Non-Pareto Optimality of MPTCP: Performance Issues and a Possible Solution

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ABSTRACT

MPTCP has been proposed as a mechanism to support transparently multiple connections to the application layer and is under discussion at the IETF [1]. It can effectively use the available bandwidth and it improves throughput and fairness, compared to independent TCP flows in many scenarios [2–4]. However we show, by measurements over our testbed and analytically, that MPTCP still suffers from two problems: (P1) upgrading some TCP users to MPTCP can reduce the throughput of others without any benefit to the upgraded users, which is a symptom of non-Pareto optimality; (P2) MPTCP users could be excessively aggressive towards TCP users. We attribute these problems to the "Linked Increases" Algorithm (LIA) of MPTCP [1], and more specifically, to an excessive amount of traffic transmitted over congested paths.

The design of LIA forces a tradeoff between optimal resource pooling and responsiveness. Hence, to provide good responsiveness MPTCP's current implementation must depart from Pareto-optimality. We revisit the problem and show that it is possible to simultaneously provide these two properties. We implement the resulting algorithm, called Opportunistic "Linked Increases" Algorithm (OLIA), in the Linux kernel and study its performance over our testbed by simulations and by theoretical analysis. We prove that OLIA is Pareto-optimal, hence avoids the problems (P1) and (P2). Our measurements and simulations indicate that MPTCP with OLIA is as responsive and non-flappy as MPTCP with LIA, while solving problems (P1) and (P2).

1. INTRODUCTION

The regular TCP uses a window-based congestion-control mechanism to adjust the transmission rate of users [5]. It always provides a Pareto-optimal allocation of resources: it is impossible to increase the throughput of one user without decreasing the throughput of another or increasing the congestion cost [6]. It also guarantees a fair allocation of bandwidth among the users but favors the connections with lower RTT [7].

Various mechanisms have been used to build a multipath transport protocol compatible with the regular TCP. [8–10] propose a family of algorithms inspired by utility maximization frameworks. These algorithms tend to use only the best paths available to users and are optimal in static settings where paths have similar RTTs. However, in practice, they suffer from several problems [2–4]. First, they might fail to quickly detect free capacity as they do not probe paths with high loss probabilities sufficiently. Second, they exhibit flappiness: when there are multiple good paths available to a user, the user will randomly flip its traffic between these paths. This is not desirable, specifically, when the achieved rate depends on RTTs, as with TCP.

MultiPath TCP (MPTCP) is a concrete proposal for multipath transport, under discussion at the IETF [1]. Because of the issues just mentioned, its congestion control part does not follow the algorithms in [8–10]. Instead, it follows an ad-hoc design based on the following goals [1]. (1) Improve throughput: a multipath TCP user should perform at least as well as a TCP user that uses the best path available to it. (2) Do no harm: a multipath TCP user should never take up more capacity from any of its path than a regular TCP user. And (3) balance congestion: a multipath TCP algorithm should balance congestion in the network, subject to meeting the first two goals. MPTCP compensates for different RTTs and solves many problems of multipath transport [2,4]: it can effectively use the available bandwidth, it improves throughput and fairness compared to independent regular TCP flows in many scenarios, and it solves the flappiness problem.

However we show, by measurements over our testbed and analytically, that MPTCP still suffers from the following problems:

(P1) Upgrading some regular TCP users to MPTCP can reduce the throughput of other users without any benefit to the upgraded users. This is a symptom of non-Pareto optimality,

(P2) MPTCP users could be excessively aggressive towards TCP users.

We attribute these problems to the "Linked Increases" Algorithm (LIA) of MPTCP [1] and specifically to an excessive amount of traffic transmitted over congested paths. These problems indicate that MPTCP fails to fully satisfy its design goals, especially goal (3).

The design of LIA forces a tradeoff between optimal resource pooling and responsiveness, it cannot provide both in the same time. Hence, to provide good responsiveness LIA's current implementation must depart from Pareto-optimality, which leads to problems (P1) and (P2). We revisit the design and show that it is possible to simultaneously provide both properties. We introduce OLIA, the "Opportunistic Linked Increases" Algorithm, as an alternative to LIA. Based on utility maximization frameworks, we prove that OLIA is Pareto-optimal, hence avoids the problems (P1) and (P2). Furthermore, its construction makes it as responsive and non-flappy as LIA.

OLIA is a window-based congestion control mechanism. Similarly to LIA, it couples the additive increases and uses unmodified TCP behavior in the case of a loss. OLIA's increase part (Eq. (5)) has two terms:

- The first term is an adaptation of the increase term of Kelly and Voice's algorithm [8]. This term is essential to provide Pareto-optimality.
- The second term, which contains parameter α_r, guarantees responsiveness and non-flappiness of OLIA. By measuring the number of transmitted bits since the last loss, it reacts to events within the current window and adapts to changes faster than the first term.

By adapting the window increases as a function of RTTs, OLIA also compensates for different RTTs.

We implement OLIA in the Linux kernel and study its performance over our testbed, by simulations and by theoretical analysis. Using a fluid model of OLIA based on differential inclusion, we prove that OLIA is Pareto-optimal (Theorem 3) and satisfies the design goals of MPTCP (Corollary 2). Our measurements and simulations indicate that MPTCP with OLIA is as responsive and non-flappy as MPTCP with LIA. Moreover, it solves problems (P1) and (P2).

The paper is structured as follows. In the next section, we briefly introduce MPTCP and related work. In Section 3, we provide a number of examples and scenarios in which MPTCP with LIA exhibits problems (P1) and (P2). Section 4 introduces OLIA and details its Linux implementation. In Section 5, we prove that our algorithm is Pareto-optimal and satisfy the design goals of MPTCP. In Section 6, we study the performance of OLIA through measurements and by simulations.

2. MPTCP AND RELATED WORK

Multipath TCP (MPTCP) is a set of extensions to the regular TCP that allows users to spread their traffic across potentially disjoint paths [1]. MPTCP discovers the number of paths available to a user, establishes the paths, and distributes traffic across these paths through creation of separate subflows [11,12].

MPTCP's congestion control algorithm forces a tradeoff between optimal resource pooling and responsiveness [3]. The idea behind the algorithm is to transmit over a path r at a rate proportional to $p_r^{-1/\varepsilon}$, where p_r is the loss probability over this link and $\varepsilon \in [0, 2]$ is a design parameter. The choice ε=0 corresponds to the fully coupled algorithm of [8–10]: the traffic is sent only other the best paths, it is Pareto-optimal but is flappy. The choice ε=2 corresponds to having uncoupled TCP flow on each path: it is very responsive and non flappy but does not balance congestion. MPTCP's implementation uses ε=1 to provide a compromise between optimal resource pooling and responsiveness. This algorithm is called "Linked Increases" Algorithm (LIA) [1].

Let w_r and rtt_r be the window size and the estimated round-trip time on path $r \in \mathcal{R}_u$. $\mathcal{R}_u$ is the set of all paths available to users u. LIA work as follows:

- For each ACK on subflow r, increase w_r by
$$\min\left(\frac{\max_{i\in\mathcal{R}_u} w_i/\text{rtt}_i^2}{(\sum_{i\in\mathcal{R}_u} w_i/\text{rtt}_i)^2}, \frac{1}{w_r}\right). \tag{1}$$
- For each loss on subflow r, decrease w_r by $w_r/2$.

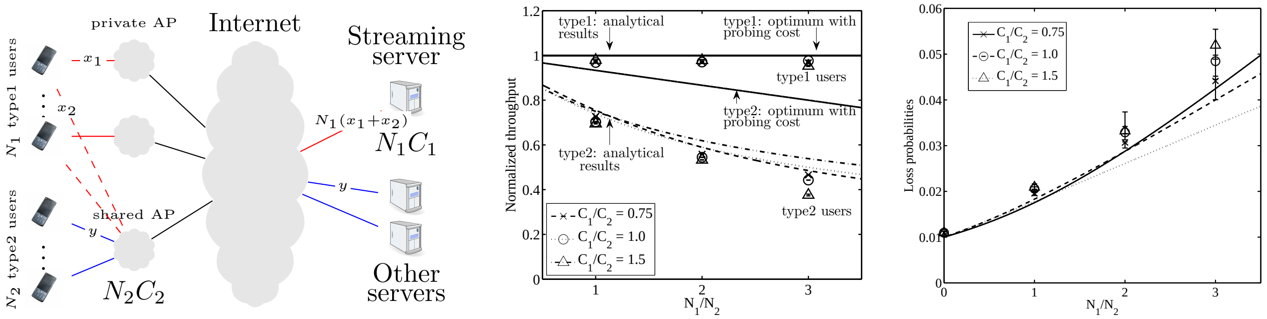
LIA increases by at most $1/w_r$ to be not more aggressive than a regular TCP on any of its paths. When the RTTs are similar, this minimum can be neglected as the first term would always be less than $1/w_r$. In this case, a fixed point analysis provides a simple loss-throughput formula for LIA [4]: LIA allocates to a path r a window w_r proportional to the inverse of the loss probability $1/p_r$ and such that the total rate $\sum_{p\in\mathcal{R}_u} w_p/\text{rtt}_p$ equals the rate that a regular TCP user would get on the best path, *i.e.* $\max_{p\in\mathcal{R}_u} \sqrt{2/p_p}/\text{rtt}_p$. Thus, the window size for the flow on a path r is given by:

$$w_r = \frac{1}{p_r} \cdot \frac{\max_{p\in\mathcal{R}_u} \sqrt{2/p_p}/\text{rtt}_p}{\sum_{p\in\mathcal{R}_u} 1/(\text{rtt}_p p_p)}. \tag{2}$$

Besides MPTCP and algorithms in [8–10], a few other algorithms have been proposed to implement multipath protocols. In [13], an opportunistic multipath scheduler measures the path conditions on time scales up to several seconds. [14] uses a mechanism to detect shared bottlenecks and to avoid the use of multiple subflows on the same bottleneck. [15] proposes uncoupled TCP with a weight depending on the congestion level. These mechanisms are complex, their robustness is not clear, and they need explicit information about congestion in the network. Note that our proposed algorithm, OLIA, differs from these works as it is implemented, proven to be Pareto optimal, and relies only on information that is available to regular TCP. It also differs from [8–10] as it is not flappy and has a better responsiveness.

3. PERFORMANCE PROBLEMS OF MPTCP

In this section, we investigate the behavior of MPTCP with LIA in three different scenarios: scenarios A, B,



(a) Scenario A

(b) Normalized throughput of users: $(x_1 + x_2)/C_1$ and y/C_2.

(c) Loss prob. p_2 at the shared AP.

Figure 1: Scenario A: type1 users are all downloading through the same streaming server and have access to both a private high speed access point and a shared access point. Type2 users have access only to the shared access point. The performance of MPTCP with LIA obtained by measurement (points) or numerical analysis (lines) is shown on figures (b) and (c). We observe that it is not Pareto-optimal, penalizes type2 users, and its performance is far from the theoretical optimum with probing cost. It also fails to balance the congestion.

and C. Using scenarios A and B, we show that upgrading some regular TCP users to MPTCP could reduce the throughput of other users in the network without any benefit to the upgraded users (problem P1). Scenario C discusses the aggressiveness of MPTCP users that compete with regular TCP users (problem P2). Our conclusions are based on analytical results and measurements over a testbed.

3.1 Testbed Setup

To investigate the behavior of the algorithms, testbed topologies are created representing our scenarios. Server-client PCs run MPTCP (with LIA or OLIA) enabled Linux kernels. In all our scenarios laptop PCs are used as routers. We install "Click Modular Router" software [16] to emulate topologies with different characteristics. We emulate links with configurable bandwidth and delay with RED queuing (drop-tail queuing is also studied in htsim simulation, see Section 6.2). Motivation for testbed configuration represented in Figure 2 might be the scenario described in Figure 1(a).

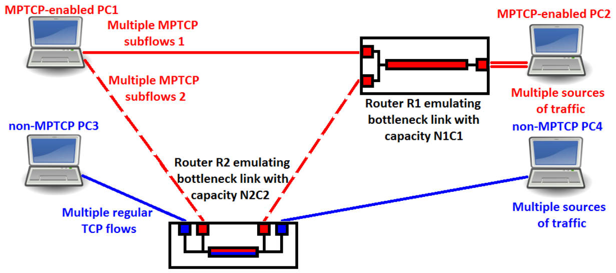

MPTCP-enabled PC1
Multiple MPTCP subflows 1
Multiple MPTCP subflows 2
MPTCP-enabled PC2
Router R1 emulating bottleneck link with capacity N1C1
Multiple sources of traffic
non-MPTCP PC3
non-MPTCP PC4
Router R2 emulating bottleneck link with capacity N2C2
Multiple regular TCP flows
Multiple sources of traffic

Figure 2: Testbed implementation of scenario A: router R_1 emulates the streaming server and router R_2 the shared AP. Iperf is used to emulate multiple connections. The red PCs use MPTCP and the blue PCs use regular TCP.

3.2 Scenario A: MPTCP is not Pareto-optimal and penalizes regular TCP users

Consider a network with two types of users as shown in Figure 1(a). There are N_1 users of type1, each with a high-speed private connection, accessing different files on a media streaming server. The server has an accessing rate limit of N_1C_1 Mbps. These users can activate a second connection through a shared AP by using MPTCP. There are also N_2 users of type2 in the network that only have connections to the shared AP. They download their contents from the Internet. The shared AP has a capacity of N_2C_2 Mbps.

Let x_1 be the rate that a type1 user receives over its private connection (by symmetry, every user of type1 will receive the same rate x_1). Similarly, let x_2 (resp. y) be the rate that a type1 (resp. type2) user receives over the shared connection. We denote by p_1 and p_2 the loss probability at the link connected to the streaming server and the shared AP, respectively (the loss probabilities at the Internet backbone and the private APs are negligible).

When type1 users uses only their own private AP, we have $x_1{=}C_1$, $x_2{=}0$, and $y{=}C_2$. In this case the normalized throughput for both type1 and type2 users is 1. In the other case, when all type1 users activate their public connections and use MPTCP with LIA to balance load between their connections, we have

$$
\begin{aligned}
&(a) \quad N_1(x_1{+}x_2) = N_1C_1 \qquad && N_1x_2 + N_2y = N_2C_2 \\
&(b) \quad x_1 + x_2 = \frac{1}{\text{rtt}}\sqrt{\frac{2}{p_1}} \qquad && x_2 = \frac{1}{2{+}p_2/p_1}\frac{1}{\text{rtt}}\sqrt{\frac{2}{p_1}} \\
&(c) \quad y = \frac{1}{\text{rtt}}\sqrt{2/p_2} &&
\end{aligned}
$$

where (a) are the capacity constraints at the two bottlenecks, (b) comes from the loss-throughput formula for LIA (Eq.(2)), and (c) follows the TCP loss-throughput formula [17]. This system has a unique solution (see Appendix A). Figure 1(b) depicts the normalized throughput of type1 and type2 users, i.e. $(x_1 + x_2)/C_1$ and y/C_2. As shown in Appendix A, these values depend only on the ratios C_1/C_2 and N_1/N_2.

A theoretically optimal algorithm will allocate a normalized throughput of 1 to both type1 and type2 users. However, as the values of the congestion windows are bounded below by 1 MSS, we should consider that a minimum probing traffic of 1 MSS per RTT will be transmitted over a path. Hence, in this paper, we introduce the *theoretical optimum with probing cost* that takes into account this minimum probing cost.

We measure the performance of LIA in Scenario A, using the testbed, as shown in Figure 2. The measurements are taken for $N_2 = 10$ and three values of $N_1 = 10, 20, 30$. The capacities of R1 and R2 are N_1C_1 and N_2C_2 Mbps, where we set $C_2 = 1$Mbps and $C_1 = 0.75$, 1, 1.5 Mbps. All paths have similar RTTs (≈ 150 ms). For each case, we took 5 measurements. The results are reported on Figure 1(b). Note that in all cases we present confidence intervals, but in many cases they are too small to be visible. The loss probability p_1 depends only on C_1 and is 0.02, 0.009, 0.004 for $C_1 = 0.75$, 1, 1.5 Mbps. We also show our analytical analysis of LIA, as well as theoretical optimum with probing cost as defined above.

These figures have multiple implications. First, they show that MPTCP with LIA exhibits problem (P1) from the introduction: upgrading type1 users to MPTCP penalizes type2 users without any gain for type1 users. As the number of type1 users increases, the throughput of type2 users decreases, but the throughput of type1 users does not change as it is limited by the capacity of the streaming server. For $N_1{=}N_2$, type2 users see a decrease of about 30%. When $N_1{=}3N_2$, this decrease is between 50% to 60%. This is explained by the fact that LIA does not fully balance congestion, as shown in Figure 1(c). It excessively increases congestion on the shared AP (not in compliance with goal 3). We observe that LIA performs far from how an optimal algorithm with probing cost would perform. Furthermore, these figures show that the fixed point analysis predicts accurately the behavior of the algorithm: the two curves (theoretical and experimental) exhibit the same trend. As a summary for this section, we conclude that MPTCP with LIA is not Pareto-optimal and could penalize TCP users without any benefit for anybody.

3.3 Scenario B: MPTCP is not Pareto-optimal and can penalize other MPTCP users.

Consider the multi-homing scenario depicted in Figure 3. We have four Internet Service Providers, ISPs, X, Y, Z, and T. Y is a local ISP in a small city, which connects to the Internet through Z. X, Z, and T are nation-wide service providers and are connected to each other through high speed links. X provides Internet services to users in the city and is a competitor of Y. They have access capacity limits of C_X, C_Y, C_Z, and C_T.

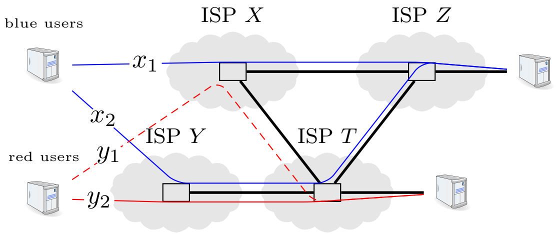
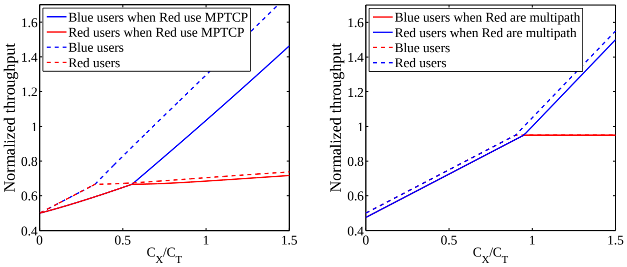


Figure 3: Scenario B. Thick lines represent peering agreements. Blue users are downloading from servers in ISP Z and Red users from servers in ISP T. By default, Blue users use multi-homing and have access to ISPs X and Y. Initially, Red users have access only to ISP Y but upgrade to MPTCP and connect to both ISPs X and Y (by activating dashed connection).

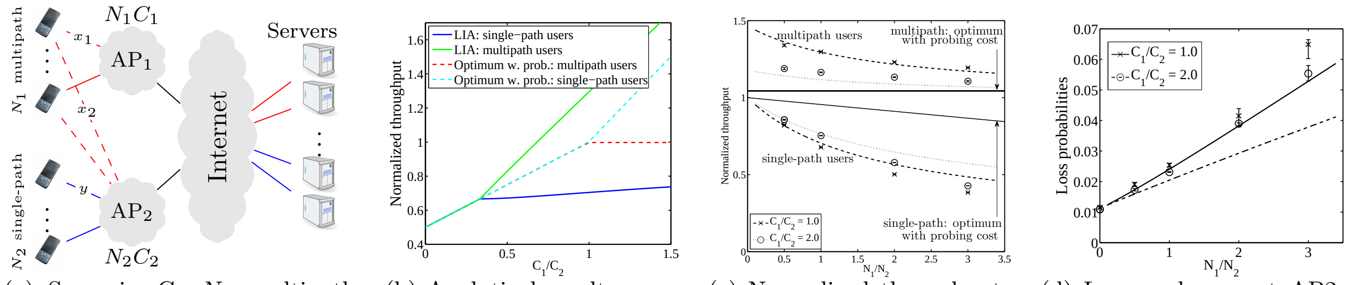


(a) Performance of LIA. (b) Optimum w. probing cost

Figure 4: Scenario B: analytical results for 15 Blue, 15 Red users and C_T=36 Mbps. We show the normalized throughput $(15(x_1 + x_2)/C_T$ and $15(y_1 + y_2)/C_T)$ as a function of C_X/C_T. Dashed curves: normalized throughput when Red users connect only to ISPY. Solid curves: the case when Red users upgrade to multipath. For all values of C_X/C_T, the throughput of all users decreases when Red users upgrade to MPTCP.

Z and T are hosts of different video streaming servers. There are two types of users: Blue users download contents from a server in Z and Red users download from a server in ISP T. Blue users use multi-homing and are connected to both ISPs X and Y to increase their reliability. Red users can connect either only to Y or to both X and Y. We assume that only ISPs X and T are bottlenecks and denote by p_X and p_T the loss probabilities. We assume that all paths have similar RTTs.

We first present a theoretical analysis of the rate that each user would achieved. There are two possible cases. When Red users connect only to Y, the analysis is the same as the one of scenario C, given in Section 3.4. Here, we analyze the case when Red users upgrade to MPTCP. The loss throughput formula (Eq.(2)) shows



(a) Scenario C: N_1 multipath users and N_2 single-path users are connected to two APs with capacities N_1C_1 and N_2C_2 Mbps

(b) Analytical results: normalized throughput of all users using LIA (solid) or optimum with probing cost (dashed) for $N_1 = N_2$.

(c) Normalized throughputs using LIA, obtained by measurement (points) or analysis (lines).

(d) Loss prob. p_2 at AP2: LIA fails to balance the congestion.

Figure 5: Scenario C: MPTCP with LIA excessively penalize TCP users (when $C_1/C_2 \geq 1$, for any fairness criterion, MPTCP users should not impact TCP users). We show the normalized throughputs ($(x_1+x_2)/C_1$ and y/C_2) received by the users, as well as p_2. The performance of LIA is far from the theoretical optimum with probing cost.

that the throughput of the different connections are:

$$\begin{cases} y_1 = \dfrac{1/\text{rtt}}{2+\frac{p_X}{p_T}}\sqrt{\dfrac{2}{p_T}} \\ y_2 = \dfrac{p_X}{p_T}y_1 \end{cases}, \begin{cases} x_1 = \dfrac{1/\text{rtt}}{1+p_X/p_T}\sqrt{\max\dfrac{2}{p_X},\dfrac{2}{p_T}} \\ x_2 = \dfrac{1/\text{rtt}}{1+p_T/p_X}\sqrt{\max\dfrac{2}{p_X},\dfrac{2}{p_T}} \end{cases}$$

As shown in Appendix B, this set of equations has a unique positive solution. A numerical evaluation of these formulas is depicted in Figure 4(a). Figure 4(b) shows the theoretical optimum with probing cost for $C_T = 36$ Mbps and RTT=150 ms (see Appendix B). The result shows that upgrading Red users to MPTCP with LIA decreases the performance for everyone. As an example, when $C_X/C_T \approx 0.75$, by upgrading the Red users we reduce the throughput of the Blue user up to 21%. This decreases is around 3% when we use a Pareto-optimal algorithm (Fig. 4(b)).

Red users	Rate/user		Aggregate
	Blue users	Red users	
Single-path	2.5	1.5	59.8
Multipath	2.0	1.4	52.0

Table 1: Measurement results for scenario B, showing the effect of upgrading the Red users from regular TCP to MPTCP with LIA. The number of Red and Blue users is 15 and all values are recorded in Mbps. By upgrading Red users to MPTCP, the throughput drops for all users and the aggregate throughput falls by 13%.

We emulate this scenario in our testbed in a similar manner as for Scenario A. The measurement results are reported in Table 1 for a setting with $C_X = 27$, $C_T = 36$, and $C_Z = 100$, all in Mbps, and where we have 15 Red and 15 Blue users. RTTs are 100ms over all paths. We observe that when Red users only connect to ISP Y, the aggregate throughput of users is close to the cut-set bound, 63 Mbps. However, Blue users get a higher share of the network bandwidth. Now consider that Red users upgrade to MPTCP by establishing a second connection through X (showed by dashed line in Figure 3). Our results in Table 1 show that Red users do not receive any higher throughput. However, the average rate of Blue users drops by 20%, which results in a drop of 13% in aggregate throughput. This confirms our analytical observation and shows that MPTCP with LIA is not Pareto-optimal and could penalize other MPTCP users without any benefit for anybody.

3.4 Scenario C: MPTCP users could be excessively aggressive towards TCP users.

We consider a scenario with N_1 multipath users, N_2 single-path users, and two APs with capacities N_1C_1 and N_2C_2 Mbps (see Figure 5). Multipath users connect to both APs and they share AP2 with single-path users.

If the allocation of rates was proportionally fair, multipath users should use AP2 only if $C_1 < C_2$ and all users would receive $(N_1C_1+N_2C_2)/(N_1+N_2)$. When $C_1 > C_2$, a fair multipath user should not transmit over AP2. This fair allocation is represented by dashed lines in Figure 5(b) when we take into account the minimum probing cost. However, using MPTCP with LIA, multipath users get a larger share of bandwidth as soon as $C_1 \geq C_2/(2+N_1/N_2)$. We show this analytically. Let p_1 and p_2 be the loss probabilities at APs, x_1 and x_2 be rates that a multipath user receives over its paths, and y be the rate of a single-path user. Assume all RTTs are the same. When $C_1/C_2 < 1/(2+N_1/N_2)$, we have $p_1 > p_2$ and all users receive the same rate: $x_1+x_2 = y = (C_1+C_2)/2$. When $C_1/C_2 > 1/(2+N_1/N_2)$, we have $p_1 < p_2$ and the fixed point formula of LIA gives:

$$x_1 = \frac{p_2}{p_1+p_2}\frac{1}{\text{rtt}}\sqrt{\frac{2}{p_1}} \quad \text{and} \quad x_2 = \frac{p_1}{p_1+p_2}\frac{1}{\text{rtt}}\sqrt{\frac{2}{p_1}}.$$

Moreover, both the APs are bottlenecks and we have $x_1 = C_1$ and $x_2+y = C_2$. As shown in Appendix C, this set of equations has a unique positive solution that only depends on the ratio N_1/N_2 and C_1/C_2. Figure 5(b) reports a numerical evaluation of these fixed point equations for the case $N_1 = N_2$. We observe that LIA is fair with regular TCP users, as long as $C_1 < C_2/3$. However, as C_1 exceeds $C_2/3$, it takes most of the capacity of AP2 for itself.

We emulate the scenario in our testbed and measure the performance of MPTCP with LIA. The results are reported on Figures 5(c) and 5(d) for C_2=1Mbps and C_1=1, 2Mbps, with N_2=10 and N_1=5, 10, 20, 30. Similarly as for scenario A, we also present the theoretical optimum with probing cost in Figure 5(c).

As $C_1/C_2 \geq 1$, multipath users should not use AP2 at all. However, our results show that, MPTCP users are disproportionately aggressive and exhibit problem (P2). Figure 5(d) shows the loss probability at AP2. We observe that LIA excessively increases congestion on AP2 and is unable to fully balance congestion in the network. Also, we have p_1=0.01, 0.003 if C_1=1, 2Mbps. These results confirm our analytical observation and show that LIA is overly aggressive towards TCP users.

4. OLIA: THE OPPORTUNISTIC LINKED INCREASES ALGORITHM

In this section, we introduce OLIA as an alternative for MPTCP's LIA. OLIA is a window-based congestion-control algorithm that couples the increase of congestion windows and uses unmodified TCP behavior in the case of a loss. The increase part of OLIA has two terms. The first term is an adaptation of Kelly and Voice's increase term and provides the Pareto-optimality. The second term, with α, guarantees responsiveness and non-flappiness. We first present the algorithm and its Linux implementation. Then, we illustrate with an example its operation and its difference with LIA.

4.1 Detailed Description of OLIA

Let $\mathcal{R}_u$ be the set of paths available to user u and let $r \in \mathcal{R}_u$ be a path. We denote by $\ell_{1r}(t)$ the number of bits that have been successfully transmitted by u over path r between the last two losses seen on r, and by $\ell_{2r}(t)$ the number of bits that are successfully transmitted over r after the last loss. If no losses have been observed on r, then $\ell_{1r}(t) = 0$ and $\ell_{2r}(t)$ is the total numbers of bits transmitted on r. Also, let $\ell_r(t) = \max\{\ell_{1r}(t), \ell_{2r}(t)\}$ and let $\mathrm{rtt}_r(t)$ and $w_r(t)$ be respectively RTT and the window on r at time t. We define

$$\mathcal{M}(t) = \left\{ i(t) \,|\, i(t) = \arg\max_{p \in \mathcal{R}_u} w_p(t) \right\} \quad (3)$$

$$\mathcal{B}(t) = \left\{ j(t) \,|\, j(t) = \arg\max_{p \in \mathcal{R}_u} \frac{\ell_p(t)}{\mathrm{rtt}_p(t)^2} \right\} \quad (4)$$

$\mathcal{M}(t)$ is the set of the paths of u with largest window sizes at time t. $\mathcal{B}(t)$ is the set of the paths at time t that are presumably the best paths for u ($1/\ell_r(t)$ can be considered as an estimate of packet loss probability on path r at time t, and the rate that path r can provide to a TCP user can be estimated by $\sqrt{2\ell_r(t)}/\mathrm{rtt}_r$ [17]).

Our algorithm is as follows[1]:

- For each ACK on path r, **increase** w_r by:

$$\frac{w_r/\mathrm{rtt}_r^2}{(\sum_{p \in \mathcal{R}_u} w_p/\mathrm{rtt}_p)^2} + \frac{\alpha_r}{w_r}, \quad (5)$$

- For each loss on path r, **decrease** w_r by $\frac{w_r}{2}$,

α_r is calculated as follows:

$$\alpha_r = \begin{cases} \dfrac{1/|\mathcal{R}_u|}{|\mathcal{B} \setminus \mathcal{M}|} & \text{if}\, r \in \mathcal{B} \setminus \mathcal{M} \neq \emptyset \\ -\dfrac{1/|\mathcal{R}_u|}{|\mathcal{M}|} & \text{if}\, r \in \mathcal{M} \,\text{and}\, \mathcal{B} \setminus \mathcal{M} \neq \emptyset \\ 0 & \text{otherwise.} \end{cases} \quad (6)$$

where $\mathcal{B}\backslash\mathcal{M}$ is the set of elements in $\mathcal{B}$ but not in $\mathcal{M}$, $\emptyset$ is the empty set, and $|\mathcal{R}_u|$ is the number of paths available to u at the time. Note that $\sum_{r \in \mathcal{R}_u} \alpha_r$=0.

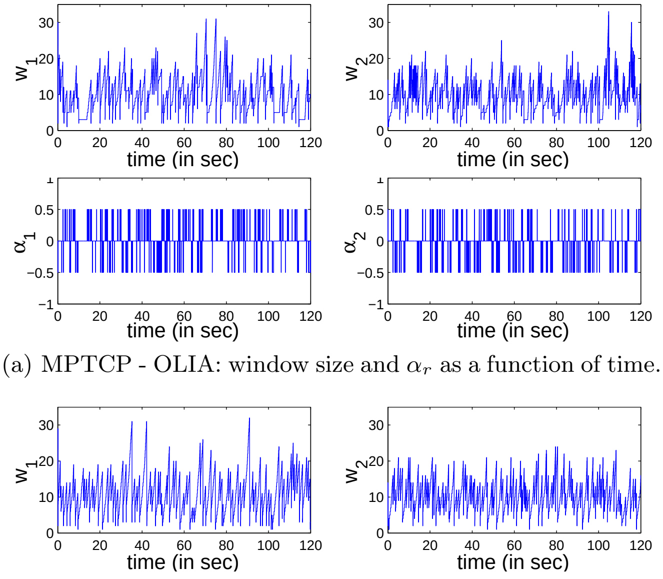
Remark 1. Kelly and Voice's algorithm is based on scalable TCP. The first term of Equation (5) is a TCP compatible version of their algorithm that compensates also for different RTTs.

Remark 2. We can see from (3), (4), and (6) that if the presumably best paths have maximum window size, then α_r=0 for any $r \in \mathcal{R}_u$. However, if there is any best path with small window size, *i.e.* if $\mathcal{B}\backslash\mathcal{M} \neq \emptyset$, then α_r would be positive if $r \in \mathcal{B}\backslash\mathcal{M}$, negative if $r \in \mathcal{M}$, and zero otherwise. Hence, OLIA increases windows faster on the paths that are presumably the best but have small windows, and slower on the paths with maximum windows.

4.2 Linux implementation of OLIA

We implemented OLIA in the MPTCP release supported on the Linux kernel 3.0.0 [18]. Similarly to LIA, our algorithm only applies to the increase part of the congestion avoidance phase. The fast retransmit and fast recovery algorithms, as well as the multiplicative decrease of the congestion avoidance phase, are the same as in TCP [5]. We also use a similar slow start algorithm as in TCP with the modification that we set the ssthresh (slow start threshold) to be 1 MSS if multiple paths are established. In the case of a single path flow, we use similar minimum ssthresh as in TCP (2 MSS). The purpose of this modification is to avoid transmitting unnecessary traffic over congested paths when multiple paths are available to a user. The minimum congestion windows size is 1 MSS as in TCP. Our implementation is available online [19].

[1] To simplify notations, we drop time argument t, however, note that w_r, rtt_r, ℓ_r, $\mathcal{M}$, and $\mathcal{B}$ are all functions of time.



(a) MPTCP - OLIA: window size and α_r as a function of time.

(b) MPTCP - LIA: window size.

Figure 6: Evolution of w and α values for a two paths flow. Each path is shared with 5 regular TCP users. OLIA uses both of the paths and there is no sign of flappiness.

One important part of our implementation is the measurement of ℓ_r on a path r. This can be done easily by using information that is already available to a regular TCP user. Our algorithm to compute ℓ_r is as follows:

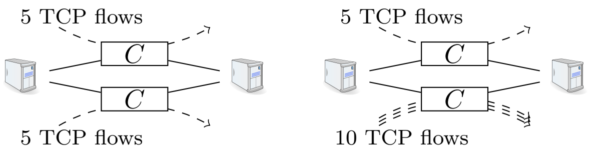
- For each ACK on r: $\ell_{2,r} \leftarrow \ell_{2,r}+$ (number of bits that are acknowledged by ACK)
- For each loss on r: $\ell_{1,r} \leftarrow \ell_{2,r}$ and $\ell_{2r} \leftarrow 0$

where $\ell_r = \max\{\ell_{1,r}, \ell_{2,r}\}$. $\ell_{1,r}$ and $\ell_{2,r}$ are initially set to zero when the connection is established. To compute a smoothed estimate of rtt_r, we use the algorithm, proposed in [20], and implemented in the Linux kernel.

4.3 Illustrative example of OLIA's behavior

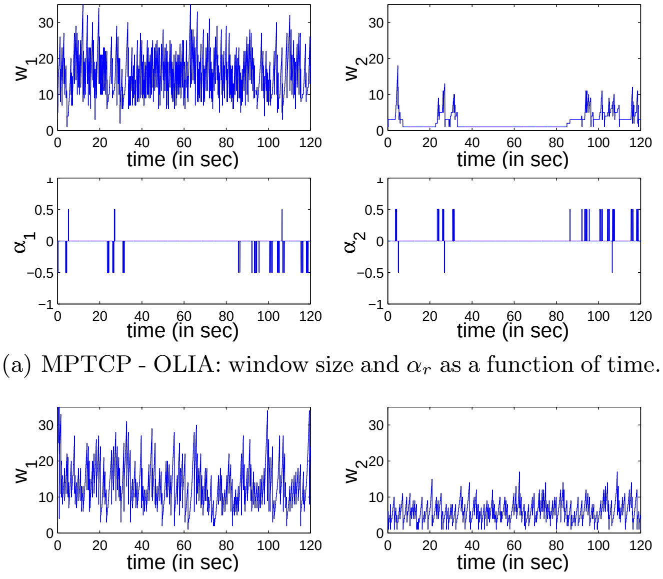
To give more insight into how OLIA performs, we show the evolution of window sizes and α values for a two-path flow (Fig. 7). The measurement results on our testbed are reported in Figures 6 and 8.

We first consider a symmetric case, depicted on Figure 7(a). As both of the paths are equally good, a multipath user should use both of them. Figure 6(a) shows the evolution of w_r and α_r as a function of time. We observe that OLIA simultaneously uses both of the paths, similarly to LIA (Fig. 6(b)), which is desired behavior. There is no sign of flappiness as α_1 and α_2 react quickly and adjust w_1 and w_2 accordingly.



(a) Symetric scenario (b) Asymetric scenario

Figure 7: A multipath user sharing two bottlenecks of same capacity C with single-path users.



(a) MPTCP - OLIA: window size and α_r as a function of time.

(b) MPTCP - LIA: window size.

Figure 8: Evolution of w and α for a two paths flow. The first path is shared with 5 TCP flows and the second with 10. OLIA uses only the good path. LIA transmits significant traffic over the congested path and less over the good.

We now study the asymmetric scenario of Figure 7(b). In this case, the second path is shared with 10 TCP flows and the multipath users should only use the first path, which is what we observe in Figure 8(a). The window on the congested path is 1 most of the time (because of the first increase term). However, due to α, the window increases from time to time over the congested path whenever the path has the largest inter-loss distance ℓ_r. This increase is brief since losses occur more frequently on this path. LIA, however, transmits significant traffic over the congested paths and lower traffic, compared to OLIA, over the good path (Fig. 8(b)).

5. PARETO-OPTIMALITY OF OLIA

In this section, we prove with a fluid model that OLIA provides a Pareto-optimal allocation of rates. We build a fluid model of OLIA using differential inclusions. We show that this model provides a Pareto-optimal allocation (Theorem 3) that satisfies the three design goals suggested by the RFC [1]. Finally, we show that MPTCP with OLIA is fair with TCP: if all routes of a user have the same RTT, then OLIA maximizes the same fairness criteria as the regular TCP (Theorem 4).

5.1 Fluid Model of OLIA

We consider a network model similar to [7]. The network is static and composed of a set $\mathcal{L}$ of links (or resources). We denote by $\mathcal{R}_u$ the set of paths available to a user u, each path being a set of links. If the route r is available to user u, we write $r \in R_u$. If a route r uses a resource ℓ, we write $\ell \in r$. Similarly, we refer to all routes that cross ℓ as $r \ni \ell$.

Let $x_r(t) \geq 0$ be the rate of traffic transmitted by the user u on a path $r \in \mathcal{R}_u$. We assume that the RTT of a route r is fixed in time and we denote it by rtt_r. In the fluid model, the rate x_r is an approximation of the window size divided by the RTT, i.e. $x_r = w_r/\mathrm{rtt}_r$.

Let $p_\ell(\sum_{\ell \in r} x_r)$ be the loss rate at link ℓ. p_ℓ depends on the capacity of the link, C_ℓ, and the total amount of traffic sent through the link, $\sum_{\ell \in r} x_r$. We assume that p_ℓ is an increasing function of the total traffic. To simplify the notation, we omit the dependence on x and write only p_ℓ. However, note that if x varies with time, p_ℓ will also vary. We assume that the loss probabilities of links are independent and small; hence, the loss probability on a route r is $p_r{=}1{-}\prod_{\ell \in r}(1{-}p_\ell) \approx \sum_{\ell \in r} p_\ell$.

When p_r is small, a user u receives acknowledgments on a route $r \in \mathcal{R}_u$ at rate x_r and increases the window w_r as Equation (5). Losses occur at rate $p_r x_r$ on r, and the user decreases w_r by half whenever it detects a loss. We consider a fluid approximation of OLIA in which we replace the stochastic variations of rates by their expectaion. This leads to the differential equation:

$$\frac{dx_r}{dt} = x_r^2 \left(\frac{1/\mathrm{rtt}_r^2}{(\sum_{p \in \mathcal{R}_u} x_p)^2} - \frac{p_r}{2} \right) + \frac{\alpha_r}{\mathrm{rtt}_r^2}, \qquad (7)$$

α_r depends on the values p_p and w_p for all paths $p \in \mathcal{R}_u$ of users u. It is defined by Equation (6). To compute α_r, we approximate ℓ_r by its average: $l_r = 1/p_r$.

For a user u, the set of best paths $\mathcal{B}_u$ and the set of paths with maximum window size $\mathcal{M}_u$ depend noncontinuously on the probability of loss on each route, as well as on the various window sizes of the routes of this user. This implies that the right-hand side of Equation (7) is not a continuous function of x_r. Therefore, this differential equation is not well defined and could have no solutions. A natural way to deal with a differential equation with a discontinuous right-hand size is to replace the differential equation (7) by a differential inclusion $dx/dt \in F(x)$ where the discontinuous α_r of (7) is replaced by the convex closure of the possible values of α in a small neighborhood of x. Differential inclusions have been proven to be a good approximation for stochastic system with discontinuous dynamics [21].

We show in Appendix D, that the differential inclusion corresponding to the Equation (7) is:

$$\frac{dx_r}{dt} = x_r^2 \left(\frac{1/\mathrm{rtt}_r^2}{(\sum_{p \in \mathcal{R}_u} x_p)^2} - \frac{p_r}{2} \right) + \frac{\bar{\alpha}_r}{\mathrm{rtt}_r^2} \qquad (8)$$

where $\bar{\alpha} = (\bar{\alpha}_1 \dots \bar{\alpha}_{|\mathcal{R}_u|})$ is such that:

$$\bar{\alpha}_r \in \begin{cases} [\alpha \mathbf{1}_{|\mathcal{B}_u|=1}, \alpha] & \text{if } r \in \mathcal{B}_u \setminus \mathcal{M}_u \\ [-\alpha, -\alpha \mathbf{1}_{|\mathcal{M}_u|=1}] & \text{if } r \in \mathcal{M}_u \setminus \mathcal{B}_u \\ [-\alpha \mathbf{1}_{|\mathcal{B}_u| \geq 2}, \alpha \mathbf{1}_{|\mathcal{M}_u| \geq 2}] & \text{if } r \in \mathcal{M}_u \cap \mathcal{B}_u \\ \{0\} & \text{if } r \notin \mathcal{M}_u \cup \mathcal{B}_u \end{cases} \qquad (9)$$

with $\sum_{r \in \mathcal{R}_u} \bar{\alpha}_r = 0$ and $\sum_{r \in \mathcal{B}_u} \bar{\alpha}_r = a$ if $\mathcal{B}_u \cap \mathcal{M}_u = \emptyset$. The notation $\mathbf{1}_{|\mathcal{B}_u|=1}$ means that this term is equal to 1 if $|\mathcal{B}_u| = 1$ and 0 otherwise. For example, when there is only one best path (*i.e.* $|\mathcal{B}| = 1$), $\alpha_r = a$ for $r \in \mathcal{B}_u \setminus \mathcal{M}_u$. If there are two or more best paths (*i.e.* $|\mathcal{B}| \neq 1$), then $\alpha_r \in [0, a]$ for $r \in \mathcal{B}_u \setminus \mathcal{M}_u$.

Note that there are multiple $\bar{\alpha}$ that correspond to definition (9). The differential inclusion might have multiple solutions, but this does not affect our analysis [19].

5.2 Pareto Optimality of OLIA

A fixed point of the congestion control algorithm (8) is a vector of rates $x = (x_1 \dots x_{|\mathcal{R}|})$ such that there exists $\bar{\alpha}$ satisfying (9) and such that, Eq.(8) is equal to zero for any route r. We say that x is a non-degenerate allocation of rates if each user transmits with a non-zero rate on at least one of its paths. In practice, due to reestablishment routines in traditional TCP, the allocation of rates will not be degenerate. Hence, in our analysis, we consider only the non-degenerate fixed points and analyze their properties.

Theorem 1. *Any non degenerate fixed point x of OLIA congestion control algorithm, given by Eq.*(8)*, has the following properties:*

(i) Only the best paths will be used, i.e. *paths r with maximum $\sqrt{2/p_r}/\mathrm{rtt}_r$.*

(ii) The total rate obtained by a user u is equal to the rate that a regular TCP user would receive on the best path available to u:

$$\sum_{r \in \mathcal{R}_u} x_r = \max_{r \in \mathcal{R}_u} \frac{1}{\mathrm{rtt}_r} \sqrt{\frac{2}{p_r}}.$$

Proof. The proof is given in Appendix E. □

This theorem implies the following corollary:

Corollary 2. *OLIA satisfies the three design goals suggested by the RFC [1].*

Proof. The proof is given in Appendix F. □

The following theorem gives a global optimality property of OLIA. For a rate allocation x, we define the total congestion cost by $C(x) = \sum_\ell \int_0^{\sum_{r \ni \ell} x_r} p_\ell(y) dy$.

Theorem 3. *Any non-degenerate fixed point x of our congestion control algorithm* (8) *is Pareto optimal, i.e.:*

- *It is impossible to increase the quantity $\sum_{r \in \mathcal{R}_u} x_r/\mathrm{rtt}_r^2$ for some users without decreasing it for others or increasing the congestion cost $C(x)$.*

Proof. The proof is given in Appendix G. □

Remark 1. If the probability p_ℓ is sharp around C_ℓ, i.e. if $p_\ell(y) \approx 0$ when $y < C_\ell$ and p_ℓ grows rapidly when y exceeds C_ℓ, then the cost C is a binary function: it is very small if the capacity constraints $\sum_{r\in\ell} x_r \leq C_\ell$ are respected and grows rapidly otherwise. In this case, Theorem 3 shows that if x is a fixed point of our algorithm, it is impossible to increase the quantity $\sum_{r\in\mathcal{R}_u} x_r/\text{rtt}_r^2$ for some users without decreasing it for others while respecting the capacity constraints.

Remark 2. As pointed out by Kelly [6], as $C(x)$ is an increasing function of rates, single-path congestion control mechanisms are always Pareto optimal and the choice of an allocation of rates is only a matter of fairness. However, if we have multiple paths, it is likely that an algorithm will lead to a non-Pareto optimal allocation [6]. Theorem 3 guarantees that this cannot happen with OLIA. As a consequence, our algorithm will not exhibit either problem P1 nor P2.

Remark 3. Although the utility function of each user $\sum_{r\in\mathcal{R}_u} x_r/\text{rtt}_r^2$ might appear an ad-hoc utility function, it reflects the fact that as TCP, OLIA favors paths with low rtt.

5.3 TCP Compatibility

We show that our algorithm is fair with the regular TCP under the assumption (A): all the paths belonging to a user u have the same RTT rtt_u. Let

$$V(x) = \sum_{u\in\mathcal{U}} -\frac{1}{\text{rtt}_u^2 \sum_{r\in\mathcal{R}_u} x_r} - \frac{1}{2}\sum_{l\in\mathcal{L}} \int_0^{\sum_{r\ni l} x_r} p_\ell(x)dx,$$

where x is the set of all the rates of the users.

Theorem 4. *Under the assumption (A), the congestion control algorithm defined by Eq.*(8) *converges to a maxima of the utility function V:*

$$\lim_{t\to\infty} V(x(t)) = \max_{x\geq 0} V(x).$$

Proof. The proof is given in Appendix H. □

This implies that OLIA maximizes the same utility function as the regular TCP of [22] where we replace the rate of a connection by the total rate that a user achieves on all its paths. If the probabilities of losses p_ℓ are sharp around C_ℓ, then our algorithm converges to an optimum of the following global maximization problem:

$$\max \sum_{u\in\mathcal{U}} -\frac{1}{\text{rtt}_u^2 \sum_{r\in\mathcal{R}_u} x_r} \quad \text{subject to} \quad \begin{cases} \sum_{r\ni\ell} x_r \leq C_\ell \\ x_r \geq 0. \end{cases}$$

This is analog to the TCP maximization problem.

6. OLIA EVALUATION: MEASUREMENTS AND SIMULATIONS

In this section, we study the performance of MPTCP with OLIA through measurements and by simulations. We first perform measurements on our testbed to show that OLIA outperforms LIA in all the scenarios from Section 3, as an evidence that OLIA solves Problems (P1) and (P2). Results from this section are in line with our theoretical analysis from Section 5. We then study the performance of OLIA in a data center using htsim simulator [2].

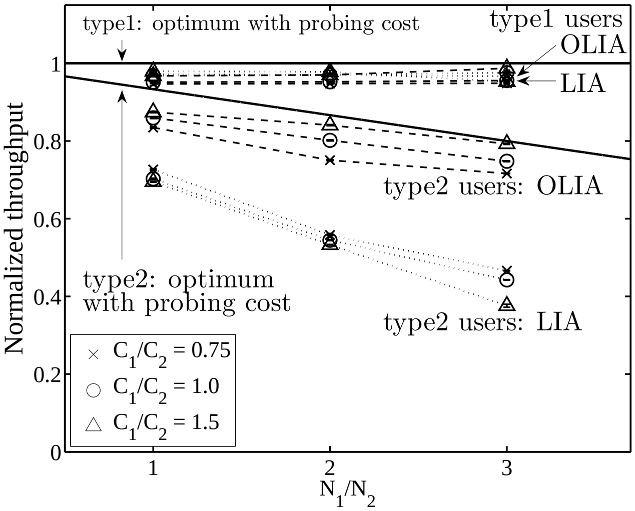


Figure 9: Scenario A - Normalized throughput of type1 and type2 users: we compare performance of LIA and OLIA. By using OLIA, type2 users achieve up to 2 times higher rates. OLIA performs close to the theoretical optimum with probing cost.

6.1 Performance of OLIA in Scenarios A,B,C

In this section, we study the performance of MPTCP with OLIA in the scenarios A,B and C described in Sections 3.2 to 3.4. We show that in practice, OLIA is very close to theoretical optimum with probing cost. These results are obtained through measurements over our testbed, by using our Linux implementation of OLIA.

6.1.1 Scenario A

We showed in Section 3.2 that when the addition of an extra link does not help (like in Scenario A), using MPTCP with LIA might reduce the throughput of competing TCP users. In this section, we show by measurements that MPTCP with OLIA significantly outperforms MPTCP with LIA and comes close to the theoretical optimum with probing cost.

Figures 9 and 10 report measurements obtained on the testbed shown in Figure 2. Figure 9 depicts the normalized throughput of type1 and type2 users using LIA or OLIA. The results show that OLIA performs close to an optimal multipath algorithm that transmits the minimum traffic over congested paths (theoretical optimum with probing cost). OLIA significantly out-

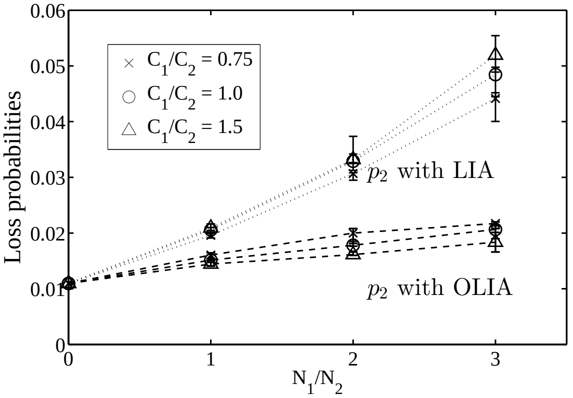


Figure 10: Scenario A - Loss probability p_2 at shared AP: we observe that OLIA significantly reduce the congestion level at this bottleneck and improve the congestion balancing. p_1 is almost the same using LIA or OLIA.

performs LIA: by using OLIA, type2 users achieve up to rates two times higher than with LIA, with no reduction for type1 users.

Figure 10 depicts the measured loss probability p_2 on the shared access point. We observe that OLIA balances congestion much better than LIA. When we use OLIA, p_2 increases only by a factor of 1.3 in the worst case, whereas with LIA, p_2 increases by a factor of 5. p_1 is almost the same using LIA or OLIA.

6.1.2 Scenario B

We now show the performance of OLIA in the scenario B described in Section 3.3. As we shown, OLIA is Pareto optimal. Hence, taking into account the minimum probing cost, we expect only 3% reduction in the Blue users' rates and in the aggregate throughput when we upgrade Red users to OLIA (see Figure 4(b)).

Table 2 presents the measurements for the scenario described in Section 3.3 using OLIA. We set C_X=27, C_T=36, C_Z=100, all in Mbps. We have 15 Red and 15 Blue users. We set RTTs to 100 ms over all paths. Our results show that there is 3.5% decrement in aggregate throughput when we update Red users to OLIA, which is much smaller than the 13% reduction we observed when we used LIA (see Table 1). This 3.5% reduction in the aggregate throughput is due to the minimum traffic transmitted by users over congested paths and cannot be reduced as it is bounded below by 1/rtt packets/sec.

Red users	Rate/user		Aggregate
	Blue users	Red users	
Single-path	2.2	1.8	59.3
Multipath	2.2	1.7	57.8

Table 2: Measurement results for scenario B showing the effect of upgrading the Red users from regular TCP to MPTCP with OLIA. We observe a small drop of 3.5% in the aggregate throughput, which is due to the overhead of minimum traffic (1/rtt) over the congested path. Compared to LIA (see Table 1), we see significant improvement.

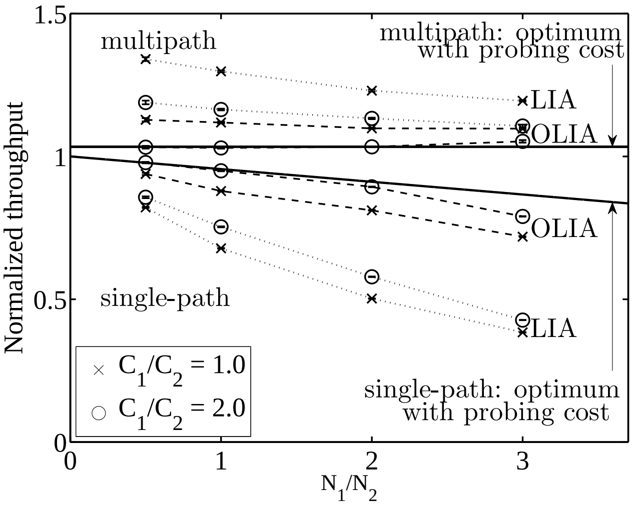


Figure 11: Scenario C - Normalized throughput single-path and multipath users: we compare the performance of LIA and OLIA. We observe that by using OLIA, type2 users achieve up to 2 times higher rates. OLIA performs close to the theoretical optimum with probing cost.

6.1.3 Scenario C

Finally, we study the performance of MPTCP with OLIA in scenario C described in Section 3.4. From theorems 1 and 4, we can show that using our algorithm, multipath users will not send any traffic on their paths crossing AP2. Hence, in theory, OLIA provides a fair allocation among the users and performs similarly as shown in Figure 5(b) (dashed lines). Next, we show by measurements that OLIA is also fair in practice.

Figure 11 depicts the normalized throughput of single-path and multipath users, as a function of N_1/N_2 and for C_1/C_2=1, 2. We show the results for LIA, OLIA, as well as for an optimal algorithm with minimum probing cost. This figure shows that with OLIA multipath users transmit only one packet per RTT over AP2. Compared to LIA, type2 users receive up to 2 times higher throughput. Hence, OLIA is less aggressive than LIA towards regular TCP users.

Figure 12, shows the measured loss probability p_2. The results show again that OLIA balances congestion in the network and reduces the loss probability in bottlenecks much better than LIA. In particular, we observe that by increasing N_1 from 0 to $3N_2$, p_2 increases by a factor of 2 using OLIA, whereas the increase is in order

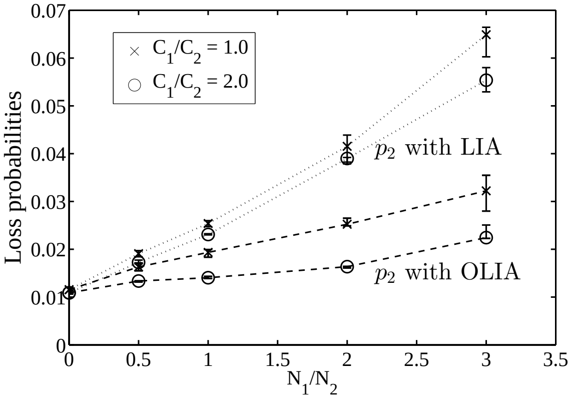
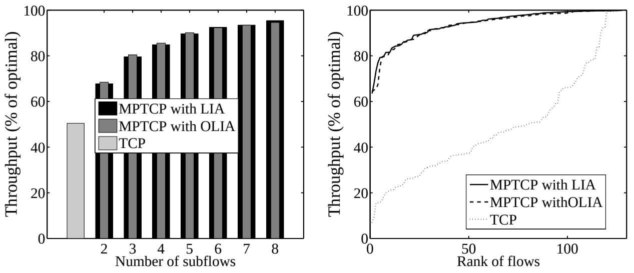


Figure 12: Scenario C - Loss probability p_2 at shared AP: we observe that OLIA significantly reduce the congestion level at this bottleneck (4 to 6 times lower compared tp LIA). p_1 is almost the same using LIA or OLIA.



(a) Aggregated throughput. (b) Throughput of users.

Figure 13: Performance of OLIA in a FatTree with many possible parallel paths between users. It successfully explores the path diversity that exists in the network and uses the available capacity (a sign of non-flappiness). LIA performs similarly, as in this scenario, it can successfully balance the congestion.

of 4 to 6 times when using LIA. p_1 is almost the same using OLIA or LIA.

6.2 Performance of OLIA in Data Center and Dynamic Scenarios

The three preceding examples shows that by providing a better congestion balance, MPTCP with OLIA outperforms the MPTCP with LIA in Scenarios A, B, and C. In this section, we show that OLIA can fully use the multiple paths available in a data center, by being non-flappy and as responsive as LIA.

Our study is based on a series of scenarios in which MPTCP with LIA has been studied in [2]. Because of space constraints, we present the results for only two of the cases where it was shown to be very efficient. We observe that OLIA performs as well or better than LIA in these two scenarios. This indicates that it is not flappy, and has a very good responsiveness. These results are obtained using `htsim` simulator used in [2], provided by Raiciu et al. We implemented OLIA in the simulator and used the same scenarios as [2].

6.2.1 *Static FatTree Topology*

We first study exactly the same scenario as in [2], Section 4.2-Throughput: the network is a FatTree with 128 hosts, 80 eight-port switches, 100Mb/s links. Each host sends a long-lived flow to another host chosen at random. Figure 13(a) shows the aggregate throughput achieved by long-lived TCP and MPTCP (LIA and OLIA) flows. We show the results for different numbers of subflows used. Our results show that OLIA can successfully exploit the multiple paths that exist in the network and can use the available capacity. This is a sign that it is not flappy. Moreover, we also measure the loss probabilities of links, and we observe that most of the users have multiple equally good paths. This shows that both LIA and OLIA successfully balance the congestion in this scenario and explains why they exhibit similar performances (OLIA is slightly better). Regular TCP shows a poor performance.

Figure 13(b) shows the throughput of individual users ranked in order of PDF of achieved throughputs, for LIA and OLIA with 8 subflows per user and with TCP. We see that both LIA and OLIA provide similar fairness among users and are more fair than TCP. The reason, as stated before, is that many paths have similar qualities.

6.2.2 *Dynamic Setting with Short Flows*

We study the same scenario as the one described in Section 4.3.4-ShowFlows of [2]. The scenario is a 4:1 oversubscribed FatTree where each host sends to one other host. One third of the hosts sends a continuous flow using either TCP, MPTCP with LIA (8 subflows) or with OLIA (8 subflows). The remaining hosts send short flows of size 70Kbyte every 200ms on average (they generate these flows according to a poisson process). They use regular TCP. This is a highly dynamic setting in which changes occur in order of milliseconds.

Table 3 shows the average completion time for short flows and the network core usage. Figure 14 shows the

algorithm	Short flow finish time (mean/stdev)	Network core utilization
MPTCP - LIA	98 ± 57 ms	63.2%
MPTCP - OLIA	90 ± 42 ms	63%
Regular TCP	73 ± 57 ms	39.3%

Table 3: Performance of OLIA in a highly dynamic setting. It uses the available capacity as efficient as LIA, but decreases the average completion time of short flows by 10%.

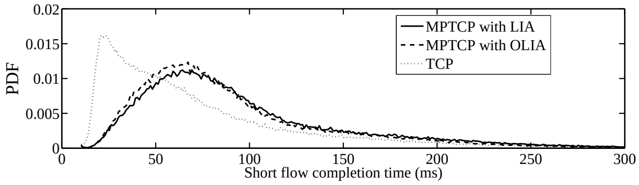


Figure 14: Completion time of short flows competing with long-lived TCP, MPTCP with LIA, or MPTCP with OLIA flows in a highly dynamic setting. OLIA reacts faster to the changes in the network and is more fair toward short flows.

distribution of short flows completion time. Our results show that although OLIA uses the available capacity as efficiently as LIA, the average completion time of short flows decreases by 10% using OLIA.

Moreover, we also observe in Figure 14 that OLIA decreases the completion time of both fast and slow short flows. With TCP, we have a lower average completion time for short flows but very low network utilization. This shows that OLIA has a better responsiveness than LIA and uses capacity quickly when it is available.

7. CONCLUSION

We have shown that MPTCP with LIA suffers from performance problems and that an alternative algorithm like OLIA should be considered. Our theoretical results show that OLIA is Pareto-optimal and satisfies the three design goals of the RFC [1]. Moreover, we have shown through measurements and by simulation that OLIA is as responsive and non-flappy as LIA, while it solves identified problems with LIA.

As we discussed in this paper, the minimum probing traffic (1 MSS per RTT) is a constraining factor. It can be reduced even further by adjusting the retransmit timer of TCP. We relegate this issue to the future.

8. REFERENCES

[1] C. Raiciu, M. Handly, and D. Wischik. Coupled congestion control for multipath transport protocols. *RFC 6356 (Experimental)*, 2011.

[2] C. Raiciu, S. Barre, C. Pluntke, A. Greenhalgh, D. Wischik, and M. Handly. Improving datacenter performance and robustness with multipath tcp. *ACM Sigcomm*, 2011.

[3] D. Wischik, M. Handly, and C. Raiciu. Control of multipath tcp and optimization of multipath routing in the internet. *NetCOOP*, 2009.

[4] D. Wischik, C. Raiciu, A. Greenhalgh, and M. Handly. Design, implementation and evaluation of congestion control for multipath tcp. *Usenix NSDI*, 2011.

[5] M. Allman, V. Paxon, and E. Blanton. Tcp congestion control. In *RFC 5681*, September 2009.

[6] F.P. Kelly. Mathematical modelling of the internet. *Mathematics unlimited-2001 and beyond.*

[7] F.P. Kelly, A.K. Maulloo, and D.K.H. Tan. Rate control for communication networks: shadow prices, proportional fairness and stability. *Journal of the Operational Research society*, 49, 1998.

[8] F. Kelly and T. Voice. Stability of end-to-end algorithms for joint routing and rate control. *ACM SIGCOMM CCR*, 35, 2005.

[9] H. Han, S. Shakkottai, CV Hollot, R. Srikant, and D. Towsley. Multi-path tcp: a joint congestion control and routing scheme to exploit path diversity in the internet. *ToN*, 14, 2006.

[10] W.H. Wang, M. Palaniswami, and S.H. Low. Optimal flow control and routing in multi-path networks. *Performance Evaluation*, 52, 2003.

[11] A. Ford, C. Raiciu, M. Handley, S. Barre, and J.Iyengar. Architectural guidelines for multipath tcp development. *RFC 6182 (informational)*, 2011.

[12] A. Ford, C. Raiciu, M. Handley, and O. Bonaventure. Tcp extensions for multipath operation with multiple addresses. *IETF Internet Draft*, 2011.

[13] C. Cetinkaya and E.W. Knightly. Opportunistic traffic scheduling over multiple network paths. In *INFOCOM*, 2004.

[14] M. Zhang, J. Lai, A. Krishnamurthy, L. Peterson, and R. Wang. A transport layer approach for improving end-to-end performance and robustness using redundant paths. In *USENIX*, 2004.

[15] M. Honda, Y. Nishida, L. Eggert, P. Sarolahti, and H. Tokuda. Multipath congestion control for shared bottleneck. In *PFLDNeT workshop*, 2009.

[16] E. Kohler, R. Morris, B. Chen, J. Jannotti, and M. F. Kaashoek. The click modular router. *ACM Trans. Comput. Syst.*, 18, 2000.

[17] V. Misra, W.-B. Gong, and D. Towsley. Fluid-based analysis of a network of AQM routers supporting TCP flows with an application to RED. In *SIGCOMM*, 2000.

[18] http://mptcp.info.ucl.ac.be/.

[19] R. Khalili, N. Gast, M. Popovic, U. Upadhyay, and J.-Y. Le Boudec. Non pareto-optimality of mptcp: Performance issues and a possible solution. *EPFL Technical report. Available at http://infoscience.epfl.ch/record/177901*, 2012.

[20] V. Jacobson. Congestion avoidance and control. In *ACM SIGCOMM CCR*, volume 18, 1988.

[21] N. Gast and B. Gaujal. Mean field limit of non-smooth systems and differential inclusions. *ACM SIGMETRICS*, 2010.

[22] S. Kunniyur and R. Srikant. End-to-end congestion control schemes: Utility functions, random losses and ecn marks. *ToN*, 11, 2003.

[23] M. Kunze. *Non-smooth dynamical systems.* Number 1744 in Lecture Notes in Mathematics. Springer Verlag, 2000.

APPENDIX

These appendix are divided in two parts. The first part (Appendix A to C) focuses on the proofs of the analytical results for LIA. It contains the fixed point analysis and the computation of the optimal allocation with probing cost for scenarios A, B and C. The second part (Appendix D to H) contains the proofs related to the Pareto optimality of OLIA.

A. FIXED POINT ANALYSIS FOR SCEN. A

In this appendix, we present a fixed point analysis of the scenarios A of Section 3. For more clarity, we represent the scenario A in Figure 15.

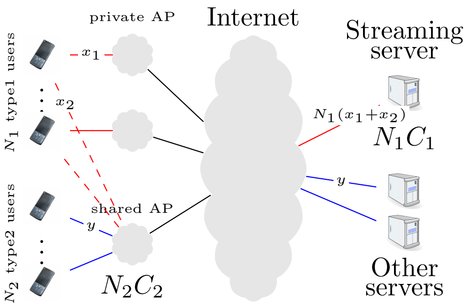


Figure 15: The scenario A of Section 3

Let us denote by p_1 and p_2 the loss probability for the streaming server and the shared AP. As we assume that the private APs are not the bottlenecks, the loss probabilities at the private APs are negligible. We assume that the RTT are the same over all connections and equal to rtt. Let x_1 be the rate of a type1 user over the path crosses its private AP and x_2 be its through over the shared AP. Let y be the rate of a type2 user.

The type1 users use MPTCP with LIA on two paths that have loss probabilities p_1 for the path through private AP and $p_1 + p_2$ for the path through the shared AP. Thus, the fixed point formula (Eq.(2)) of LIA gives:

$$x_1+x_2 = C_1 = \frac{1}{\text{rtt}}\sqrt{\frac{2}{p_1}} \quad \text{and} \quad x_2 = \frac{1}{2+p_2/p_1}\frac{1}{\text{rtt}}\sqrt{\frac{2}{p_1}}$$

Users of type 2 are using the regular TCP over a link with probability of loss p_2, they get a throughput

$$y = \frac{1}{\text{rtt}}\sqrt{2/p_2}.$$

This comes from the loss-throughput formula for TCP.

As the streaming server and shared AP are the bottlenecks, the capacity constraints give:

$$N_1(x_1+x_2) = N_1C_1 \quad \text{and} \quad N_1x_2 + N_2y = N_2C_2$$

Let $Z := \sqrt{p_1/p_2}$. A direct computation shows that Z is a root of

$$Z + \frac{Z^2}{1+2Z^2}\frac{N_1}{N_2} = \frac{C_2}{C_1} \tag{10}$$

As $Z^2/(1+2Z^2)$ is an increasing function of Z, this equation has a unique positive solution. Although this solution has no simple closed-form solution (it is the root of a third-order polynomial), it can be easily computed numerically. Hence, It provides a numerical scheme for computing x_1, x_2 and y.

Type1 users always receives a rate of C_1; hence, their normalized throughput, $(x_1+x_2)/C_1$, is always 1. The normalized throughput of type2 users, y/C_2, is equal to $\sqrt{p_1/p_2}\sqrt{2/p_1} = ZC_1$, where Z is the unique positive solution of Equation (10). In particular, this shows that y/C_2 only depends on the ratios C_1/C_2 and N_1/N_2.

A.1 Optimal with probing cost for scenario A

In scenario A, the throughput of type1 users is bounded by the streaming server. Using the shared AP can reduce the throughput of type2 users but cannot bring any gain to type1 users. Thus, an optimal algorithm should put as low traffic as possible on the second path.

Assuming that the minimum traffic sent over a link is one packet of size MSS per round trip time, this leads to the following allocation of rate:

$$x_1 + x_2 = \frac{C_1}{N_1} \quad \text{and} \quad x_2 = \frac{\text{MSS}}{\text{rtt}}$$
$$y = \frac{C_2}{N_2} - \frac{N_1}{N_2}\frac{\text{MSS}}{\text{rtt}}$$

This allocation is represented by the solid lines on Figure 1(b).

B. FIXED POINT ANALYSIS FOR SCEN. B

We present a theoretical analysis of the rate of blue and red users in scenario B when multipath users use MPTCP with LIA. We represent scenario B in Figure 16.

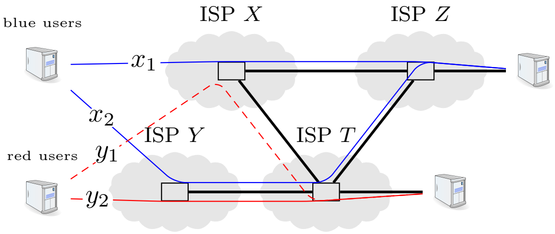


Figure 16: The scenario B of Section 3

We assume that the capacity of link Y and link Z are greater than $C_X + C_T$. This ensures that only links X and T are bottlenecks and we denote by p_X and p_T the probabilities of loss over them. If Red users are only connected to Y, the theoretical analysis is the same as the one of scenario C, and we refer to Section C for more details. In the case where all paths are activated, *i.e.* when Red users upgrade to MPTCP users, the loss throughput formula (Eq.(2)) for LIA shows that the throughput of the different connections are:

$$\begin{cases} y_1 = \dfrac{1/\text{rtt}}{2+\frac{p_X}{p_T}}\sqrt{\dfrac{2}{p_T}} \\ y_2 = \dfrac{p_X}{p_T} y_1 \end{cases}, \begin{cases} x_1 = \dfrac{1/\text{rtt}}{1+p_X/p_T}\sqrt{\max\dfrac{2}{p_X},\dfrac{2}{p_T}} \\ x_2 = \dfrac{1/\text{rtt}}{1+p_T/p_X}\sqrt{\max\dfrac{2}{p_X},\dfrac{2}{p_T}} \end{cases}$$

When $C_X/C_T < 5/9$, we have $p_X > p_T$. In that case, p_X/p_T is the root of the second order polynomial

$$2x^2 + x(5 - 2\frac{C_T}{C_X}) + 2 - 3\frac{C_T}{C_X}$$

When $C_X/C_T > 5/9$, we have $p_T > p_X$. In that case, $\sqrt{p_X/p_T}$ is the unique positive root of the fifth order polynomial:

$$Z^5+Z^4+Z^3(3-\frac{C_T}{C_X})+Z^2(2-\frac{C_T}{C_X})+Z(2-\frac{C_T}{C_X})-2\frac{C_T}{C_X}$$

These equation provide an efficient numerical method to evaluate the rate sent over the various links and therefore evaluate the performance of LIA. Note that he solutions of these equation only depend on C_T/C_X.

B.1 Optimal with probing cost for scenario B

To simplify the notations, we present the analysis for $N_1 = N_2 = N$, which is the case in the scenarios studied in Section 3. The analysis is similar when $N_1 \neq N_2$.

We distinguish two cases: first when red users use the regular TCP, then when red users uses an optimal multipath algorithm and activate the dashed connection.

B.1.1 Optimal when red users are single-path (dashed connections not activated)

Let us first assume that red users are only connected to ISPY. As ISP Y and Z are not bottlenecks, we have $x_1 = C_X/N$. Moreover, the capacity constraint for ISP T implies that $N(x_2 + y_2) = C_T$.

Assuming that $x_2 \geq$ MSS/rtt, there are two cases:

- When $C_X \leq C_T - N$MSS/rtt, a fair allocation will allocate the same rate, i.e. $(C_X+C_T)/(2N)$, to all users .
- When $C_X > C_T - N$MSS/rtt, blue users will get more than red users. Thus, blue users should only transmit the minimal traffic $x_2 =$ MSS/rtt over the second link.

This shows that using an optimal algorithm with probing, each blue user will get a rate $x_1 + x_2$ and each red user will get a rate y_2, where:

$$x_1 + x_2 = \max\left(\frac{C_X}{N} + \frac{\text{MSS}}{\text{rtt}}, \frac{C_T + C_X}{2N}\right) \qquad (11)$$

$$y_2 = \min\left(\frac{C_T}{N} - \frac{\text{MSS}}{\text{rtt}}, \frac{C_X + C_T}{2N}\right). \qquad (12)$$

B.1.2 Optimal when red users are multipath (dashed connections activated)

As y_1 and y_2 share the same bottleneck, ISP T, the Red users should only transmit the minimum traffic over the dashed path, i.e. $y_1 =$ MSS/rtt. Similarly to scenario A, if the Red users transmit over the dashed path they will penalize the other users without any benefit for themselves. This implies that $x_1 = C_X/N -$ MSS/rtt. Also, the capacity constraints for ISP T gives $N(x_2 + y_1 + y_2) = C_T$. Therefore, we have: $N(x_1 + x_2 + y_1 + y_2) = C_T + C_X - N$MSS/rtt.

As $x_2 \geq$ MSS/rtt, a fair allocation should allocate x_2 such that:

- if $C_X \leq C_T - N$MSS/rtt, we should have $x_1 + x_2 = y_1 + y_2 = (C_T + C_X - N\text{MSS/rtt})/(2N)$
- if $C_X \geq C_T - N$MSS/rtt, Blue users should transmit the minimal traffic $x_2 =$ MSS/rtt over their second link.

Thus, using this optimal algorithm with probing cost, each Blue user will get a rate $x_1 + y_2$ and each Red user will get a rate $y_1 + y_2$ where

$$x_1{+}x_2 = \max\left(\frac{C_X}{N}, \frac{C_T + C_X}{2N} - \frac{\text{MSS}}{2\text{rtt}}\right) \qquad (13)$$

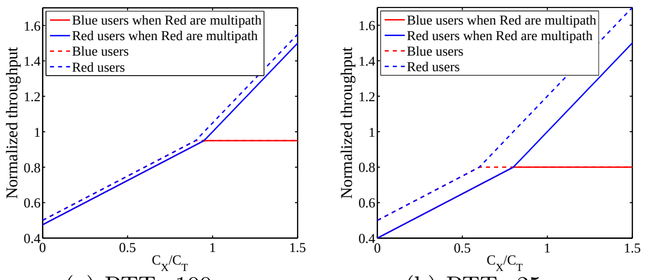
$$y_1{+}y_2 = \min\left(\frac{C_T}{N} - \frac{\text{MSS}}{\text{rtt}}, \frac{C_X{+}C_T}{2N} - \frac{\text{MSS}}{2\text{rtt}}\right) \qquad (14)$$

Compared to Equations (11) and (12), the rates obtained by (13) and (14) are strictly smaller. The agregate throughput of all users decreases by NMSS/rtt.

B.1.3 Illustrations for two values of RTT

Figure 17 depicts the throughput reduction when upgrading Red users to multipath for an optimal algorithm with probing cost. The values are shown for $C_X = 27$ Mbps, $C_T = 36$ Mbps and $N_1 = N_2 = 15$ users. The values of the MSS is 1500 Bytes. Since the minimal probing traffic sent over a link is MSS/rtt, a lower value of the RTT means a higher reduction of throughput.

When $N_1 = N_2 = N$, the total rate of users decreases by NMSS/rtt. If we take the values of our testbet (MSS = 1500B, a RTT of 100ms, $C_X = 27$ Mbps, $C_T = 36$ Mbps and $N_1 = N_2 = 15$), this corresponds to 2.8% of reduction of the agregate throughput. This is much lower than the 13% reduction caused by MPTCP with LIA (Table 1) but comparable with the 3.5% reduction caused by OLIA (Table 2).



(a) RTT=100ms (b) RTT=25ms

Figure 17: Illustration of the optimal allocation with probing for scenario B for two values of the RTT. We set $C_T = 36$ Mbps and N_1=N_2=15 users.

C. FIXED POINT ANALYSIS FOR SCEN. C

Let us assume that multipath users use LIA to balance their traffic over the two paths of scenario C. We represent scenario C in Figure 18.

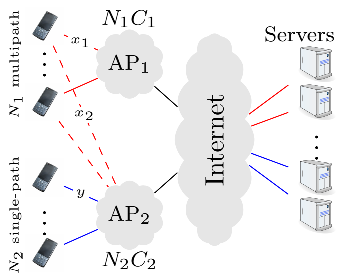


Figure 18: The scenario C of Section 3

Let p_1 and p_2 be the packet loss probability at access points, x_1 and x_2 be rates that a multipath user receives over its connections, and y be the rate of a single-path user. Assume that all RTT are rtt.

When $C_1/C_2 < 1/(2 + N_1/N_2)$, we have $p_1 > p_2$ and all users receive the same rate: $x_1 + x_2 = y = (C_1 + C_2)/2$. The normalized throughputs of users are $y/C_2 = (1+C_1/C_2)/2$ and $(x_1+x_2)/C_1 = (1+C_2/C_1)/2$

When $C_1/C_2 > 1/(2 + N_1/N_2)$, we have $p_1 < p_2$ and the fixed point formula of LIA (Eq.(2)) is:

$$x_1 = \frac{p_2}{p_1 + p_2}\frac{1}{\text{rtt}}\sqrt{\frac{2}{p_1}}$$

$$x_2 = \frac{p_1}{p_1 + p_2}\frac{1}{\text{rtt}}\sqrt{\frac{2}{p_1}} = \frac{p_1}{p_2}x_1.$$

Moreover, both links are bottlenecks and we have $x_1 = C_1$ and $x_2 + y = C_2$. Let $Z := p_1/p_2$. Using that the TCP loss throughput formula, $y = \sqrt{2/p_2}$, the quantity Z is the unique positive root of:

$$Z^3 + \frac{N_1}{N_2}Z^2 + Z - \frac{C_2}{C_1}.$$

Hence, the normalized throughputs of multipath users are $(x_1+x_2)/C_1 = 1+Z$. The single path users receive a rate of $y/C_2 = 1 - \frac{C_1}{C_2}Z$. Again, this quantity only depends on the ratio N_1/N_2 and C_1/C_2. Moreover, it provides an efficient way to evaluate numerically the performance of LIA.

C.1 Optimal with probing cost for scenario C

The analysis is similar to what we proposed in Section B.1.2.

D. CONSTRUCTION OF THE DIFFERENTIAL INCLUSION

D.1 Brief introduction on differential inclusions

In this section, we briefly recall some definitions and results about differential inclusions and their relation to stochastic systems that have discontinuous drifts.

A set valued function $F : \mathbb{R}^d \to \mathcal{S}(\mathbb{R}^d)$ is a function that associates to each vector $x \in \mathbb{R}^d$ a set of vectors $F(x) \subset \mathbb{R}^d$. We say that a function $x : [0, T] \to \mathbb{R}^d$ is a solution of the differential inclusion $dx/dt \in F(x)$ on the interval $[0, T]$ if there exists a function $f : [0, T] \to \mathbb{R}^d$ such that

$$\forall t \in [0, T] : x(t) = x(0)+\int_0^t f(s)ds \quad \text{with} \quad f(t) \in F(t).$$

In particular, this implies that x is differentiable for almost every t and its derivative x' satisfies $x'(t) \in F(x(t))$.

In general, a differential inclusion may have multiple solutions and verifying that it has only one solution can be difficult. Nevertheless, proving the existence of solutions is easier. In particular, if for all x, $F(x) \neq \emptyset$ is compact and convex and the graph of F, the set of all $\{(x, y)$ with $x \in \mathbb{R}^d$ and $y \in F(x)\}$, is a closed set, then the differential inclusion $dx/dt \in F(x)$ has at least one solution on some interval $[0, T]$ [23].

Differential inclusion provide a natural way to represent differential equation with discontinuous right-hand side. Let $f : \mathbb{R}^d \to \mathbb{R}^d$ be a single-valued function. If f is Lipschitz continuous, then the differential equation $dx/dt = f(x)$ has a unique solution. However, when f is not continuous, it often has no solutions. Following [21], we define the set-valued function F corresponding to f:

$$F(x) = \bigcap_{\varepsilon\to 0} \text{convex\_closure}\{f(x) : \|x - w\| \leq \varepsilon\}.$$

This definition guarantees that $F(x)$ is non-empty, compact, convex and that the graph of F is closed. Therefore, it shows that $dx/dt \in F(x)$ has at least one solution. Moreover, it has been shown in [21] that the solution of differential inclusions are a good approximation of the stochastic systems with discontinuous drift, such as Eq.(7).

For example, let $f(x) = -1$ for $x > 0$ and 1 for $x \leq 0$. The set-valued F corresponding to f is $F(x) = \{-1\}$

if $x > 0$, $F(x) = \{1\}$ for $x < 0$ and $F(0) = [-1, 1]$. While the differential equation $dx/dt = f(x)$ has no solution starting in 0, the differential inclusion has a unique solution starting in 0.

D.2 Computation of Equation (9)

In this section, we show how to obtain the conditions on α given by Eq.(9) and how to compute the differential inclusion (8) from the differential equation (7).

The only non-continuous part of the ODE (7) is due to α_r. The set-valued function $\bar{\alpha}$ corresponding to α is

$$\bar{\alpha}(w) = \bigcap_{\varepsilon \to 0} \text{convex\_closure}\left\{\alpha(x) : \|x - w\| \leq \varepsilon\right\}.$$

The computation of $\bar{\alpha}$ can be done by a careful inspection of Figure 19. For a route r, the set $\bar{\alpha}_r$ corresponds to the convex closure of the values that α_r can take when all the points $(w_r, p_r/\text{rtt}_r)$ move in a small neighborhood. We detail the computation for a link $r \in \mathcal{B}_u \setminus \mathcal{M}_u$. The other cases ($r \in \mathcal{M}_u \setminus \mathcal{B}_u$ and $r \in \mathcal{M}_u \cap \mathcal{B}_u$ and $r \notin \mathcal{M}_u \cup \mathcal{B}_u$) are similar.

Let r be a route in $\mathcal{B}_u \setminus \mathcal{M}_u$. Let us first assume that there are two or more best paths (*e.g.* this is the case for the route r_4 of Figure 19), then if all points move in a small neighborhood (represented by the dotted circles around nodes on Figure 19), then there are some situations for which this route will be the only route in $\mathcal{B}_u \setminus \mathcal{M}_u \neq \emptyset$ and α_r will be equal to a in that case. In other situations, the only best route can be route r_1 and in that case $\alpha_r = 0$. Since this route cannot become a route with maximum window size, α_r can take any value in $[0, a]$.

On the other hand, if there is only one best paths and if $r \in \mathcal{B}_u \setminus \mathcal{M}_u$, then r is the best path. (this would be the case for the route r_4 on Figure 19 if the node r_1 did not exist). In that case, r will always be in $\mathcal{B}_u \setminus \mathcal{M}_u$ and $\alpha_r = a$.

This shows the first line of Equation (9): for $r \in \mathcal{B}_u \setminus \mathcal{M}_u$:

$$\alpha_r \in [0, a] \text{ if } |\mathcal{B}_u| \neq 1 \quad \text{and} \quad \alpha_r = a \text{ otherwise.}$$

We omit the proofs of the other cases of Eq.(9) since they are very similar.

E. PROOF OF THEOREM 1

Let x be a non-degenerate fixed point of our algorithm. Recall that a fixed point of the congestion control algorithm (8) is a rate allocation vector x such that there exists α_r satisfying Eq.(9) such that the quantity dx_r/dt defined by Eq. (8) is null.

Proof of *(i)*. We prove *(i)* by contradiction.

Let us consider a user u and assume that there exists a route $r \notin \mathcal{B}_u$ such that $x_r > 0$. Since x is a fixed point, for every route r, we have:

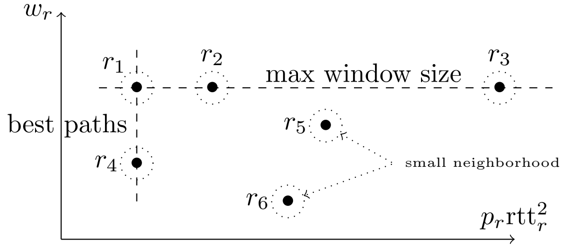


Figure 19: State of the routes of a user that has 6 routes at a given time. Each route is represented by a point •, its x-coordinate being the ratio $p_r\text{rtt}_r^2$ of a link (the inverse of its hypothetical rate for a single regular TCP flow) and its y-coordinate being its window size. The dotted circles around each node represent a small neighborhood of the points. In this example, the routes r_1, r_2, r_3 are routes with maximum window size: $r_1, r_2, r_3 \in \mathcal{M}_u$ while the routes r_1, r_4 are best routes: $r_1, r_4 \in \mathcal{B}_u$. Finally, we have $r_5, r_6 \notin \mathcal{B}_u \cup \mathcal{M}_u$.

$$\frac{dx_r}{dt} = x_r^2 \left(\frac{1/\text{rtt}_r^2}{(\sum_{p \in \mathcal{R}_u} x_p)^2} - \frac{p_r}{2} \right) + \frac{\bar{\alpha}_r}{\text{rtt}_r^2} = 0. \qquad (15)$$

By definition of $\bar{\alpha}$, and since $r \notin \mathcal{B}_u$, we have $\bar{\alpha}_r \leq 0$. As x satisfies Eq. (15), this implies that

$$\frac{1}{(\sum_{p \in \mathcal{R}_u} x_p)^2} - \text{rtt}_r^2 \frac{p_r}{2} \geq 0 \quad \text{for } r. \qquad (16)$$

We show that Eq. (16) leads to a contradiction. For any path $p \in \mathcal{R}_u$, the equation dx_p/dt contains two terms, denoted term A and term B in the next equation:

$$0 = \frac{dx_p}{dt} = \underbrace{x_p^2 \left(\frac{1/\text{rtt}_p^2}{(\sum_{s \in \mathcal{R}_u} x_s)^2} - \frac{p_p}{2} \right)}_{\text{term } A} + \underbrace{\bar{\alpha}_p}_{\text{term } B}. \qquad (17)$$

Because of (16), the term A is non-negative for any path p such that $\text{rtt}_p^2 p_p \leq \text{rtt}_r^2 p_r$ which is in particular the case for all best-paths. Moreover, by definition of $\bar{\alpha}$, α_p is non-negative for a best-path.

Now, if there exists $p \in \mathcal{B}_u \cap \mathcal{M}_u \neq \emptyset$, we necessarily have $x_p \neq 0$ which means that the term A is strictly positive and therefore $dx_p/dt > 0$. If $\mathcal{B}_u \cap \mathcal{M}_u = \emptyset$, then there exists $p \in \mathcal{B}_u$ such that $\alpha_p > 0$ which implies that term B is strictly positive and thus $dx_p/dt > 0$. In both cases, we have $dx_p/dt > 0$ which contradicts that $dx_p/dt = 0$.

This shows that Eq.(16) leads to a contradiction which means that $x \notin \mathcal{B}_u$ is not possible.

Proof of *(ii)*. Because of *(i)*, for all routes $r \notin \mathcal{B}_u$, we have $x_r = 0$. This means that for all routes $r \notin \mathcal{B}_u$, we have $r \notin \mathcal{M}_u$ and $\bar{\alpha}_r = 0$. The best paths are the set of paths p with minimum $p_p\mathrm{rtt}_p^2$. Therefore, the term A of Eq. (17) is of the same sign for all best paths. This implies that the term $\bar{\alpha}_p$ is of the same sign for all p. Since $\sum_{p\in u}\bar{\alpha}_p = \sum_{p\in\mathcal{B}_u}\bar{\alpha}_p = 0$, this implies that $\bar{\alpha}_p = 0$ for all paths $p \in \mathcal{R}_u$.

This implies that the fixed point x satisfies

$$x_r = 0 \quad \text{or} \quad \sum_{p\in\mathcal{R}_u} x_p = \frac{1}{\mathrm{rtt}_r}\sqrt{\frac{2}{p_r}}. \tag{18}$$

By assumption, x is non-degenerate, which means that there exists a route $r \in \mathcal{R}_u$ such that $x_r \neq 0$. Because of *(i)*, r is a necessarily a best path. Therefore, we have

$$\sum_{p\in\mathcal{R}_u} x_p = \frac{1}{\mathrm{rtt}_r}\sqrt{\frac{2}{p_r}} = \max_p \frac{1}{\mathrm{rtt}_p}\sqrt{\frac{2}{p_p}}.$$

This concludes the proof of *(ii)*. □

F. PROOF OF COROLLARY 2

Point *(i)* of Theorem 1 implies that OLIA improves the throughput compared to regular TCP in the sense that the total rate that OLIA gets ($\sum_{r\in\mathcal{R}_u} x_r$) is the same as the rate that a regular TCP would get on its best link ($\max_{r\in\mathcal{R}_u}\sqrt{2/p_r}/\mathrm{rtt}_r$).

Moreover, as OLIA only uses its best paths, the rate sent over a path $r \notin \mathcal{B}_u$ is equal to 0 and the rate sent over a path $r \in \mathcal{B}_u$ is $x_r \leq \sum_{i\in\mathcal{R}_u} x_i = \max_{i\in\mathcal{R}_u}\sqrt{2/p_i}/\mathrm{rtt}_i = \sqrt{2/p_r}/\mathrm{rtt}_r$. This shows that OLIA does not transmit more than regular TCP on any of its paths. Thus, it satisfies the goal 2. Finally, since OLIA only uses its best path, it does perfect congestion balancing and satisfies goal 3.

G. PROOF OF PARETO-OPTIMALITY (THM.3)

Let x^* be a fixed point of the algorithm and define the utility function $V^*(x)$ as

$$\sum_{u\in\text{users}} -\frac{1}{\tau_u^2\sum_{r\in\mathcal{R}_u}\frac{x_r}{\mathrm{rtt}_r^2}} - \frac{1}{2}\sum_{\ell\in\text{links}}\int_0^{\sum_{r\in\ell}x_r} p_\ell(x)dx,$$

where τ_u is defined by: $\tau_u = (\sum_{r\in\mathcal{R}_u} x_r^*)/(\sum_{r\in\mathcal{R}_u} x_r^*/\mathrm{rtt}_r^2)$.

The function V^* is a non-positive function. Moreover, using that $p_\ell(x)$ is increasing, it goes to $-\infty$ when $x \to \infty$. Therefore, it has a maximum, attained for a finite x. By concavity of V^*, a necessary and sufficient condition for a point x to be a maximizer of $\mathcal{U}$ is that for every route r:

$$\frac{\partial V^*}{\partial x_r}(x) \leq 0 \quad \text{and} \quad \frac{\partial V^*}{\partial x_r}(x) = 0 \text{ or } x_r = 0.$$

By definition of V^*, this is means that for every r:

$$\frac{1}{\tau_u^2}\frac{1/\mathrm{rtt}_r^2}{(\sum_{r\in\mathcal{R}_u} x_r/\mathrm{rtt}_r^2)^2} - \frac{p_r}{2} \leq 0, \tag{19}$$

$$\frac{1}{\tau_u^2}\frac{1/\mathrm{rtt}_r^2}{(\sum_{r\in\mathcal{R}_u} x_r/\mathrm{rtt}_r^2)^2} - \frac{p_r}{2} = 0 \text{ or } x_r = 0. \tag{20}$$

The definition of τ_u and the fact that x^* satisfies point *(i)* of Theorem 1 implies (19). The fact that x^* satisfies (18) implies (20).

This shows that x^* is a maximum of the function V^*. Since $V^*(x)$ is an increasing function of $\sum_{r\in\mathcal{R}_u} x_r/\mathrm{rtt}_r^2$ and a decreasing function of the congestion cost $C(x)$, it is impossible to increasing $\sum_{r\in\mathcal{R}_u} x_r/\mathrm{rtt}_r^2$ for some users without decreasing it for others or increasing the cost. □

H. PROOF OF TCP-COMPATIBILITY (THM.4)

Theorem 4 assumes that all the paths belonging to user u have the same round trip time rtt_u. Recall that V is:

$$V(x) = \sum_{u\in\mathcal{U}} -\frac{1}{\mathrm{rtt}_u^2\sum_{r\in\mathcal{R}_u} x_r} - \frac{1}{2}\sum_{l\in\mathcal{L}}\int_0^{\sum_{r\in l}x_r} p_\ell(x)dx.$$

By construction of F, there exists at least one solution of the differential inclusion given by Eq.(8) (see D.1). Let x be one of these solutions. In fact there could be multiple solutions. In that case, x denote any of them. Then, there exists a function $\bar{\alpha}(t) = (\bar{\alpha}_1(t)\dots\bar{\alpha}_{|\mathcal{R}|}(t))$ satisfying Eq.(9) for all t and such that

$$\frac{dx_r}{dt} = x_r^2\left(\frac{1/\mathrm{rtt}_r^2}{(\sum_{p\in\mathcal{R}_u} x_p)^2} - \frac{p_r}{2}\right) + \frac{\bar{\alpha}_r(t)}{\mathrm{rtt}_r^2}.$$

Let us compute the derivative of the utility $V(x(t))$ with respect to time when running the algorithm:

$$\frac{d}{dt}V = \sum_{u\in\mathcal{U}}\sum_{r\in\mathcal{R}_u}\frac{\partial V}{\partial x_r}\frac{dx_r}{dt}$$

$$= \sum_{u\in\mathcal{U}}\sum_{r\in\mathcal{R}_u} x_r^2\left(\frac{1}{\mathrm{rtt}_u^2(\sum_{p\in\mathcal{R}_u} x_p)^2} - \frac{p_r}{2}\right)^2 \tag{21}$$

$$+ \sum_{u\in\mathcal{U}}\sum_{r\in\mathcal{R}_u}\left(\frac{1}{\mathrm{rtt}_u^2(\sum_{p\in u} x_p)^2} - \frac{p_r}{2}\right)\frac{\bar{\alpha}_r}{\mathrm{rtt}_u^2} \tag{22}$$

By definition of $\bar{\alpha}$, we have $\sum_{r\in\mathcal{R}_u}\alpha_r = 0$. Moreover, when all rtt are equal, the best paths are the paths with minimal probability loss and $\alpha_r \leq 0$ for such paths. Thus:

$$\sum_{r\in\mathcal{R}_u}\alpha_r p_r = \sum_{r\in\mathcal{B}_u}\alpha_r p_r + \sum_{r\notin\mathcal{B}_u}\alpha_r p_r \leq \sum_r \alpha_r p_{\min} = 0.$$

These two properties together show that the term (22) is non-negative. Since (21) is also non-negative, this shows that $dV(x(t))/dt \geq 0$ for all t. Thus, the function V is non decreasing. Since V is non-positive, this shows that $\lim_{t\to\infty} dV(x(t))/dt = 0$.

Let x^* be a limit point of $x(t)$, which exists since $x(t)$ remains in a compact set. Since $\lim_{t\to\infty} dV(x(t))/dt = 0$, this implies that (21) and (22) are equal to 0 for this

x^*. In particular, this implies that for all $r \in \mathcal{R}_u$:

$$\frac{1}{\mathrm{rtt}_p^2 (\sum_{p \in \mathcal{R}_u} x_p^*)^2} = \frac{p_r}{2} \text{ or } (x_r = 0 \text{ and } \alpha_r = 0).$$

This shows that x^* is a fixed point of the algorithm. When the RTT of all paths of a user u are equal to rtt_u, the quantity τ_u defined in the proof of Theorem 3 is equal to rtt_u^2. Thus, the function V^* of the proof of Theorem 3 is equal to V. In particular, V^* does not depend on x^*. Since x^* is a fixed point of the algorithm, this implies that x^* is a maximizer of V. □