ratio of the integrals is almost a constant: the average is 0.36 and the standard deviation is 0.02. Despite a slight dependance on E_0, there is no significant influence of the pressure p_∞. Therefore, we conclude that our method is suitable for the estimation of E_{SW}.

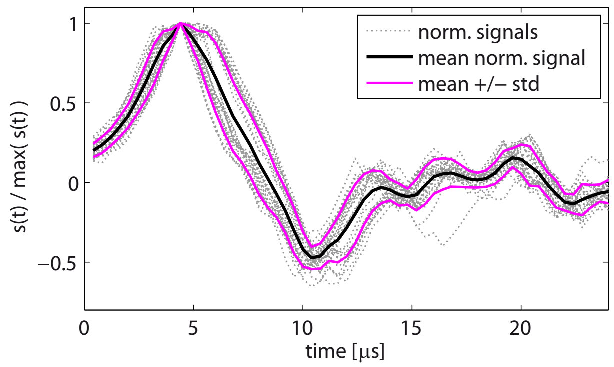

FIG. 1. (Color online) Superposition of all the normalized signals $s(t)/max(s(t))$ (dotted grey curves), the mean normalized signal (solid black curve) and the mean normalized signal $\pm$ the standard deviation (solid magenta curve).

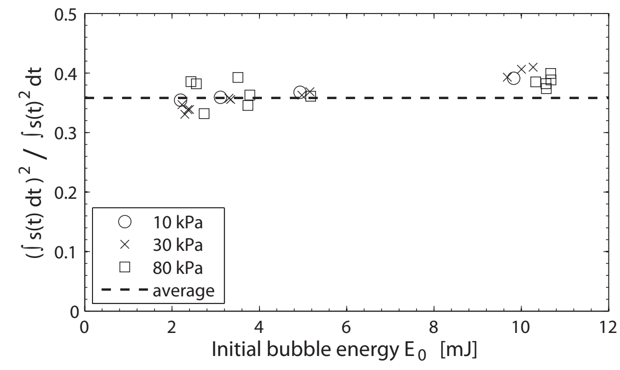

FIG. 2. Confirmation of the validity of the assumptions for the estimation of E_{SW} : $(\int s(t)\mathrm{d}t)^2/\int s^2(t)\mathrm{d}t \approx$ const.