Sequestering by global symmetries in Calabi-Yau string models

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Abstract

We study the possibility of realizing an effective sequestering between visible and hidden sectors in generic heterotic string models, generalizing previous work on orbifold constructions to smooth Calabi-Yau compactifications. In these theories, genuine sequestering is spoiled by interactions mixing chiral multiplets of the two sectors in the effective Kähler potential. These effective interactions however have a specific current-current-like structure and can be interpreted from an M-theory viewpoint as coming from the exchange of heavy vector multiplets. One may then attempt to inhibit the emergence of generic soft scalar masses in the visible sector by postulating a suitable global symmetry in the dynamics of the hidden sector. This mechanism is however not straightforward to implement, because the structure of the effective contact terms and the possible global symmetries is a priori model dependent. To assess whether there is any robust and generic option, we study the full dependence of the Kähler potential on the moduli and the matter fields. This is well known for orbifold models, where it always leads to a symmetric scalar manifold, but much less understood for Calabi-Yau models, where it generically leads to a non-symmetric scalar manifold. We then examine the possibility of an effective sequestering by global symmetries, and argue that whereas for orbifold models this can be put at work rather naturally, for Calabi-Yau models it can only be implemented in rather peculiar circumstances.

1 Introduction

In supergravity models, it is natural to imagine that supersymmetry breaking occurs at an intermediate scale in a hidden sector and is dominantly mediated to the visible sector by gravitational interactions, with the net effect of inducing soft breaking terms of a size close to the electroweak scale. These soft terms are however induced through higher-dimensional operators mixing visible and hidden sector fields in the effective theory, with a structure that depends on the details of the underlying microscopic theory and is therefore a priori generic. In particular, one naturally expects soft scalar masses with a generic flavor structure, while the non-observation of certain flavor changing processes instead requires these to be approximately universal. This leads to the so-called supersymmetric flavor problem, which consists in finding a natural and robust explanation for the approximate flavor universality that soft scalar masses need to enjoy.

One of the most interesting proposals for solving this problem is the idea of sequestering the visible and the hidden sectors by localizing them on two distinct branes at different positions along an extra dimension [1]. In the basic situation where these two sectors interact only through minimal gravity in the bulk, which corresponds to the so-called no-scale models [2], local contact terms between the two brane sectors are guaranteed to be absent. Moreover, contact terms between each brane sector and the additional radion chiral multiplet arising in the bulk, which can also participate to supersymmetry breaking, turn out to be absent too. As a consequence, scalar masses vanish at the classical level and are induced only by non-local loop effects of various kinds, like for instance anomaly mediation [1, 3], radion-mediation [4] or brane-to-brane mediation [5, 6], which have the crucial common characteristic of being approximately flavor-universal. Thanks to this property, this minimal setup allows to construct phenomenologically acceptable and satisfactory effective models based on 5D supergravity theories with one compact dimension.

In string models, which are supposed to be the microscopic theories underlying supergravity models, the framework that is needed to implement sequestering arises very naturally, since the emergence of extra dimensions and localized matter sectors is almost unavoidable. It has however been emphasized in [7] that there is an endemic difficulty in realizing the minimal setup needed for sequestering. As a matter of fact, in most of the string models where the 4D low-energy effective theory has been worked out, there appear non-trivial contact terms between matter sectors in the effective Kähler potential, even when these are sequestered at distinct points in the internal compact space, as well as couplings between each matter sector and the non-minimal moduli sector. As a result, non-vanishing and non-universal soft scalar masses generically arise at the classical level. From the perspective of the 5D intermediate effective theory obtained by retaining only the compact dimension separating the visible and the hidden sectors, these effects were interpreted in [7] as being induced by additional vector multiplets propagating in the bulk and coupling non-minimally to the localized brane sectors. Since these vector multiplets appear very generically, one is then forced to conclude that sequestering is rather unnatural to realize in string models.

For heterotic models based on a compact manifold X with a vector bundle V over it [8, 9], the above phenomenon can be visualized very clearly. Indeed, these models have a simple interpretation within M-theory, where the additional extra dimension is a segment connecting two branes supporting charged sectors [10]. These two brane sectors are then naturally identified with visible and hidden sectors. In the weak-coupling regime, which corresponds to a small size for the extra segment, the heterotic and M-theory pictures becomes equivalent, the former being obtained by integrating out the heavy KK modes in the latter. After compactifying on X, this implies a similar relation between the 4D effective theory and a 5D theory compactified on a segment connecting two 4D branes. In the bulk of this theory one obtains one vector multiplet for each non-trivial Kähler structure deformation of X, with couplings to the brane sectors that are determined by the choice of V through a shift in its Bianchi identity [11, 12, 13]. From the 4D point of view, each of these multiplets contains one chiral multiplet zero mode describing a non-universal modulus of X in the low-energy effective theory and one tower of vector multiplet KK modes inducing non-trivial effective interactions when integrated out. The non-trivial contact terms of the 4D effective Kähler potential are then in one-to-one correspondence with the presence of non-minimal Kähler moduli for X, besides the one controlling its overall volume, and have a structure that depends on the choice of V.

For orientifold models based on a compact manifold X with D-branes wrapped on it (see for example [14, 15] for recent reviews), the situation is similar. Visible and hidden sectors may naturally arise from D-branes wrapping on two non-intersecting cycles of X. It is however less straightforward to relate the 4D effective theory to a higher-dimensional theory and reinterpret the contact terms as being induced by the exchange of heavy fields. Nonetheless, it turns out that in all the cases where it has been worked out, the 4D effective theory displays a structure that is very similar to the one arising in heterotic models. In particular, the contact terms arising in the 4D effective Kähler potential again seem to be in one-to-one correspondence with non-minimal Kähler moduli of X, suggesting that in this case too one should be able to interpret these as due to the exchange of corresponding heavy vector multiplets. A precise argumentation justifying this conclusion was presented in [7] for the special case of toroidal orientifolds, where one can make use of T-duality to reach a situation where the two sectors are again separated by a single extra dimension, and it is plausible that it indeed holds more in general.

In summary, we see that in string models one may naturally achieve the situation where the visible and hidden sectors are split along an extra dimension, but this is not enough to really achieve sequestering. Nevertheless, the situation is still better that in a generic supergravity model, because the non-vanishing contact terms that arise in the Kähler potential have a very specific form, as a consequence of the fact that they are induced by the exchange of heavy vector multiplets. More precisely, these contact terms consist of products of two or more of the current superfields $J_{\rm v}^a$ and $J_{\rm h}^a$ that act as sources for the heavy vector superfields. One may then hope to be able to exploit the structure of these classical contact terms to devise situations where they actually give a satisfactory contribution to soft masses. In playing this game, one may take the point of view of [16, 17] that the effective Kähler potential, which controls through the contact interactions mixing visible and hidden sectors the general structure of soft scalar masses, is known and therefore fixed, whereas the superpotential, which controls the size of the supersymmetry breaking auxiliary fields of the hidden sector fields, is not known and a priori generic. For generality, one should moreover consider the situation where both the moduli fields and the hidden brane fields participate to supersymmetry breaking. Finally one may also take into account the fact that there are constraints from the condition that the supersymmetry breaking sector should admit a metastable de Sitter vacuum with sufficiently small energy and sufficiently long life time. For a given Kähler potential, this constrains the acceptable directions for the Goldstino vector of auxiliary fields and therefore the acceptable superpotentials [18].

A first appealing possibility is to assume that the moduli fields dominate supersymmetry breaking and that for some reason the contact terms between these fields and the visible sector fields are flavor universal [16, 17]. In that case one would get a non-vanishing but flavor-universal classical contribution to soft scalar masses, and loop contributions would only represent a small correction. This scenario would for instance naturally occur if the dilaton could dominate supersymmetry breaking on its own, since its couplings are automatically universal at the classical level [16]. But unfortunately, it turns out that due to the leading order form of the Kähler potential for the dilaton, the assumption that it dominates supersymmetry breaking is actually incompatible with the existence of a metastable de Sitter vacuum, at least under the assumption that the string coupling is weak [19, 20, 18].

Another appealing possibility is to imagine that the hidden brane fields dominate supersymmetry breaking and that their dynamics enjoys a set of global symmetries ensuring the conservation of the hidden-sector current superfields $J_{\rm h}^a$, which appear together with visible-sector current superfields $J_{\rm v}^a$ in the contact terms [21]. In such a situation, the classical contribution to the soft scalar masses would cancel out, at least at leading order in the hidden scalar expectation values, and flavor-universal loop corrections would represent the dominant effect. The basic point behind this idea was already explained in [22], although in a different context and in the approximation of rigid supersymmetry, and rests on the fact that the conservation of the superfields $J_{\rm h}^a$ implies that both their F and D components vanish. The consequent vanishing of classical soft scalar masses can then also be viewed as a cancellation between the various contributions coming from the hidden sector fields, which is determined by the constraints put on the ratios of their auxiliary fields by the invariance of the superpotential under the global symmetries. In our previous paper [23], we studied how this nice framework may be implemented in supergravity models. We showed that the cancellation mechanism is generically spoiled by non-linear effects coming from terms with more than two currents in the contact interactions, as well as by gravitational effects in the Ward identity of the global symmetries. We however also argued that both of these effects become small in the limit of small expectation values for the hidden-sector matter scalar fields, and can in practice be safely neglected. In this situation, one would thus recover a milder form of the sequestering mechanism, working thanks to global symmetries.

The aim of this work is to understand whether it is be possible to implement the above mechanism of sequestering by global symmetries within generic string models and with both the moduli and the matter fields participating to supersymmetry breaking. More specifically, we want to clarify the circumstances under which it is possible to find suitable global symmetries ensuring the conservation of the currents building up the contact terms. In fact, it is a priori not automatic that such symmetries exist, because the couplings of the heavy vector multiplets to the brane fields are not minimal gauge couplings, but rather dictated by modified Bianchi identities, and one may then wonder how natural it is that they arise. For concreteness and simplicity, we shall focus on the case of heterotic models, but we expect that it should be possible to perform a similar study for orientifold models. In [23] we examined the special subclass of models based on orbifolds, and found that in that case the needed global symmetries naturally arise. Our goal here is to study what happens in the more general case of models based on smooth Calabi-Yau manifolds, and in particular whether the needed global symmetries still arise in a natural way. One major difficulty in this generalization concerns the knowledge of the effective Kähler potential. For orbifold models, the exact dependence on both the moduli fields and the brane fields is known [24, 25], and the structure of contact terms is therefore well under control. For generic Calabi-Yau manifolds, on the other hand, only the dependence on the moduli fields is known exactly [26, 27, 28], whereas the knowledge of the dependence on the brane fields is mostly limited to the leading quadratic order [29]. An interesting claim on the structure of the exact dependence on the matter fields has however recently appeared in the literature, based on the higher-dimensional M-theory interpretation of these models [30]. This generalizes the result of [34] applying to the special case of Calabi-Yau manifolds possessing only a minimal volume Kähler modulus. It moreover has a structure that is qualitatively similar to the one derived in [31, 32, 33] for orientifold models. One of our main tasks will then be to assess this result from the standard heterotic string point of view and to study the resulting structure of contact terms.

The rest of the paper is organized as follows. In section 2 we consider the heterotic string compactified on a smooth Calabi-Yau manifold and study the general structure of the effective Kähler potential. In section 3 we consider similarly the heterotic string compactified on a toroidal orbifold and show how the effective Kähler potential for the untwisted sector can be understood in similar terms. In section 4 we comment on the M-theory interpretation of these models and the way in which the contact terms arising in the effective Kähler potential can be understood as emerging from the exchange of heavy vector multiplets. In section 5 we study the scalar manifolds emerging in these models and discuss a canonical parametrization that is particularly convenient to describe the neighborhood of the reference point where only the universal volume modulus has a scalar expectation value. In section 6 we study the structure of soft scalar masses at this reference point and examine under which circumstances they may be made to vanish by imposing some global symmetry. In section 7 we present our conclusions. Finally, in appendix A we summarize some basic facts about Calabi-Yau manifolds and vector bundles over them, and in appendix B we record some useful facts about the symmetric spaces emerging in orbifold models.

2 The heterotic string on a Calabi-Yau manifold

Let us consider the heterotic string compactified on a generic Calabi-Yau manifold X with a generic stable holomorphic vector bundle V over it [8]. The 4D low-energy effective supergravity theory describing such a model can be obtained by starting from the 10D supergravity effective theory and working out its reduction on X. We shall start by reviewing the general structure of these models. We shall next describe how the effective Kähler potential can be derived by computing the form of the bosonic kinetic terms. We shall focus on the Kähler moduli and the matter fields, and study the full dependence of the Kähler potential on these fields, generalizing the known results for the exact dependence on the moduli and the leading dependence on the matter fields.

2.1 General structure

In the original 10D effective supergravity theory, the bosonic fields are the metric G_{MN}, the antisymmetric tensor B_{MN}, the dilaton Φ and the $E_8 \times E_8$ gauge fields A_M^X. It is convenient to describe B_{MN} in terms of a 2-form B and A_M^X in terms of a Lie-algebra-valued 1-form A. At the two-derivative order, the effective action for these fields reads:

$$\begin{aligned} S_{10} = & \frac{1}{\kappa_{10}^2} \int d^{10}x \sqrt{-G} e^{-2\Phi} \left[\frac{1}{2}R + 2\,\partial_M \Phi\, \partial^M \Phi - \frac{1}{4}|H|^2\right] \\ & + \frac{1}{g_{10}^2} \int d^{10}x \sqrt{-G} e^{-2\Phi} \left[-\frac{1}{2}\mathrm{tr}|F|^2\right]. \end{aligned} \tag{2.1}$$

The 10D gravitational and gauge couplings κ_{10}^2 and g_{10}^2 are related to the string slope α' through the formula $\kappa_{10}^2/g_{10}^2 = \alpha'/4$. The 2-form F denotes the usual field-strength of the non-Abelian gauge field A and the 3-form Γ the Chern-Simons form associated to it, whereas the 3-form H is a modified field-strength for the Abelian antisymmetric field B:

$$F = dA + A \wedge A\,, \quad \Gamma = \mathrm{tr}\Big(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\Big)\,, \tag{2.2}$$

$$H = dB - \frac{\kappa_{10}^2}{g_{10}^2}\Gamma\,. \tag{2.3}$$

At higher order in the derivative expansion, there appear other terms involving the curvature 2-form $R = d\omega + \omega \wedge \omega$ related to the spin connection 1-form ω, as well as the Chern-Simons 3-form $\Xi = \mathrm{tr}(\omega \wedge d\omega + 2/3\,\omega \wedge \omega \wedge \omega)$ associated to it. In particular, at the four-derivative level one gets extra terms that essentially correspond to substituting $\mathrm{tr}|F|^2$ with $\mathrm{tr}|F|^2 - \mathrm{tr}|R|^2$ and Γ with $\Gamma - \Xi$. These two kinds of new terms are related by supersymmetry, and turn out to be relevant for the consistency of the microscopic theory. Most importantly, the Bianchi identity for the 3-form H becomes

$$dH = \frac{\kappa_{10}^2}{g_{10}^2}\Big(\mathrm{tr}(R \wedge R) - \mathrm{tr}(F \wedge F)\Big)\,. \tag{2.4}$$

Consistent supersymmetric backgrounds must not only lead to vanishing supersymmetry transformations of the fermions, but also solve the above Bianchi identity. In particular, the right-hand side of (2.4) must vanish in cohomology. This represents a topological relation between the tangent bundle TX of the compactification manifold X and the vector bundle V over it, which restricts the possible choices of V for a given X. One simple and universal possibility, called standard embedding, is to take V to be isomorphic to TX. This means that V has structure group $SU(3)$ and that the background values of the gauge connection A and the spin connections ω are identified. In such a case the right-hand side of (2.4) vanishes identically and the background is a Calabi-Yau geometry for G_{MN}. A more general possibility, called non-standard embedding, is to require that V should have the same second Chern character as TX. This allows V to have more general structure groups and the background values of A and ω to differ [35, 36]. In this more general case, however, the right-hand side of (2.4) does not vanish identically but only modulo an exact form. As a result, the background is no-longer a simple Calabi-Yau geometry for G_{MN} and also involves a non-trivial profile for B_{MN} and Φ. However, it has been argued in [35] that such a background exists and that it can be understood as a deformation of the standard case in a large volume or small α' expansion. Some of the leading corrections have been worked out in [37, 38, 39].

To characterize the models resulting from this construction, one can start by classifying the relevant modes in terms of representations under the holonomy group $SU(3)$ of X and the structure group S of V. The 10D Lorentz group $SO(1,9)$ is broken to $SO(1,3) \times U(1) \times SU(3)$, where the $SO(1,3)$ factor survives as 4D Lorentz symmetry. The fundamental representation splits as $\mathbf{10} \to \mathbf{4} \oplus \mathbf{3} \oplus \bar{\mathbf{3}}$. We correspondingly split the 10D Lorentz indices M into 4D Lorentz indices μ and internal $SU(3)$ indices $i, \bar{\imath}$. The 10D gauge group $E_8 \times E_8$ is broken to $G \times S$, where G is the commutant of S and survives as 4D gauge group. One actually gets $S = S_{\rm v} \times S_{\rm h}$ and $G = G_{\rm v} \times G_{\rm h}$, where $G_{\rm v}$ and $G_{\rm h}$ are the commutants of $S_{\rm v}$ and $S_{\rm h}$ within the two E_8 factors, but for the moment we shall treat the two sectors together. The adjoint representation splits pretty generically as $\mathbf{496} \to (\mathbf{Adj}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{Adj}) \oplus (\mathbf{R}, \mathbf{r}) \oplus (\bar{\mathbf{R}}, \bar{\mathbf{r}})$, where $\mathbf{R}$ and $\mathbf{r}$ are complex and generically reducible representations of G and S (except for a few special cases that we shall disregard for notational simplicity). We correspondingly split the 10D gauge indices X of the adjoint representation of $E_8 \times E_8$ into 4D gauge indices x in the adjoint representation of G, indices ρ in the adjoint representation of S and indices $\alpha\epsilon$ and $\bar{\alpha}\bar{\epsilon}$ in the representations that are left over. Using the above decompositions, one may now classify the bosonic fields in terms of representations of $SU(3) \times S$. In the neutral sector, the fields transform non-trivially only under $SU(3)$ but are all singlets under S. $G_{\mu\nu}$ gives a symmetric tensor in the $\mathbf{1}$, $G_{\mu i}$, $G_{\mu\bar{\imath}}$ give vectors in the $\mathbf{3} \oplus \bar{\mathbf{3}}$, and $G_{i\bar{\jmath}}$, G_{ij}, $G_{\bar{\imath}\bar{\jmath}}$ give scalars in the $\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{6} \oplus \bar{\mathbf{6}}$. Similarly $B_{\mu\nu}$ gives an antisymmetric tensor dual to a scalar in the $\mathbf{1}$, $B_{\mu i}$, $B_{\mu\bar{\imath}}$ give vectors in the $\mathbf{3} \oplus \bar{\mathbf{3}}$, and $B_{i\bar{\jmath}}$, B_{ij}, $B_{\bar{\imath}\bar{\jmath}}$ give scalars in the $\mathbf{1} \oplus \mathbf{8} \oplus \mathbf{3} \oplus \bar{\mathbf{3}}$. Finally Φ gives just a scalar in the $\mathbf{1}$. In the charged sector, on the other hand, the fields transform non-trivially under $SU(3) \times S$. A_μ^x gives vectors in the $(\mathbf{1}, \mathbf{1})$, A_μ^ρ gives vectors in the $(\mathbf{1}, \mathbf{Adj})$, $A_\mu^{\alpha\epsilon}$ and $A_\mu^{\bar{\alpha}\bar{\epsilon}}$ give vectors in the $(\mathbf{1}, \mathbf{r}) \oplus (\mathbf{1}, \bar{\mathbf{r}})$, A_i^x, $A_{\bar{\imath}}^x$ give scalars in the $(\mathbf{3}, \mathbf{1}) \oplus (\bar{\mathbf{3}}, \mathbf{1})$, A_i^ρ, $A_{\bar{\imath}}^\rho$ give scalars in the $(\mathbf{3}, \mathbf{Adj}) \oplus (\bar{\mathbf{3}}, \mathbf{Adj})$, and finally $A_i^{\alpha\epsilon}$, $A_{\bar{\imath}}^{\alpha\epsilon}$, $A_i^{\bar{\alpha}\bar{\epsilon}}$, $A_{\bar{\imath}}^{\bar{\alpha}\bar{\epsilon}}$ give scalars in the $(\mathbf{3}, \mathbf{r}) \oplus (\bar{\mathbf{3}}, \mathbf{r}) \oplus (\mathbf{3}, \bar{\mathbf{r}}) \oplus (\bar{\mathbf{3}}, \bar{\mathbf{r}})$.

The spectrum of light fields entering the 4D low-energy effective theory is determined by figuring out all the zero-modes admitted by the above 10D bosonic fields. This is done by associating these modes to differential forms on X taking values in appropriate vector bundles constructed out of TX or V, and looking for all the possible independent harmonic components of these forms. The linear space of such harmonic forms is known to be in one-to-one correspondence with non-trivial equivalence classes of the Dolbeault cohomology groups, and one may then use bases of such spaces to parametrize the various light fields. For neutral fields, what matters are the tangent and cotangent bundles TX and T^*X with structure group $SU(3)$, and the components of the relevant harmonic forms fill representations of $SU(3)$. There is 1 harmonic $(3,0)$ form Ω and its conjugate filling the $\mathbf{1} \oplus \mathbf{1}$, $h^{1,2} = \dim(H^1(X, TX))$ harmonic $(1,2)$ forms σ_Z filling the $\mathbf{6} \oplus \bar{\mathbf{6}}$, and finally $h^{1,1} = \dim(H^1(X, T^*X))$ harmonic $(1,1)$ forms ω_A filling the $\mathbf{1} \oplus \mathbf{8}$. For charged fields, what matter are the bundles $V_{\rm Adj}$, $V_{\rm r}$ and $V_{\bar{\rm r}}$ obtained by lifting V to the representations $\mathbf{Adj}$, $\mathbf{r}$ and $\bar{\mathbf{r}}$ of its structure group S, and the components of the relevant harmonic forms fill representations of $SU(3) \times S$. There are $n_1 = \dim(H^1(X, V_{\rm Adj}))$ harmonic $(1,0)$ forms σ_Θ and their conjugate filling the $(\mathbf{3}, \mathbf{Adj}) \oplus (\bar{\mathbf{3}}, \mathbf{Adj})$, $n_R = \dim(H^1(X, V_{\rm r}))$ harmonic $(1,0)$ forms u_P and their conjugate filling the $(\mathbf{3}, \mathbf{r}) \oplus (\bar{\mathbf{3}}, \bar{\mathbf{r}})$, and $n_{\bar{R}} = \dim(H^1(X, V_{\bar{\rm r}}))$ harmonic $(1,0)$ forms v_K and their conjugate filling the $(\mathbf{3}, \bar{\mathbf{r}}) \oplus (\bar{\mathbf{3}}, \mathbf{r})$. Using the above set of harmonic forms, one finally finds the following spectrum of light 4D bosonic fields. In the neutral sector, there is 1 symmetric tensor coming from $G_{\mu\nu}$ and belonging to the gravitational multiplet G, 1 universal complex scalar coming from Φ and $B_{\mu\nu}$ and belonging to the dilaton chiral multiplet S, $h^{1,1}$ complex scalars coming from the decomposition of the forms associated to $G_{i\bar{\jmath}}$ and $B_{i\bar{\jmath}}$ onto the basis ω_A and belonging to Kähler moduli chiral multiplets T^A, and finally $h^{1,2}$ complex scalars coming from the decomposition of the forms associated to G_{ij} and $G_{\bar{\imath}\bar{\jmath}}$ onto the basis σ_Z and belonging to complex structure moduli chiral multiplets U^Z. In the charged sector, there is 1 set of vectors coming from A_μ^x and belonging to vector multiplets V^x in the $\mathbf{Adj}$ of G, n_1 complex scalars coming from A_i^ρ and $A_{\bar{\imath}}^\rho$ and belonging to vector bundle moduli chiral multiplets E^Θ in the $\mathbf{1}$ of G, n_R sets of complex scalars coming from $A_i^{\alpha\xi}$ and $A_{\bar{\imath}}^{\alpha\xi}$ and belonging to matter chiral multiplets $\Phi^{P\alpha}$ in the $\mathbf{R}$ of G, and finally $n_{\bar{R}}$ sets of complex scalars coming from $A_i^{\bar{\alpha}\bar{\xi}}$ and $A_{\bar{\imath}}^{\bar{\alpha}\bar{\xi}}$ and belonging to matter chiral multiplets $\Psi^{K\bar{\alpha}}$ in the $\bar{\mathbf{R}}$ of G.

The models with the simplest gauge quantum numbers are obtained by choosing bundles whose structure group involves factors that are either trivial or equal to $SU(3)$ in each of the two sectors. In the first case one has $E_8 \to E_8$ with $\mathbf{248} \to \mathbf{248}$, and the gauge group in unbroken. In the second case one has $E_8 \to E_6 \times SU(3)$ with $\mathbf{248} \to (\mathbf{78}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}) \oplus (\mathbf{27}, \mathbf{3}) \oplus (\overline{\mathbf{27}}, \bar{\mathbf{3}})$, but nothing from the $SU(3)$ factor survives and the gauge group is thus broken to E_6. A first type of model can be constructed by making the asymmetric choice $S_{\rm v} = SU(3)$, $S_{\rm h} =$ trivial. One then finds $G_{\rm v} = E_6$ and $n_1^{\rm v} = \dim(H^1(X, V_{\rm v} \otimes V_{\rm v}^*))$, $n_{27}^{\rm v} = \dim(H^1(X, V_{\rm v}))$, $n_{\overline{27}}^{\rm v} = \dim(H^1(X, V_{\rm v}^*))$ in the visible sector, and just $G_{\rm h} = E_8$ in the hidden sector. The standard embedding where V is isomorphic to TX is a particular case of this class of models where the Bianchi identity is automatically satisfied, with the special property that $n_{27} = h^{1,2}$ and $n_{\overline{27}} = h^{1,1}$. A second type of model can be constructed by making the symmetric choice $S_{\rm v} = SU(3)$, $S_{\rm h} = \mathrm{SU}(3)$. One then finds $G_{\rm v} = E_6$ and $n_1^{\rm v} = \dim(H^1(X, V_{\rm v} \otimes V_{\rm v}^*))$, $n_{27}^{\rm v} = \dim(H^1(X, V_{\rm v}))$, $n_{\overline{27}}^{\rm v} = \dim(H^1(X, V_{\rm v}^*))$ in the visible sector and similarly $G_{\rm h} = E_6$ and $n_1^{\rm h} = \dim(H^1(X, V_{\rm h} \otimes V_{\rm h}^*))$, $n_{27}^{\rm h} = \dim(H^1(X, V_{\rm h}))$, $n_{\overline{27}}^{\rm h} = \dim(H^1(X, V_{\rm h}^*))$ in the hidden sector. Notice that in this case $V_{\rm v}$ and $V_{\rm h}$ are not allowed to be isomorphic to TX, because this would violate the Bianchi identity.

2.2 Effective Kähler potential

The 4D effective Kähler potential can be determined by performing the reduction of the 10D effective kinetic terms for the bosonic fields by integrating over the compact Calabi-Yau X and comparing the result with the standard general form of the Lagrangian of 4D supergravity theories. To perform this computation, we will make two approximations which are commonly done and which crucially simplify the task. The first approximation is that we will ignore the higher-derivative corrections to the 10D effective action and the deformations of the background, and therefore simply consider the reduction of the action

(2.1) onto a generic Calabi-Yau X with a generic stable holomorphic vector bundle V over it. This implies that the result will only be accurate for terms involving arbitrary powers of the moduli fields and arbitrary powers of the combination of α' times two matter fields, and will miss corrections involving powers of α' that are not accompanied by two matter fields, but this is not a big limitation for our purposes. The second approximation is that we will ignore the effect of properly integrating out massive Kaluza-Klein modes and restrict to the truncation of the action to the 4D massless zero-modes. This would generically imply that the result is accurate only for terms involving an arbitrary number of moduli but at most two matter fields, since terms with four and more matter fields can receive corrections induced by the exchange of heavy neutral modes, and this would represent a dramatic limitation for our purposes. We will therefore imagine to restrict to those models for which these effects happen to be absent, at least for the term involving four matter fields in which we are primarily interested. This is guaranteed to happen if there is no cubic coupling between two light matter modes and one heavy moduli mode. Finally, we shall for simplicity restrict our attention to the $h^{1,1}$ Kähler moduli and the n_R families of charged matter fields in the representation $\mathbf{R}$, and instead completely discard the dilaton, the $h^{1,2}$ complex structure moduli, the n_1 vector bundle moduli and the $n_{\bar{R}}$ families of matter fields in the representation $\bar{\mathbf{R}}$.

To compute the 4D effective kinetic terms, we now proceed as follows. We start from eq. (2.1) restricted to the modes associated to $G_{i\bar{\jmath}}$, $B_{i\bar{\jmath}}$ and A_i and integrate over the internal manifold X. We then express the result in terms of the 4D gravitational and gauge couplings. These are defined as $\kappa_4^2 = \kappa_{10}^2/V$ and $g_4^2 = g_{10}^2/V$, where V denotes the background value of the volume of the manifold X, and are again related as $\kappa_4^2/g_4^2 = \alpha'/4$. In the following, we shall set $\kappa_4 = 1$ by a choice of units. Moreover we shall effectively set $g_4 = 1$ in the scalar sector of the Lagrangian by suitably rescaling the charged matter fields. This corresponds to setting $\alpha' = 4$. In this way, one finds the following result:

$$\begin{aligned}\mathcal{L}_4 = \frac{1}{V}\int d^6y\sqrt{G}\bigg[&-\frac{1}{4}G^{i\bar{\jmath}}G^{p\bar{q}}\,\partial_\mu G_{i\bar{q}}\partial^\mu G_{p\bar{\jmath}} \\ &+\frac{1}{4}G^{i\bar{\jmath}}G^{p\bar{q}}\Big(\partial_\mu B_{i\bar{q}} + \mathrm{tr}(A_i\overleftrightarrow{\partial_\mu}\bar{A}_{\bar{q}})\Big)\Big(\partial^\mu B_{p\bar{\jmath}} + \mathrm{tr}(A_p\overleftrightarrow{\partial^\mu}\bar{A}_{\bar{\jmath}})\Big) \\ &- G^{i\bar{\jmath}}\,\mathrm{tr}(\partial_\mu A_i\partial^\mu\bar{A}_{\bar{\jmath}})\bigg]\,. \end{aligned} \tag{2.5}$$

To proceed, we associate the 10D fields $G_{i\bar{\jmath}}$, $B_{i\bar{\jmath}}$ and A_i to differential forms J, B and A, which are defined as follows in local complex coordinates z^i:

$$J = iG_{i\bar{\jmath}}\,dz^i\wedge d\bar{z}^{\bar{\jmath}}\,, \tag{2.6}$$
$$B = B_{i\bar{\jmath}}\,dz^i\wedge d\bar{z}^{\bar{\jmath}}\,, \tag{2.7}$$
$$A = A_i dz^i\,. \tag{2.8}$$

We then decompose these forms onto suitable bases of harmonic forms, with coefficients identified with the 4D light fields. Some basic notation and results concerning harmonic forms on compact Calabi-Yau manifolds X and stable holomorphic bundles V over them are recorded for convenience in appendix A. To define the moduli fields, we shall need to introduce a basis of harmonic $(1,1)$ forms $\omega_A = \omega_{Ai\bar{\jmath}}\,dz^i\wedge d\bar{z}^{\bar{\jmath}}$ on X, which can also be viewed as 1 forms with values in T^*X over X:

$$\omega_A\,,\;\; A=0,\cdots,h^{1,1}-1\;:\;\; \text{basis of}\;\; H^{1,1}(X)\simeq H^1(X,T^*X)\,. \tag{2.9}$$

To define the matter fields, we shall also need a basis of Lie-algebra-valued harmonic 1-forms $u_P = u_{Pi}\,dz^i$ on V_r over X:

$$u_P\,,\;\; P=1,\cdots,n_R\;:\;\; \text{basis of}\;\; H^1(X,V_r)\,. \tag{2.10}$$

We observe now that the forms constructed by taking the product of one u_P and one conjugate $\bar u_Q$ and tracing over the representation $\mathbf{r}$ yield $(1,1)$ forms on X. These $(1,1)$ forms are related to the description of the gauge invariant composite field that can be formed out of two charged matter fields. Since they play an important role, we shall define a dedicated symbol for them:

$$c_{PQ} = i\,\mathrm{tr}(u_P\wedge\bar u_Q)\;:\;\; \text{generic}\;(1,1)\;\text{forms on}\;X\,. \tag{2.11}$$

A crucial observation is that these $(1,1)$ forms are however generically not harmonic. As a result, their scalar product with the non-harmonic $(1,1)$ forms describing massive neutral modes is in general non-vanishing.

It turns out that the low-energy effective Kähler potential always depends on the volume V of X, which is given by the following expression in terms of the Kähler form J:

$$V = \frac{1}{6}\int_X J\wedge J\wedge J\,. \tag{2.12}$$

More explicitly, when rewritten in terms of the 4D fields describing the moduli and matter fields, this will depend on two quantities characterizing X and V. The first is given by the integral of three harmonic $(1,1)$ forms ω_A, ω_B and ω_C, which defines the intersection numbers of X:

$$d_{ABC} = \int_X \omega_A\wedge\omega_B\wedge\omega_C\,. \tag{2.13}$$

The second is given by the integral of the $(1,1)$ forms c_{PQ} and a dual harmonic $(2,2)$ form ω^A, which defines the component of the harmonic part of c_{PQ} along ω_A and encodes therefore the overlap between the traced product of the 1-forms u_P and $\bar u_Q$ with the $(1,1)$ forms ω_A:

$$c^A_{PQ} = \int_X \omega^A\wedge c_{PQ}\,. \tag{2.14}$$

It should be emphasized that (2.13) is a topological invariant, as a result of the fact that the forms ω_A are harmonic, whereas (2.14) is a priori not, because the forms c_{ST} are in general not harmonic.

In the following, we shall restrict to the special case where the forms c_{PQ} are harmonic and c^A_{PQ} is a constant topological invariant, and derive the low-energy effective Kähler potential under these assumptions. We believe that this is a priori necessary to guarantee that the result obtained by truncating to the massless modes, without properly integrating out the massive modes, is reliable. But as matter of fact, we will also crucially exploit these assumptions to be able to obtain a simple result. We shall discuss in subsection 2.3 what may happen in the more general case where c_{PQ} is not harmonic and c^A_{PQ} is not a topological invariant. For notational simplicity, we shall from now on omit to write any trace over the representation $\mathbf{R}$ of the gauge group, since the way in which these traces appear can be reconstructed in an unambiguous way at any stage of the derivation.

2.2.1 Kähler moduli space

The effective Kähler potential for the Kähler moduli, ignoring matter fields, is well known [29, 28]. It can be derived in a quite straightforward way by retaining only the terms depending quadratically on space-time derivatives of the fields $G_{i\bar{\jmath}}$ and $B_{i\bar{\jmath}}$ in (2.5). To work out the reduction, one considers the real $(1,1)$ forms J and B associated to these two fields and decomposes the complex combination $J + iB$ onto the basis of real harmonic $(1,1)$ forms ω_A, with complex coefficients T^A defining the 4D complex moduli fields:

$$J + iB = 2\,T^A \omega_A\,. \tag{2.15}$$

In components this means $G_{i\bar{\jmath}} = -i(T^A + \bar{T}^A)\omega_{Ai\bar{\jmath}}$ and $B_{i\bar{\jmath}} = -i(T^A - \bar{T}^A)\omega_{Ai\bar{\jmath}}$. Plugging these decompositions into the first two terms of (2.5), one then finds a kinetic term for the complex scalar fields T^A of the form

$$\mathcal{L}_4 = -g^{\rm mod}_{A\bar{B}}\,\partial_\mu T^A \partial^\mu \bar{T}^B\,, \tag{2.16}$$

where

$$\begin{aligned} g^{\rm mod}_{A\bar{B}} &= -\frac{1}{V}\int d^6y\,\sqrt{G}\,G^{i\bar{\jmath}}G^{p\bar{q}}\omega_{Ai\bar{q}}\,\omega_{Bp\bar{\jmath}} \\ &= \frac{1}{V}\int_X \omega_A \wedge *\omega_B\,. \end{aligned} \tag{2.17}$$

This metric does not depend at all on the forms c_{PQ}, and the issue of whether these are harmonic or not is therefore trivially irrelevant here. Using the decomposition $J = J^A\omega_A$ with $J^A = T^A + \bar{T}^A$, which implies that $\partial_A J^B = \delta_A^B$, and the relation (A.19), one can rewrite (2.17) in the following form:

$$\begin{aligned} g^{\rm mod}_{A\bar{B}} &= \frac{1}{V}\int_X \omega_A \wedge *\omega_B \\ &= -\partial_A \partial_{\bar{B}} \log V\,. \end{aligned} \tag{2.18}$$

From this expression we deduce that the Kähler potential is given, up to a Kähler transformation, by:

$$K = -\log V\,. \tag{2.19}$$

This can finally be rewritten more explicitly in terms of the chiral multiplets T^A and the intersection numbers d_{ABC} as

$$K = -\log\Big[\frac{1}{6}d_{ABC}J^AJ^BJ^C\Big]\,,\ \ \text{with}\ \ J^A = T^A + \bar{T}^A\,. \tag{2.20}$$

This result has the special property of being special-Kähler and also of the no-scale type, with the property:

$$K_A K^A = 3\,. \tag{2.21}$$

Notice finally that in geometrical terms the quantities K_A and K^A have the following simple expressions:

$$K_A = -\frac{1}{V}\int_X \omega_A \wedge *J\,,\quad K^A = -\int_X \omega^A \wedge J\,. \tag{2.22}$$

2.2.2 Matter field metric

Let us next consider the addition of matter fields, under the simplifying assumption that their background value vanishes or is very small. In this situation, all the terms involving the fields A_i without space-time derivatives can be neglected in (2.5), and the only term to be added is therefore the last one. In this limit the matter sector can be considered as a small perturbation to the moduli sector, and one can neglect the interference between these two sectors. To work out the reduction, one may consider the 1-forms A taking values in the representation $(\mathbf{R},\mathbf{r})$ of $G\times S$, and decompose them onto the basis of harmonic 1-forms u_P taking values in the representation $\mathbf{r}$ of S with complex coefficients Φ^P taking values in the representation $\mathbf{R}$ of G and defining the 4D matter fields:

$$A = \Phi^P u_P\,. \tag{2.23}$$

In components this means $A_i = \Phi^P u_{Pi}$. Plugging this decomposition into the last term of (2.5), one then finds a kinetic term for the complex scalar fields Φ^P of the form

$$\mathcal{L}_4 = -g^{\rm mat}_{P\bar Q}\,\partial_\mu \Phi^P \partial^\mu \bar\Phi^Q\,, \tag{2.24}$$

where

$$\begin{aligned} g^{\rm mat}_{P\bar Q} &= -\frac{i}{V}\int d^6y\,\sqrt{G}\,G^{i\bar\jmath}\,c_{PQi\bar\jmath} \\ &= \frac{1}{V}\int_X c_{PQ}\wedge *J\,. \end{aligned} \tag{2.25}$$

This metric depends on the forms c_{PQ}, but only through their scalar product with the Kähler form J, which is harmonic. As a result, only the harmonic component of the Hodge decomposition of c_{PQ} matters, and the issue of whether the whole forms c_{PQ} are harmonic or not is therefore again irrelevant. Using the decomposition $J = J^A\omega_A$ with $J^A = T^A + \bar T^A$, which as before implies that $\partial_A J^B = \delta_A^B$, as well as the decomposition of $*J$ on the dual basis ω^A and the relation (A.18), one may rewrite (2.25) in the following form:

$$\begin{aligned} g^{\rm mat}_{P\bar Q} &= \frac{1}{V}\int_X \omega_A\wedge *J\int_X c_{PQ}\wedge\omega^A \\ &= \partial_A \log V\, c^A_{PQ}\,. \end{aligned} \tag{2.26}$$

This means that the matter metric is linked to the moduli Kähler potential by the relation $g^{\rm mat}_{P\bar Q} = -K_A c^A_{PQ}$ [40, 30]. This in turn implies that the leading matter-dependent correction to the Kähler potential is given by this metric contracted with two matter fields.

It should however be emphasized that this not only reproduces the matter kinetic terms analyzed in this subsection, but also induces a kinetic mixing between matter and moduli fields proportional to one matter field, as well as a correction to the moduli metric proportional to two matter fields. These terms do indeed occur, as will be clarified in next subsection, but they are negligible under the assumptions made here, and the leading correction to the Kähler potential is indeed

$$\Delta K = -K_A c^A_{PQ} \Phi^P \bar{\Phi}^Q \,. \tag{2.27}$$

Notice finally that one can write the following simple geometric expressions for the contractions $K_A c^A_{PQ}$ and $K_{AB} c^B_{PQ}$:

$$K_A c^A_{PQ} = -\frac{1}{V} \int_X c_{PQ} \wedge *J \,, \tag{2.28}$$

$$K_{AB} c^B_{PQ} = \frac{1}{V} \int_X \omega_A \wedge *c_{PQ} \,. \tag{2.29}$$

2.2.3 Full scalar manifold

Let us finally consider the full dependence on both the Kähler moduli and the matter fields, which is relevant when the matter fields have a non-vanishing and sizable VEV. In this case, one has to consider all the terms in (2.5). The relevant fields are as before $G_{i\bar{\jmath}}$, $B_{i\bar{\jmath}}$ and A_i. The first two can be combined to form a complex $(1,1)$ form $J + iB$, and decomposed onto the basis of harmonic $(1,1)$ forms ω_A. The second can be viewed as matrix-valued 1-forms A, and decomposed onto the basis of harmonic 1-forms u_P. It however turns out that that the precise definition of the 4D moduli fields T^A and matter fields Φ^S that allows to recast the action in a manifestly supersymmetric form involves a non-trivial shift. The form of this shift may be guessed by generalizing the results applying in the two special cases of Calabi-Yau manifolds with a single modulus and of orbifolds, which are also the only two cases where a derivation of the full effective Kähler potential is already known, respectively from [34] and [24]. The only quantity that can possibly enter in the non-trivial shift is c^A_{PQ}, and the appropriate definitions turn out to be

$$J + iB = 2\Big(T^A - \frac{1}{2} c^A_{PQ} \Phi^P \bar{\Phi}^Q\Big)\omega_A \,, \quad A = \Phi^P u_P \,. \tag{2.30}$$

In components this means $G_{i\bar{\jmath}} = -i(T^A + \bar{T}^A - c^A_{PQ}\Phi^P\bar{\Phi}^Q)\omega_{Ai\bar{\jmath}}$, $B_{i\bar{\jmath}} = -i(T^A - \bar{T}^A)\omega_{Ai\bar{\jmath}}$ and $A_i = \Phi^P u_{Pi}$. Plugging these decompositions into (2.5), one then finds kinetic terms for the complex scalar fields T^A and Φ^P of the form

$$\mathcal{L}_4 = -g^{\rm mod}_{A\bar{B}}\, \partial_\mu T^A \partial^\mu \bar{T}^B - g^{\rm mat}_{P\bar{Q}}\, \partial_\mu \Phi^P \partial^\mu \bar{\Phi}^Q - \big(g^{\rm mix}_{A\bar{Q}}\, \partial_\mu T^A \partial^\mu \bar{\Phi}^Q + \text{c.c.}\big) \,, \tag{2.31}$$

where

$$\begin{aligned} g^{\rm mod}_{A\bar{B}} &= -\frac{1}{V} \int d^6y \sqrt{G}\, G^{i\bar{\jmath}} G^{p\bar{q}}\, \omega_{Ai\bar{q}}\, \omega_{Bp\bar{\jmath}} \\ &= \frac{1}{V} \int_X \omega_A \wedge *\omega_B \,, \end{aligned} \tag{2.32}$$

$$\begin{aligned}
g^{\rm mat}_{P\bar Q} &= -\frac{i}{V}\int d^6y\,\sqrt{G}\,G^{i\bar\jmath}\,c_{PQi\bar\jmath} - \frac{1}{V}\int d^6y\,\sqrt{G}\,G^{i\bar\jmath}G^{m\bar n}\,c_{PSi\bar n}\,c_{RQm\bar\jmath}\,\Phi^R\bar\Phi^S \\
&= \frac{1}{V}\int_X c_{PQ}\wedge *J + \frac{1}{V}\Big\{\int_X c_{PS}\wedge *c_{RQ}\Big\}\Phi^R\bar\Phi^S\,, \qquad (2.33)\\
g^{\rm mix}_{A\bar Q} &= \frac{1}{V}\int d^6y\,\sqrt{G}\,G^{i\bar\jmath}G^{m\bar n}\,\omega_{Ai\bar n}\,c_{RQm\bar\jmath}\,\Phi^R \\
&= -\frac{1}{V}\Big\{\int_X \omega_A\wedge *c_{RQ}\Big\}\Phi^R\,. \qquad (2.34)
\end{aligned}$$

This metric now significantly depends on the forms c_{PQ}, not only through their scalar product with the Kähler form J or the basis forms ω_A, which are harmonic, but also through their scalar products among themselves. As a result, not only the harmonic part but also the exact and coexact parts of the Hodge decomposition of c_{PQ} matter, and the issue of whether c_{PQ} is harmonic or not is therefore crucial in this case. As already said, we shall for the moment assume that c_{PQ} is harmonic and c^A_{PQ} is constant, so that one can use the decomposition $c_{PQ} = c^A_{PQ}\omega_A$. Taking into account the new decomposition $J = J^A\omega_A$ with $J^A = T^A + \bar T^A - c^A_{PQ}\Phi^P\bar\Phi^Q$, which still implies that $\partial_A J^B = \delta^B_A$ since c^A_{PQ} is constant, and using the relations (A.18) and (A.19), the metric components (2.32), (2.33) and(2.34) can be rewritten as

$$\begin{aligned}
g^{\rm mod}_{A\bar B} &= \frac{1}{V}\int_X \omega_A\wedge *\omega_B \\
&= -\partial_A\partial_{\bar B}\log V\,, \qquad (2.35)\\
g^{\rm mat}_{P\bar Q} &= \frac{1}{V}\Big\{\int_X \omega_A\wedge *J\Big\}c^A_{PQ} + \frac{1}{V}\Big\{\int_X \omega_A\wedge *\omega_B\Big\}c^A_{PS}c^B_{RQ}\Phi^R\bar\Phi^S \\
&= \partial_A\log V\,c^A_{PQ} - \partial_A\partial_{\bar B}\log V\,c^A_{PS}c^B_{RQ}\Phi^R\bar\Phi^S \\
&= -\partial_P\partial_{\bar Q}\log V\,, \qquad (2.36)\\
g^{\rm mix}_{A\bar Q} &= -\frac{1}{V}\Big\{\int_X \omega_A\wedge *\omega_B\Big\}c^B_{RQ}\Phi^R \\
&= \partial_A\partial_{\bar B}\log V\,c^B_{RQ}\Phi^R \\
&= -\partial_A\partial_{\bar Q}\log V\,. \qquad (2.37)
\end{aligned}$$

From these expressions we see that, modulo an arbitrary Kähler transformation, the Kähler potential is simply given by:

$$K = -\log V\,. \qquad (2.38)$$

More explicitly, this reads in this case:

$$K = -\log\Big[\frac{1}{6}d_{ABC}J^AJ^BJ^C\Big]\,,\ \ \text{with}\ \ J^A = T^A + \bar T^A - c^A_{PQ}\Phi^P\bar\Phi^Q\,. \qquad (2.39)$$

This result coincides with the one proposed in [30] on the basis of an M-theory argumentation. It manifestly reproduces the result (2.20) for the moduli and the leading order correction (2.27) at quadratic order in the matter fields. Moreover its satisfies a no-scale property generalizing (2.21). This is easily demonstrated as follows [30]. Since e^{-K} is homogenous of degree 3 in J^A, we have $J^A \partial K/\partial J^A = -3$. Denoting the fields by $Z^I = T^A, \Phi^P$, we then compute $K_I = \partial K/\partial J^A \partial J^A/\partial Z^I$. In particular, $K_A = \partial K/\partial J^A$ so that $K_A J^A = -3$. Taking a derivative of this relation with respect to $\bar{Z}^{\bar{J}}$, it follows that $K_{\bar{J}A} J^A + K_A \partial J^A/\partial \bar{Z}^{\bar{J}} = 0$, or $K_{\bar{J}A} J^A + K_{\bar{J}} = 0$. Finally, acting on this with the inverse metric $K^{I\bar{J}}$ one deduces that $K^I = -\delta^I_A J^A$. It finally follows that $K_I K^I = -K_A J^A = 3$. Splitting again the two kinds of indices, this means:

$$K_A K^A + K_P K^P = 3\,. \tag{2.40}$$

Notice finally that K_A, K_P, K^A and K^P can be written in the following simple geometrical terms:

$$K_A = -\frac{1}{V}\int_X \omega_A \wedge *J\,, \quad K^A = -\int_X \omega^A \wedge J\,, \tag{2.41}$$

$$K_P = \frac{1}{V}\int_X c_{PS}\bar{\Phi}^S \wedge *J\,, \quad K^P = 0\,. \tag{2.42}$$

Moreover, from the assumption that the forms c_{PQ} are harmonic it follows that also the contraction $K_{AB} c^A_{PQ} c^B_{RS}$ admits a simple geometrical expression:

$$K_{AB} c^A_{PQ} c^B_{RS} = \frac{1}{V}\int_X c_{PQ} \wedge *c_{RS}\,. \tag{2.43}$$

Similarly one also finds that

$$d_{ABC} c^A_{PQ} c^B_{RS} c^C_{MN} = \int_X c_{PQ} \wedge c_{RS} \wedge c_{MN}\,. \tag{2.44}$$

2.3 Range of validity

The simple derivation presented in last subsection is manifestly valid in those cases where the forms c_{PQ} are harmonic and the quantities c^A_{PQ} are constant topological invariants. One special situation in which this is certainly true is when all the involved forms ω_A and u_P are actually not only harmonic but actually covariantly constant. As we shall see more explicitly in next section, this is for instance the case for toroidal orbifold models. But we believe that it could be true also in a less trivial fashion. We will imagine that this is indeed the case for some subset of smooth Calabi-Yau models. For further use, let us then explore a few simple consequences of the above assumptions. Recall that $A = 0, \cdots, h^{1,1} - 1$ labels the different Kähler moduli and $P, Q = 1, \cdots, n_R$ label the different matter fields. By definition, for each of the $h^{1,1}$ values of A the quantity c^A_{PQ} is a Hermitian $n_R \times n_R$ matrix. This means that even when $h^{1,1} > n_R^2$, the number of these matrices that are linearly independent can not exceed n_R^2. In fact, the $h^{1,1}$ matrices c^A_{PQ} can always be rewritten as linear combinations of the n_R^2 independent transposed Hermitian matrices $\lambda^{A'}_{QP}$, with $A' = 0, \cdots, n_R^2 - 1$ and where the transposition is included for later convenience. Notice that whereas the matrices c^A_{PQ} do a priori not satisfy any completeness relation and do not generate any closed algebra, the matrices $\lambda^{A'}_{PQ}$ do instead satisfy an obvious completeness relation since they form a basis of Hermitian matrices and generate a closed algebra, which is that of $U(n_R)$. We therefore know that under the assumptions that we made

$$c^A_{PQ} \;:\; \text{linear combinations of } \lambda^{A'}_{QP}\,, \tag{2.45}$$

$$\lambda^{A'}_{PQ} \;:\; n_R \times n_R \text{ matrices representing the generators of } U(n_R)\,. \tag{2.46}$$

The extension to more general situations where instead c_{PQ} is not harmonic and the quantities c^A_{PQ} are not constant topological invariants is clearly more challenging, and one may wonder whether a result similar to (2.39) could hold true. One first major change arising for a non-harmonic c_{PQ} is that since its Hodge decomposition contains now not only a harmonic piece but also an exact piece and a coexact piece, eq. (2.43) does no longer hold true. More precisely, its left-hand side acquires extra terms matching the contributions to the right-hand side coming from the non-harmonic parts of c_{PQ}, which are clearly more difficult to deal with. In particular, when going from (2.33) to (2.36), one would get additional terms that clearly have to do with the effect of heavy non-zero modes. In fact, these heavy modes must be related to the 10D B field. Indeed, using a democratic formulation of the original 10D theory involving not only the 2 form B but also its magnetic dual 6 form $\tilde{B}$, the contact term from which the problem originates can be deconstructed and the seed for its origin is then reduced to a linear coupling between $\tilde{B}$ and $d\Gamma = \mathrm{tr}(F\wedge F)$. When reducing on X, one then gets a direct coupling between two light matter modes coming from A and one heavy mode coming from $\tilde{B}$ whenever c_{PQ} is not harmonic, and this must be responsible form the extra contributions to the contact terms. A second source of difficulty arising for a non-constant c^A_{PQ} is that this quantity may then be expected to depend on continuous deformations of both the vector bundle V and the manifold X. The first of these dependences, which was already mentioned in [30], does not concern us since it would be related to vector bundle moduli, which we have ignored from the beginning. But the second of these dependences, which we believe should also be a priori feared, is instead directly relevant for our derivation, since it is related to the Kähler moduli that we want to keep in the effective theory. Now, a moduli dependence c^A_{PQ} would imply additional terms in (2.32)–(2.34). Moreover, it would also affect the simple relation $\partial_A J^B = \delta^B_A$ that was used to rewrite these metric in the form (2.35)–(2.37). At first one might hope that these two sources of complications could compensate each other, but things do not seem to be so simple. One may then perhaps have to generalize the decomposition (2.30) through a more complicated and implicit definition of the moduli and matter fields. We were however not able to reach a conclusive assessment of this possibility.

We believe that subtleties very similar to those explained here for heterotic models may actually arise also for orientifold models. More precisely, it seems to us that the results derived in [32, 33] concerning the higher-order dependence of the Kähler potential on the matter fields arising from D-brane sectors should a priori also be correct and reliable only for those special models were massive non-zero modes do not induce non-trivial corrections. We attribute the fact that this is not directly signaled by a technical difficulty in the derivation of [32, 33] to the use of a democratic formulation in terms of all the Ramond-Ramond forms, which deconstructs the original 10D contact term and hides the subtlety.

2.4 Standard embedding

The concerns raised in previous subsection may be illustrated more concretely by considering in some detail the special case of Calabi-Yau manifolds X with a generic number of moduli but standard embedding for the vector bundle V. In this case the situation is somewhat simpler and there exist an alternative way of performing the dimensional reduction for the matter fields. Indeed, recall that in this case V is identified with TX, so that $S = SU(3)$ and $G = E_6 \times E_8$. As a consequence, the additional index in the representation $\mathbf{r} = \bar{\mathbf{3}}$ can be reinterpreted as a cotangent space index, and one may exploit this to construct the $SU(3)$-valued harmonic 1 forms u_A in terms of the harmonic $(1,1)$ forms ω_A.

In the approximation where one works at leading order in the matter fields and neglects the interference between moduli and matter fields, as in subsection 2.2.2, the way in which this decomposition can be done has been explained in [41] (see also [42]). In the end, it essentially amounts to describe the matter modes in terms of a standard $(1,1)$ form $\tilde{A}$ and decompose it on the basis of harmonic $(1,1)$ forms ω_A with $h^{1,1}$ complex coefficients Φ^A taking values in the representation $\mathbf{R} = (\overline{\mathbf{27}}, \mathbf{1})$ of $E_6 \times E_8$ and defining the 4D matter fields. It has been shown in [41] that one must however include a suitable power of the norm of the covariantly constant holomorphic $(3,0)$ form of X in this decomposition, in order to be able to express the potential coming from the non-derivative part of the action in terms of a holomorphic superpotential. Here, since we are considering the case of absent or frozen complex structure moduli, this simply implies some extra power of the volume V, and the correct definition turns out to be

$$\tilde{A} = V^{1/6} \Phi^A \omega_A \,. \tag{2.47}$$

One then finds a kinetic term of the form

$$\mathcal{L}_4 = -g^{\rm mat}_{A\bar{B}} \partial_\mu \Phi^A \partial^\mu \bar{\Phi}^B \,, \tag{2.48}$$

where

$$\begin{aligned} g^{\rm mat}_{A\bar{B}} &= -\frac{1}{V^{2/3}} \int d^6 y \, \sqrt{G} \, G^{i\bar{\jmath}} G^{p\bar{q}} \, \omega_{Ai\bar{q}} \, \omega_{Bp\bar{\jmath}} \\ &= \frac{1}{V^{2/3}} \int_X \omega_A \wedge * \omega_B \,. \end{aligned} \tag{2.49}$$

Through the usual manipulations, this metric can be rewritten as

$$g^{\rm mat}_{A\bar{B}} = -V^{1/3} \partial_A \partial_{\bar{B}} \log V \,. \tag{2.50}$$

This implies that the matter metric is in this case linked to the moduli metric by the relation $g^{\rm mat}_{A\bar{B}} = e^{-K/3} g^{\rm mod}_{A\bar{B}}$, which was first derived in [29] by matching an actual string scattering amplitude computation. The leading matter-dependent correction to the moduli Kähler potential must then have the form

$$\Delta K = e^{-K/3} K_{A\bar{B}} \Phi^A \bar{\Phi}^B \,. \tag{2.51}$$

Comparing the result (2.51) with the general expression (2.27) and requiring them to be equal, we deduce that in the case of standard embedding the matrices c^A_{BC} must have a special form. This is indeed the case. The components of the $(1,1)$ form c_{AB} are found to be given by

$$c_{ABi\bar{\jmath}} = -i\, V^{1/3} G^{p\bar{q}}\, \omega_{Ai\bar{q}}\, \omega_{Bp\bar{\jmath}}\,. \tag{2.52}$$

It is a straightforward exercise to verify that the forms c_{AB} defined by these components are generically not harmonic, except for the particular case where ω_A and/or ω_B is identified with the Kähler form J or happen more in general to be a covariantly constant $(1,1)$ form. Since by eq. (2.22) one has $K^A \omega_A = -J$, this means that:

$$c_{AB} \text{ not harmonic}\,,\ \text{ but } K^A c_{AB} \text{ and } K^B c_{AB} \text{ harmonic}\,. \tag{2.53}$$

One may nevertheless compute the quantity c^A_{BC} by using the expression (2.52) for the components of c_{PQ}. The result depends on the metric and is thus a function of $T^D + \bar{T}^D$. It might be possible to express this function in terms of derivatives of the Kähler potential K for the moduli. But even without writing an explicit expression, one can observe that the factor $V^{1/3} G^{p\bar{q}}$ appearing in the expression (2.52) is a homogenous function of degree 0 in the components of the metric, and therefore in the geometric moduli fields. More precisely, one finds that $c^0_{00} = 1$ when $h^{1,1} = 1$ and there is a single modulus T^0, whereas $c^A_{BC} = c^A_{BC}((T^D + \bar{T}^D)/(T^E + \bar{T}^E))$ when $h^{1,1} > 1$ and there are several moduli T^A. Since by eq. (2.22) one has $K^D = -(T^D + \bar{T}^D)$, this means that

$$c^A_{BC} \text{ not constant}\,,\ \text{ but } K^D \partial_D c^A_{BC} = 0\,. \tag{2.54}$$

Finally, using the relation (2.28) and the expression (2.52), one easily verifies that c^A_{BC} does indeed satisfy an identity ensuring that the two expressions (2.27) and (2.51) are identical:

$$-K_A c^A_{BC} = e^{-K/3} K_{BC}\,. \tag{2.55}$$

One can demonstrate analytically that the above relation forces c^A_{BC} to be constant in the special case $h^{1,1} = 1$ and non-constant when instead $h^{1,1} > 0$. To do so, one starts by assuming that (2.55) is satisfied with a constant c^A_{BC}. One may then take a derivative of (2.55), use $\partial_D c^A_{BC} = 0$ and act with the inverse of the moduli metric to derive the expression $c^A_{BC} = -e^{-K/3} K^{AD} \big(K_{BCD} - \frac{1}{3} K_D K_{BC}\big)$. Finally, one may compute the derivative of this expression to check whether it is really zero, as assumed. In particular, using the identity $\partial_A K^B = -\delta^B_A$ one finds rather easily that $\partial_A c^A_{BC} K^B K^C = -3\, e^{-K/3}\big(h^{1,1} - 1\big)$, which vanishes when $h^{1,1} = 1$ but not when $h^{1,1} > 1$, contradicting in this last case the hypothesis that c^A_{BC} was constant.

When attempting to go on and work out the result at higher orders in the matter fields, one can no longer neglect the interference between matter and moduli fields. One then needs to properly change the definition of the moduli fields. The natural guess based on our general derivation is that the definition of the moduli fields should be shifted by a term that is quadratic in the matter fields and involves c^A_{BC}. Indirect evidence in favor of this has been found in [41] (whose quantity σ_{ABC} is seen to be proportional to our c^A_{BC} specified by (2.52) with the upper index lowered with the moduli metric) by studying the interference of this redefinition and the possible emergence of a non-trivial superpotential. It is however not obvious how one should proceed to work out the full result, as both of the subtleties discussed in section 2.3, namely the non-harmonicity of c_{BC} and the non-constancy of c^A_{BC}, have been manifestly shown to arise in this case, except for the particular situations where $h^{1,1} = 1$, for which the result (2.39) holds true and reduces to the result derived in [34].

3 The heterotic string on an orbifold

It is interesting to compare the general situation occurring for compactifications on a smooth Calabi-Yau manifold X with that of compactifications on toroidal orbifolds of the type T^6/Z_N [9], which represent singular limits of them from the geometrical point of view. We shall briefly review the structure of these models and the derivation of the effective Kähler potential. We shall as before focus on the Kähler moduli and the matter fields, restricting to the untwisted sector for which a simple derivation based on dimensional reduction is possible, and show how the known exact results for the dependence of the Kähler potential on the Kähler moduli and matter fields can be rephrased in the same language as in the previous section.

3.1 General structure

The Z_N orbifold action that is used to define the background is specified by a first twist vector $\alpha_i = (\alpha_1, \alpha_2, \alpha_3)$ in the $SU(3)$ internal space-time group and a second twist vector $\beta_\alpha = (\beta_1, \cdots, \beta_8; \beta_1, \cdots, \beta_8)$ in the $E_8 \times E_8$ gauge group. These should satisfy the following consistency condition for some integer n, which comes from the level-matching condition [9]:

$$\frac{n}{N} = \sum_i \alpha_i(\alpha_i + 1) - \sum_\alpha \beta_\alpha(\beta_\alpha + 1)\,. \tag{3.1}$$

From a geometric perspective, the choice of α_i defines the structure group of the tangent space to be a discrete subgroup of $SU(3)$, whereas the choice of β_α corresponds to a choice of vector bundle. The condition (3.1) is the analogue of the Bianchi identity (2.4) that must be imposed for smooth Calabi-Yau compactifications and constrains the choice of vector bundle for a given tangent bundle. This leaves as before several possibilities, among which one again finds the special possibility of the standard embedding, which corresponds to the choice $\beta_\alpha = (\alpha_1, \alpha_2, \alpha_3, 0, \cdots, 0)$ and trivially satisfies (3.1) with $n = 0$.

The states arising in the untwisted sector are associated to the subset of harmonic forms on T^6 that are left invariant by the Z_N twist. As a result, the low-energy effective theory can easily be computed and turns out to be a projection of what would be obtained by compactifying on T^6. The spectrum of neutral fields can be understood by looking at the transformation properties of the various harmonic forms under the discrete structure group $Z_N \subset SU(3)$ of TX. One in particular sees that the **1** is always kept and the **3** is always lost, whereas $h^{1,2}$ forms in the **6** and $h^{1,1} - 1$ forms in the **8** survive the projection, with $h^{1,1}$ and $h^{1,2}$ being the effective Hodge numbers pertaining to the untwisted sector. We will restrict to the prototypical cases based on $N = 3, 6, 7$, which lead to $h^{1,1} = 9, 5, 3$ and $h^{1,2} = 0$. The spectrum of charged fields can similarly be understood by looking at the transformation properties of the various forms not only under the discrete structure group $Z_N \subset SU(3)$ of TX, but also under the discrete structure group S of V.

The simplest models are obtained by choosing bundles whose structure group is either trivial or a discrete Z_N subgroup of $SU(3)$ in each of the two sectors. In the first case one has $E_8 \to E_8$ with $\mathbf{248} \to \mathbf{248}$, and the gauge group is unbroken. In the second case on has $E_8 \to E_6 \times SU(3)$ with $\mathbf{248} \to (\mathbf{78}, \mathbf{1}) \oplus (\mathbf{1}, \mathbf{8}) \oplus (\mathbf{27}, \mathbf{3}) \oplus (\overline{\mathbf{27}}, \overline{\mathbf{3}})$, and further $SU(3) \to H$ with $\mathbf{3} \to \mathbf{h}$, so that the gauge group is broken to $E_6 \times H$, where the enhanced gauge symmetry $H \subset SU(3)$ arises as the non-trivial commutant of the discrete structure group Z_N within $SU(3)$. In the three models under consideration, one respectively finds the three possible maximal-rank subgroups $H = SU(3), SU(2) \times U(1), U(1) \times U(1)$, with $\mathbf{h} = \mathbf{3}, \mathbf{2} \oplus \mathbf{1}, \mathbf{1} \oplus \mathbf{1} \oplus \mathbf{1}$. The various generations of untwisted matter fields in the $\mathbf{27}$ and $\overline{\mathbf{27}}$ of E_6 must then arrange into the representations $\mathbf{h}$ and $\bar{\mathbf{h}}$ of H descending form the $\mathbf{3}$ and $\overline{\mathbf{3}}$ of $SU(3)$. In order to compare with the case of smooth Calabi-Yau manifolds and make it simpler, let us for a moment count the total numbers n_{27} and $n_{\overline{27}}$ of $\mathbf{27}$ and $\overline{\mathbf{27}}$ without caring about the H quantum numbers. A first type of model can be constructed by making the asymmetric choice $S_{\rm v} = Z_N$, $S_{\rm h} =$ trivial. One then finds $G_{\rm v} = E_6 \times H$ and $n_1^{\rm v} = 0$, $n_{27}^{\rm v} = 0$, $n_{\overline{27}}^{\rm v} = h^{1,1}$ in the visible sector, and just $G_{\rm h} = E_8$ in the hidden sector. The standard embedding is a particular case of this class of models where the level matching condition is trivially satisfied. A second type of model can be constructed by making the symmetric choice $S_{\rm v} = Z_N$, $S_{\rm h} = Z_N$. One then finds $G_{\rm v} = E_6 \times H$ and $n_1^{\rm v} = 0$, $n_{27}^{\rm v} = 0$, $n_{\overline{27}}^{\rm v} = h^{1,1}$ in the visible sector, and similarly $G_{\rm h} = E_6 \times H$ and $n_1^{\rm h} = 0$, $n_{27}^{\rm h} = 0$, $n_{\overline{27}}^{\rm h} = h^{1,1}$ in the hidden sector. In addition, there always are $h^{1,1}$ Kähler moduli.

3.2 Effective Kähler potential

The 4D effective Kähler potential for the untwisted sector of orbifold models is most easily computed by simply retaining those fields that are invariant under the Z_N projection in (2.5). One can then compute the metric, guess the appropriate definition of the chiral multiplets that makes this manifestly Kähler, and finally find out the form of the Kähler potential. This last step can be done by relying on some basic properties of square matrices, which are described at the end of appendix A. Here we would like to emphasize that the same result can be obtained by proceeding exactly as we did in section 2 for compactifications on smooth Calabi-Yau manifolds. We shall briefly summarize how this is done for the three different kind of models under consideration, in order to make contact with the results of [23]. As before, for notational simplicity we shall omit to write explicitly the traces over the representation $\mathbf{R}$ of the gauge group G. We also omit any detail about the trace over the representation $\mathbf{r}$ of the structure group S, since this is discrete. Finally, we shall here restrict for concreteness to the particular models discussed at the end of the previous subsection.

3.2.1 Models with $H = SU(3)$

Let us first consider the case of the Z_3 orbifold, where $H = SU(3)$. In this case, $h^{1,1} = 9$ and $n_{(\overline{\mathbf{3}}, \overline{\mathbf{27}})} = 3$, so that in total $n_{\overline{\mathbf{27}}} = 9$. There are 9 harmonic $(1,1)$ forms ω_{ij} and 3 Z_3-valued harmonic 1-forms u_i, with $i = 1, 2, 3$:

$$\omega_{ij} = i\, dz^i \wedge d\bar{z}^j \,, \tag{3.2}$$

$$u_i = dz^i \,. \tag{3.3}$$

The intersection numbers are found to be:

$$d_{ijpqrs} = \epsilon_{ipr}\epsilon_{jqr}\,. \tag{3.4}$$

The forms $c_{ij} = i\,u_i \wedge \bar{u}_j$ are found to be given by $c_{ij} = \omega_{ij}$, and their components on the basis ω_{mn} read

$$c_{ij}^{mn} = \delta_i^m \delta_j^n\,. \tag{3.5}$$

The moduli fields T^{ij} and the matter fields Φ^i are then defined by the following expansions:

$$J + iB = 2\Big(T^{ij} - \frac{1}{2}\Phi^i\bar{\Phi}^j\Big)\omega_{ij}\,, \tag{3.6}$$

$$A = \Phi^i u_i\,. \tag{3.7}$$

The Kähler potential is finally found to be given by [24, 25]:

$$K = -\log\Big[\det\Big(T^{ij} + \bar{T}^{ij} - \Phi^i\bar{\Phi}^j\Big)\Big]\,. \tag{3.8}$$

3.2.2 Models with $H = SU(2) \times U(1)$

Let us next consider the case of the Z_6 orbifold, where $H = SU(2) \times U(1)$. In this case, $h^{1,1} = 5$ and $n_{(\mathbf{2},\overline{\mathbf{27}})} = 2$, $n_{(\mathbf{1},\overline{\mathbf{27}})} = 1$, so that in total $n_{\overline{\mathbf{27}}} = 5$. There are 5 harmonic $(1,1)$ forms $\omega_{\underline{ij}}$, ω_{33} and 3 Z_6-valued harmonic 1-forms $u_{\underline{i}}$, u_3, with $\underline{i} = 1, 2$:

$$\omega_{\underline{ij}} = i\,dz^{\underline{i}} \wedge d\bar{z}^{\underline{j}}\,,\ \ \omega_{33} = i\,dz^3 \wedge d\bar{z}^3\,, \tag{3.9}$$

$$u_{\underline{i}} = dz^{\underline{i}}\,,\ \ u_3 = dz^3\,. \tag{3.10}$$

The non-vanishing entries of the intersection numbers are:

$$d_{\underline{ijpq}33} = \epsilon_{\underline{ip}3}\epsilon_{\underline{jq}3}\,. \tag{3.11}$$

The forms $c_{ij} = i\,u_i \wedge \bar{u}_j$ are easily computed and one finds $c_{\underline{ij}} = \omega_{\underline{ij}}$, $c_{33} = \omega_{33}$, while the other vanish. The non-vanishing components of these forms on the basis ω_{mn} are

$$c_{\underline{ij}}^{\underline{mn}} = \delta_{\underline{i}}^{\underline{m}}\delta_{\underline{j}}^{\underline{n}}\,,\ \ c_{33}^{33} = 1\,. \tag{3.12}$$

The moduli fields $T^{\underline{ij}}$, T^{33} and the matter fields $\Phi^{\underline{i}}$, Φ^3 are then defined by the following expansions:

$$J + iB = 2\Big(T^{\underline{ij}} - \frac{1}{2}\Phi^{\underline{i}}\bar{\Phi}^{\underline{j}}\Big)\omega_{\underline{ij}} + 2\Big(T^{33} - \frac{1}{2}\Phi^3\bar{\Phi}^3\Big)\omega_{33}\,, \tag{3.13}$$

$$A = \Phi^{\underline{i}}u_{\underline{i}} + \Phi^3 u_3\,. \tag{3.14}$$

The Kähler potential is finally found to be given by [24, 25]:

$$K = -\log\Big[\det\Big(T^{\underline{ij}} + \bar{T}^{\underline{ij}} - \Phi^{\underline{i}}\bar{\Phi}^{\underline{j}}\Big)\Big(T^{33} + \bar{T}^{33} - \Phi^3\bar{\Phi}^3\Big)\Big]\,. \tag{3.15}$$

3.2.3 Models with $H = U(1) \times U(1)$

Let us finally consider the case of the Z_7 orbifold, where $H = U(1) \times U(1)$. In this case, $h^{1,1} = 3$ and $n_{(\mathbf{1},\overline{\mathbf{27}})} = 3$, so that in total $n_{\overline{\mathbf{27}}} = 3$. There are 3 harmonic $(1,1)$ forms ω_{11}, ω_{22}, ω_{33} and 3 Z_7-valued harmonic 1-forms u_1, u_2, u_3:

$$\omega_{11} = i\, dz^1 \wedge d\bar{z}^1\,, \quad \omega_{22} = i\, dz^2 \wedge d\bar{z}^2\,, \quad \omega_{33} = i\, dz^3 \wedge d\bar{z}^3\,, \tag{3.16}$$

$$u_1 = dz^1\,, \quad u_2 = dz^2\,, \quad u_3 = dz^3\,. \tag{3.17}$$

The non-vanishing entries of the intersection numbers are found to be:

$$d_{112233} = 1\,. \tag{3.18}$$

The forms $c_{ij} = i\, u_i \wedge \bar{u}_j$ are found to be given by $c_{11} = \omega_{11}$, $c_{22} = \omega_{22}$, $c_{33} = \omega_{33}$, while the other vanish. The non-vanishing components of these c_{ij} on the basis ω_{mn} read:

$$c_{11}^{11} = 1\,, \quad c_{22}^{22} = 1\,, \quad c_{33}^{33} = 1\,. \tag{3.19}$$

The moduli fields T^{11}, T^{22}, T^{33} and the matter fields Φ^1, Φ^2, Φ^3 are then defined by the following expansions:

$$J + iB = 2\Big(T^{11} - \frac{1}{2}\Phi^1\bar{\Phi}^1\Big)\omega_{11} + 2\Big(T^{22} - \frac{1}{2}\Phi^2\bar{\Phi}^2\Big)\omega_{22} + 2\Big(T^{33} - \frac{1}{2}\Phi^3\bar{\Phi}^3\Big)\omega_{33}\,, \tag{3.20}$$

$$A = \Phi^1 u_1 + \Phi^2 u_2 + \Phi^3 u_3\,. \tag{3.21}$$

The Kähler potential is finally found to be given by [24, 25]:

$$K = -\log\bigg[\Big(T^{11} + \bar{T}^{11} - \Phi^1\bar{\Phi}^1\Big)\Big(T^{22} + \bar{T}^{22} - \Phi^2\bar{\Phi}^2\Big)\Big(T^{33} + \bar{T}^{33} - \Phi^3\bar{\Phi}^3\Big)\bigg]\,. \tag{3.22}$$

3.2.4 General structure

The above results can be rewritten in a more convenient and unified way by performing a suitable change of basis for the harmonic $(1,1)$ forms [23], which clarifies their similarity with the results derived for Calabi-Yau compactifications. To perform this change of basis, we can proceed in parallel for all the three models considered above and introduce the 3×3 Hermitian matrices λ^A representing the generators of $U(1) \times H$ and normalized in such a way that $\mathrm{tr}(\lambda^A\lambda^B) = \delta^{AB}$. More precisely, λ^0 denotes the generator of $U(1)$ proportional to the identity matrix and λ^a the generators of H associated to a subset of the Gell-Mann matrices spanning the fundamental representation of $SU(3)$ ($a = 1, \cdots, 8$ for $H = SU(3)$, $a = 1,2,3,8$ for $H = SU(2) \times U(1)$, $a = 3,8$ for $H = U(1) \times U(1)$):

$$\lambda_{ij}^A \;:\;\; 3\times3 \text{ matrices representing the generators of } U(1) \times H\,. \tag{3.23}$$

We then define the new basis of harmonic $(1,1)$ forms $\omega_A = \lambda_{ij}^A \omega_{ij}$. The corresponding new moduli fields then read $T^A = \lambda_{ji}^A T^{ij}$, and since the matrices λ^A are Hermitian, one finds $\bar{T}^A = \lambda_{ji}^A \bar{T}^{ij}$, where $\bar{T}^{ij}$ denotes as in the previous formulae the Hermitian conjugate of T^{ij} as a matrix. In this new basis, the intersection numbers are given by $d_{ABC} = \lambda_{ij}^A \lambda_{pq}^B \lambda_{rs}^C d_{ijpqrs}$, which yields

$$d_{ABC} = 2\,\mathrm{tr}\big(\lambda^{(A}\lambda^B\lambda^{C)}\big) - 3\,\mathrm{tr}\big(\lambda^{(A}\big)\mathrm{tr}\big(\lambda^B\lambda^{C)}\big) + \mathrm{tr}\big(\lambda^{(A}\big)\mathrm{tr}\big(\lambda^B\big)\mathrm{tr}\big(\lambda^{C)}\big)\,. \tag{3.24}$$

The components c_{ij}^A of c_{ij} are instead given by $c_{ij}^A = \lambda_{nm}^A c_{ij}^{mn}$, which simply gives:

$$c_{ij}^A = \lambda_{ji}^A \,. \tag{3.25}$$

In this basis, the fields are defined as

$$J + iB = 2\Big(T^A - \frac{1}{2} c_{ij}^A \Phi^i \bar{\Phi}^j\Big)\omega_A \,, \quad A = \Phi^i u_i \,, \tag{3.26}$$

and the Kähler potential takes the form:

$$K = -\log\left[\frac{1}{6} d_{ABC} J^A J^B J^C\right] \,, \quad J^A = T^A + \bar{T}^A - c_{ij}^A \Phi^i \bar{\Phi}^j \,. \tag{3.27}$$

For the untwisted sector of these orbifolds, one thus finds exactly the same kind of result as for smooth Calabi-Yau manifolds, with the peculiarity, however, that the intersection numbers d_{ABC} and the quantities c_{ij}^A admit a group-theoretical interpretation. This corresponds to the fact that the scalar manifold becomes a symmetric space. More precisely, in the three kinds of models under consideration the scalar manifolds are given by:

$$\mathcal{M}_{SU(3)} = \frac{SU(3,3+n)}{U(1) \times SU(3) \times SU(3+n)} \,, \tag{3.28}$$

$$\mathcal{M}_{SU(2)\times U(1)} = \frac{SU(2,2+n)}{U(1) \times SU(2) \times SU(2+n)} \times \frac{SU(1,1+n)}{U(1) \times SU(1+n)} \,, \tag{3.29}$$

$$\mathcal{M}_{U(1)\times U(1)} = \frac{SU(1,1+n)}{U(1) \times SU(1+n)} \times \frac{SU(1,1+n)}{U(1) \times SU(1+n)} \times \frac{SU(1,1+n)}{U(1) \times SU(1+n)} \,. \tag{3.30}$$

3.3 Range of validity

For the untwisted sector of orbifold models, we see that the low-energy effective Kähler potential can always be derived in an exact way, without any limitation. From the perspective of the more general study that we performed for smooth Calabi-Yau manifolds, this reflects the fact that untwisted orbifold sectors automatically satisfy the assumptions that we made in section 2. More specifically, we see that the forms c_{ij} are harmonic and the quantities c_{ij}^A are constants. This can be traced back to the fact that in this case the forms ω_A and u_i are not only harmonic, but actually covariantly constant, which is a much stronger property.

4 M-theory interpretation

The structure of the Kähler potential characterizing the 4D low-energy effective theories of heterotic string models admits a simple interpretation in terms of a 5D effective theory compactified on a segment S^1/Z_2, which describes the M-theory lift of these models. In particular, the definition of the chiral multiplets and the structure of the Kähler potential can be understood quite naturally and intuitively within this framework. As we shall briefly review in this section, this is a consequence of the fact that the matter contact terms arising from the non-trivial shift in the field-strength of the 2 form B in the heterotic picture arises in the M-theory picture from the exchange of the heavy Kaluza-Klein modes of the 3 form C reduced on S^1/Z_2, whose couplings to the brane fields are ruled by a Bianchi identity of the schematic form

$$dC = -\mathrm{tr}(F \wedge F)\delta(y - y_0)\,. \tag{4.1}$$

Here and in the following, we shall implicitly understand the splitting of the charged fields over the two brane sectors located at different positions y_0, but for notational simplicity we shall not display this explicitly in the formulae.

4.1 General structure

The content of light bosonic fields of the 5D supergravity theory obtained by compactifying 11D supergravity on a Calabi-Yau manifold X consists of 1 symmetric tensor from G_{MN}, $h^{1,1}$ scalars from the $(1,1)$ components of $G_{i\bar{\jmath}}$, $h^{1,2}$ complex scalars from the $(1,2)$ and $(2,1)$ components of G_{ij} and $G_{\bar{\imath}\bar{\jmath}}$, 1 scalar from the dualization of C_{MNP}, 1 complex scalar from the $(3,0)$ and $(0,3)$ components of C_{ijk} and $C_{\bar{\imath}\bar{\jmath}\bar{k}}$, $h^{1,1}$ vectors from the $(1,1)$ components of $C_{Mi\bar{\jmath}}$ and $h^{1,2}$ complex scalars from the $(1,2)$ and $(2,1)$ components of $C_{i\bar{\jmath}\bar{k}}$ and $C_{\bar{\imath}jk}$. In total this yields 1 symmetric tensor, $h^{1,1} + 4\,h^{1,2} + 3$ real scalar fields and $h^{1,1}$ vector fields, which corresponds to the bosonic content of 1 gravitational multiplet $\mathcal{G}$ and 1 universal hypermultiplet $\mathcal{H}$ plus $h^{1,1} - 1$ vector multiplets $\mathcal{V}^a$ associated to the harmonic $(1,1)$ forms arising in addition to the Kähler form and $h^{1,2}$ hyper multiplets $\mathcal{H}^Z$ associated to the harmonic $(1,2)$ forms [11, 12, 13].

When this 5D theory is further compactified on S^1/Z_2 and reinterpreted from a 4D viewpoint, one finds $N = 2$ supersymmetry projected to $N = 1$ supersymmetry. To understand the spectrum of neutral fields, one can then think in terms of $N = 2$ multiplets and figure out their content in terms of $N = 1$ multiplets with definite Z_2 parities. Listing the even and odd multiplets separated by a semicolon, one finds that $\mathcal{G} = (G, T^0; \Gamma)$ where G is the gravitational multiplet, T^0 a chiral multiplet and Γ is a spin-3/2 multiplet, $\mathcal{H} = (S; S^c)$ where S and S^c are chiral multiplets, $\mathcal{V}^a = (T^a; V^a)$ where T^a are chiral multiplets and V^a vector multiplets, and finally $\mathcal{H}^Z = (U^Z; U^{cZ})$ where U^Z and U^{cZ} are chiral multiplets. The spectrum of light neutral multiplets thus consists of the graviton G, the dilaton S, the overall volume modulus T^0, $h^{1,1} - 1$ relative Kähler moduli T^a and $h^{1,2}$ complex structure moduli U^Z. The spectrum of charged fields is instead determined as in the weakly coupled heterotic string, except that the fields coming from the two E_8 factors are now localized at the two different branes at the ends of the S^1/Z_2 segment. Altogether they fill a number of $N = 1$ chiral multiplets Φ^P, Ψ^K and vector multiplets V^x, in the representations $\mathbf{R}$, $\bar{\mathbf{R}}$ and $\mathbf{Adj}$ of the gauge group.

4.2 Effective Kähler potential

The 4D effective effective Kähler potential can be determined by performing the reduction of the 11D theory on the Calabi-Yau manifold X, and then further reducing the resulting 5D theory on S^1/Z_2. In this case, it is possible to do the last step by using superfields to directly compute the Kähler potential, rather than working with the components and looking at the bosonic kinetic terms. To perform this computation, we shall do the same approximations as in section 2. We shall first neglect the effects of higher-derivative corrections to the 11D effective theory and deformations of the basic background, and simply consider the reduction of the two-derivative 11D effective theory on $X \times S^1/Z_2$. We shall then also discard the effects of massive Kaluza-Klein modes on X, although we will retain the effects of massive Kaluza-Klein modes on S^1/Z_2, which turn out to be crucial to understand the contact terms. Correspondingly, we will also make the same assumptions as in section 2, namely that the $(1,1)$ forms c_{PQ} associated to composites of two matter fields are harmonic and that the quantities c^A_{PQ} are constant topological invariants. Finally, we shall again restrict to the Kähler moduli T^A and the charged matter fields Φ^P.

The starting point is the 5D intermediate theory, where we retain not only the Z_2-even submultiplets T^0, T^a, Φ^P, which contain the light 4D moduli and matter modes, but also the Z_2-odd submultiplets V^a, which contain the heavy Kaluza-Klein modes that have non-trivial linear couplings to the other fields and therefore need to be properly integrated out. It is convenient to work with $N = 1$ superfields T^0, T^a, Φ^P and V^a depending also on the internal coordinate y, and integrate out the heavy modes associated to the V^a directly at the superfield level and in a clever way, by solving their equations of motion by neglecting space-time derivatives to determine their wave-function profile. In the limit where gravity is decoupled, this can be done with usual superfields within rigid supersymmetry along the lines of [43, 44, 45], with T^0 playing the role of the radion superfield. Taking into account gravitational effects is slightly more complicated, but can actually be done in a very similar way by using a superconformal superfield formalism within supergravity, where half of the supersymmetry is manifestly realized off-shell. This formalism has been developed in [46, 47] and further elaborated in [48, 49]. It has the nice feature of allowing to describe the graviphoton A^0_M on the same footing as the other odd gauge fields A^a_M, and the volume modulus T^0 on the same footing as the other Kähler moduli T^a, through vector multiplets V^A and chiral multiplets T^A with $A = 0, a$, at the price of introducing also some constraints. The relevant 5D Lagrangian turns out to be

$$\begin{aligned}\mathcal{L}^{\rm local}_{\rm 5D} = &\int d^2\theta \left[-\frac{1}{4}\mathcal{N}_{AB}(T^A)\, W^{A\alpha} W^B_\alpha + \frac{1}{48}\mathcal{N}_{ABC}\, \bar{D}^2\big(V^A \overleftrightarrow{D}^\alpha \partial_y V^B\big) W^C_\alpha\right] + \text{c.c.} \\ &+ \int d^4\theta\, (-3)\, \mathcal{N}^{1/3}(J^A_y)\,. \end{aligned} \tag{4.2}$$

In this expression, the quantity $\mathcal{N}$ is a norm function playing the role of real prepotential, which is identified with the cubic polynomial defined by the intersection numbers d_{ABC} of the Calabi-Yau manifold X:

$$\mathcal{N}(Z^A) = \frac{1}{6}\, d_{ABC} Z^A Z^B Z^C\,. \tag{4.3}$$

The quantity W^A_α denotes the usual super-field-strength associated to V^A, namely

$$W^A_\alpha = -\frac{1}{4}\bar{D}^2 D_\alpha V^A\,. \tag{4.4}$$

Finally, the quantity J^A_y is a current defined in terms of the quantities c^A_{PQ} characterizing the vector bundle V over X and given by:

$$J^A_y = -\partial_y V^A + T^A + \bar{T}^A - c^A_{PQ}\Phi^P\bar{\Phi}^Q\,\delta(y - y_0)\,. \tag{4.5}$$

In the above expressions, the bosonic modes of T^A come from the decomposition of the 2 forms J and C_y with components $iG_{i\bar{\jmath}}$ and $C_{yi\bar{\jmath}}$ on the basis of harmonic $(1,1)$ forms

ω_A, the bosonic modes of Φ^P come from the decomposition of the Lie-algebra-valued 1 forms A, $\bar{A}$ with components A_i, $\bar{A}_{\bar{\imath}}$ on the basis of harmonic 1 forms u_P, and finally the bosonic modes of V^A come from the decomposition of the 2 forms C_μ with components $C_{\mu i\bar{\jmath}}$ on the basis ω_A. The correct definition of the chiral multiplets in terms of the above modes turns out to be [30]

$$T^A = \frac{1}{2}\Big(J^A + iC_y^A + c^A_{PQ}A^P\bar{A}^Q\delta(y-y_0)\Big)\,, \tag{4.6}$$

$$\Phi^P = A^P\,. \tag{4.7}$$

We see that these definitions reproduce the ones we have introduced in the component derivation of section 2 based on the weakly coupled heterotic string when averaged over the extra dimension. Here these definitions ensure that the lowest component of J_y^A simply reduces to the metric components, as required in order to reproduce an Einstein gravitational kinetic term coming entirely from the bulk and not from the branes, whereas the $\theta\sigma^\mu\bar{\theta}$ component of J_y^A correctly reproduces the modified version of the mixed components of the field strength implied by the reduction of the Bianchi identity (4.1):

$$J_y^A| = J^A\,, \tag{4.8}$$

$$J_y^A|_{\theta\sigma^\mu\bar{\theta}} = \partial_\mu A_y^A - \partial_y A_\mu^A + ic^A_{PQ}\Phi^P\overleftrightarrow{\partial}_\mu\bar{\Phi}^Q\delta(y-y_0)\,. \tag{4.9}$$

This provides a nice superfield interpretation on the need for the shift in the definition of the moduli chiral multiplets.

Integrating out the heavy modes of the vector multiplets V^A effectively amounts to replacing the currents J_y^A with their zero modes in the term of the action that does not involve the vector fields. This is easy to show in the rigid limit, where only the V^a matter [23], but actually holds true also in the supergravity regime where all the V^A appear but suffer from non-trivial constraints [49]. One finds the following expression, written within the usual superconformal superfield formalism,

$$\mathcal{L}_{4\mathrm{D}}^{\mathrm{local}} = \int d^4\theta\,(-3)\,\mathcal{N}^{1/3}(J^A)\,, \tag{4.10}$$

where now

$$J^A = T^A + \bar{T}^A - c^A_{PQ}\Phi^P\bar{\Phi}^Q\,. \tag{4.11}$$

The effective Kähler potential can finally be deduced by matching the integrand of this expression with $-3\,e^{-K/3}$. This gives $K = -\log\mathcal{N}(J^A) = -\log V$, which is the same result as we obtained directly from the heterotic string:

$$K = -\log\Big[\frac{1}{6}d_{ABC}J^AJ^BJ^c\Big]\,,\ \ \text{with}\ \ J^A = T^A + \bar{T}^A - c^A_{PQ}\Phi^P\bar{\Phi}^Q\,. \tag{4.12}$$

A component version of this five-dimensional derivation is also possible, and was presented in [50] for the particular case where $h^{1,1}=1$ with standard embedding.

The effective Kähler potential for the untwisted sector of orbifold compactifications can be similarly derived from an M-theory perspective. The only changes are the same as those already emphasized in section 3, namely that the intersection numbers d_{ABC}

and the quantities c_{PQ}^A acquire a simple group-theoretical interpretation. Moreover, in this case the forms c_{PQ} are automatically harmonic and the quantities c_{PQ}^A are always constant, as already discussed in section 3. Further details on a component version of this five-dimensional derivation can be found in [51, 52, 53, 54].

4.3 Range of validity

We have seen in the previous subsection that the results derived in section 2 for the low-energy effective Kähler potential admit a simple 5D interpretation, in which the non-trivial contact terms spoiling the sequestered structure arise from the exchange of heavy 4D Kaluza-Klein modes of the light 5D vector multiplets coming from the harmonic components of the M-theory 3-form C on X. This interpretation was however derived under the same restrictive assumptions as in section 2, namely that the forms c_{PQ} are harmonic and that the quantities c_{PQ}^A are constants. It is then natural to wonder once again what would be the situation if these assumptions were to be relaxed.

The relevance of the assumptions about c_{PQ} and c_{PQ}^A within the M-theory perspective must obviously be very similar to that already discussed within the heterotic perspective. But it turns out to offer a slightly sharper perspective. The harmonicity of c_{PQ} is as before needed to ensure the trivial decoupling of heavy neutral modes from pairs of light charged modes. More specifically, we see here that when c_{PQ} is not harmonic a direct danger comes from the heavy 5D vector multiplets that arise from the non-harmonic components of the 3 form C on X. Indeed, such heavy modes can be brutally truncated away only when they are not sourced by light fields, and from the reduction of the solution of the Bianchi identity (4.1) we see that this is the case only when the non-harmonic parts of C describing the heavy 5D vector modes have no overlap with the forms c_{PQ} describing the composite of two light matter modes, that is when c_{PQ} is harmonic. In the opposite case, one would have to properly integrate out these heavy 5D vector modes too, and this would give extra contributions to the contact terms in the 4D effective Kähler potential. These additional effects must correspond to the additional terms that would arise in the left-hand side of eq. (2.43) within the heterotic perspective. The constancy of c_{PQ}^A is again needed to ensure a simple determination of the right definition of the chiral multiplets containing the moduli. More specifically, we see here that for moduli-dependent c_{PQ}^A it is not clear how one should modify the definitions (4.6) and (4.7) to arrange that (4.8) and (4.9) hold true.

5 General structure of the scalar manifold

We have seen that for compactifications on both smooth Calabi-Yau manifolds and singular orbifolds the Kähler potential for the Kähler moduli and matter fields takes the same general form, at least under the already explained assumptions. We will now study in some more detail the general geometric features of this scalar manifold, which will be relevant for the structure of the soft scalar masses induced in the presence of a non-trivial superpotential. We will introduce for this purpose a new parametrization of the scalar manifold, which will turn out to be very convenient at some special reference point.

5.1 Canonical parametrization

The general class of scalar manifolds we want to study is defined by the following Kähler potential, which only depends on the two symmetric and Hermitian but otherwise arbitrary constants d_{ABC} and c^A_{PQ}:

$$K = -\log\Big[\frac{1}{6}d_{ABC}J^A J^B J^C\Big]\,,\ \ \text{with}\ \ J^A = T^A + \bar{T}^A - c^A_{PQ}\Phi^P\bar{\Phi}^Q\,. \tag{5.1}$$

The fields T^A and Φ^P define a specific parametrization of the scalar manifold defined by this Kähler potential, which naturally emerges from string theory. We are however free to make holomorphic change of coordinates as well as Kähler transformations to define other equivalent parametrizations. It turns out that this freedom can be used to define a particularly convenient kind of parametrization. We shall call this the canonical parametrization, because it is a natural generalization including the $N = 1$ matter sector of the one that was introduced in [55, 56] for the very special manifolds describing the $N = 2$ moduli sector.

The main idea is to think of some reference point of particular interest on the scalar manifold, and then to perform a field redefinition that allows to simplify things as much as possible around that point. This reference point can for instance be thought of as the one defined by the VEVs $\langle T^A\rangle$ and $\langle\Phi^P\rangle$ that the scalar fields would eventually acquire in the presence of a non-trivial superpotential. Since our primary goal is to study situations where the moduli have sizable VEVs whereas the matter fields have a small VEVs, we shall start by considering the situation where

$$\langle T^A\rangle \neq 0\,,\ \ \langle\Phi^P\rangle = 0\,. \tag{5.2}$$

We may now reparametrize the fields in such a way to simplify the metric and the curvature tensor at such a point. To this aim, we shall consider the following linear field redefinitions:

$$\hat{T}^A = U^A_{\ B}T^B\,,\ \ \hat{\Phi}^P = V^P_{\ Q}\Phi^Q\,. \tag{5.3}$$

In addition, we may also perform a Kähler transformation on K. In particular, we may perform a trivial constant shift of the type

$$\hat{K} = K - \log|\alpha|^2\,. \tag{5.4}$$

For our purposes, it will be enough to take $U^A_{\ B}$ to be a real matrix, $V^P_{\ Q}$ to be a complex matrix, and α to be a real number. Under such transformations, the new Kähler potential in terms of the new fields has the same form as the original Kähler potential in terms of the original fields, but with new numerical coefficients given by:

$$\hat{d}_{ABC} = \alpha^2\,U^{-1D}_{\ \ \ A}U^{-1E}_{\ \ \ B}U^{-1F}_{\ \ \ C}d_{DEF}\,,\ \ \hat{c}^A_{PQ} = U^A_{\ B}V^{-1R}_{\ \ \ P}\bar{V}^{-1S}_{\ \ \ Q}c^B_{RS}\,. \tag{5.5}$$

At this point, we may choose $U^A_{\ B}$ and $V^P_{\ Q}$ in such a way that the VEVs of the fields are aligned along just one direction, the VEV of the metric becomes diagonal, and the overall scale of one of these two quantities (but not both) is set to some reference value. We may furthermore choose α to set the overall scale of the intersection numbers to a convenient value. More specifically, we shall require that in the new basis the reference point should be at

$$\langle \hat{T}^A \rangle = \frac{\sqrt{3}}{2}\,\delta^A_0\,, \quad \langle \hat{\Phi}^P \rangle = 0\,, \tag{5.6}$$

the metric at that point should take the form

$$\langle \hat{g}_{AB} \rangle = \delta_{AB}\,, \quad \langle \hat{g}_{PQ} \rangle = \delta_{PQ}\,, \quad \langle \hat{g}_{AQ} \rangle = 0\,, \tag{5.7}$$

and finally the Kähler frame should be such that at that point

$$\langle \hat{K} \rangle = 0\,. \tag{5.8}$$

It is easy to get convinced by a counting of parameters that it is indeed always possible to impose this kind of conditions. Moreover, by comparing the transformed expressions for the VEVs of the fields, the metric and the Kähler potential with the values required in the previous equations, we deduce that the new values of the numerical coefficients $\hat{d}_{ABC}$ and $\hat{c}^A_{PQ}$ must satisfy the following properties:

$$\hat{d}_{000} = \frac{2}{\sqrt{3}}\,, \quad \hat{d}_{00a} = 0\,, \quad \hat{d}_{0ab} = -\frac{1}{\sqrt{3}}\,\delta_{ab}\,, \quad \hat{d}_{abc} = \text{generic}\,, \tag{5.9}$$

$$\hat{c}^0_{PQ} = \frac{1}{\sqrt{3}}\,\delta_{PQ}\,, \quad \hat{c}^a_{PQ} = \text{generic}\,. \tag{5.10}$$

The new form of the Kähler potential after the change of basis is then

$$\hat{K} = -\log\left[\frac{1}{6}\Big(\frac{2}{\sqrt{3}}\hat{J}^0\hat{J}^0\hat{J}^0 - \sqrt{3}\,\hat{J}^0\hat{J}^a\hat{J}^a + \hat{d}_{abc}\hat{J}^a\hat{J}^b\hat{J}^c\Big)\right], \tag{5.11}$$

where now

$$\hat{J}^0 = \hat{T}^0 + \bar{\hat{T}}^0 - \frac{1}{\sqrt{3}}\delta_{PQ}\hat{\Phi}^P\bar{\hat{\Phi}}^Q\,, \tag{5.12}$$

$$\hat{J}^a = \hat{T}^a + \bar{\hat{T}}^a - \hat{c}^a_{PQ}\hat{\Phi}^P\bar{\hat{\Phi}}^Q\,. \tag{5.13}$$

The above canonical parametrization has a nice interpretation from the point of view of the properties of the Calabi-Yau manifold X and the holomorphic vector bundle V over it, on which the model is based. It essentially corresponds to a particular choice of bases for the harmonic forms $\hat{\omega}_A$ and $\hat{u}_P$ at the reference point defined by the VEVs. More specifically, the sets of harmonic forms $\hat{\omega}_A$ and $\hat{u}_P$ can be chosen to be orthonormal with respect to the natural positive definite metrics defined by $\hat{g}_{AB} = V^{-1}\int_X \hat{\omega}_A \wedge *\hat{\omega}_B$ and $\hat{g}_{PQ} = V^{-1}\int_X \hat{c}_{PQ}\wedge *J$, and one can moreover orient them in such a way that $\hat{\omega}_0$ is aligned with the Kähler form J. In this way the multiplets $\hat{T}^0$ and $\hat{T}^a$ describe respectively the overall volume and the relative Kähler moduli, and the fields $\hat{\Phi}^P$ are canonically defined. In this new basis, the VEV of the metric is the identity matrix, with $\hat{g}_{AB} = \delta_{AB}$ and $\hat{g}_{PQ} = \delta_{PQ}$, and as shown in appendix A the intersection numbers $\hat{d}_{ABC}$ and the quantities $\hat{c}^A_{PQ}$ do indeed take the structure of (5.9) and (5.10), after effectively setting the volume V to unity by a rescaling. It is worth remarking that if the traceful part of $\hat{c}_{PQ}$ were parallel to J and thus proportional to $\hat{\omega}^0$, whereas the remaining traceless part of $\hat{c}_{PQ}$

were orthogonal to J and thus a linear combination of the $\hat{\omega}^a$'s, all the matrices $\hat{c}^a_{PQ}$ would be traceless. This turns out to be the case for orbifolds, and it is not unconceivable that it might actually also hold true for most if not all of the Calabi-Yau's subject to the stringent restriction that the $(1,1)$ forms c_{PQ} are harmonic. We were not able to verify this, but we find it rather suggestive that the trace part of $\hat{c}_{PQ}$ indeed has positive-definite components, like J.

Notice that the new coordinates that have been introduced do not exactly coincide with normal coordinates at the reference point. Indeed, some of the components of the Christoffel connection have non-trivial values:

$$\langle \Gamma_{00\bar{0}} \rangle = -\frac{2}{\sqrt{3}}\,, \;\; \langle \Gamma_{0a\bar{b}} \rangle = -\frac{2}{\sqrt{3}}\delta_{ab}\,, \;\; \langle \Gamma_{ab\bar{0}} \rangle = -\frac{2}{\sqrt{3}}\delta_{ab}\,, \;\; \langle \Gamma_{ab\bar{c}} \rangle = -\hat{d}_{abc}\,, \tag{5.14}$$

$$\langle \Gamma_{AP\bar{Q}} \rangle = -\hat{c}^A_{PQ}\,. \tag{5.15}$$

Nevertheless, they turn out to lead to rather simple expressions for the Riemann curvature tensor at the reference point.

5.2 Curvature for Calabi-Yau models

In the general case of compactifications on a smooth Calabi-Yau manifold, the scalar manifold $\mathcal{M}$ on which the low-energy effective theory is based is a generic Kähler manifold. The curvature of such a manifold depends on the point. Let us then consider the special reference point introduced above, assuming that it is dynamically selected by the superpotential, and let us switch to the canonical parametrization. After a simple computation, one finds the following results for the VEV of the Riemann tensor:

$$\langle R_{A\bar{B}C\bar{D}} \rangle = \delta_{AB}\delta_{CD} + \delta_{AD}\delta_{BC} - \hat{d}_{ACE}\hat{d}_{BDE}\,, \tag{5.16}$$

$$\langle R_{P\bar{Q}R\bar{S}} \rangle = \frac{1}{3}\big(\delta_{PQ}\delta_{RS} + \delta_{PS}\delta_{RQ}\big) + \hat{c}^a_{PQ}\hat{c}^a_{RS} + \hat{c}^a_{PS}\hat{c}^a_{RQ}\,, \tag{5.17}$$

$$\langle R_{P\bar{Q}0\bar{0}} \rangle = \frac{1}{3}\delta_{PQ}\,, \;\; \langle R_{P\bar{Q}a\bar{b}} \rangle = \frac{2}{3}\delta_{PQ}\delta_{ab} + (\hat{d}_{abc}\hat{c}^c - \hat{c}^a\hat{c}^b)_{PQ}\,, \;\; \langle R_{P\bar{Q}0\bar{b}} \rangle = \frac{1}{\sqrt{3}}\hat{c}^b_{PQ}\,. \tag{5.18}$$

These expressions are valid only around the point under consideration. In particular, they get deformed if one switches on a non-vanishing VEV for the matter fields.

5.3 Curvature for orbifold models

In the special case of orbifold compactifications, the scalar manifold $\mathcal{M}$ on which the low-energy effective theory is based is a symmetric Kähler manifold. The curvature of such a manifold does not depend on the point. Let us nevertheless consider the special reference point introduced above and switch as before to the canonical parametrization. It is straightforward to verify that the new parametrization described in section 3.2.4 actually coincides with the canonical one. To do so, one simply needs to recall that c^0 is equal to $\mathbb{1}/\sqrt{3}$, whereas the c^a are a subset of the transposed of the Gell-Mann matrices λ^a. One then verifies that the expressions (3.24) and (3.25) do indeed take the canonical forms defined by (5.9) and (5.10), with:

$$\hat{d}_{abc} = 2\,\mathrm{tr}(\lambda^{(a}\lambda^b\lambda^{c)})\,, \;\; \hat{c}^a_{ij} = \lambda^a_{ji}\,. \tag{5.19}$$

We see that in this case $\hat{d}_{abc}$ is the symmetric invariant symbol of the group H, whereas the $\hat{c}^a_{ij}$ are the transposed of the generators of H in the representation $\mathbf{h}$ descending from the $\mathbf{3}$ of $SU(3)$ in terms of 3×3 matrices. In this case the transposed of the matrices $\hat{c}^a_{ij}$ possess the non-trivial property of being traceless and generating the Lie algebra of H, whose structure constants can be written as

$$f_{abc} = -2i \,\mathrm{tr}(\lambda^{[a}\lambda^b\lambda^{c]})\,. \tag{5.20}$$

Moreover, for all the three kinds of models one finds:

$$[\lambda^a, \lambda^b] = i f_{abc}\lambda^c\,,\ \ \{\lambda^a, \lambda^b\} = d_{abc}\lambda^c + \frac{2}{3}\delta_{ab}\mathbb{1}\,. \tag{5.21}$$

Using these properties of the matrices λ^a, the components of the Riemann tensor are then seen to simplify and can entirely be rewritten in terms of these matrices:

$$\langle R_{A\bar{B}C\bar{D}}\rangle = \mathrm{tr}(\hat{c}^A\hat{c}^B\hat{c}^C\hat{c}^D) + \mathrm{tr}(\hat{c}^A\hat{c}^D\hat{c}^C\hat{c}^B)\,, \tag{5.22}$$

$$\langle R_{P\bar{Q}R\bar{S}}\rangle = \hat{c}^A_{PQ}\hat{c}^A_{RS} + \hat{c}^A_{PS}\hat{c}^A_{RQ}\,, \tag{5.23}$$

$$\langle R_{P\bar{Q}C\bar{D}}\rangle = (\hat{c}^D\hat{c}^C)_{PQ}\,. \tag{5.24}$$

These expressions are actually valid at any point of the scalar manifold, as already said. Their simple form reflects the fact that the curvature of symmetric manifolds is completely determined by the structure constants of their isometry group. This is explained in some detail in appendix B, where we also summarize some basic results about the geometry of such symmetric coset manifolds.

6 Soft scalar masses and sequestering

Let us now come to the crucial question of what are the properties of soft scalar masses in the effective theories for heterotic string models compactified on a generic Calabi-Yau manifold with a generic stable holomorphic vector bundle over it, in the presence of some source of supersymmetry breaking. We shall restrict our analysis to the Kähler moduli and matter fields, for which we know the form of the Kähler potential, and to the neighborhood of the reference point introduced last section, by assuming that the superpotential that induces supersymmetry breaking is such that the scalar VEVs of the moduli and matter scalar fields are respectively generic and vanishing. We will first work out the general structure of the soft scalar masses and then study the possibility of ensuring the vanishing of these masses with the help of some kind of global symmetry.

6.1 Structure of scalar masses

Our starting point is the effective Kähler potential (5.1), which is characterized by the two constants d_{ABC} and c^A_{PQ}. Since we want to study soft terms at the particular reference point introduced in last section, it will be convenient to switch to the canonical parametrization that we defined there. From now on, we shall for simplicity drop all the hats on the redefined parameters and fields, and also the brackets denoting VEVs at the reference point. It will moreover be convenient to further redefine $T = T^0/\sqrt{3}$ and correspondingly $J = J^0/\sqrt{3}$, and to explicitly split the matter fields Φ^P into two sets Q^α and

X^i respectively coming from the two E_8 factors, in such a way to match the notation that was adopted in [23] for orbifold models. The visible sector is then identified with the fields Q^α and the hidden sector generically contains all the remaining fields X^i, T, T^a, and the Kähler potential becomes

$$K = -\log\Big(J^3 - \frac{1}{2}JJ^aJ^a + \frac{1}{6}d_{abc}J^aJ^bJ^c\Big)\,, \tag{6.1}$$

where

$$J = T + \bar{T} - \frac{1}{3}Q^\alpha \bar{Q}^\alpha - \frac{1}{3}X^i \bar{X}^i\,, \tag{6.2}$$

$$J^a = T^a + \bar{T}^a - c^a_{\alpha\beta}Q^\alpha \bar{Q}^\beta - c^a_{ij}X^i \bar{X}^j\,. \tag{6.3}$$

Let us now study this expression around the point under consideration. In the new coordinates, this corresponds to:

$$T = \frac{1}{2}\,,\;\; T^a = 0\,,\;\; Q^\alpha = 0\,,\;\; X^i = 0\,. \tag{6.4}$$

The metric takes a simple diagonal result, with non-vanishing entries given by

$$g_{T\bar{T}} = 3\,,\;\; g_{a\bar{b}} = \delta_{ab}\,,\;\; g_{\alpha\bar{\beta}} = \delta_{\alpha\beta}\,,\;\; g_{i\bar{\jmath}} = \delta_{ij}\,. \tag{6.5}$$

For the Christoffel connection, the non-vanishing components are given by

$$\Gamma_{TT\bar{T}} = -6\,,\;\; \Gamma_{Ta\bar{b}} = -2\,\delta_{ab}\,,\;\; \Gamma_{ab\bar{T}} = -2\,\delta_{ab}\,,\;\; \Gamma_{ab\bar{c}} = -d_{abc}\,, \tag{6.6}$$

$$\Gamma_{TP\bar{Q}} = -\delta_{PQ}\,,\;\; \Gamma_{aP\bar{Q}} = -c^a_{PQ}\,. \tag{6.7}$$

The components of the Riemann tensor that are relevant for soft scalar terms, with a pair of indices along the visible sector fields and the other pair along the hidden sector fields, are then found to be

$$R_{\alpha\bar{\beta}i\bar{\jmath}} = \frac{1}{3}\delta_{\alpha\beta}\delta_{ij} + c^a_{\alpha\beta}c^a_{ij}\,, \tag{6.8}$$

$$R_{\alpha\bar{\beta}T\bar{T}} = \delta_{\alpha\beta}\,,\;\; R_{\alpha\bar{\beta}a\bar{b}} = \frac{2}{3}\delta_{\alpha\beta}\delta_{ab} + (d_{abc}c^c - c^a c^b)_{\alpha\beta}\,,\;\; R_{\alpha\bar{\beta}T\bar{b}} = c^b_{\alpha\beta}\,. \tag{6.9}$$

We are now in position to compute the soft scalar masses induced for the visible-sector fields Q^α when the hidden-sector fields $\Phi^\Theta = X^i, T, T^a$ get non-vanishing auxiliary fields, at the reference point under consideration. This can be done by using the following standard geometrical expression

$$m^2_{\alpha\bar{\beta}} = -\Big(R_{\alpha\bar{\beta}\Theta\bar{\Gamma}} - \frac{1}{3}g_{\alpha\bar{\beta}}g_{\Theta\bar{\Gamma}}\Big)F^\Theta \bar{F}^{\bar{\Gamma}}\,. \tag{6.10}$$

Using the results (6.5) and (6.8)-(6.9) for the metric and the Riemann tensor at the point under consideration, this gives:

$$m^2_{\alpha\bar{\beta}} = -c^a_{\alpha\beta}c^a_{ij}F^i\bar{F}^{\bar{\jmath}} - \Big(\frac{1}{3}\delta_{\alpha\beta}\delta_{ab} + (d_{abc}c^c - c^a c^b)_{\alpha\beta}\Big)F^a\bar{F}^{\bar{b}}$$
$$- c^a_{\alpha\beta}F^a\bar{F}^T + \text{c.c.}\,. \tag{6.11}$$

The structure of the soft scalar masses (6.11) can also be understood in terms of ordinary superfields. To do this, one considers the kinetic function $\Omega = -3\,e^{-K/3}$, which is the gravitational analogue of the rigid Kähler potential. At the considered reference point, it is sufficient to expand it at cubic order in $J^a \ll J$. In this way one finds:

$$\Omega \simeq -3\,J + \frac{1}{2}\frac{J^a J^a}{J} - \frac{1}{6} d_{abc} \frac{J^a J^b J^c}{J^2}\,. \tag{6.12}$$

More precisely, the relevant terms are selected by decomposing the fields in scalar VEVs plus fluctuations, so that $J = 1 + \tilde{J}$ and $J^a = \tilde{J}^a$, and retaining up to cubic terms in an expansion in powers of the fluctuations. This yields $\Omega = -3 + \tilde{\Omega}$ with:

$$\tilde{\Omega} \simeq -3\,\tilde{J} + \frac{1}{2}\tilde{J}^a\tilde{J}^a - \frac{1}{2}\tilde{J}\,\tilde{J}^a\tilde{J}^a - \frac{1}{6}d_{abc}\tilde{J}^a\tilde{J}^b\tilde{J}^c\,. \tag{6.13}$$

The soft scalar masses can the be computed by looking at the quadratic part of the contribution to the scalar potential from $\tilde{\Omega}$: $\mathcal{L}_{m^2} = -\tilde{\Omega}|_{D,q^2}$. The various terms in (6.11) then emerge as follows from $\tilde{\Omega}|_D$, after splitting the currents into visible-sector and hidden-sector parts. The term $-c^a_{\alpha\beta}c^a_{ij}F^i\bar{F}^{\bar{\jmath}}$ comes from $\tilde{J}^a_{\rm v}|\tilde{J}^a_{\rm h}|_D$, the term $-1/3\,\delta_{\alpha\beta}\delta_{ab}F^a\bar{F}^{\bar{b}}$ comes from $-\tilde{J}_{\rm v}|\tilde{J}^a_{\rm h}|_F\tilde{J}^a_{\rm h}|_{\bar{F}}$, the term $-c^a_{\alpha\beta}F^a\bar{F}^T + \text{c.c.}$ comes from $-\tilde{J}_{\rm h}|_{\bar{F}}\tilde{J}^a_{\rm v}|\tilde{J}^a_{\rm h}|_F + \text{c.c.}$, the term $(c^a c^b)_{\alpha\beta}F^a\bar{F}^{\bar{b}}$ comes from the combination of $-3\,\tilde{J}_{\rm v}|_D$ and $\tilde{J}^a_{\rm v}|_F\tilde{J}^a_{\rm h}|_{\bar{F}} + \text{c.c.}$, and finally the term $-d_{abc}c^a_{\alpha\beta}F^b\bar{F}^{\bar{c}}$ comes from $-d_{abc}\tilde{J}^a_{\rm v}|\tilde{J}^b_{\rm h}|_F\tilde{J}^c_{\rm h}|_{\bar{F}}$.

6.2 Sequestering by global symmetries

From the form of the expression (6.11), we can deduce the following observations. In the particular case where $h^{1,1} = 1$, the soft scalar masses vanish identically, even in the presence of generic non-vanishing values for F^T and F^i. This is the well known situation arising in sequestered models. In the general case where $h^{1,1} > 1$, one the contrary, the soft scalar masses receive non-trivial contributions in the presence of generic non-vanishing values of F^T, F^i and F^a. However, these contributions involve very special combinations of these auxiliary fields, controlled by the quantities d_{abc} and the matrices $c^a_{\alpha\beta}$ and c^a_{ij}. One may then wonder whether it is possible to ensure that these combinations of auxiliary fields vanish, so that the soft scalar masses would again vanish, by assuming that some approximate global symmetry of the Kähler potential K is extended to constrain also the superpotential W and therefore the Goldstino direction. It would also be interesting to study what constraints are put on the Goldstino direction by the requirement that there should exist a metastable supersymmetry breaking vacuum, generalizing the results derived in [57] for Kähler moduli to include also matter fields, but we shall not attempt to do this here.

From the results derived in the previous subsection, and taking into account that the scalar VEVs of the fields T^a and X^i are assumed to be negligible, we see that a simple and general possibility to get vanishing soft scalar masses is to require that:

$$c^a_{ij}F^i\bar{F}^{\bar{\jmath}} = 0 \;\Leftrightarrow\; J^a_{\rm h}|_D = 0\,, \tag{6.14}$$

$$F^a = 0 \;\Leftrightarrow\; J^a_{\rm h}|_F = 0\,. \tag{6.15}$$

These two relations clearly have the form of the two D and F type Ward identities that would be implied by the conservation of the currents

$$J_{\rm h}^a = T^a + \bar T^a - c_{ij}^a X^i \bar X^j \,. \tag{6.16}$$

Notice however that one might also view the two relations (6.14) and (6.15) as emerging from the conservation of the following two independent currents, which each lead to only one non-trivial Ward identity, respectively the D and F type one:

$$J_{{\rm h}X}^a = -c_{ij}^a X^i \bar X^j \,, \tag{6.17}$$
$$J_{{\rm h}T}^a = T^a + \bar T^a \,. \tag{6.18}$$

This follows form the observation that at the considered vacuum reference point one finds $J_{{\rm h}X}^a|_D = J_{\rm h}^a|_D$, $J_{{\rm h}X}^a|_F = 0$, $J_{{\rm h}T}^a|_D = 0$ and $J_{{\rm h}T}^a|_F = J_{\rm h}^a|_F$.

To understand which global symmetry would lead to this conserved current, let us now recall that the general form of the conserved Nöther current superfield J^a for a globally supersymmetric non-linear sigma model with a global symmetry $\delta \Phi^I = k_a^I \delta \epsilon^a$ is given, in the general case where the Kähler potential is allowed to undergo a Kähler transformation parametrized by some holomorphic functions f_a, by the following expression:

$$J^a = {\rm Im}(k_a^I K_I - f_a) \,. \tag{6.19}$$

The D and F type Ward identities following from the conservation of this current take the following form:

$$J^a|_D = 0 \;\Leftrightarrow\; \nabla_I k_{a\bar J} F^I \bar F^{\bar J} = 0 \,, \tag{6.20}$$
$$J^a|_F = 0 \;\Leftrightarrow\; \bar k_{aI} F^I = 0 \tag{6.21}$$

Somewhat surprisingly, gravitational effects complicate the situation [23]. Although it is not totally trivial to generalize the superfield expression (6.19), it is rather straightforward to show that the two component Ward identities (6.20) and (6.21) are deformed to $\nabla_I k_{a\bar J} F^I \bar F^{\bar J} = -2iD_a m_{3/2}^2$ and $k_{aI} F^I = -iD_a m_{3/2}$, where $D_a = {\rm Im}(k_a^I K_I - f_a)$. This is due to the fact that the auxiliary fields F^I receive a gravitational contribution involving derivatives of K, in addition to the usual contribution involving derivatives of W. Notice however that at the particular reference point that we have considered, the only non-vanishing component of K_I is along the T direction, so that $K_\alpha = 0$, $K_i = 0$ and $K_a = 0$. Under the mild restriction that the considered symmetry should not act on T and should not involve a Kähler transformation, meaning that $k_a^T = 0$ and $f_a = 0$, one would then get $D_a = 0$. Under this assumption, one can then use the rigid version of the Ward identities.

To get an idea of the situation, we may now start by naively applying the expression (6.19) with a Kähler potential K given by the leading quadratic part of Ω, namely

$$K \simeq \frac{1}{2}(T^a + \bar T^a)(T^a + \bar T^a) + X^i \bar X^i \,. \tag{6.22}$$

To match (6.19) with the two partial currents (6.17) and (6.18), we would then respectively need to take $k_a^i \simeq -i c_{ji}^a X^j$ for the matter fields X^i and $k_a^b \simeq i \delta_a^b$ for the moduli fields

T^a. These Killing vectors define two sets of transformations that indeed leave the leading Kähler potential (6.22) independently invariant:

$$\delta_a X^i \simeq -i c^a_{ji} X^j \,, \tag{6.23}$$

$$\delta_a T^b \simeq i \delta^b_a \,. \tag{6.24}$$

The crucial question is now whether the transformations (6.23) and (6.24) are eligible to represent an approximate global symmetry of K around the vacuum reference point under consideration or not. A first condition is that the matrices c^a should form a closed algebra with $[c^a, c^b] = -i f_{abc} c^c$. In this way the transformations (6.23) would form an algebra with structure constants f_{abc} associated to a group H, while the transformations (6.24) automatically form an Abelian algebra associated to $U(1)^{h^{1,1}-1}$. A second condition is that higher order terms in K should have an unimportant effect and that it should somehow be meaningful to impose to W a symmetry that leaves a priori invariant only the leading quadratic part of K. One possibility is that the corrections spoil the symmetries (6.17) and (6.18) but only in a parametrically suppressed way. It is however not clear whether this can robustly happen. A more appealing possibility is that (6.23) and (6.24) can be extended to exact symmetries of the full scalar manifold, thereby guaranteeing the existence of exactly conserved currents which reduce to (6.17) and (6.18) in the vicinity of the point under consideration. We see however from the form (6.1) of K that (6.23) can be generalized to an exact symmetry only by extending it to act linearly also on the T^a in the adjoint representation of H and only if d_{abc} corresponds to an invariant of the group H, while (6.24) is always an exact symmetry, without the need of any modification and for any values of d_{abc}. The exact conserved currents differ from (6.17) and (6.18), on one hand because of the extension in the symmetry action and on the other because of the non-linearities in the Kähler potential. The Ward identities (6.20) and (6.21) are then correspondingly deformed. However, taken together they still ensure that $c^a_{ij} F^i \bar{F}^{\bar{j}} = 0$ and $F^a = 0$, which guarantee the vanishing of the soft scalar masses.

In addition to the general possibility that we just explored, there might also be other options that arise in specific situations. For instance, the three terms of the second piece in (6.11) may conspire to give a simpler structure, and one might try to exploit this in the search for a different global symmetry that could ensure the vanishing of soft masses by constraining the F^a's but without setting them all to zero. In such a case one would however have to assume that F^T vanishes to get rid of the last piece in (6.11). Let us then study more specifically what are the options for general Calabi-Yau models and for orbifold models, focusing for simplicity on models with a symmetric embedding in the visible and hidden sectors, for which the set of matrices $c^a_{\alpha\beta}$ and c^a_{ij} are identical.

6.3 Calabi-Yau models

For generic Calabi-Yau models, the intersection numbers d_{abc} and the Hermitian matrices $c^a_{\alpha\beta}$ or equivalently c^a_{ij} are a priori generic, with $a = 1, \cdots, h^{11}-1$ and $\alpha, \beta, i, j = 1, \cdots, n_R$. The only thing that we know for sure from the discussion of section 2.3 is that the matrices c^a and c^0 can always be written as transposed linear combinations of the n_R^2 matrices $\lambda^{A'}$ representing the generators of $U(n_R)$ in the fundamental representation. As remarked at the end of section 5, a further property that could conceivably arise with some naturalness and generality is that these matrices might be traceless. In that case they could then be expressed in terms of the $n_R^2 - 1$ traceless generators of $SU(n_R)$. On the other hand, further restrictions leading to yet smaller subgroups H' seem less likely, and the minimal case where the matrices c^a themselves generate a group H of dimension $h^{1,1} - 1$ appears to be very special.

Consider first the brane-mediated effect corresponding to the first term of (6.11). If the matrices c^a happen to be transposed linear combinations of the generators $\lambda^{a'}$ of some group $H' \subset U(n_R)$, we may ensure the vanishing of this contribution by imposing the global symmetry H' that acts as in (6.23) but with c^a_{ji} replaced by $\lambda^{a'}_{ij}$: $\delta_{a'} X^i = -i\,\lambda^{a'}_{ij} X^j$. This is still an approximate symmetry of K and leads to the conservation of the larger set of currents $J^{a'}_{\mathrm{h}X} = -\lambda^{a'}_{ji} X^i \bar{X}^j$, which implies the stronger Ward identity $\lambda^{a'}_{ji} F^i \bar{F}^j = 0$. The maximal choice $H' = U(n_R)$ is available for any generic model, but has the drawback that it would actually imply $F^i = 0$, due to the completeness relation $\lambda^{a'}_{ij} \lambda^{a'}_{pq} = \delta_{iq}\delta_{pj}$. Other non-maximal choices $H' \subset U(n_R)$ are instead available only in particular models, but have the advantage of allowing $F^i \neq 0$. Notice finally that such an approximate symmetry group H' can in general not be extended to an exact symmetry of the full scalar manifold. The only very special case where this is possible is when the c^a generate by themselves a minimal group H of dimension $h^{1,1} - 1$ and the intersection numbers d_{abc} are invariant under this group H.

Consider next the moduli-mediated effect corresponding to the remaining terms of (6.11). In general one may ensure that these vanish by imposing the independent Abelian global symmetry $U(1)^{h^{1,1}-1}$ acting as in (6.24): $\delta_a T^b = i\,\delta_a^b$. This symmetry leads to the conservation of the currents $J^a_{\mathrm{h}T} = T^a + \bar{T}^a$, and the corresponding F type Ward identity implies that $F^a = 0$. Moreover it always corresponds to an exact symmetry of the full scalar manifold. Notice finally that in this case it is rather unlikely that the second piece of (6.11) could simplify dramatically enough to allow for other options.

We conclude that for smooth Calabi-Yau compactifications there generically exists the possibility of ensuring the vanishing of soft scalar masses at points with negligible VEVs for X^i and T^a by imposing the approximate global symmetry $U(n_R) \times U(1)^{h^{1,1}-1}$, where the first factor acts linearly on the X^i and the second acts as a shift on the T^a. However, this forces both the F^i and the F^a to vanish, meaning that there is actually no breaking of supersymmetry at all. Moreover, it is not a true symmetry of the full scalar manifold. A more interesting situation may be obtained in the special cases where the matrices c^a generate some non-maximal subgroup $H \subset U(n_R)$. In such a situation, the F^i would be constrained but not forced to vanish, although the F^a would still vanish, and supersymmetry can be broken. Moreover, this symmetry can be extended to a true symmetry of the full scalar manifold that still implies the vanishing of the scalar masses.

6.4 Orbifold models

For orbifold models, the intersection numbers d_{abc} and the matrices $c^a_{\alpha\beta}$ or equivalently c^a_{ij}, with $a = 1, \cdots, h^{1,1} - 1$ and $\alpha, \beta, i, j = 1, 2, 3$, are a respectively the symmetric invariant symbol and the transposed tridimensional representation of the generators of a group $H \subset SU(3)$. Moreover, one can easily verify that the second term in (6.11) simplifies to $1/3\,\delta_{\alpha\beta}\delta_{ab} + (d_{abc}c^c - c^a c^b)_{\alpha\beta} = (c^b c^a)_{\alpha\beta} - 1/3\,\delta_{ab}\delta_{\alpha\beta}$, which is traceless. As a result, the mass matrix (6.11) is traceless and depends only on $h^{1,1}-1$ independent parameters, which can be taken to be $c^a_{ji} m^2_{ij}$.

Consider first the first brane-mediated term in (6.11). In this case, this can be ensured to vanish by imposing the global symmetry H acting as in (6.23): $\delta_a X^i = -i\lambda^a_{ij} X^j$. This leads to the conservation of the currents $J^a_{\mathrm{h}X} = -\lambda^a_{ji} X^i \bar{X}^j$, which implies the D type Ward identity $\lambda^a_{ji} F^i \bar{F}^j = 0$. Moreover, this approximate symmetry can be extended to an exact symmetry of the full manifold, as explained in appendix B, by assigning a non-trivial linear transformation law to the fields T^a in the adjoint representation of H. Notice finally that in this case one does not have the option of enlarging the symmetry to a bigger group $H' \subset U(n_R)$, because the various generations are grouped into triplets transforming in the fundamental representation of the gauge group enhancement factor, which happens to coincide with H.

Consider next the remaining moduli-mediated terms in (6.11). In general, we may again ensure the vanishing of these terms by imposing an independent Abelian global symmetry $U(1)^{h^{1,1}-1}$ acting as in (6.24): $\delta_a T^b = i\,\delta^b_a$. This leads to the conservation of the currents $J^a_{\mathrm{h}T} = T^a + \bar{T}^a$, which implies the F type Ward identity $F^a = 0$. Moreover, this symmetry is actually as before an exact symmetry of the full scalar manifold. Notice finally that in this case the second piece of (6.11) actually simplifies to $(d_{abc} + i f_{abc}) F^b \bar{F}^c$. One may then wonder whether the vanishing of this moduli-mediated contribution could perhaps be achieved together with the brane-mediated contribution with a single exact global symmetry H, acting on both the X^i and the T^a respectively in the fundamental and in the adjoint representations. Comparing with the structure (6.20) of the Ward identity, we however see that this does not work.

We conclude that for toroidal orbifold compactifications there always exists the possibility of ensuring the vanishing of soft scalar masses at points with negligible VEVs for X^i and T^a by imposing the approximate global symmetry $H \times U(1)^{h^{1,1}}$, where the first factor acts linearly on the X^i and the second factor acts as a shift on the T^a. In this situation, the F^i would be constrained but not forced to vanish, although the F^a would still vanish, and supersymmetry can be broken. Moreover, this symmetry can be extended to a true symmetry of the full scalar manifold that still implies the vanishing of the scalar masses.

7 Conclusions

In this paper, we have attempted a general study of the structure of soft scalar masses in heterotic string models obtained by compactification on a Calabi-Yau manifold X with a stable holomorphic vector bundle V over it. We investigated in particular the possibility of ensuring that such masses vanish at the classical level, by an effective sequestering mechanism based on global symmetries, and are then dominated by approximately universal quantum effects, so that the supersymmetric flavor problem could be naturally solved. Our main goal was to generalize a similar study previously done in [23] for the special case of singular orbifolds, and to assess how much of the structure allowing for an interesting implementation of this mechanism survives in the general case of smooth Calabi-Yau manifolds. We focused for simplicity on the low-energy effective theory restricted to the Kähler moduli T^A and the charged matter fields Q^α and X^i coming from the two E_8 sectors, with the Q^α defining the visible sector and the X^i and T^A the hidden sector. We then studied the terms in the effective Kähler potential K that mix the visible matter fields Q^α with either the hidden moduli fields T^A or the hidden matter fields X^i, and the moduli-mediated and brane-mediated contributions to soft scalar masses for Q^α that these operators induce when T^A and X^i acquire some non-vanishing auxiliary fields due to a superpotential W of unspecified origin.

We were able to derive the full dependence of K on both T^A and Q^α, X^i, by using the standard method of working out the reduction of the kinetic terms of the bosonic fields, but only under an a priori strong assumption on X and V. This assumption consists in some non-trivial properties of the harmonic 1-forms u_P on X with values in V, which define the charged matter zero-modes, relative to the harmonic $(1,1)$ forms ω_A on X, which define the neutral moduli zero-modes. More precisely, the assumption is that the $(1,1)$ forms $c_{PQ} = i\,\mathrm{tr}(u_P \wedge \bar{u}_Q)$ are harmonic and can be expanded onto the basis ω_A with some constant coefficients c_{PQ}^A. For models where X and V are such that this is true, K can be derived in closed form, with a moduli dependence controlled by the intersection numbers d_{ABC} and a matter dependence controlled by the quantities c_{PQ}^A, which are constant by assumption. The result that we derived precisely matches the general form proposed in [30] by an M-theory argumentation. We however believe that its validity is restricted to the situations satisfying the above mentioned assumptions, which we argued to be needed also from the M-theory viewpoint to be able to safely discard the effect of non-zero modes. Unfortunately we have no clear idea on how restrictive the above assumption really is. We however showed that compactifications based on orbifolds do automatically satisfy it, as a consequence of the fact that the forms u_P and ω_A are in this case not only harmonic but actually covariantly constant, and explained how the known result for K in these models [24, 25] emerges from the more general expression that we derived.

Our main conclusions concerning the possibility of implementing an effective sequestering mechanism based on a global symmetry are the following. For simplicity we focused on the reference point corresponding to scalar VEVs that are negligible for all the matter fields and sizable only for the moduli fields, where gravitational effects to the global symmetry Ward identities trivialize. In the special case of the untwisted sector of singular orbifolds, d_{abc} and c_{PQ}^a can be identified with the symmetric invariant symbol and the transposed fundamental representation generators of some group $H \subset SU(3)$, and the scalar manifold is a symmetric Kähler manifold. It then turns out that there exists an exact global symmetry $H \times U(1)^{h^{1,1}-1}$ of K which, if extended also to W, implies the vanishing of all the contributions to soft terms, with constrained but non-trivial F^i although vanishing F^a. In the more general case of smooth Calabi-Yau's, on the other hand, d_{abc} and c_{PQ}^a have no particular properties, other than being respectively symmetric and Hermitian, and the scalar manifold is a generic Kähler manifold. It then turns out that a similar mechanism can be at work only in the special case where the intersection numbers d_{abc} and the matrices c^a are respectively the symmetric invariant and the transposed fundamental generators of some group H. In such a situation there exists an exact global symmetry $H \times U(1)^{h^{1,1}-1}$ of K which, if extended also to W, implies the vanishing of all the contributions to soft terms, with constrained but non-trivial F^i although vanishing F^a.

In summary, it emerges rather clearly that an effective mechanism of sequestering based on a global symmetry seems to be naturally possible only whenever the scalar manifold is a very particular space with properties that resemble those of symmetric spaces. From an effective theory point of view, the analysis that we have done for this presumably larger class of models is then somewhat similar in spirit to the analysis that was done in [58] for models based on symmetric spaces. More precisely, the authors of [58] studied the possibility of achieving degenerate boson and fermion masses in some arbitrary sector of the model but at arbitrary points by suitably dialing the Goldstino direction, whereas here we studied the possibility of achieving vanishing scalar masses in a visible matter sector and at a particular reference point as a robust result of imposing a global symmetry on the hidden matter and moduli sector to suitably constrain the Goldstino direction.

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A Calabi-Yau manifolds and vector bundles over them

In this appendix, we review some notation and results concerning compact Calabi-Yau manifolds X and holomorphic vector bundles V over them. We will focus on those results that concern more directly $(1,1)$ forms on X and 1 forms on X with values in V, since these are the ingredients that we need to work out the results we are interested in.

Consider first a compact Calabi-Yau manifold X. The tangent and cotangent bundles TX and T^*X have structure group $SU(3)$, since this is the holonomy group characterizing this kind of manifolds. We can introduce a basis of $h^{1,1}$ independent harmonic $(1,1)$ forms ω_A on X, which provide a basis for the cohomology group $H^{1,1}(X) \simeq H^1(X, T^*X)$:

$$\{\omega_A\} = \text{basis of } H^{1,1}(X)\,. \tag{A.1}$$

We next consider the dual basis of $(2,2)$ harmonic forms ω^A and the corresponding basis of 4-cycles γ_A, defined in such a way that

$$\int_X \omega_A \wedge \omega^B = \int_{\gamma_A} \omega^B = \delta_A^B\,. \tag{A.2}$$

We may then define the intersection numbers d_{ABC}, which are topological invariants of X counting how many times a triplet of 4 cycles γ^A, γ^B and γ^C intersect each other, as

$$d_{ABC} = \int_X \omega_A \wedge \omega_B \wedge \omega_C = \text{intersections}(\gamma_A, \gamma_B, \gamma_C)\,. \tag{A.3}$$

Any harmonic $(1,1)$ form σ can be decomposed on the basis ω_A as

$$\sigma = \sigma^A \omega_A\,, \tag{A.4}$$

with real components σ^A given by

$$\sigma^A = \int_X \omega^A \wedge \sigma\,. \tag{A.5}$$

The Hodge dual $*\sigma$ is a harmonic $(2,2)$ form, and can therefore be decomposed onto the basis of ω^A as

$$*\sigma = \sigma_A\, \omega^A\,, \tag{A.6}$$

with real components σ_A given by

$$\sigma_A = \int_X \omega_A \wedge *\sigma\,. \tag{A.7}$$

There always exist at least one harmonic $(1,1)$ form defining the Kähler structure:

$$J = \text{Kähler form}\,. \tag{A.8}$$

In fact, it turns out that the volume form $*1$ on X can be expressed as the exterior product of three Kähler forms J:

$$*1 = \frac{1}{6} J \wedge J \wedge J\,. \tag{A.9}$$

Integrating this expression over X one deduces that the volume V of X can be expressed as follows:

$$V = \frac{1}{6} \int_X J \wedge J \wedge J\,. \tag{A.10}$$

As a consequence of the existence and the properties of J, the Hodge dual of any harmonic $(1,1)$ form σ on X can be expressed in the following way in terms of J [26]:

$$*\sigma = -J \wedge \sigma + \frac{1}{4V} \bigg\{ \int_X \sigma \wedge J \wedge J \bigg\}\, J \wedge J\,. \tag{A.11}$$

In particular, one has:

$$*J = \frac{1}{2} J \wedge J\,. \tag{A.12}$$

Taking the exterior product of (A.11) with any other harmonic $(1,1)$ form ρ and integrating over X, one further deduces that the natural positive-definite scalar product on the space of all the harmonic $(1,1)$ forms can be rewritten as:

$$\int_X \rho \wedge *\sigma = -\int_X \rho \wedge \sigma \wedge J + \frac{1}{4V} \int_X \rho \wedge J \wedge J \int_X \sigma \wedge J \wedge J\,. \tag{A.13}$$

In particular, one finds:

$$\int_X J \wedge *J = 3V\,, \tag{A.14}$$

$$\int_X \omega_A \wedge *J = \frac{1}{2} \int_X \omega_A \wedge J \wedge J\,, \tag{A.15}$$

$$\int_X \omega_A \wedge *\omega_B = -\int_X \omega_A \wedge \omega_B \wedge J + \frac{1}{4V} \int_X \omega_A \wedge J \wedge J \int_X \omega_B \wedge J \wedge J\,. \tag{A.16}$$

Dividing by V and using the decomposition $J = J^A\omega_A$, which implies that $\omega_A = \partial J/\partial J^A$, these relations can also be rewritten in the following more compact form:

$$\frac{1}{V}\int_X J \wedge *J = 3\,, \tag{A.17}$$

$$\frac{1}{V}\int_X \omega_A \wedge *J = \frac{\partial}{\partial J^A}\log V\,, \tag{A.18}$$

$$\frac{1}{V}\int_X \omega_A \wedge *\omega_B = -\frac{\partial^2}{\partial J^A \partial J^B}\log V\,. \tag{A.19}$$

Consider now a holomorphic vector bundle V over X, with structure group S. Out of this we can define a whole family of vector bundles V_r associated to any representation $\mathbf{r}$ of S, by promoting the transition functions of V, which are matrices in the fundamental representation of S, to the corresponding matrices in the representation $\mathbf{r}$ of S. We can then introduce a basis of n_R harmonic 1-forms u_P taking values in the representation $\mathbf{r}$ of the Lie algebra of S, associated to the cohomology group $H^1(X,V_{\rm r})$:

$$\{u_P\} = \text{basis of } H^1(X,V_{\rm r})\,. \tag{A.20}$$

By taking the exterior product of such a u_P with a conjugate $\bar{u}_Q$ and tracing over the indices of the representation $\mathbf{r}$, one may construct $(1,1)$ forms on the Calabi-Yau manifold X, which are however generically not harmonic:

$$c_{PQ} = i\,\mathrm{tr}\big(u_P \wedge \bar{u}_Q\big)\,. \tag{A.21}$$

One may then define the following quantities, which are a priori not topological invariants and depend in general on the geometry:

$$c^A_{PQ} = \int_X \omega^A \wedge c_{PQ}\,, \tag{A.22}$$

In the particular cases where the $(1,1)$ forms c_{PQ} are harmonic, the quantities c^A_{PQ} represent their components on the basis defined by the ω_A, and one may then write $c_{PQ} = c^A_{PQ}\omega_A$. More in general, one may write a Hodge decomposition with exact and coexact terms parametrized by generic $(1,0)$ and $(1,2)$ forms α_{PQ} and β_{PQ}:

$$c_{PQ} = c^A_{PQ}\omega_A + \bar{\partial}\alpha_{PQ} + \bar{\partial}^\dagger\beta_{PQ}\,. \tag{A.23}$$

Notice that by performing general linear transformations one may choose convenient special bases $\{\hat{\omega}_A\}$ and $\{\hat{u}_P\}$ for harmonic $(1,1)$ forms and Lie-algebra-valued 1 forms. For instance, one may define canonical bases by requiring that the $\hat{\omega}_A$ and $\hat{u}_P$ should form orthonormal sets with respect to the positive definite scalar products that can be defined on them. More precisely, we can impose that

$$\hat{\omega}_A:\ \ \frac{1}{V}\int_X \hat{\omega}_A \wedge *\hat{\omega}_B = \delta_{AB}\,, \tag{A.24}$$

$$\hat{u}_P:\ \ \frac{1}{V}\int_X \hat{c}_{PQ} \wedge *J = \delta_{PQ}\,. \tag{A.25}$$

One may moreover orient these bases with respect to the Kähler form, in such a way that $\hat{\omega}_0 = J/\sqrt{3}$ and thus $*J = \sqrt{3}\,V\hat{\omega}^0$. By using eqs. (A.14)-(A.16) it then follows that in such a basis the intersection numbers $\hat{d}_{ABC}$ and the quantities $\hat{c}^A_{PQ}$ have the following special structure:

$$\hat{d}_{000} = \frac{2}{\sqrt{3}} \cdot V\,, \;\; \hat{d}_{00a} = 0 \cdot V\,, \;\; \hat{d}_{0ab} = -\frac{1}{\sqrt{3}}\delta_{ab} \cdot V\,, \;\; \hat{d}_{abc} = \text{generic} \cdot V\,, \tag{A.26}$$

$$\hat{c}^0_{PQ} = \frac{1}{\sqrt{3}}\delta_{PQ}\,, \;\; \hat{c}^a_{PQ} = \text{generic}\,. \tag{A.27}$$

We would like to conclude this appendix by making a few comments concerning the particular case of orbifolds, where the Calabi-Yau manifold X degenerates to the projection of a flat torus and the holomorphic vector bundle V over it is correspondingly constructed as the projection of a trivial bundle. In that case, the whole technology simplifies and most of the relations listed above map to simple identities in linear algebra. Recall for instance that for any invertible square matrix M, the definitions of determinant, cofactor and inverse imply that:

$$\begin{aligned} M^{-1}_{ij} &= \frac{\text{cofactor}_{ji} M}{\det M} = \frac{\partial_{M_{ji}} \det M}{\det M} \\ &= \partial_{M_{ji}} \log\det M\,. \end{aligned} \tag{A.28}$$

Moreover, starting from $M_{ik} M^{-1}_{kj} = \delta_{ij}$, taking a derivative and multiplying by the inverse, one also deduces that:

$$\begin{aligned} M^{-1}_{ij} M^{-1}_{pq} &= -\partial_{M_{jp}} M^{-1}_{iq} = -\partial_{M_{qi}} M^{-1}_{pj} \\ &= -\partial_{M_{jp}} \partial_{M_{qi}} \log\det M\,. \end{aligned} \tag{A.29}$$

Applying these relations to the matrix formed by the components of the metric, one then sees that (A.28) and (A.29) essentially correspond to (A.18) and (A.19).

B Symmetric coset manifolds

In this appendix, we summarize some basic facts about the geometry of the symmetric scalar manifolds appearing in the low energy effective theories of orbifold compactifications. These have the form $\mathcal{M} = \mathcal{G}/\mathcal{H}$, where the isometry group $\mathcal{G}$ is a non-compact Lie group and the isotropy group $\mathcal{H}$ is a maximal compact subgroup of it. Rather than studying separately the three kinds of spaces (3.28), (3.29) and (3.30), we shall focus on their basic building block, which is the following Grassmannian coset space for $p = 1, 2, 3$ and arbitrary integer n, which has complex dimension $p(p+n)$:

$$\mathcal{M} = \frac{SU(p, p+n)}{U(1) \times SU(p) \times SU(p+n)}\,. \tag{B.1}$$

The canonical parametrization of the above space involves a rectangular $p \times (p+n)$ matrix of complex coordinates Z^{iJ}, with $i = 1, \cdots, p$, $s = 1, \ldots, n$ and $I = i, s$. In this parametrization, the full stability group $\mathcal{H} = U(1) \times SU(p) \times SU(p+n)$ acts linearly on Z^{iJ}, in the bifundamental representation $(\mathbf{p}, \mathbf{p+n})_1$. Moreover, at the reference point $Z^{iJ} = 0$ these canonical coordinates correspond to normal coordinates, with trivial metric and vanishing Christoffel symbols. The Kähler potential reads [59]:

$$K = -\log\det\big(1 - Z\bar{Z}\big)\,. \tag{B.2}$$

The parametrization that naturally emerges in the string setting is however a slightly different one. It involves a $p \times p$ matrix of moduli coordinates T^{ij} and a $p \times n$ matrix Φ^{is} of matter coordinates. These are related as follows to the $p \times p$ and $p \times n$ sub-blocks Z^{ij} and Z^{is} of the above canonical coordinates Z^{iJ}:

$$Z^{ij} = \left(\frac{1 - 2\,T}{1 + 2\,T}\right)^{ij}, \quad Z^{is} = \left(\frac{2\,\Phi}{1 + 2\,T}\right)^{is}. \tag{B.3}$$

In this new parametrization, the action of $\mathcal{H}$ is more complicated. However, the subgroup $U(1) \times SU(p)_{\rm diag} \times SU(n) \subset \mathcal{H}$ still acts linearly on T^{ij}, Φ^{is}, in the adjoint and bifundamental representations $(\mathbf{1} \oplus \mathbf{p^2} - \mathbf{1}, 1)_0$ and $(\mathbf{p}, \mathbf{n})_1$. In particular, under the universal subgroup $U(p) \simeq U(1) \times SU(p)_{\rm diag}$ that is independent of n, T^{ij} and Φ^{is} transform in the adjoint and the fundamental representations $\mathbf{n^2}$ and $\mathbf{n}$. Moreover, at the reference point $T^{ij} = 1/2\,\delta^{ij}$, $\Phi^{is} = 0$ these new coordinates are only almost normal coordinates, with trivial metric but some non-vanishing Christoffel symbols. The Kähler potential becomes, up to a Kähler transformation [24]:

$$K = -\log\det\left(T + \bar{T} - \Phi\bar{\Phi}\right). \tag{B.4}$$

The manifold under consideration is not only homogeneous but actually symmetric, since the Lie algebra g of $\mathcal{G}$ is the sum of the Lie algebra h of $\mathcal{H}$ and a normal component n associated to $\mathcal{G}/\mathcal{H}$, $g = h \oplus n$, such that $[h,h] \subset h$, $[h,n] \subset n$ and $[n,n] \subset h$. This implies that the Riemann curvature tensor is covariantly constant, $\nabla_m R_{i\bar{j}p\bar{q}} = 0$. As a consequence, the metric and the curvature tensors with tangent space indices are both completely fixed in terms of group theoretical properties of $\mathcal{G}$ and $\mathcal{H}$. To be more precise, let us label the generators of g with T^X, those of h with T^x and finally those of n with T^θ. The metric is then given by the Killing form of g restricted to n:

$$g_{\theta\bar{\xi}} = -B_{\theta\xi}\,. \tag{B.5}$$

The Riemann tensor is instead fixed by the structure constants ruling the part $[n,n] \subset h$ of the algebra, and reads

$$R_{\theta\bar{\xi}\sigma\bar{\tau}} = f_{\theta\xi}{}^x f_{\sigma\tau}{}^y B_{xy}\,. \tag{B.6}$$

Note that although the Killing form B_{XY} on g is indefinite, its restriction $B_{\theta\xi}$ to h is negative definite, so that the above metric is positive definite, and its restriction B_{xy} to n is positive definite, so that the curvature is negative definite.

For the manifold at hand, it is a simple exercise to compute the components of the metric and the Riemann tensor. To do so, it is convenient to switch to the standard two-index labeling of the generators of unitary groups. The generators $T^{\Theta\Gamma}$ of $U(p,p{+}n)$ satisfy $[T^{\Theta\Gamma}, T^{\Sigma\Delta}] = \eta^{\Gamma\Sigma} T^{\Theta\Delta} - \eta^{\Theta\Delta} T^{\Gamma\Sigma}$. The generators T^{ij} and T^{IJ} of the subgroups $U(p)$ and $U(p{+}n)$ similarly satisfy $[T^{ij}, T^{kl}] = \delta^{jk} T^{il} - \delta^{il} T^{jk}$ and $[T^{IJ}, T^{KL}] = -\delta^{JK} T^{IL} + \delta^{IL} T^{JK}$. The remaining generators T^{iJ} and T^{Ij} in the coset $U(p,p+n)/(U(p) \times U(p+n))$, which are associated to the fields Z^{iJ} and their conjugate $\bar{Z}^{I\bar{j}}$, satisfy instead the following commutation relations: $[T^{iJ}, T^{kL}] = 0$, $[T^{Ij}, T^{kL}] = 0$, $[T^{iJ}, T^{Kl}] = -\delta^{JK} T^{il} - \delta^{il} T^{JK}$, $[T^{Ij}, T^{kL}] = \delta^{jk} T^{IL} + \delta^{IL} T^{jk}$. The metric is trivial:

$$g_{iI\bar{j}\bar{J}} = \delta_{ij}\delta_{IJ}\,. \tag{B.7}$$

The Riemann tensor is instead found to be given by the following simple expression, which can also be verified by a direct computation using canonical coordinates at the reference point as in [59]:

$$R_{iI\bar{j}\bar{J}kK\bar{l}\bar{L}} = \delta_{ij}\delta_{kl}\delta_{IL}\delta_{JK} + \delta_{il}\delta_{jk}\delta_{IJ}\delta_{KL}\,. \tag{B.8}$$

Finally, one may split the $p(p+n)$ "complex" coset generators T^{iJ} into moduli generators T^{im} and matter generators $T^{i\alpha}$. The metric then splits into

$$g_{im\bar{j}\bar{n}} = \delta_{ij}\delta_{mn}\,,\quad g_{i\alpha\bar{j}\bar{\beta}} = \delta_{ij}\delta_{\alpha\beta}\,,\quad g_{im\bar{j}\bar{\beta}} = 0\,, \tag{B.9}$$

and the Riemann tensor decomposes as

$$R_{im\bar{j}\bar{n}kp\bar{l}\bar{q}} = \delta_{ij}\delta_{kl}\delta_{mq}\delta_{np} + \delta_{il}\delta_{jk}\delta_{mn}\delta_{pq}\,, \tag{B.10}$$

$$R_{i\alpha\bar{j}\bar{\beta}k\gamma\bar{l}\bar{\delta}} = \delta_{ij}\delta_{kl}\delta_{\alpha\delta}\delta_{\beta\gamma} + \delta_{il}\delta_{jk}\delta_{\alpha\beta}\delta_{\gamma\delta}\,, \tag{B.11}$$

$$R_{im\bar{j}\bar{n}k\gamma\bar{l}\bar{\delta}} = \delta_{il}\delta_{jk}\delta_{mn}\delta_{\gamma\delta}\,. \tag{B.12}$$

At this point, one may apply the above results to the coset spaces (3.28), (3.29) and (3.30) appearing in orbifold models. The resulting expressions can be rewritten more conveniently by relabeling the generators associated to the moduli with a single index. This can be done in parallel for all the three kinds of models by making use of the 3×3 matrices λ^A representing $U(1)\times H$ for the relevant subgroup $H \subset SU(3)$. More precisely, $A = 0,\cdots,8$ for $H = SU(3)$, $a = 0,\cdots,3,8$ for $H = SU(2)\times U(1)$ and $a = 0,3,8$ for $H = U(1)\times U(1)$. Using the normalization condition $\mathrm{tr}(\lambda^A\lambda^B) = \delta^{AB}$ and the completeness properties applying to each of the three subsets of matrices, the metric is found to be

$$g_{A\bar{B}} = \delta_{AB}\,,\quad g_{i\alpha\bar{j}\bar{\beta}} = \delta_{ij}\delta_{\alpha\beta}\,,\quad g_{A\bar{j}} = 0\,, \tag{B.13}$$

and the Riemann tensor reads

$$R_{A\bar{B}C\bar{D}} = \mathrm{tr}(\lambda^A\lambda^B\lambda^C\lambda^D) + \mathrm{tr}(\lambda^A\lambda^D\lambda^C\lambda^B)\,, \tag{B.14}$$

$$R_{i\alpha\bar{j}\bar{\beta}k\gamma\bar{l}\bar{\delta}} = \lambda^A_{il}\lambda^A_{kj}\delta_{\alpha\delta}\delta_{\beta\gamma} + \lambda^A_{ij}\lambda^A_{kl}\delta_{\alpha\beta}\delta_{\gamma\delta}\,, \tag{B.15}$$

$$R_{A\bar{B}k\gamma\bar{l}\bar{\delta}} = (\lambda^B\lambda^A)_{kl}\delta_{\gamma\delta}\,. \tag{B.16}$$

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