Fluent coordination of autonomous vehicles at intersections

Laleh Makarem, Denis Gillet
School of Engineering
Ecole Polytechnique Fédérale de Lausanne (EPFL)
Lausanne, Switzerland
Laleh.makarem@epfl.ch, Denis.gillet@epfl.ch

***Abstract*—In this paper we introduce a new decentralized navigation function for coordination of autonomous vehicles at intersections. The main contribution is a navigation function designed for vehicles with predefined paths that uses expected time to intersection for collision avoidance. In such way, deadlock situations are avoided. Different inertias of the vehicles are taken into account to enable on-board energy optimization for crossing. Heavier vehicles that need more energy and time for acceleration or braking are given an indirect priority at intersections. The proposed decentralized coordination scheme shows a significant improvement in energy consumption and in motion smoothness compared to traditional crossing with human drivers.**

Keywords-Autonomous vehicles, intersection, decentralized navigation function, multi-agent systems

I. INTRODUCTION

Experimental autonomous passenger vehicles are already on the road and their commercial exploitation is envisioned in the next 10 years once coordination problems, such as platooning and intersection, will be safely and efficiently handled [1–3]. In this paper, we focus on the coordination of autonomous vehicles at intersections. Nowadays, traffic lights and stop or priority signs assist human drivers to safely cross intersections. However, in the future, with computers behind the wheels, innovative driver assistance systems or autopilots have to be designed. One of the challenges in this area of research is to find coordination methods improving vehicle performances at intersections. There are generally two different approaches to solve this problem. One approach is to design a centralized controller for the whole system of autonomous vehicles. Autonomous intersection management project is based on this approach [4]. However, decentralized control of vehicles can add more reliability and robustness to the system. At the same time, a decentralized control method decreases communication costs by reducing complexities.

The problem of coordinating autonomous vehicles at intersections in a decentralized way was first touched in [5] where a decentralized navigation function is introduced. Navigation functions are practical tools introduced in robotics for solving collision avoidance problems [6] such as formation [7], rendezvous and consensus scenarios. Decentralized navigation functions have two great benefits. First, compared with centralized approaches, navigation functions show a relatively low complexity with respect to the number of agents, in our case vehicles [8]. Second, it is possible to consider dynamic models for vehicles rather than simple kinematic ones.

Two main challenges are faced when using decentralized navigation function methods for vehicles crossing an intersection. First, in intersection scenarios, vehicles have predefined paths, from which desired speed of each vehicle along its path can be computed. Second, navigation functions might exhibit local minimums leading vehicles to stop in deadlock situations. This problem has been solved in [5] by introducing noise to the prediction of position of other vehicles. Although this method solves the problem of deadlock, it does not guarantee that all present vehicles at an intersection would not brake at the same time. Simultaneous braking is not optimal for traffic flow and energy consumption. To work out this problem, we propose a decentralized navigation function, which takes into account the expected time of arrival of the vehicles at the intersection. This approach results in a more fluent traffic at intersections.

The rest of this paper is organized as follow. In section 2, a dynamical model of the vehicles is introduced. It is simple enough to enable the handling of complex traffic situations and complex enough to capture real-world constraints. In section 3, a decentralized navigation function that takes dynamical constraints into account is proposed. The evaluation of the proposed approach is presented in section 4. It is compare with three other methods, a centralized optimal controller, the decentralized navigation function proposed in [5] and traffic lights. Section 5 briefly explores some avenues for future research and concludes.

II. PROBLEM FORMULATION

We considered a system composed of N autonomous vehicles. The goal of each vehicle is to cross an intersection with a minimal nominal speed deviation and without having any collision with other vehicles.

The position of vehicle i is known as $q_i = (x_i, y_i)$ in a global frame attached to the intersection. The path of the vehicle is predefined and is described by the parameter s_i. Therefore, the position of the vehicle in the global frame is directly derived from its location along the path using the parametric function $q_i = f_k(s_i)$, where k is the index of the chosen path. This parametric function is an injective function, which means that computing the location of a vehicle along its path is straightforward knowing its global location. The motion of each vehicle along its path is modeled using second order dynamics:

$$\ddot{s}_i = a_i \tag{1}$$

where a_i is the acceleration of the vehicle along the path. The proposed dynamic model is realistic taking into account the assumption that the vehicles follow predefined paths to reach their destinations. Additionally, using this dynamic model, it is possible to introduce real-word acceleration and braking constraints, defined as $a_{\max}$ and $b_{\max}$, respectively.

The speed limit is given by a function $v_{\max} = v_{\lim}(s_i)$ of the path parameter such that the centripetal acceleration in the bends remains below a certain value. Hence, the speed of vehicle along its path $\dot{s}_i$ is bounded in the interval $[0, v_{\max}]$.

The problem is now to find a decentralized controller that guarantees vehicle safety and high capacity at intersection under the mentioned real-world constraints related to acceleration, speed and braking. We introduce a navigation function for each vehicle to enable decentralized control.

III. Control Approach

In the literature, a navigation function is introduced as a smooth mapping from a working manifold of the vehicles to a scalar, which should be analytic in the workspace of every vehicle [9]. The gradient of navigation function is attractive to the destination point of the vehicle and repulsive from other vehicles. So, an appropriate navigation function could be combined with a proper control law in order to obtain a trajectory for every vehicle leading to the destination and avoiding collisions. We propose a new navigation, which avoids simultaneous braking of all vehicles at intersection. In addition, using this decentralized navigation function significantly improves the applicability and scalability.

A. *Decentralized navigation function*

The following navigation function is proposed:

$$\phi_i = \lambda_1 (v_i - v_{di})^2 + \lambda_2 \sum_{j \neq i} \beta(\tau_i, \tau_j, v_i) \tag{2}$$

where v_i is the speed of vehicle *i* along its path and v_{di} is its desired speed. Note that the desired speed is not necessarily the maximum speed. The maximum speed is given by the traffic regulations while the desired speed could be calculated to minimize vehicle's energy consumption along the path considering vehicle related factors. τ_i and τj are the expected time of arrival at the intersection for vehicle *i* and *j* respectively. The first term in the navigation function forces the vehicle to drive at the desired speed while the second term guarantees the crossing of the intersection without collision with other vehicles. λ_1 and λ_2 are the weights for the two terms in the navigation function. The function β is defined as follow:

$$\beta(\tau_i, \tau_j) = \begin{cases} \frac{1}{\sigma} \log\left(\frac{\sigma}{\tau_i - \tau_j}\right) v_i & 0 < \tau_i - \tau_j < \sigma \\ \frac{-1}{\sigma} \log\left(\frac{-\sigma}{\tau_i - \tau_j}\right) v_i & -\sigma < \tau_i - \tau_j < 0 \\ 0 & |\tau_i - \tau_j| > \sigma \end{cases} \tag{3}$$

where σ is the desired time difference between two vehicles reaching the intersection.

Vehicle will move toward the minimum point of the navigation function. If one vehicle is the nearest to the intersection the time differences of its arrival with others are negative. So, according to the β-function given in (3) it will accelerate and cross the intersection. On the other hand, the farthest vehicle from intersection will decelerate because the time difference is always positive. The β-function has non-zero value till the expected time of arrival for vehicles has the difference larger than σ. Tuning σ depends on the dimension of the intersection and the maximum speed allowed on the road.

B. *Decentralized control of each vehicle*

We use as control input the gradient of the navigation function presented in (2) and the dynamics of the vehicles defined in (1)

$$u_i = -\nabla_{v_i} \phi_i \tag{4}$$

As the vehicles are moving on their predefined path and the gradient is calculated along the same path, the control actions are also acting in the expected direction.

C. *Collision avoidance*

In this subsection we discuss why the proposed navigation function guarantees collision avoidance at intersections. For this purpose we first consider a situation where two vehicles are crossing an intersection. We then extend the solution for a system with a larger number of vehicles.

Let us consider two vehicles entering an intersection (Fig. 1). Without changing its speed, the first vehicle would reach the middle of intersection after time T_1, while the expected arrival time for second vehicle is T_2. We consider a case where $T_1 > T_2$. Therefore, according to β-function in (3) first vehicle will decelerate to guarantee a collision free pass for both vehicles. Collision avoidance is guaranteed if the first vehicle reaches the intersection at least σ second after the second one. This time difference allows the second vehicle to leave the intersection before the first vehicle enters.

To avoid collision in the worst-case scenario the first vehicle has to stop before reaching the intersection. So we can assume that fist vehicle has $t = T_2 + \sigma$ seconds to stop. This time interval gives a safe margin for the second vehicle to pass. The change in the vehicle's speed during this time interval can be written as:

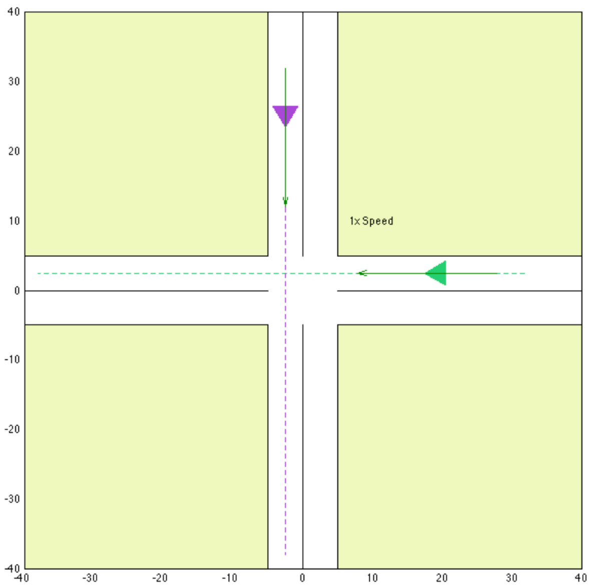

Figure 1. Intersection with two vehicles

$$dV_1 = \int_0^{T_2+\sigma} -\frac{1}{2}\lambda_1(v_1 - v_{d1}) - \lambda_2 \frac{1}{\sigma}\log(\frac{\sigma}{\tau_2 - \tau_1})dt \qquad (5)$$

The last term in the integral (5) will change over time. However we can consider it as constant to compute the lower bound of the speed difference:

$$\begin{aligned} dV_1 &= \int_0^{T_2+\sigma} -\frac{1}{2}\lambda_1(v_1 - v_{d1}) - \lambda_2 \frac{1}{\sigma}\log(\frac{\sigma}{\tau_2 - \tau_1})dt < \\ &-\lambda_2 \frac{1}{\sigma}\log(\frac{\sigma}{T_2 - T_1})(T_2 + \sigma) - \int_0^{T_j+\sigma} \frac{1}{2}\lambda_1(v_1 - v_{d1})dt < \\ &-\lambda_2 \frac{1}{\sigma}\log(\frac{\sigma}{T_2 - T_1})(T_2 + \sigma) + \frac{1}{2}\lambda_1 v_{i\max}^2 \end{aligned} \qquad (6)$$

As mentioned, for safe passing of the intersection in the most conservative case, the first vehicle should stop before the intersection (Fig. 2).

$$V_1 > \lambda_2 \frac{1}{\sigma}\log(\frac{\sigma}{T_2 - T_1})(T_2 + \sigma) - \frac{1}{2}\lambda_1 v_{i\max}^2 \qquad (7)$$

According to (7), one can tune the two parameters of λ_1 and λ_2 in order to get a safe passing.

It is worth mentioning that computing the parameters λ_1 and λ_2 using (7) does not mean that one of the vehicles stops before the intersection. Because the difference between expected arrival times is increasing along the path of vehicles, which will lead to less changes in speed of vehicles. Fig. 2 shows the speed of two vehicles crossing the intersection. A small change in the speed of two vehicles leads to the safe crossing of intersection.

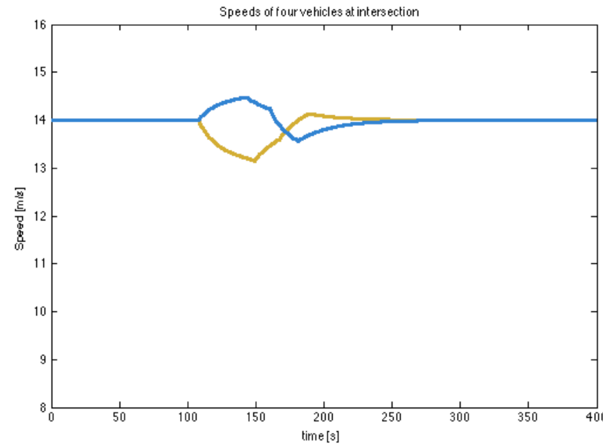

Figure 2. Change in the speeds of two vehicles at intersection, in order to safely pass the intersection. One of vehicle can accelerate to clear the intersection for the other one.

For intersections with more than one vehicle we could give the same reasoning for every pair of vehicles, and guarantee that the ones decelerating will stop before intersection. Note that this way of tuning the parameters is the most conservative one that guarantees collision avoidance at intersection.

D. Fluent traffic

Thanks to the proposed beta function we prevent deadlocks, leading in such a way to smoother trajectories for autonomous vehicles and hence more fluent traffic (Fig. 2).

E. Priority assignment

So far, all vehicles have been treated equally. However, there are some reasons to give priority to some of the vehicles. The most obvious reason is to give priority to heavier vehicles with higher inertia, thus saving energy. This can be implemented by weighting the β-function with a factor V(i,j), which correspond to a matrix of inertia.

$$\phi_i = \lambda_1(v_i - v_{di})^2 + \lambda_2 \sum_{j \neq i} V(i,j)\beta(\tau_i, \tau_j, v_i) \qquad (8)$$

$$V(i,j) = \frac{m_i}{m_j} \qquad (9)$$

m_i and m_j are the inertias of vehicle i and vehicle j respectively. Weighting the β-function with matrix of inertia is an indirect way of giving priority. Unlike direct priority assignment, it does not necessarily force lighter vehicles to decelerate, but it will put more control on them than on the heavier vehicles. The advantage of giving indirect priority is that it does not violate safety guarantees.

F. Information sharing

For constructing the navigation function, each vehicle needs information about itself and also other vehicles. Every vehicle needs its position, velocity and path. This information could be easily accessed with the today onboard sensors.

Moreover, vehicles should communicate to get information about expected time of arrival of other vehicles as well as the inertia of other vehicles.

By keeping the number of messages and the amount of information transmitted to a minimum, it is possible to put more communication reliability measures in place. Furthermore, each vehicle, as an autonomous agent, may have privacy concerns, which should be respected. On the other hand, vehicles can communicate when they are in distance less than their communication range. One of the advantages of the proposed method is to keep the communication complexity as low as possible as well as to take into account the communication range of vehicles. These points show that the proposed method is reliable considering the communication between vehicles.

IV. Simulation and Results

In this section, the performance of the proposed method is evaluated by simulation. The proposed navigation function is evaluated and compared with three other methods (i.e. intersection with traffic lights, centralized control and navigation function in [5]. The effectiveness of the proposed navigation function in coordinating the crossing of four vehicles is investigated. The convergence is obtained when vehicles leave the intersection without collision.

A. *Four vehicle scenario*

First, the control of four vehicles entering an intersection using their navigation functions given in (2) is considered (Fig. 3). The effect of taking inertia into account to get a smoother flow for vehicles is also illustrated.

In the simulation, the nominal trajectories of the vehicles are chosen in a way that without having control collision will occur. All vehicles entered the area of the intersection at the same time. So, the expected time of arrival to the intersection is more or less the same. Vehicles can communicate in a range corresponding to one third of the length of the road at each side of intersection. The chosen integration step for the simulation is 50 ms. The values of the parameters in the navigation function are $\lambda_1 = 0.5$ and $\lambda_2 = 0.8$ and $\sigma = 4$. The desired velocity for all four vehicles is 14 m/s and the maximum allowed velocity is 16 m/s.

Fig. 4 shows the velocities of the four vehicles before, during and after the crossing at the intersection. Vehicle number one is five times heavier than the other three vehicles. Thanks to the chosen navigation function, the velocities of the vehicles do not change significantly. The change in the velocity of the heavier vehicles is almost negligible. It is important to underline that, with the chosen navigation function parameters, none of the vehicles have to stop. As a consequence, the crossing is handled in a very smooth way. The navigation function gives the opportunity to the vehicles to accelerate at the intersection if they are not driving at their maximum speed. However this feature is not mandatory to guarantee collision avoidance.

In the absence of other vehicles, the first term in the navigation function is still present. So, the vehicles will adjust their speed to reach the desired speed for that part of their paths. The navigation system of the vehicle can update the desired speed in an outer loop.

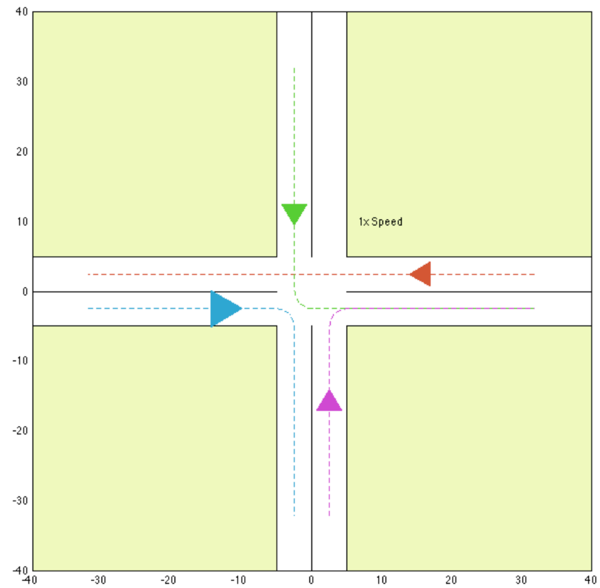

Figure 3. Intersection with four vehicles, blue vehicle is heavier than the other three.

B. *Energy efficiency*

In this subsection, the coordination of autonomous vehicles at intersections using the proposed navigation function is compared to three other control methods. First, the less effective one with drivers obeying to traffic lights. Second, the most effective one with an optimal centralized control that relies on a full knowledge of all the vehicles and their environments. Third, the decentralized navigation function introduced in [5]. The objective is to show whether this decentralized approach that relies only on local information and on limited computation power can exhibit performances close to the optimal scenario. For comparing these four scenarios, the

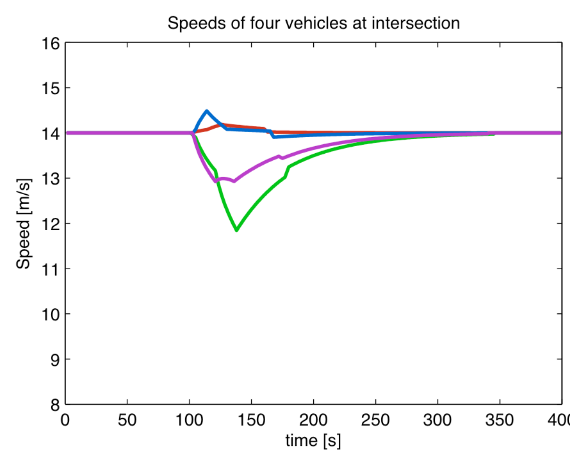

Figure 4. Change of velocities for four vehicles passing an intersection. Heavier vehicle shown in red has lesser change in velocity than others. Vehicle can accelerate to pass the intersection to give place to other vehicles to enter the intersection.

Table 1 Technical specifications of the two different types of vehicles in the system [11]

	Mass [m]	Maximum deceleration [m/s^2]	Maximum acceleration [m/s^2]
Type1	1300	80.0	30.47
Type2	20000	20.9	10.54

same four-way intersection with one lane of traffic in each direction as considered in the previous subsection (Fig. 3) is simulated.

As first control scenario we simulate traffic lights. There are two traffic lights, which have been configured such that each vehicle is given a green light for 4 seconds, a yellow one for 1 second, and a red one for 4 second. Although there are qualified works concerning the timing of the traffic lights in the literature, they cannot be considered here for two main reasons. First, most of the works have been done for the management of multiple intersections, while our focus is in solving the problem at a single intersection. Second, vehicles appear in each direction symmetrically, which very much simplifies the timing problem. As a consequence, a symmetric timing pattern is selected as mentioned before that minimizes the global energy consumption at the intersection.

As second method, the proposed navigation function is considered with two types of vehicles with different inertias. 25% of the vehicles are heavier than the others and they are uniformly distributed in the four direction of the intersection. Table 1 gives the specifications of the two different kinds of vehicles. The elements of the priority matrix introduced in (9) are supposed to be known. As the vehicles have been modeled using second order dynamics, their masses are representative of their inertias.

The third scenario corresponds to the method proposed in [10], which is an example of the application of centralized control approach for navigation. In their work, they assumed the second order dynamic as ours in (1). So the behavior of all vehicles is modeled as a linear time-invariant dynamic system in the following form:

$$\dot{X} = AX + BU \tag{10}$$

X is the state vector in (8) which is position and velocity of all vehicles. The cost function for overall system introduced in (9).

$$J = \sum_{i=1}^{N} J_i = \int_0^{\infty} q(x,u)\,dt \tag{11}$$

$$q(x,u) = \sum_{i=1}^{N} q_i(x_i, u_i) \tag{12}$$

As every vehicle's goal is to arrive at its destination point, which in our case is the other side of the intersection, they associated the quadratic cost function for every vehicle. The overall cost function will be the summation of cost functions of every vehicle:

$$q_i = \frac{1}{2}[(x_i - x_i^e)^T Q(x_i - x_i^e) + (u_i - u_i^e)^T R(u_i - u_i^e)] \tag{13}$$

x_i and x_i^e are the vectors composed of the position and velocity of the vehicle i at its current state and its goal state respectively. u_i and u_i^e also represents the current and ideal input for the vehicle, respectively.

We assumed that Q is an identity matrix and R as a diagonal matrix with the inertias of the vehicles as elements respected to every vehicle. The control law was computed using these two matrices and collision avoidance as constraints of the problem.

The forth method is the decentralized navigation function introduced in [5]. It is worth mentioning that the centralized method of navigation is not computationally efficient and hence cannot be implemented in real world applications

The four methods are compared according to two different criteria. The first criterion is the weighted average of the energy used by the vehicles passing through the intersection. The energy consumption corresponds to the control signal, i.e. the acceleration and the deceleration of the vehicles at the intersection. It is worth mentioning that it has been considered that vehicles with higher inertia are consuming more energy for acceleration and deceleration. The energy consumption index is defined as follows:

$$\bar{E} = \frac{1}{T_f N}\sum_i J_i = \frac{1}{T_f N}\sum_i \int_0^{T_f} u_i^T R_i u_i \tag{14}$$

In (14) T_f is the duration of the experiment and N is the total number of vehicles introduced in the system. The control signal is considered being zero when the vehicle exits the system.

The second criterion is the vehicle throughput or flow, which is the number of vehicles per hour that have passed through the intersection during the simulation time. It is worth mentioning that the vehicles are counted when leaving the intersection via an exit section. This means that if a blockade occurs, the flow of the vehicles would decrease significantly. The average number of vehicles that should enter the intersection could be defined using the O/D matrix of the network [12].

Table 2 shows the comparison of these four different scenarios according to the two different criteria. It is clear from this table that the presently proposed navigation function induces energy consumption even more than the previous decentralized navigation function. The proposed method is only 24% less effective than the centralized one, while being easily implementable. It also allows a higher throughput as the vehicles have the ability to accelerate to clear the intersections more rapidly.

Table 1. Comparison of four control methods for vehicles passing through an intersection by indexes as mean of energy consumption of every vehicle and maximum through put of intersection.

Control scenario	Energy criterion according to (5)	Maximum throughput
Traffic lights	45.6	1.43
Decentralized control using navigation function [5]	14.14	2.38
Proposed decentralized navigation function	12.26	2.78
Central Controller	9.86	2.59

V. CONCLUSION AND FUTURE WORKS

In this work the previous decentralized navigation function proposed in [5] has been modified in two ways. First, the navigation function is introduced as a function of the path. The attractive force is the one that regulates the speed of the vehicle toward its desired value. Second, the avoidance force from other vehicles is computed using the expected time of arrival of the vehicle itself and other vehicles in the communication range of this vehicle. Communication is active in the bounded sensing region of each individual vehicle. In this work, the navigation function has been modified in a way to optimize energy consumption taking the inertia of the vehicles into account. This paves the way towards on-board energy optimization by indirectly giving priority to heavier vehicles at intersections. The proposed method has been compared with the two centralized control approaches and a decentralized method proposed previously. The proposed method not only shows a significant improvement in comparison with the classic traffic lights from energy point of view it is also more energy efficient in comparison with previously introduced navigation function. This reduction in energy consumption is achieved thanks to the absence of local minimums in the new navigation function, which prevents simultaneous deceleration of both vehicles near the intersection.

Our future research directions include the analytical study of the convergence of the proposed coordination approach. Performance of the proposed method will be studied in multi intersection scenario. In the future we will also study the behavior of the vehicles under communication constraints and lack of energy as it could happen when using electrical vehicles.

REFERENCES

[1] S. Thron, “Toward robotic cars,” Communications of the ACM, vol. 53, no. 4, pp. 99–106, 2010.

[2] M. Buehler, K. Iagnemma, and S. Singh, The DARPA Urban Challenge: Autonomous Vehicles in City Traffic, vol. 56. Springer Verlag, 2010.

[3] R. Bishop, “Intelligent vehicle technology and trends,” Artech House, Inc. Norwood, MA2005.

[4] K. Dresner and P. Stone, “A multiagent approach to autonomous intersection management,” Journal of Artificial Intelligence Research, vol. 31, no. 1, pp. 591–656, 2008.

[5] L. Makarem and D. Gillet, “Decentralized Coordination of Autonomous Vehicles at intersections,” IFAC World Congress, 2011, vol. 18, pp. 13046–13051.

[6] D. V. Dimarogonas and K. J. Kyriakopoulos, “A feedback control scheme for multiple independent dynamic non-point agents,” International Journal of Control, vol. 79, no. 12, pp. 1613–1623, 2006.

[7] H. A. Izadi, B. W. Gordon, and C. A. Rabbath, “Decentralized control of multiple vehicles with limited communication bandwidth,” in Systems, Man and Cybernetics, 2008. SMC 2008. IEEE International Conference on, 2008, pp. 2724–2729.

[8] H. Roozbehani, S. Rudaz, and D. Gillet, “On decentralized navigation schemes for coordination of multi-agent dynamical systems,” in Systems, Man and Cybernetics, 2009. SMC 2009. IEEE International Conference on, 2009, pp. 4807–4812.

[9] H. A. Blom and J. Lygeros, “Stochastic hybrid systems,” Springer-Verlag Berlin/Heidelberg, 2006.

[10] D. M. Stipanović, P. F. Hokayem, M. W. Spong, D. D. Šiljak, and others, “Cooperative avoidance control for multiagent systems,” Journal of Dynamic Systems, Measurement, and Control, vol. 129, p. 699, 2007.

[11] G. M. H. Department for Transport, “Department for Transport,” [Online]. Available: http://www.dft.gov.uk/.

[12] J. Kim, A. Ilic, and S. Bohacek, “Realistic simulation of urban mesh networks-part I: Urban mobility,” Citeseer, 2006.