Curved Origami beams

Statuseminar SAH 2012
24 April 2012

Hani BURI
Laurent HUMBERT
Yves WEINAND

Outline

1) Origami beam geometry

- Origami principles
- Parameterization

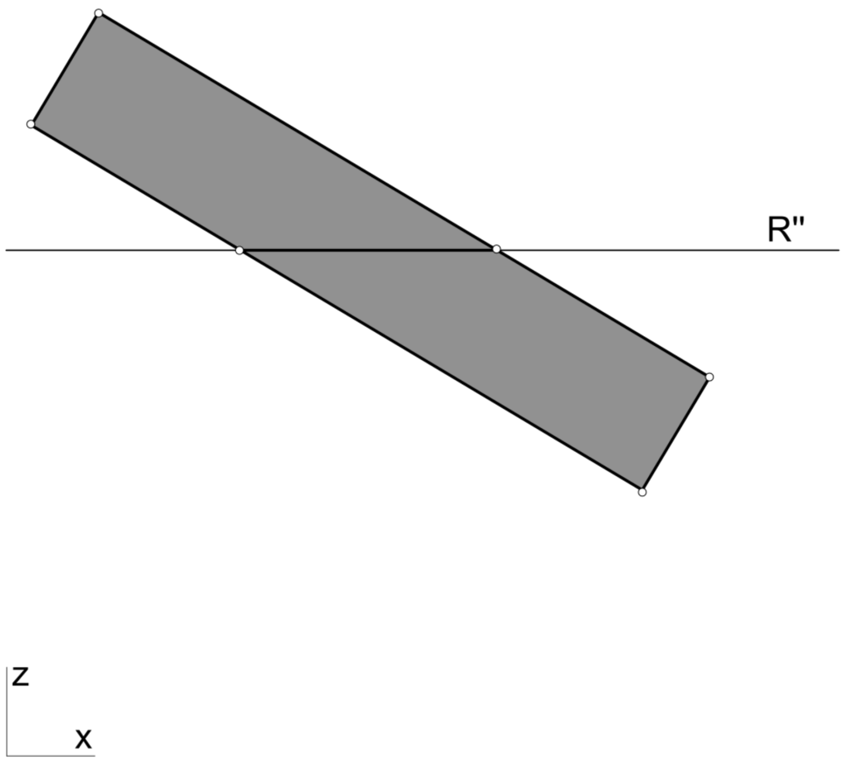
2) Experimental investigation

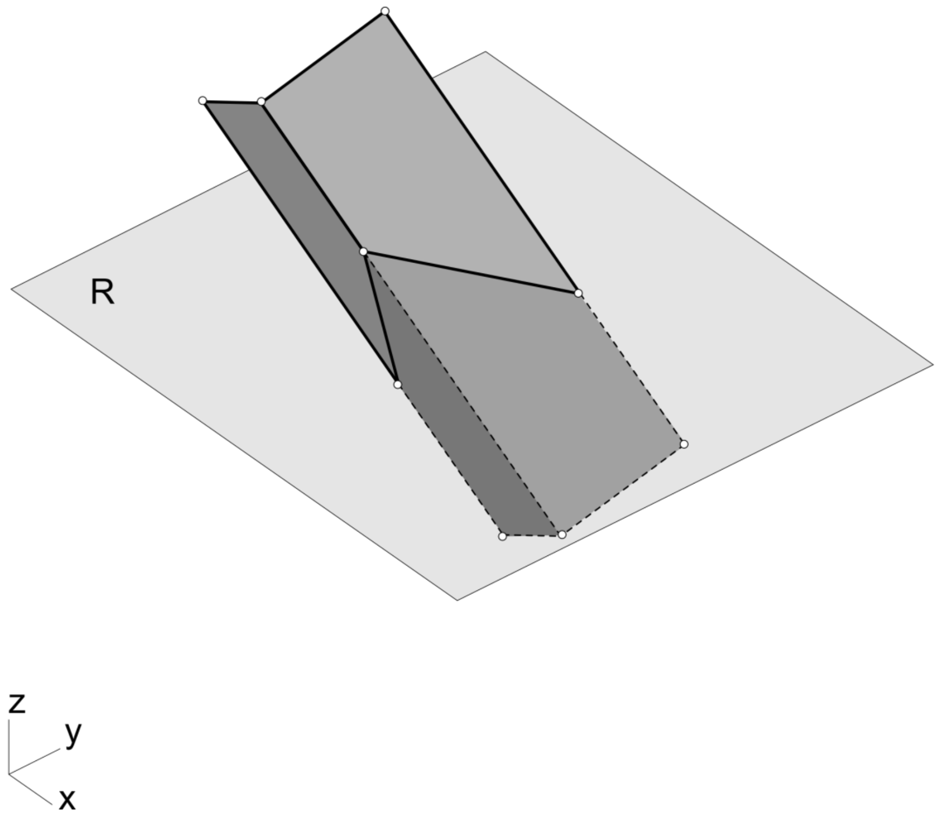
3) Numerical study

Conclusion

1) Curved prototypes geometry

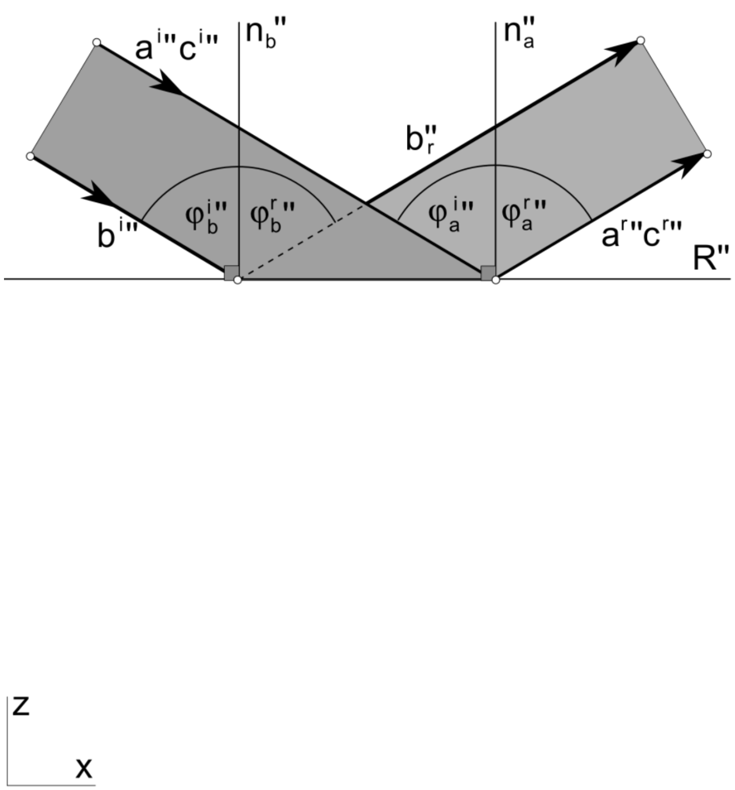
Origami principles :
Reverse fold as a reflection about a plane
a) with planar elements

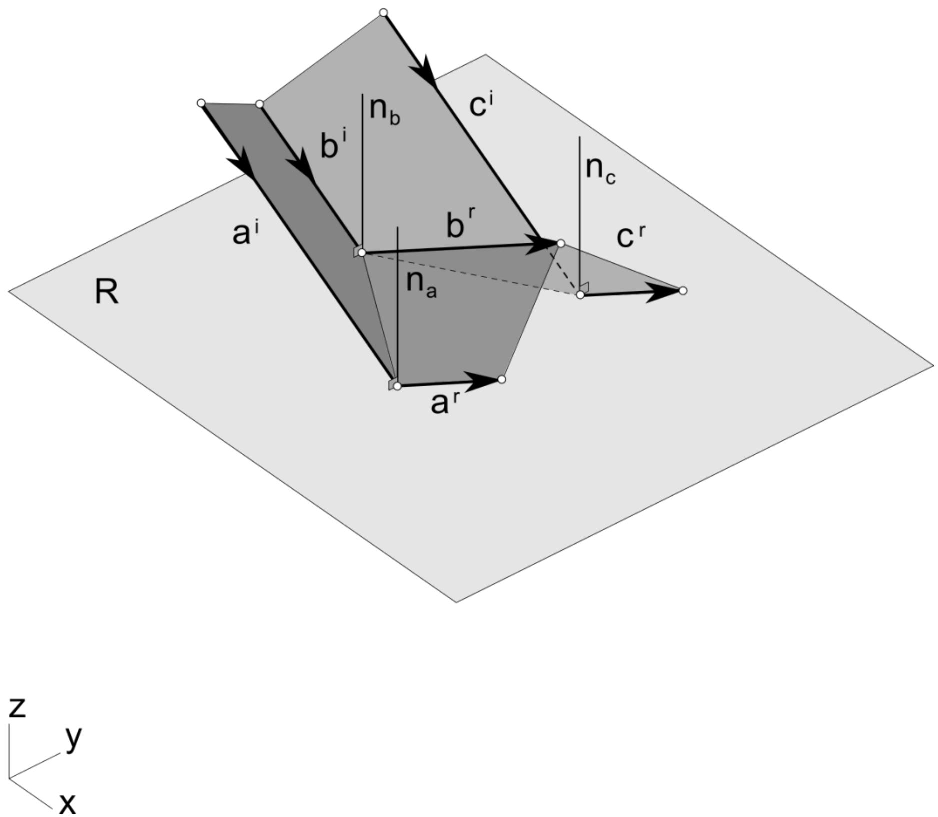




1) Curved prototypes geometry

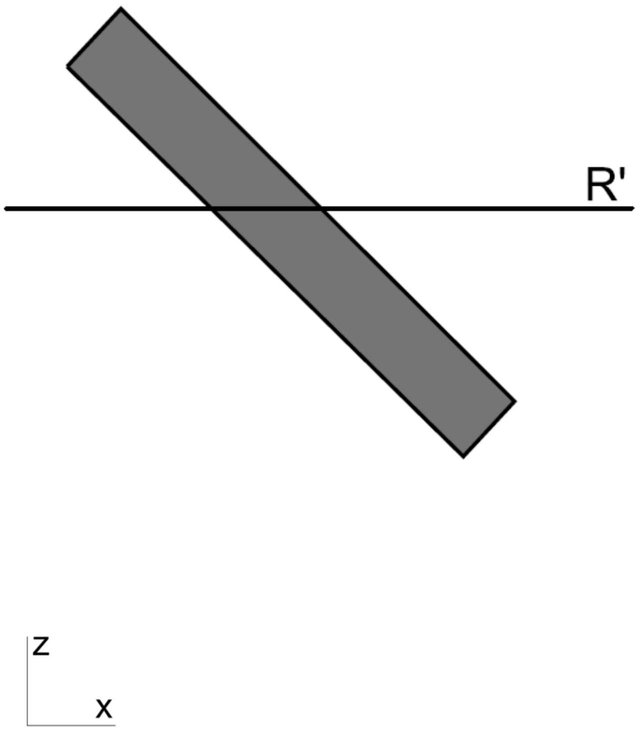
Origami principles :
Reverse fold as a reflection about a plane
a) with planar elements

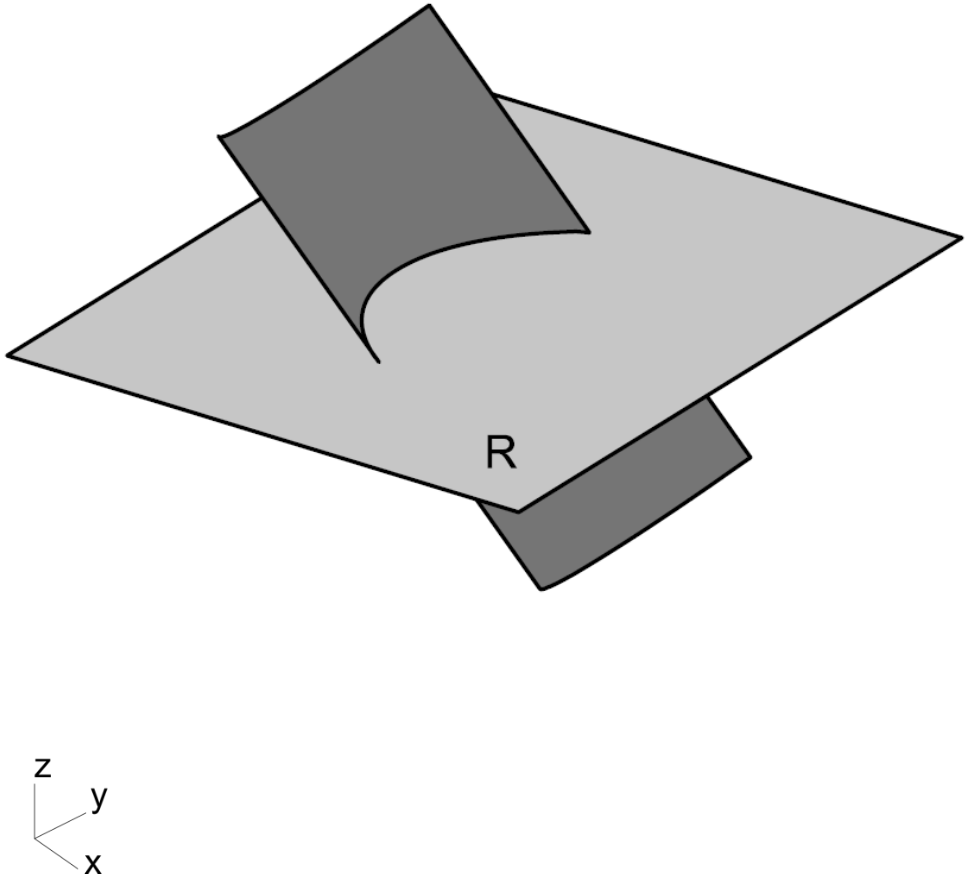




1) Curved prototypes geometry

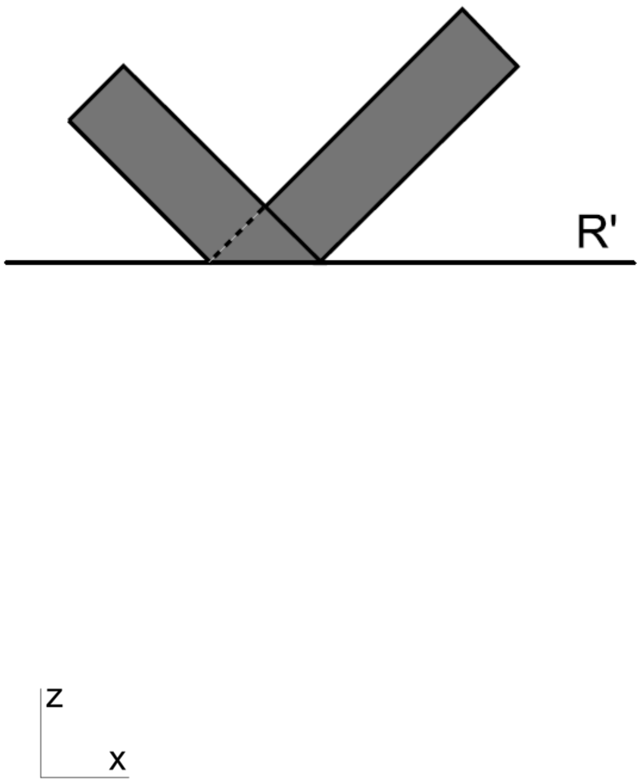
Origami principles :
Reverse fold as a reflection about a plane
b) with curved elements

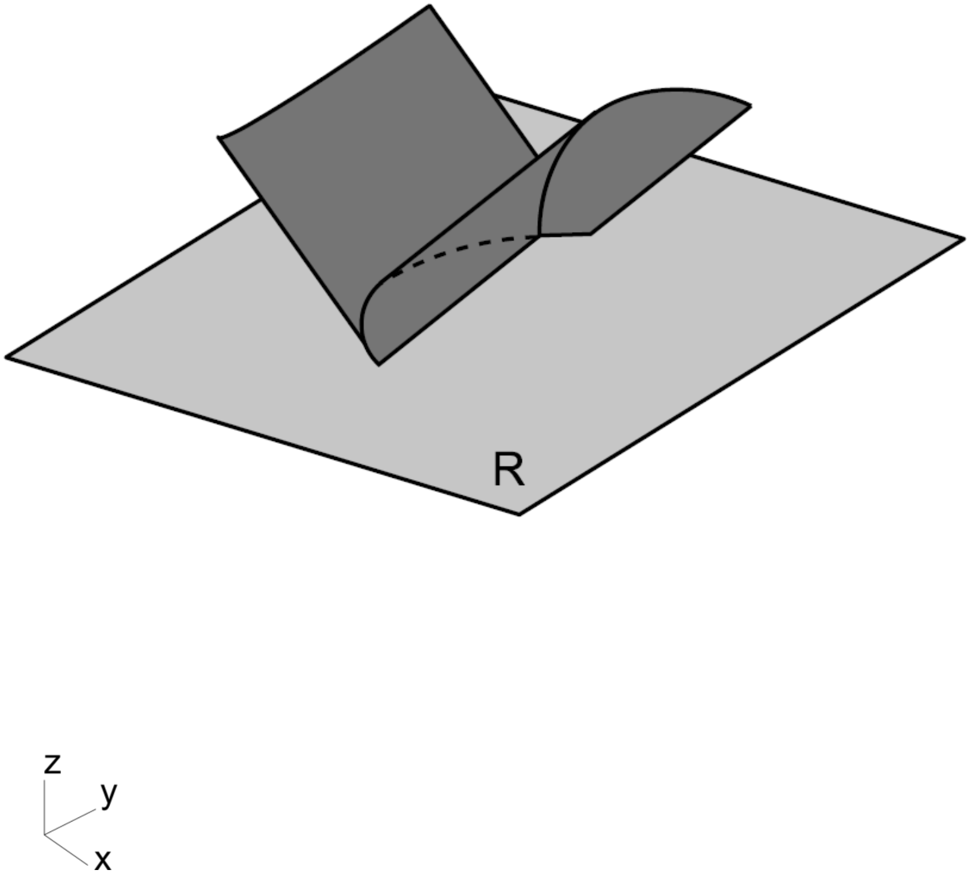




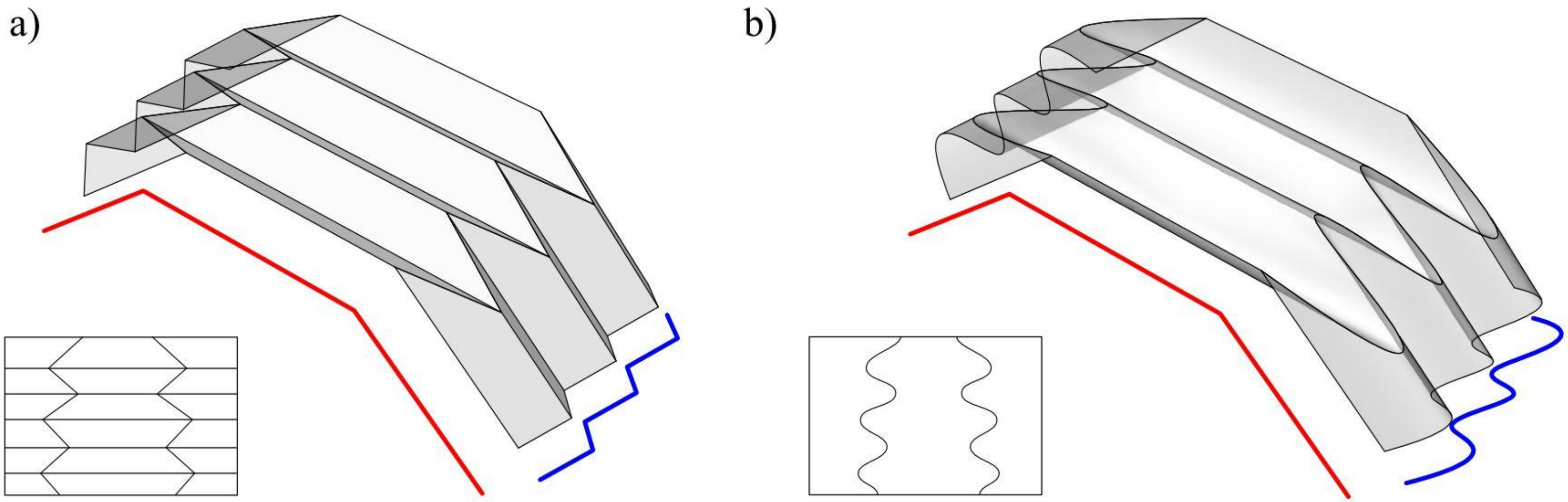
1) Curved prototypes geometry

Origami principles :
Reverse fold as a reflection about a plane
b) with curved elements





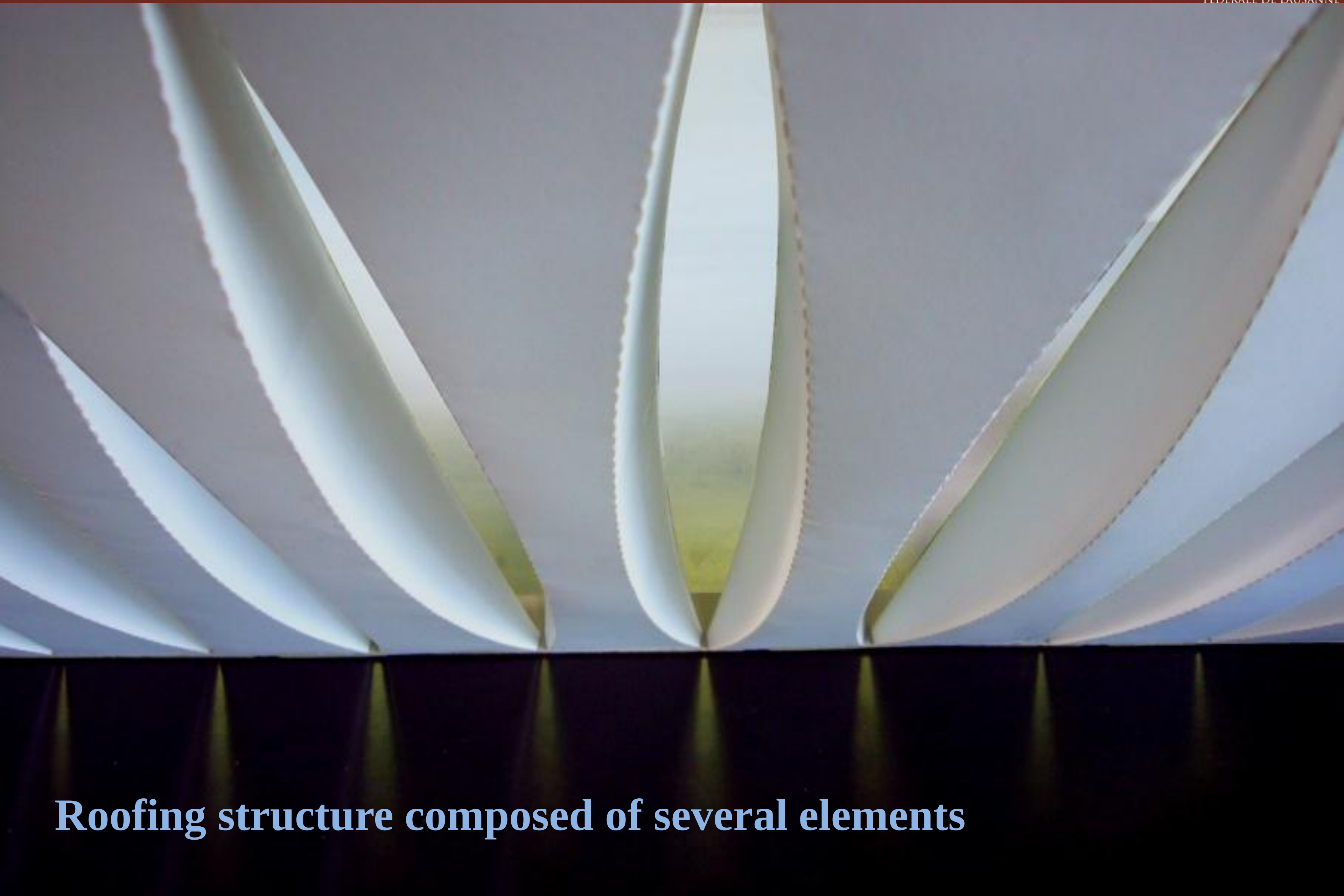
Geometric Design: Construction by two Profile Curves



a) Folded plate structures

b) Curved origami figure

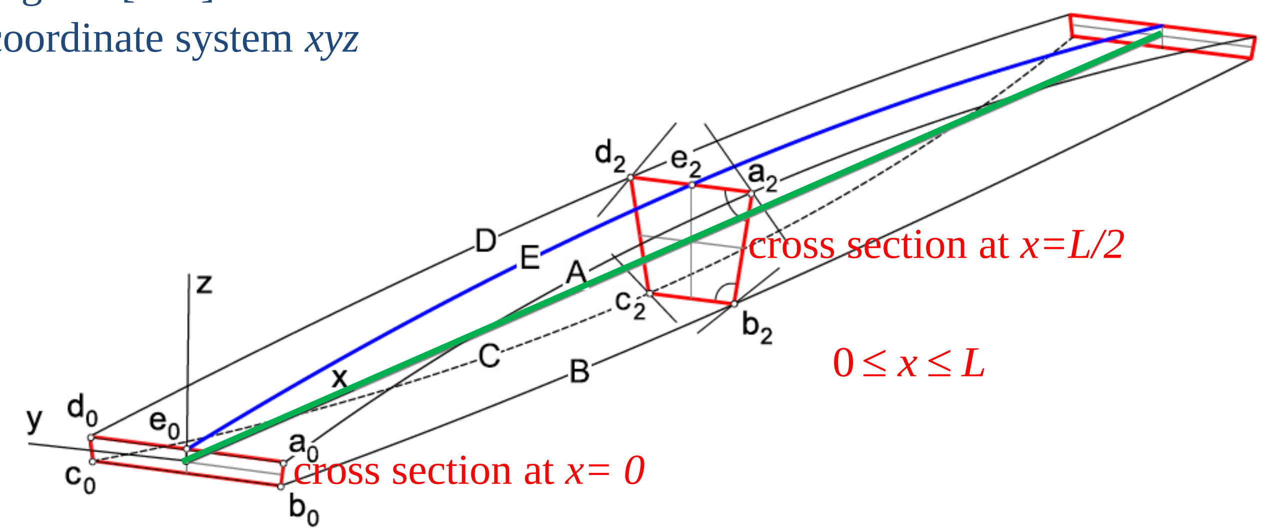
Curved origami beam



Roofing structure composed of several elements

Geometrical considerations:

Beam length L [mm]
Global coordinate system *xyz*

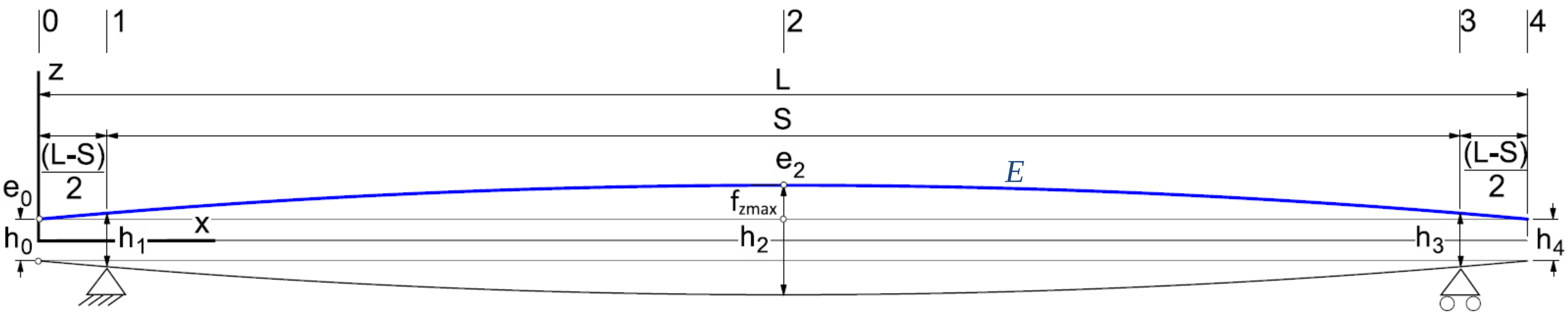


• Cross sections parallel to the *y-z* plane, delimited by the vertices ***a, b, c, d*** at the mid-thickness of the panel

• Beam axis *x* directed along the line of centroïds

• Slowly varying height along arc *E* of constant curvature (plane *x-z*), similarly for the beam width

Elevation (plane x-z):



h_0, h_1, h_2 : beam heights at $x = 0$, $(L\text{-}S)/2$, $L/2$ respectively

Radius r of arc E:

$$r^2 = \frac{L^2}{4} + \left(r - f_{z\max}\right)^2 \quad \text{with} \quad f_{z\max} = \frac{h_2 - h_0}{2}$$

solving, $$r = \frac{4 f_{z\max}^2 + L^2}{8 f_{z\max}}$$

Half-height evolution :

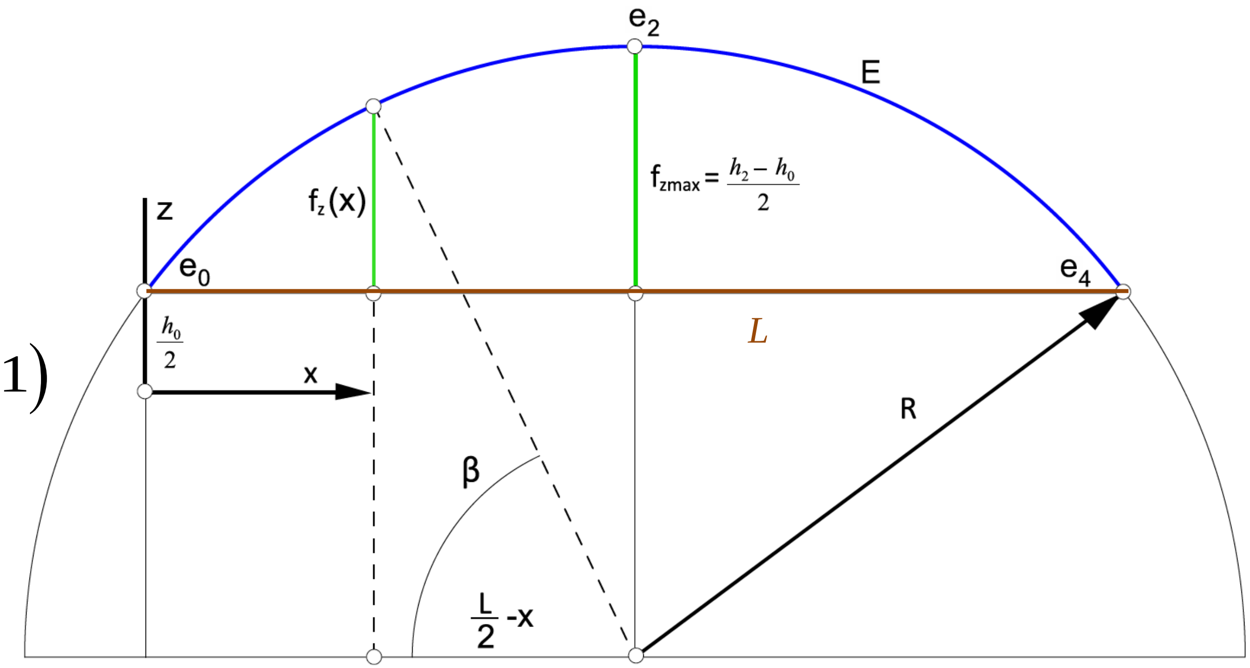
$$h(x) = f_z(x) + \frac{h_0}{2}$$

with the variable part,

$$f_z = f_{z\max} + r\left(\sin\beta(x) - 1\right)$$

where

$$\beta(x) = \operatorname{acos}\frac{L/2 - x}{r}$$



Not at scale !

such that

$$f_z(0) = 0 \qquad h(0) = h_0/2$$

$$f_z(L/2) = f_{z\max} \qquad h(L/2) = h_0/2 + f_{z\max} = h_2/2$$

Half-width evolution :

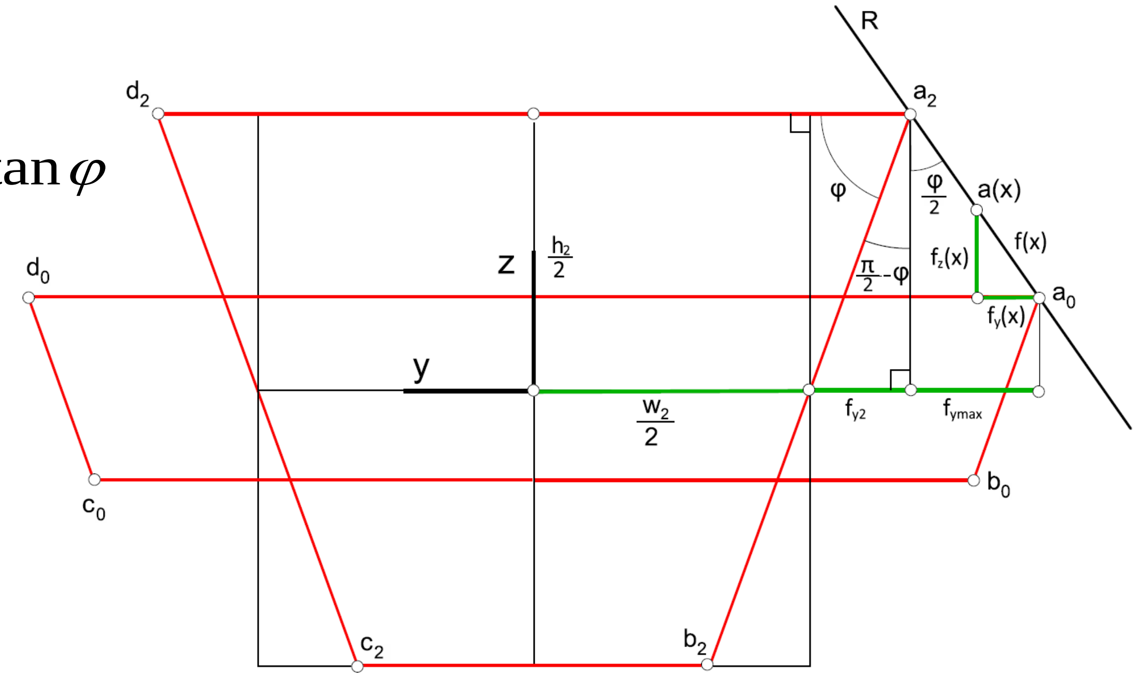
$$w(x) = \frac{w_2}{2} + f_{y2} + f_{y\max} - f_y(x)$$

with

$$f_{y2} = \frac{h_2}{2} \cdot \tan\left(\frac{\pi}{2} - \varphi\right) = \frac{h_2}{2} \cdot \operatorname{cotan} \varphi$$

$$f_{y\max} = f_{z\max} \cdot \tan \frac{\varphi}{2}$$

$$f_y(x) = f_z(x) \cdot \tan \frac{\varphi}{2}$$



such that

$$f_y(0) = 0 \qquad w(0) = w_2/2 + f_{y2} + f_{y\max}$$

$$f_y(L/2) = f_{y\max} \qquad w(L/2) = w_2/2 + f_{y2}$$

$$f_{y\max} = f_{z\max}\,, \quad f_{y2} = 0 \;\; \textit{when} \;\; \varphi = \frac{\pi}{2}$$

Vertices coordinates:

$$\boldsymbol{a}\begin{pmatrix} a_y \\ a_z \end{pmatrix} \quad \boldsymbol{b}\begin{pmatrix} b_y \\ b_z \end{pmatrix} \quad \boldsymbol{c}\begin{pmatrix} c_y \\ c_z \end{pmatrix} \quad \boldsymbol{d}\begin{pmatrix} d_y \\ d_z \end{pmatrix}$$

where,

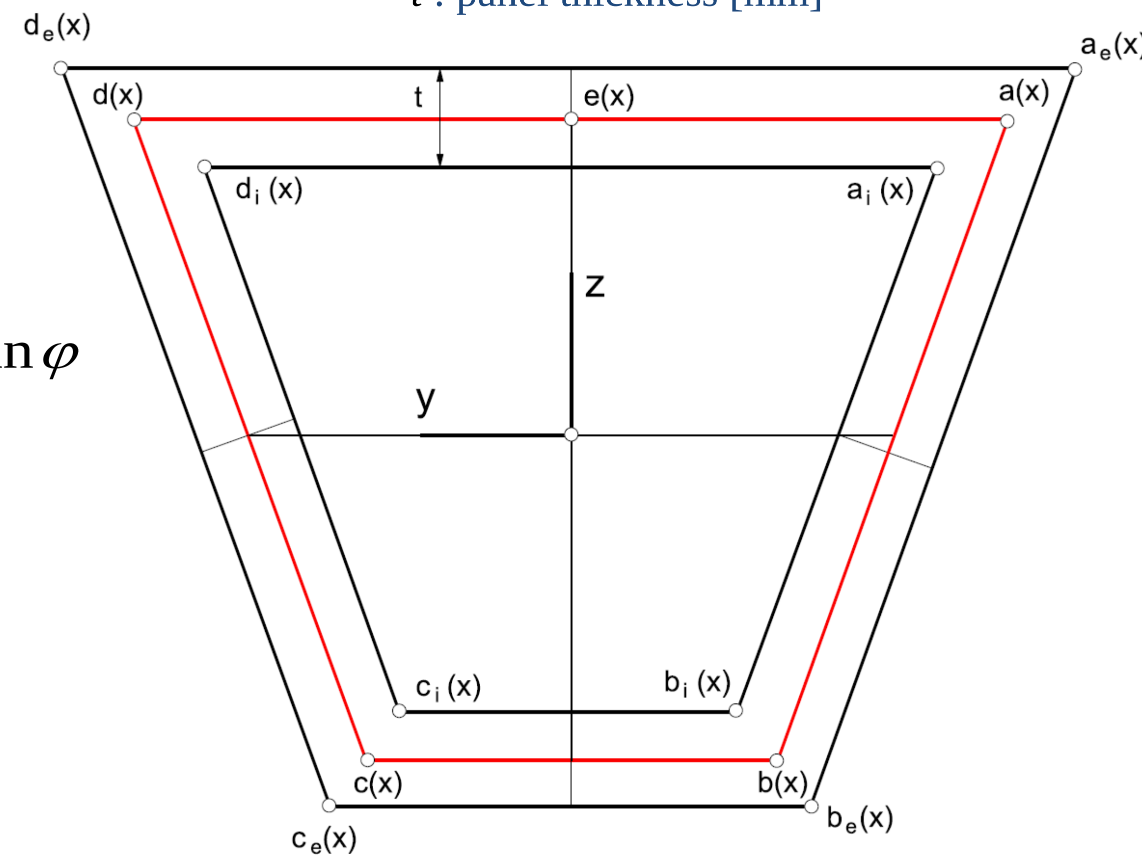
$$a_y(x) = -w(x)$$
$$a_z(x) = h(x)$$

$$b_y(x) = a_y(x) + 2a_z(x)\cdot \text{cotan}\,\varphi$$
$$b_z(x) = -a_z(x)$$

$$c_y(x) = -b_y(x)$$
$$c_z(x) = -a_z(x)$$

$$d_y(x) = -a_y(x)$$
$$d_z(x) = a_z(x)$$

t : panel thickness [mm]



- **Parametrization of the curved beam:**

- length/span ratio :

$$i = \frac{L}{S}$$

(loading consideration)

- height at end / height at mid-length ratio :

$$q = \frac{h_0}{h_2}$$

- height / width ratio at mid-length:

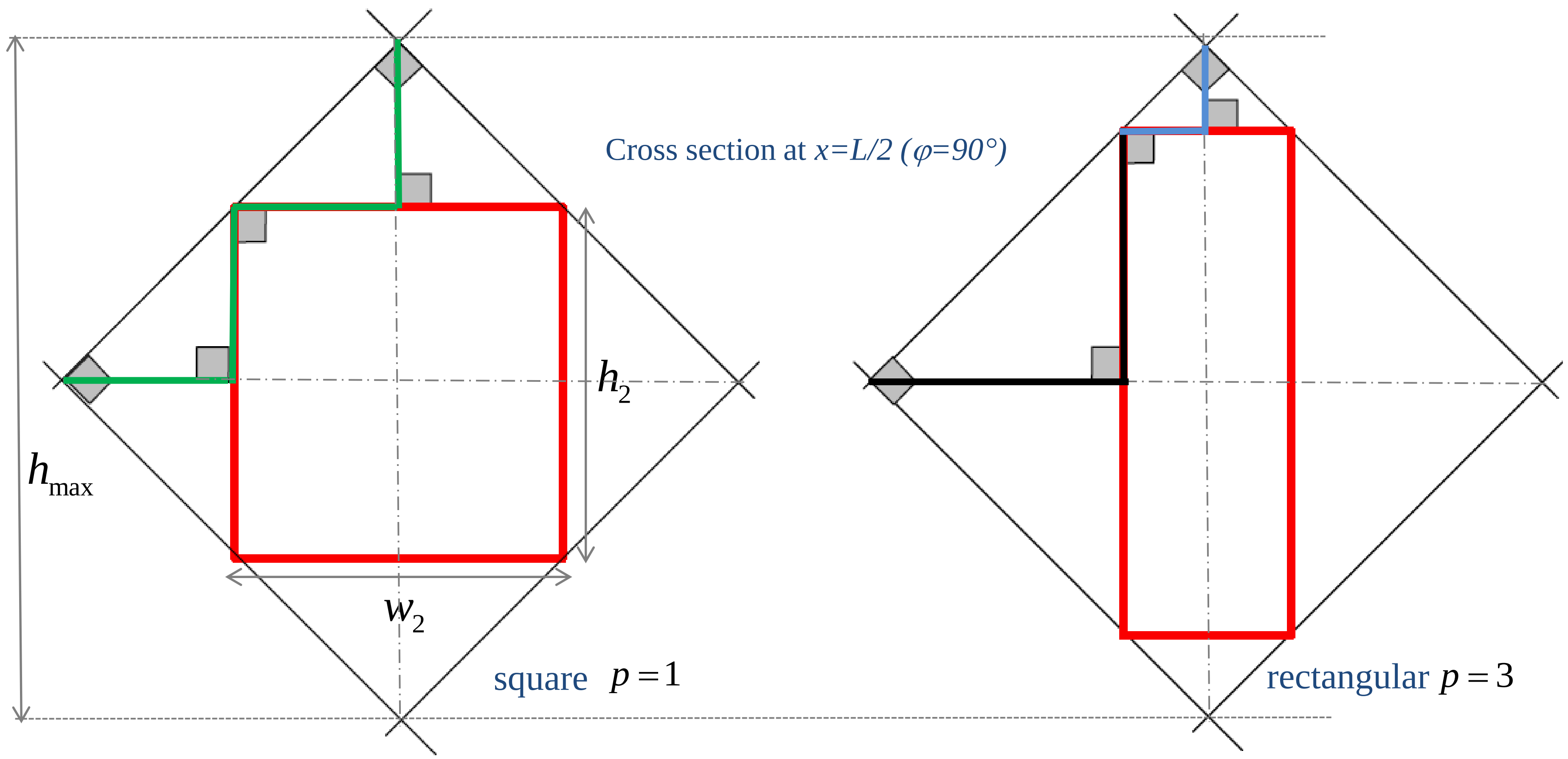
$$p = \frac{h_2}{w_2}$$

- Reflection angle φ

- “Aspect” ratio :

$p = \frac{h_2}{w_2}$

$k = \frac{h_{max}}{S}$, $h_2 + w_2 = h_{max}$ half the perimeter at mid-length



Same value of k for the two configurations

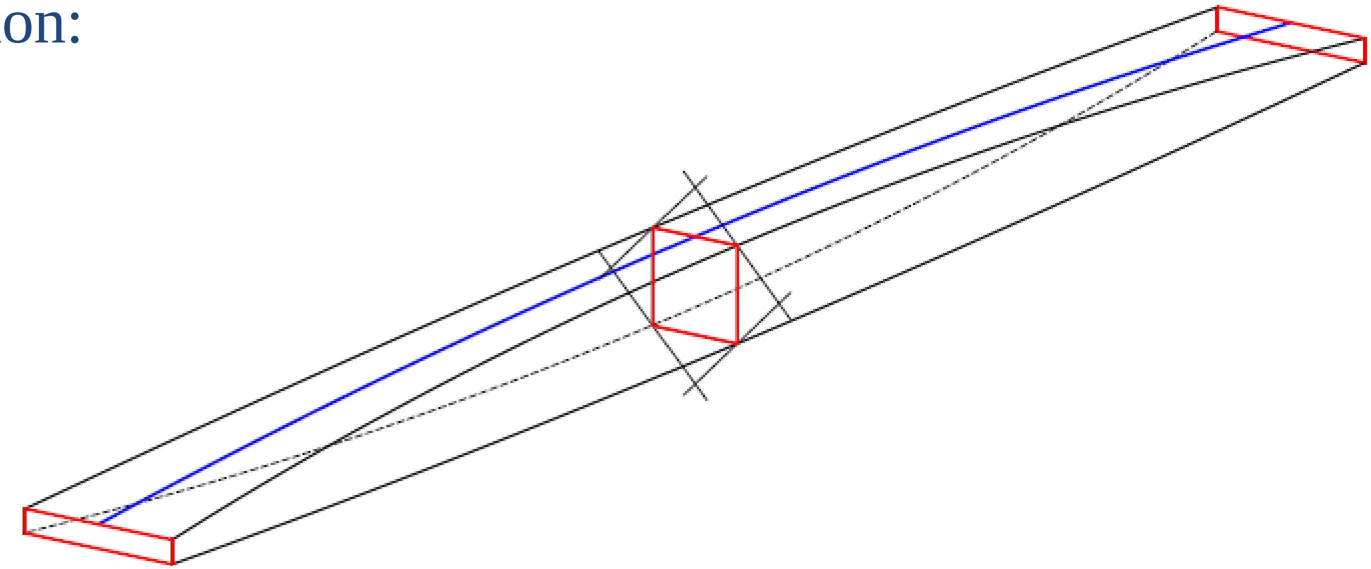
Illustration:

k=0.1

p=1

q=1/4

$\varphi = 90°$

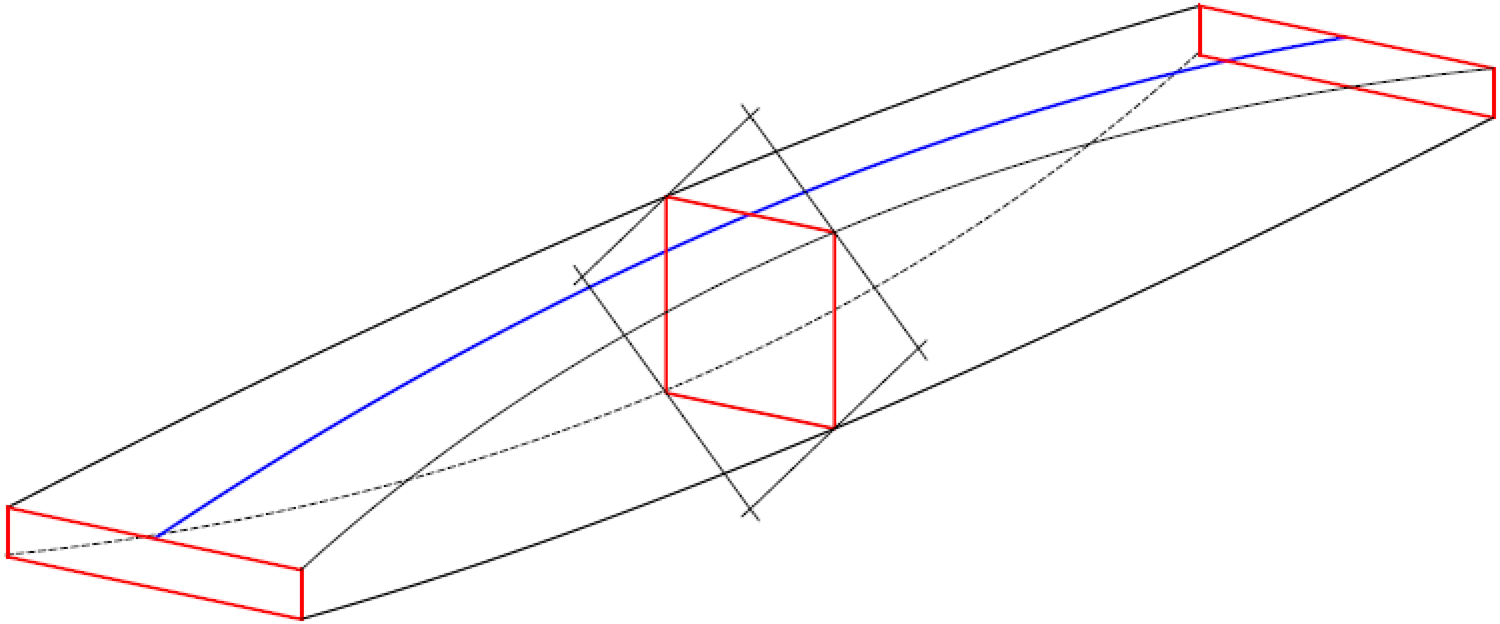


k=0.2

p=1

q=1/4

$\varphi = 90°$

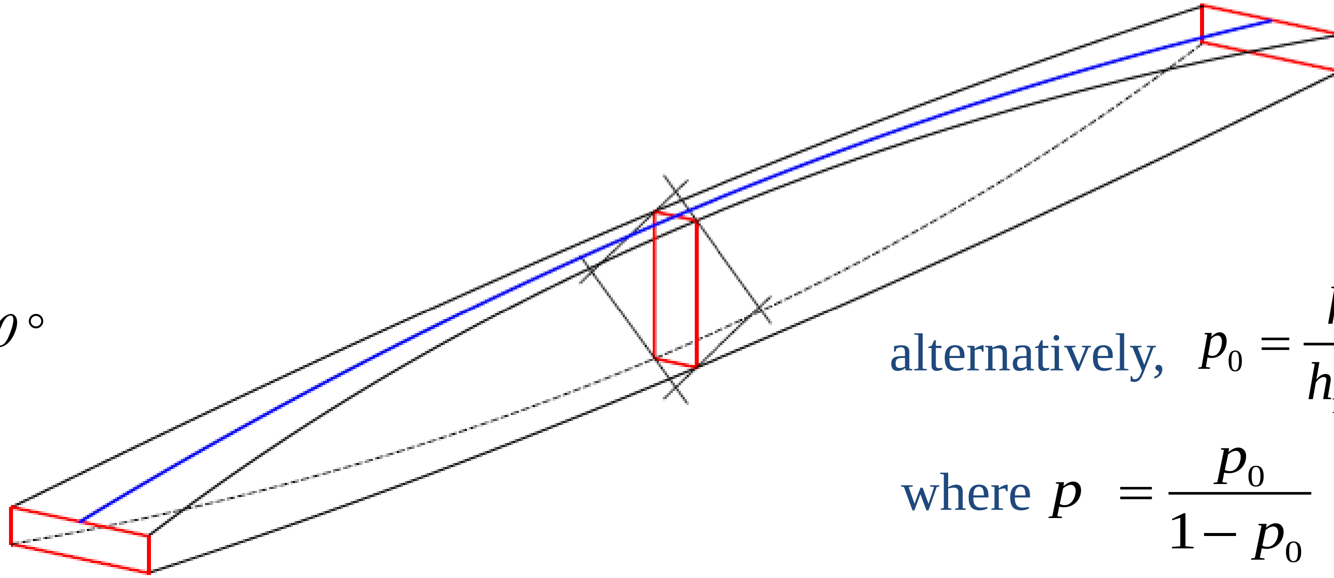


p=3

k=0.1

q=1/4

$\varphi = 90°$



alternatively, $p_0 = \frac{h_2}{h_{max}}$

where $p = \frac{p_0}{1-p_0}$ $\qquad$ $p = 3$, $p_0 = 0.75$

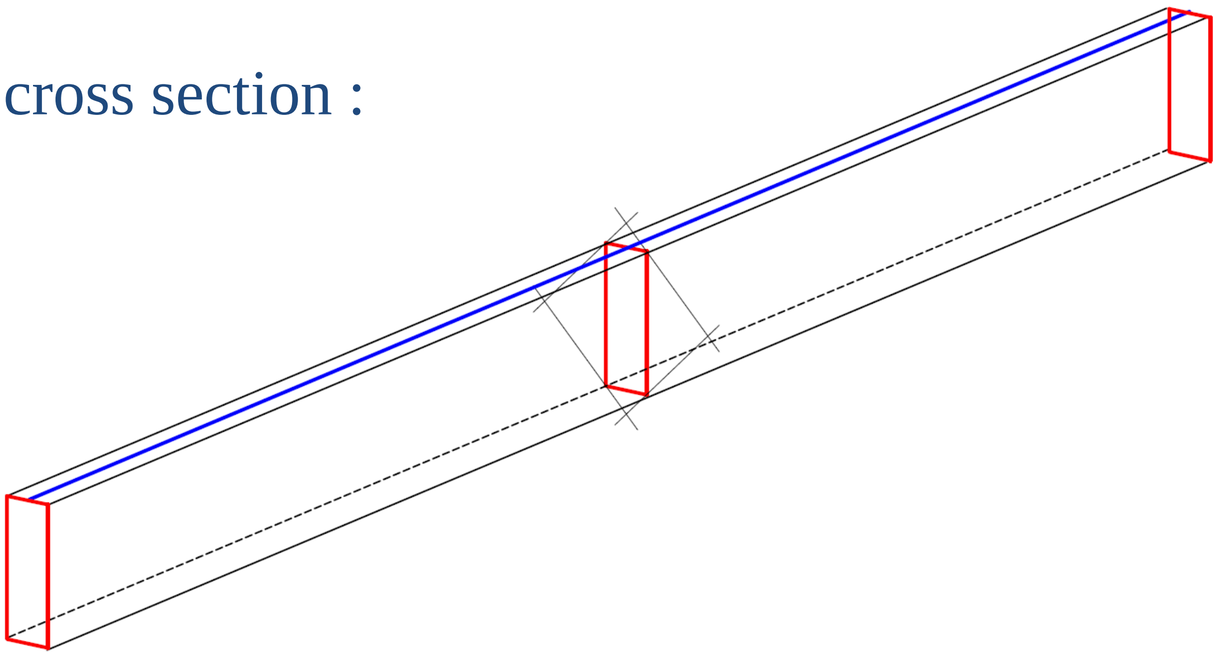
constant cross section :

p=3

k=0.1

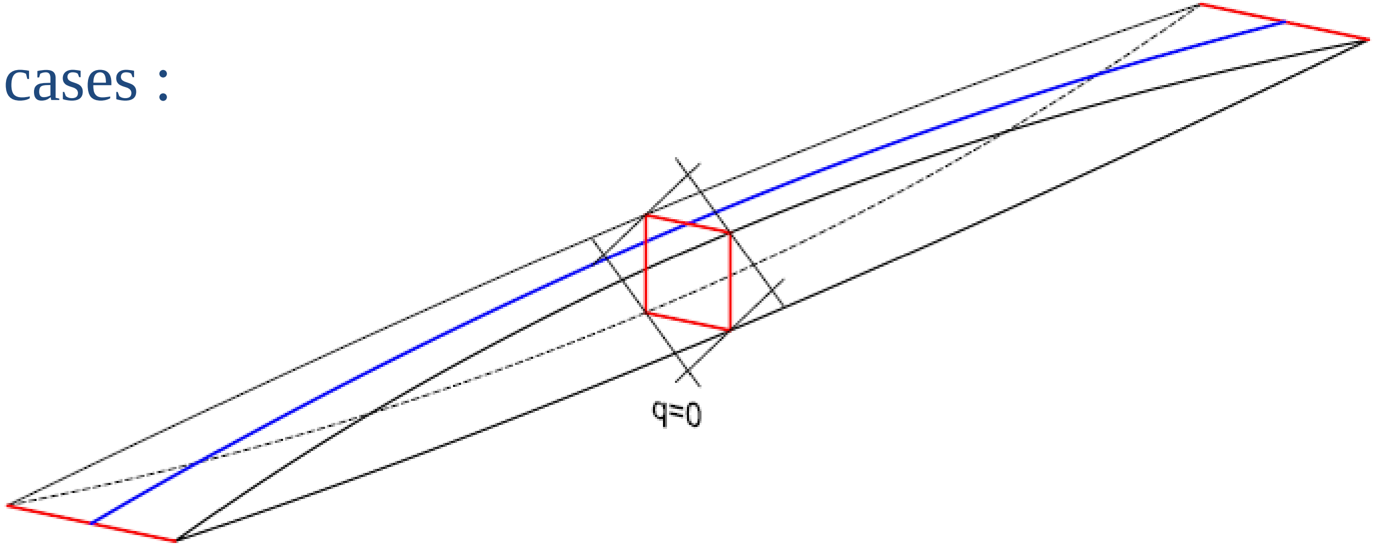
q=1

$\varphi = 90°$



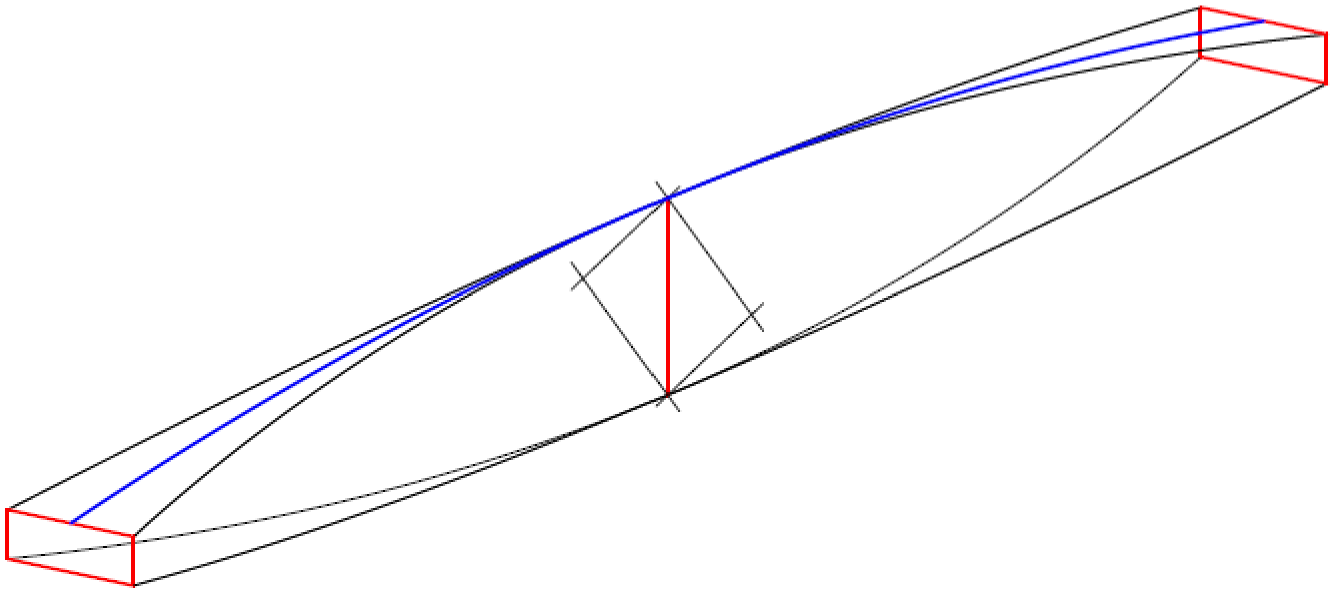
Limiting cases :

q=0



$$p=\infty$$

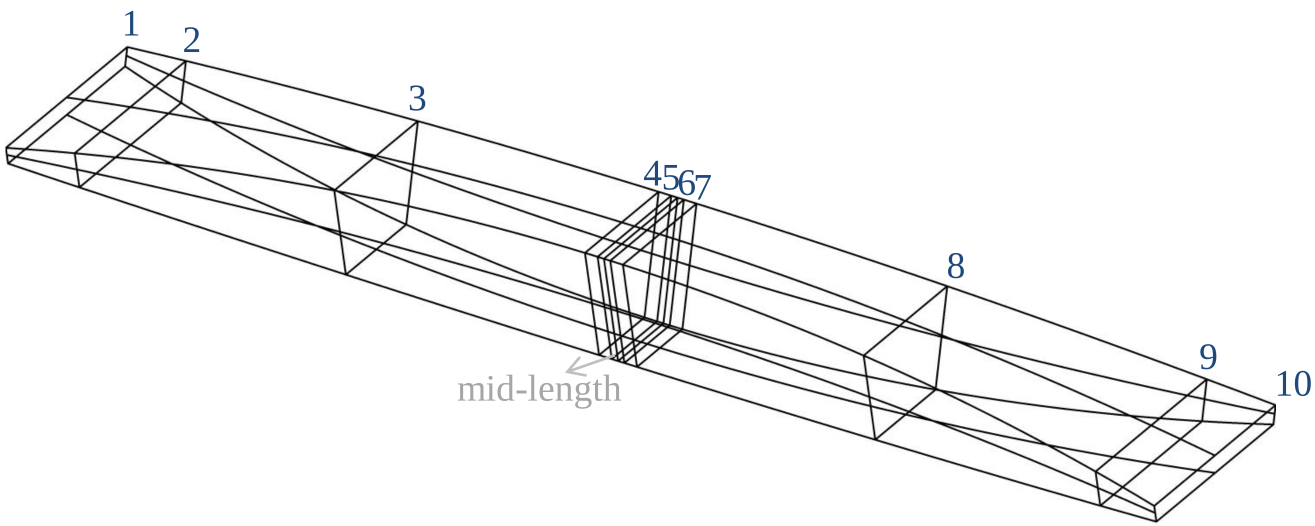
$$p_0=1$$



2) Experimental investigation

Prototypes:

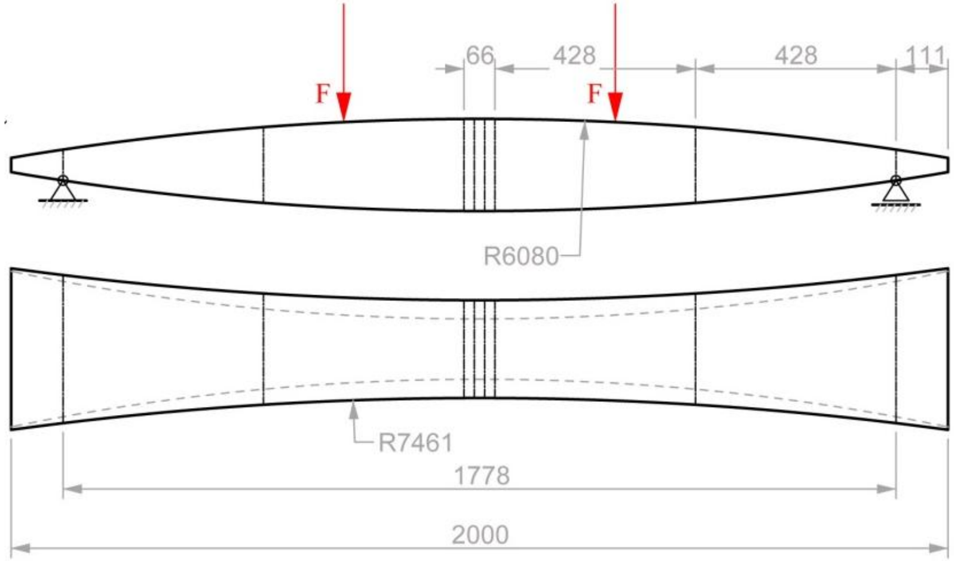
- Curved prototypes of length 2*m* :



- Okoumé plywood panels (three plies) of thickness $t = 6\ mm$
- 10 Stiffeners - Okoumé plywood of thickness *22 mm* :

  4-5-6-7 linked together
  2, 9 placed at support position

- Screwed and glued connection between the panels mid-surfaces of stiffeners and panels depicted



L = 2000 mm

S = 1778 mm

h_0 = 33.33 mm

w_0 = 340.71 mm

h_2 = 200 mm

w_2 = 170.59 mm

Parameters values:

$i = 1.1249$

$q = 0.166$

$p = 1.172$

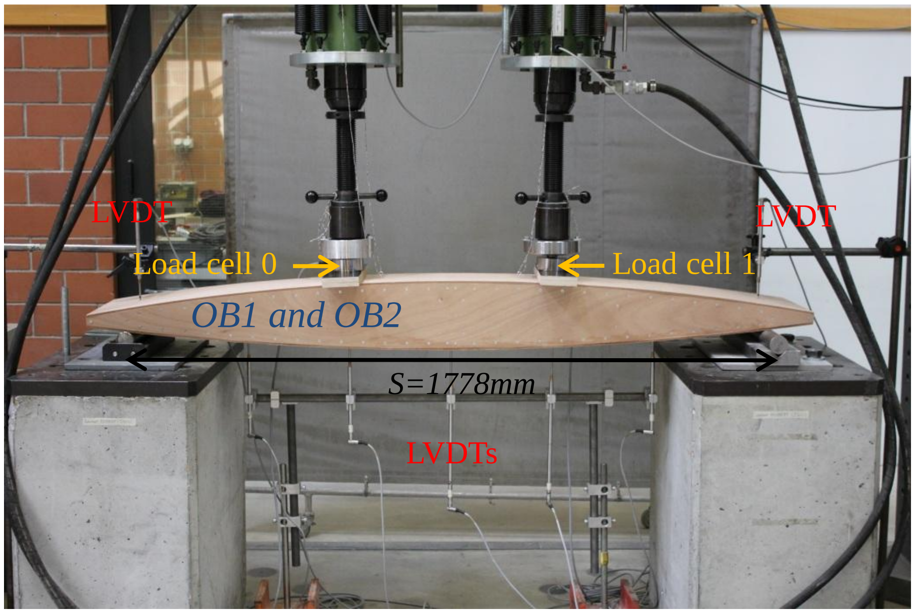
$\varphi = 78°44$

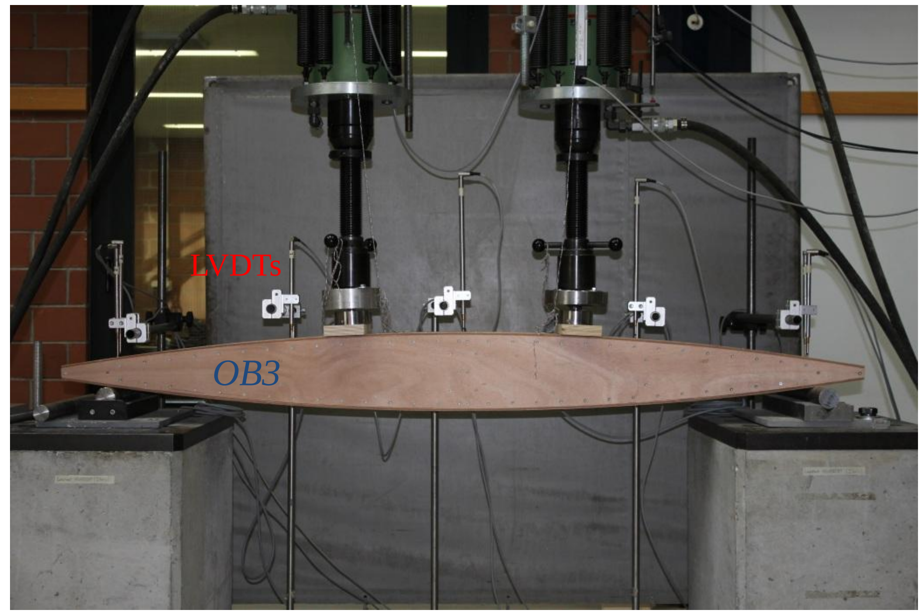
$k = 0.208$

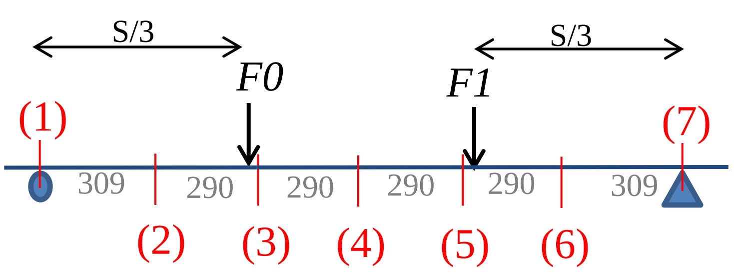
Experimental setup:

Four point bending test : W+B 300kN loading system

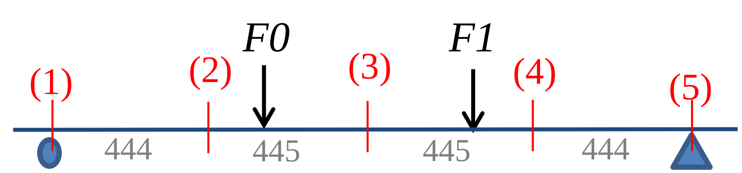
three specimens *OB1*, *OB2*, *OB3*







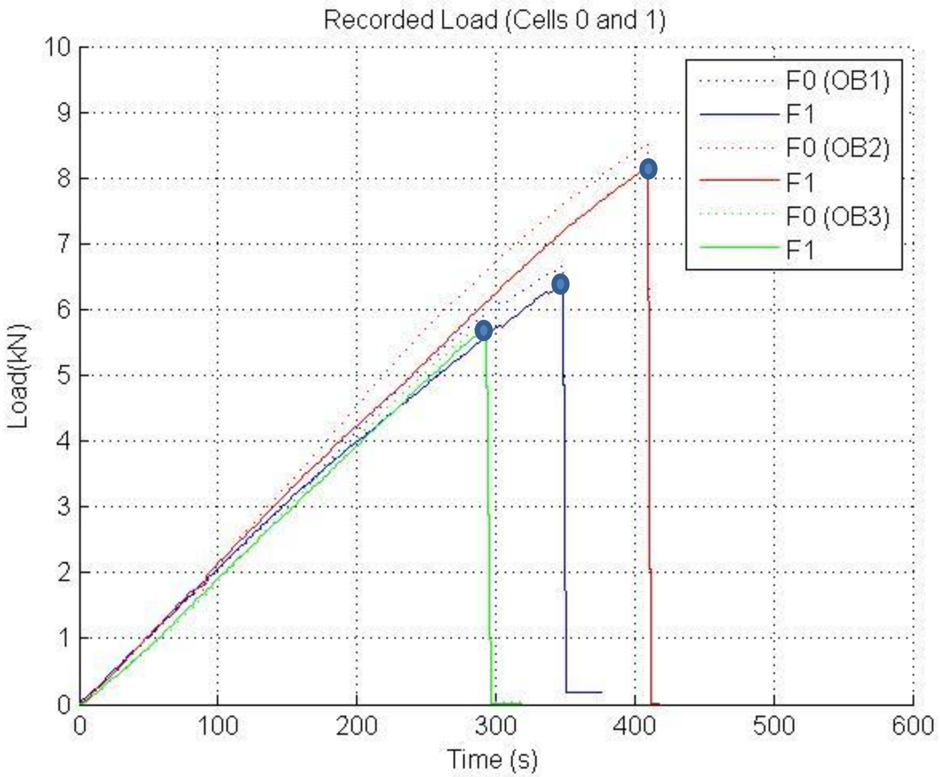
transducers locations (i=1..7)



transducers locations (i=1..5)

- Loading process:

- Imposed grip displacement at 0.05mm/s

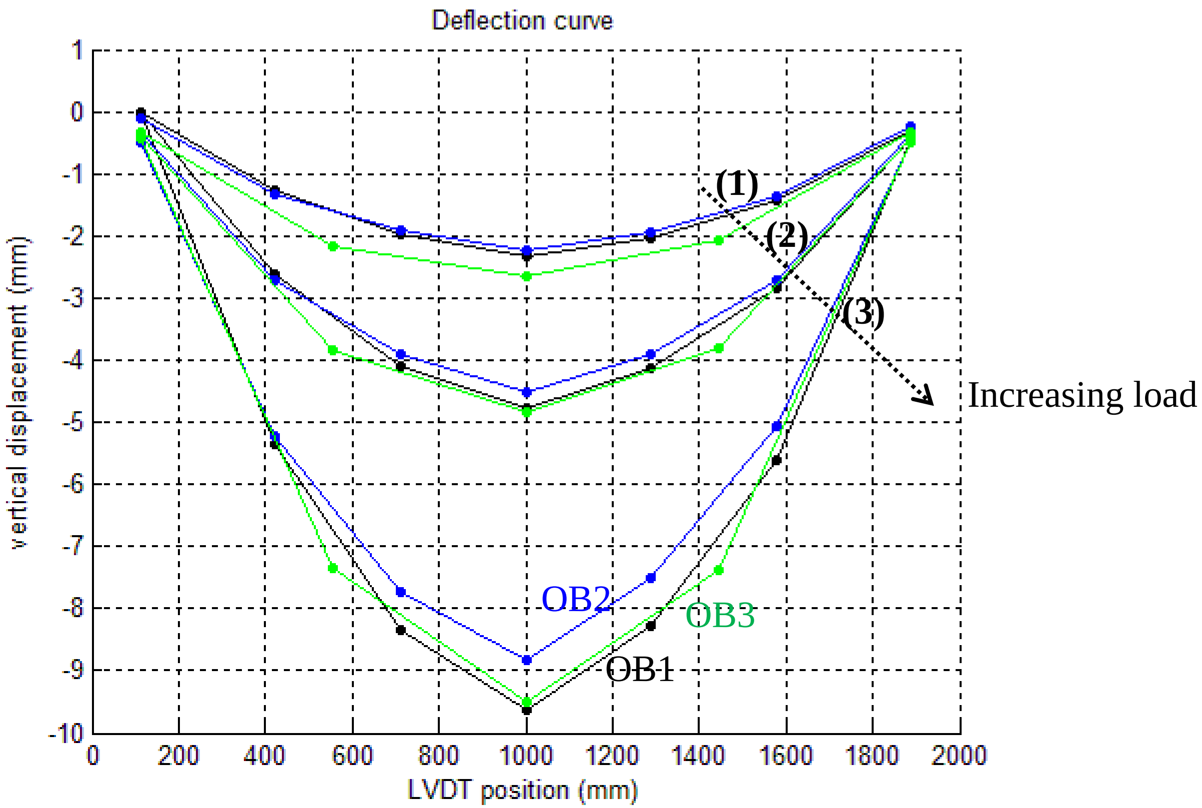


Maximum load (KN) :

	F0	*F1*	*F0+F1*
OB1	6.7	6.4	13.1
OB2	8.5	8.1	16.6
OB3	5.8	5.7	11.5

Failure :

Measurements of deflection :



	F0+F1 (KN)		
	(1)	(2)	(3)
OB1	2.02	4.10	8.08
OB2	1.99	4.11	8.04
OB3	2.07	4.00	7.96

Three levels of loads examined

3) Numerical study – 1D simplified model

- **Frame work** : bending analysis of elastic beams with general cross section variation :

  assumptions : - classical Euler beam theory
  - homogeneous material
  - rigid connections between the panels

- Model implemented in Matlab :

  - handling problems with
    - depth/width beam variations
    - any number of concentrated/linearly varying distributed loads
    - general end conditions
    - interior supports

  - Finite Element formulation *not* used here

- **Implementation**

-Use of singularity functions of order *n (=-1,0,1)*:

$$\langle x-x_0\rangle^n = \begin{cases} 0 & x < x_0 \\ (x-x_0)^n & x \ge x_0 \end{cases}$$

satisfying,

$$\int_o^x \langle x-x_0\rangle^n dx = \frac{1}{n+1}\langle x-x_0\rangle^{n+1}$$

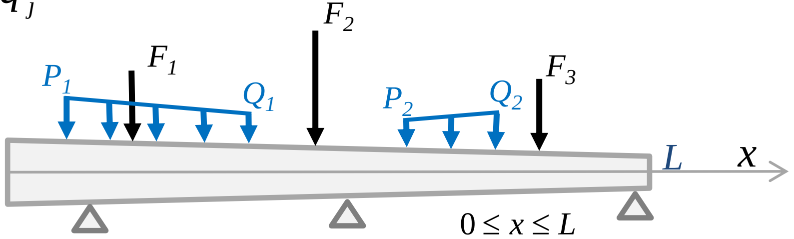
n= -1 : appropriate for describing a concentrate load

-Prescribed (general) external loads :

$$w_e(x) = \sum_{j=1}^{N_f} F_j\langle x-f_j\rangle^{-1} + \sum_{j=1}^{N_r} P_j\langle x-p_j\rangle^{0} - Q_j\langle x-q_j\rangle^{0} + \frac{Q_j-P_j}{q_j-p_j}\cdot\left(\langle x-p_j\rangle^{1} - \langle x-q_j\rangle^{1}\right)$$

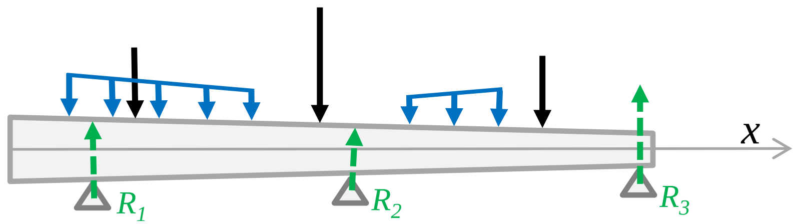
N_f concentrated loads F_j acting at positions $x = f_j$

N_r ramped loads varying (linearly) between $p_j \le x \le q_j$



- System of differential equations :

Shear & load:



$$v'(x) = q(x) = w_e(x) + \sum_{j=1}^{N_s} R_j \langle x - r_j \rangle^{-1}$$

$$v(x) = v_0 + v_e(x) + \sum_{j=1}^{N_s} R_j \langle x - r_j \rangle^{0} \quad , \quad v_e(x) = \int_0^x w_e(x)\,dx \qquad \frac{d}{dx} = (.)'$$

N_S reaction forces R_j (to be determined) at the support positions $x = r_j$

Moment & deflection:

$$m'(x) = v$$

$$m(x) = m_0 + v_0 x + m_e(x) + \sum_{j=1}^{N_s} R_j \langle x - r_j \rangle^{1} \quad , \quad m_e(x) = \int_0^x v_e(x)\,dx$$

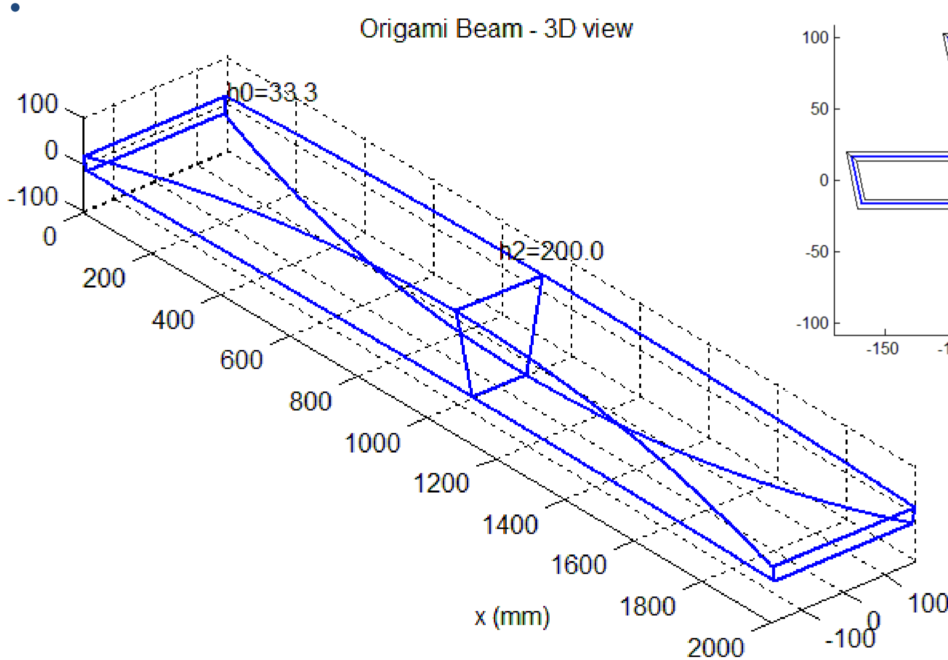
$$y''(x) = \frac{m(x)}{E \cdot I(x)}$$

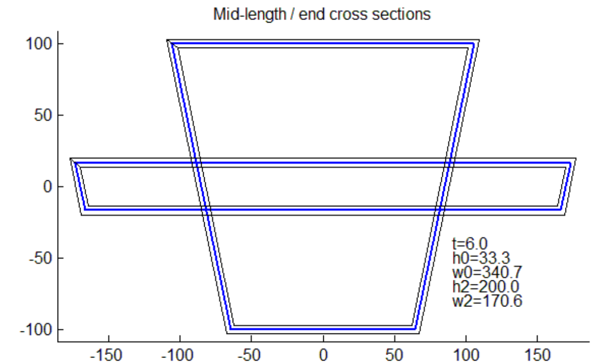
varying cross section moment of inertia *I(x)* ,
Young modulus E

+ four end conditions imposed at $x=0$ and $x=L$

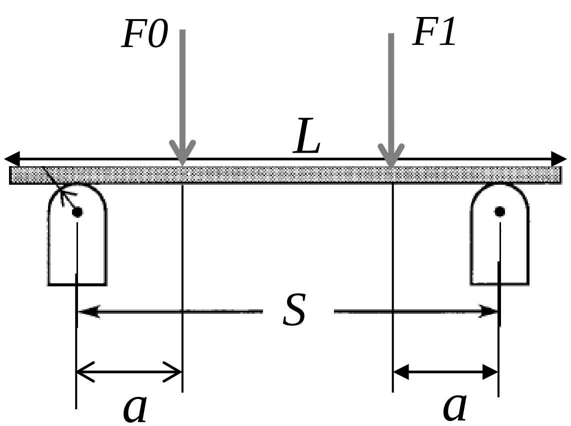
• Simulated geometry:

L (mm)	2000
i	1.125
q	0.166
p	1.172
φ	78°44
k	0.208





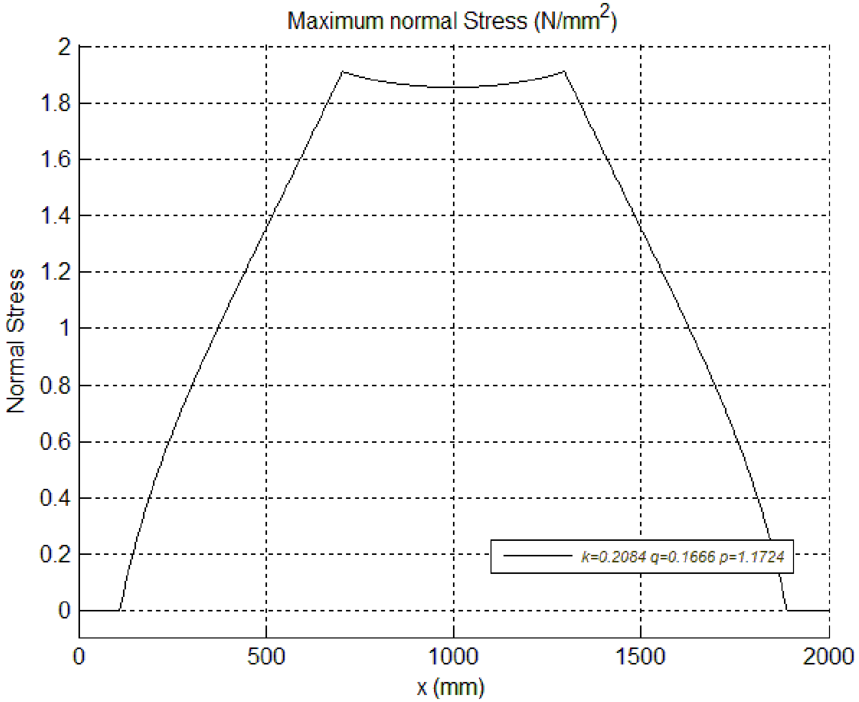
• Loading configuration (four-point bending) :



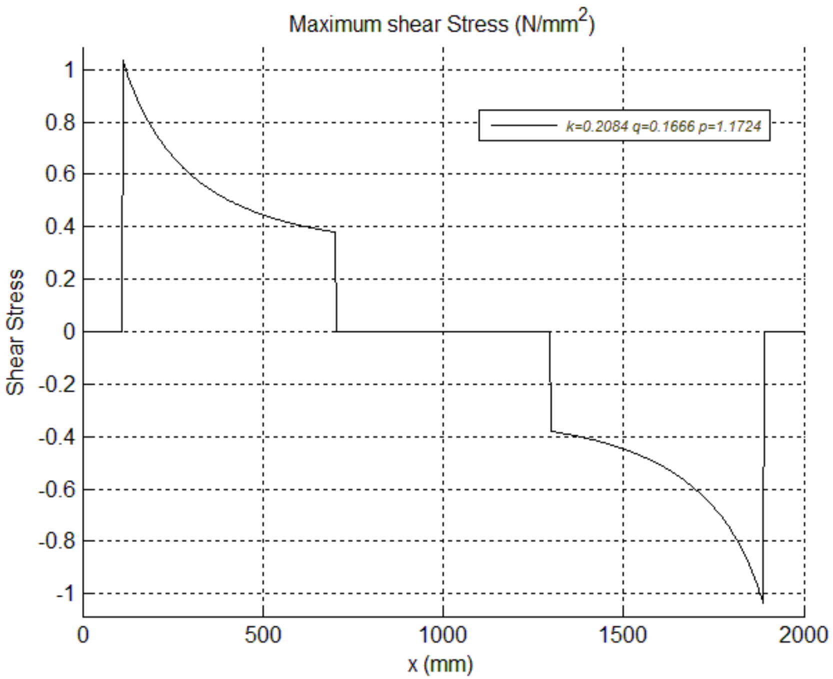
a= S/3

Young modulus E=2398 MPa
(from Thebault company)

Normal and shear maximum stresses evolution :

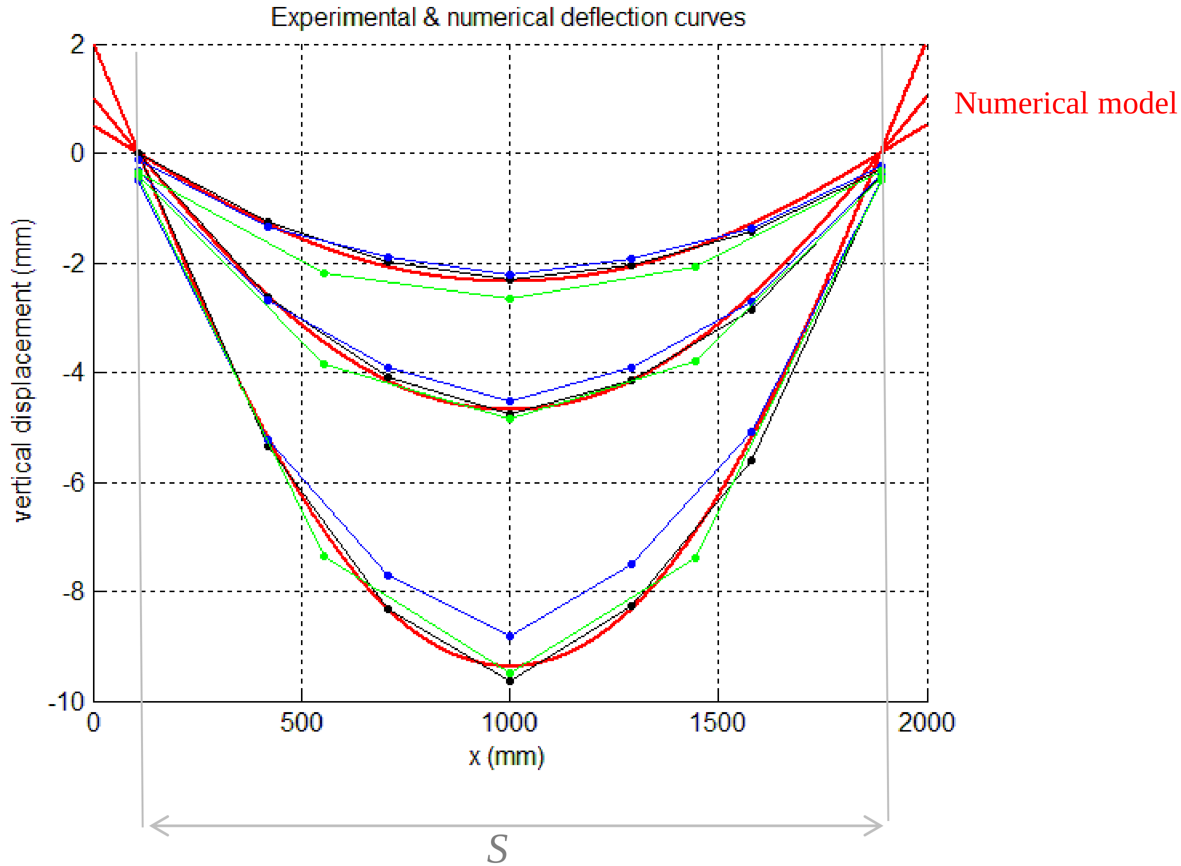


Simulation for $F0+F1=2KN$



Deflection curves

Comparison between experimental & numerical results



Conclusion

- Application of origami principles to curved beams with potential architectural interest

- Efficient parameterization of the origami beam geometry

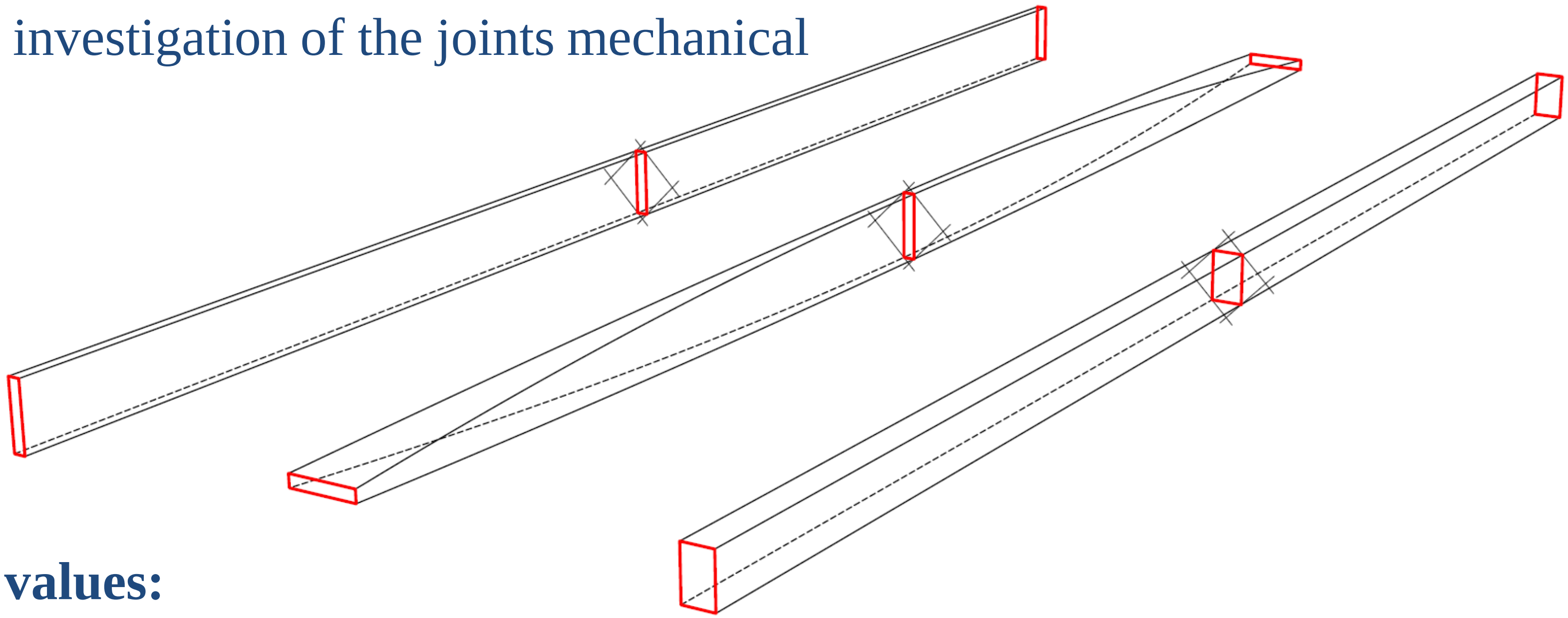
- Experimental & numerical characterization of the mechanical behavior of (small) origami beams under static bending load

• Prospects:

Larger prototypes with different section profiles and material

Introduction of semi-rigid dove-tail joints (no stiffeners)

Numerical investigation of the joints mechanical behavior



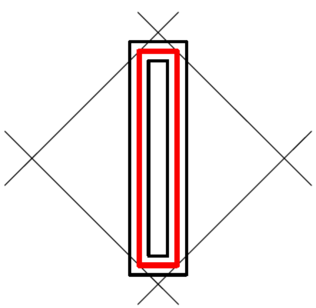
L = 4400 mm

S = 4000 mm

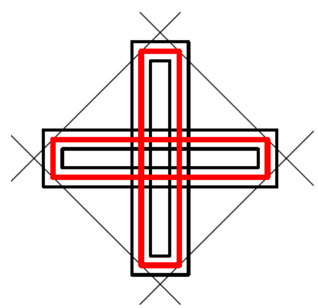
t = 15 mm

Parameters values:

i =1.1
q=1
p=5.666
φ=90
k=0.05



i =1.1
q=0.176
p=5.666
φ=90
k=0.05



i =1.1
q=1
p=1.609
φ=90
k=0.05