Unravelling cross-recurrence: coupling across timescales

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Abstract. We present an extension of cross-recurrence analysis for dual gaze analysis which is suited for complex situations where for instance the objects of interest are not all visible at all times or when stimulus exploration is not homogeneous. The typicical situation is a visual stimulus that is scrolled or that is explored sequentially. We use a recurrence simulation to illustrate how to measure the actual coupling between behavior streams without biases introduced by the complexity of the situation. Our method takes into account underlying random baselines to compute an unbiased version of the coupling.

Introduction

Cross-recurrence quantification measures has been used to measure the time dependence of dyadic gaze behavior. The measure was initially used by (Richardson and Dale, 2005) and (Richardson et al., 2007) on simple stimuli that comprise four to six areas of interest that are always visible for inspection. It is also proposed as a general method to investigate the coupling of behavioral streams by (Dale et al., ress).

A cross recurrence plot like the one in figure 1a is a representation of the time coupling between two behavioral time series produced by subjects S1 and S2. The x, y location on the plot correspond to a given time for S1 (on the horizontal axis) and a given time for S2 (on the vertical axis). Points on the line of identity (central diagonal) correspond to the synchrony. Point are black or white depending on whether the states of S1 and S2 are similar. When used to represent gaze fixations, the plot features a black pixel on the diagonal if S1 and S2 happen to look at the same element exactly at the same time. Points above the diagonal correspond to fixations of S2 that happen after S1 has fixated the element. Conversely, points below the diagonal correspond to S2 leading.

From the cross-recurrence plot, we generally compute the gaze recurrence rate at various time lags. These recurrence rates are simply the percentages of black points along several diagonals parallel to the line of identity. The plot of the successive diagonal recurrence rates results in curves as illustrated in figure1c. Each point represents the coupling of one of the subjects' behavior with the other shifted by a given lag. Because recurrence curves are sometimes quite noisy, it is possible to compute the average of the values between symetrical lags (e.g. -2 to +2 seconds) to get an average recurrence rate for small lags.

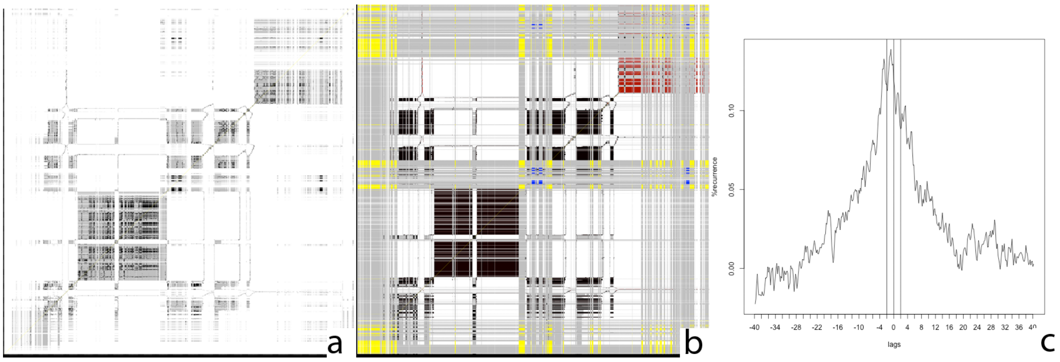

Figure 1. Recurrence plots and diagonal recurrence rate graph from a pair program understanding experiment. a) Dark dots represent gaze recurrent fixations on program tokens. Grey areas represent opportunities for recurrence (fixations on other visible tokens). White areas represent impossible recurrence lags due to stimulus scrolling. b) zone based recurrence plot. Dark areas represent recurrent fixations on program methods. c) diagonal cross-recurrence rate computed on the left recurrence plot..

(Richardson and Dale, 2005) and (Richardson et al., 2007) have shown that this recurrence rate peaks at lag zero (or at a short time lag in case of monologue) and thus indicates that gaze of discussants are coupled, in the sense that a listener's gaze tends to follow the same sequence as the speaker's gaze with a time lag of about 2 seconds. There is however a bias in the computation of such a simple diagonal recurrence rate to measure the coupling between subjects. At short time intervals (from -2 to 2 seconds represented as vertical lines in the diagonal recurrence graph (figure 1c), recurrence reflects whether collaborators fixate the same sequence of areas of interest with a small temporal delay. This phenomenon is characterized by a peak of the curve close to lag 0.

At longer time intervals, recurrence corresponds to a random baseline that is is inversely proportional to the number of workspace elements that can be fixated at a given time. If there are only 10 objects on the screen for inspection, the chance for subjects to look at one of the objects is 1/10 whereas with 100 objects, the chance becomes 1/100. When the number of objects is constant and identical for all subjects it is possible to compare the recurrence curves among pairs because they all share the same baseline and the height of the peaks can be compared.

However, in complex situations, such as collaborative programming or other groupware activities, the number of objects available for inspection changes constantlly. The objects are not explored in a temporally homogeneous way, but are explored by zones, and therefore the baseline is variable. For example, a set of 100 elements can either be explored in 5 episodes comprising fixations on 20 elements or in 10 episodes of 10 elements. In the first case, the random chance to gaze at the same element within an episode is lower (1/20=0.04) than in the second case (1/10 = 0.1). A recurrence measure is therefore artificially inflated when collaborators take into consideration smaller collections of elements at one time. The composition of episodes of varying number of elements results in a decreasing recurrence slope from around the central peak. Cross-recurrence levels in a recurrence graph like figure 1c result from the mixture of several underlying components.

When viewers return to previously visited areas or when viewers do not visit the same areas at the same time, the recurrence plots feature off-diagonal rectangular shapes. The macro strutcture of the recurrence plot reflects the organisation of exploration and is of interest by itself. However, the resulting “bumps” in the recurrence graphs away from the zero lag make the computation of a random baseline problematic (figure 3 e and f represents an extreme case of this phenomenon) and the measure of the actual coupling is impossible.

Proposed solution

We propose to compute a zone based recurrence rate that combines one recurrence rate for each zone of the stimulus that is explored. The aim is to decompose the overall recurrence rate curve and to estimate the low-level coupling peak by normalizing the curve for the number of objects in each zone as well as for the structure of exploration. A zone is a collection of objects that are explored together either because they have a logical link (e.g. lines in a method or a paragraph) or simply because they are co-located on the same region of the screen. In our application domain, computer program understanding, we define objects as code tokens (basically one word) and zones as JAVA methods.

We consider the following idealized situation of two persons who look at one zone of N objects with a uniform proability distribution of gazing at any object. However, each person also has a coupling probability C of looking at exactly the same object than the other subject some time before. We use cross-recurrence analysis to study such an artifical situation and we are interested in deducing the coupling probability from the lag-recurrence quantification analysis.

Simple case: fixed coupling lag

First of all, we have to note that the baseline probability of recurrence in case of no coupling is $\frac{1}{N}$. Indeed, the probability that both subjects look at given object O is $\frac{1}{N^2}$ and thus, the probability that both look at the same object is the sum over all objects. Concerning the coupling, it always occurs at the same time lag l: when a subject is coupled at time t, he takes the value of the other subject at time $t - l$. In such a simple situation, it is easy to compute the expected value of recurrence for a lag l. Indeed, the coupling probability defines the ratio of points that will be coupled for this lag and thus, the resulting recurrence value will simply be the combination of those coupled points and the uncoupled ones, these latter having the baseline recurrence probability. Hence, the following equation gives the recurrence value at lag l:

$$R_l = C + (1 - C) * \frac{1}{N} \tag{1}$$

And from this formula, we can derive the formula to compute the coupling from the recurrence level:

$$C = \frac{R_l - \frac{1}{N}}{1 - \frac{1}{N}} \tag{2}$$

Simple case: variable coupling lag

The difference with a fixed coupling is that the coupling may occur at different lags. More generally, we suppose that these lags follow a probability distribution, such as a normal distribution. A simple way to solve this consists in seeing it as an extension of the fixed lag case. The idea is to decompose the coupling probability C into lag coupling probability C_l, the probability of having a coupling at a specific lag l, so that $C = \sum_{l \in L} C_l$ where L is the set of lags having a non-null coupling probability. We can rewrite 1

$$R_l = C_l + (1 - C_l) * \frac{1}{N}, \forall l \in L \tag{3}$$

And from this formula, we can derive the formula for the general coupling:

$$C = \sum_{l \in L} \frac{R_l - \frac{1}{N}}{1 - \frac{1}{N}} \tag{4}$$

Composite case: normalizing coupling over zones

The approach that we take is to compute recurrence rates separately for each zone z in the set of zones that constitute the stimulus Z and then combine them by weighting their importance by the time spent in each zone t_z. This corresponds graphically to using the plot in figure 1b as a mask for the plot in figure 1a. The ratios of recurrent points on the diagonals of 1a are simply computed only on the points from 1b that fall within one zone. For each lag and for each zone, the coupling can be estimated as the relative height of the recurrence peak above the baseline $b_z$of the zone Z. Because in a non-theoretical case, the estimation of the baseline is not straightforward (the probability to gaze at elements of the zone depends on their size and layout) it is possible to empirically estimate the baseline b by averaging the recurrence rate R_l at extreme lags (-40 to -10 seconds and 10 to 40 seconds).

$$R_{lz} = C_{lz} + (1 - C_{lz}) * \frac{1}{N_z}, \forall l \in L, \forall z \in Z \tag{5}$$

Finally, the values for the coupling are averaged over L and added up with a weight proportional to the proportion of total duration that was spent in the corresponding zone t_z (the number of data points on which the diagonal R_L was computed). In such a way, it is possible to estimate an unbiased overall corrected gaze coupling.

$$C = \sum_{z \in Z} (t_z \sum \frac{R_{lz} - \frac{1}{N_z}}{1 - \frac{1}{N_z}}) \qquad (6)$$

Simulation

We use a cross-recurrence plot generator to show that the zone based coupling measure we propose above is an indicator of the actual coupling between viewers that is relatively insentitive to the macro-structure of the observation sequences.

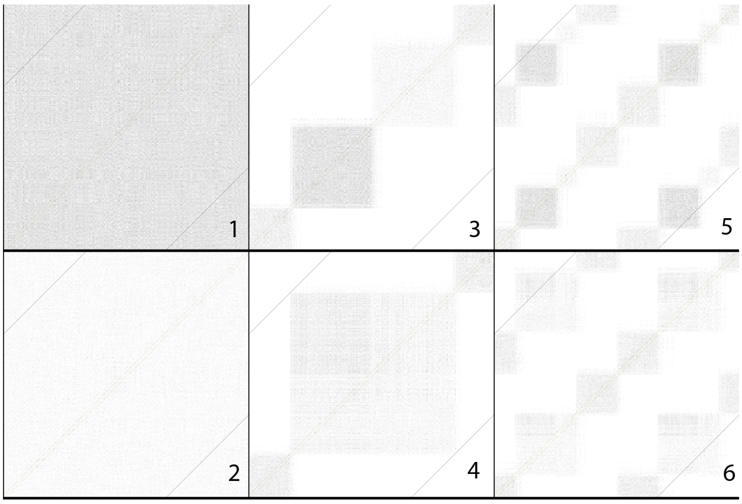

Figure 2. Cross-reccurrence plots for six scenarios. Numbers refer to the scenario ID in table. **??**.

Parameters

Four parameters determine the high level features of the cross-recurrence plot.

- Number of zones. When set to 1, the situation represents a static stimulus.
- Number of objects in each zone.
- Sequentiality of zones indicates whether viewers visit zones only once or whether they return to visit them on several occasions.
- Synchronicity indicates whether the two viewers visit zones in the same order.

Four parameters determine the type of coupling that can be generated.

- *Level* of coupling determines the probability for a fixation of one viewer to be identical to a previous fixation of the other viewer.
- *Mean lag* of coupling determines the average lag at which coupling occurs.

- *Standard deviation* of coupling lag determines variations of the lag of coupling.
- *Symmetry* of coupling determines whether subjects are equally likely to follow each other's gaze or whether one subject is leading the other.

Scenarios

We designed six scenarios to illustrate the effect of the number of zones, the sequentiality as well as the synchronicity of exploration (see table I). We set the number of zones to a maximum of 4 in the simulations. One zone is always present for inspection and the three others represent areas of the stimulus that are not visible simultaneously. For the coupling parameters, only the level of coupling was varied systematically from 0 to 1 by increments of 0.1. The lag of coupling was held constant for all simulations at 30 with a standard deviation of 5.

ID	Scenario	Z	Objects	Seq	Sync
1	uniform-10	1	10	-	-
2	uniform-70	1	70	-	-
3	sequential-sync	4	20,10,40,20	yes	yes
4	return-sync	4	20,10,40,20	no	yes
5	sequential-async	4	20,10,40,20	yes	no
6	return-async	4	20,10,40,20	no	no

Table I. Scenario parameters used in the simulation. Z stands for the number of zones, Objects refers to the size of the zones, Seq indicates whether the exploration was sequential (one pass) or with returns on previously visited zones, Sync indicates whether the sequence of zones is the same for both viewers..

Results

The macro-structure of recurrence plots (figure 2) nicely reflects the the sequentiality and synchronicity of exploration. Off diagonal squares correspond to viewers returning on previously visited zones. This effect is absent when only one zone is visited (scenarios 1 and 2). The shade of gray is proportional to the baseline of the zone. The more objects are available for inspection, the less chance for recurrence at high lags, the lighter the shade of the plot.

The diagonal recurrence rates obtained from the simulated scenarios are depicted in figure 3. In all panels, a central recurrence peak is clearly visible in the graphs. This peak corresponds to the artificial coupling of the viewers that we have introduced in the simulation. The flat baseline in panels a and b are typical of the ideal case with only one zone of objects. The combination of zones containing various number of objects (and therefore various baselines) results in a sloped baseline best visible in panels c and d. The off diagonal rectangular shapes in the recurrence

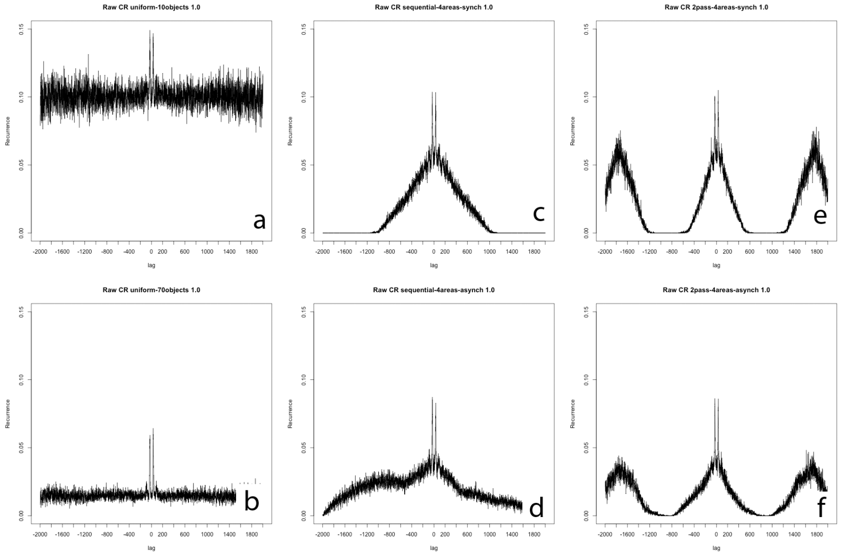

Figure 3. Diagonal recurrence rates for six scenarios. Subplots correspond to the recurrence plots in figure 2. Vertical scale is identical for all subplots..

plots 5 and 6 (figure 2) create “bumps” in the recurrence rates represented in panels e and f.

When diagonal recurrence rates are computed on a zone by zone basis, each zone features its own beaseline and peak (see figure 4) and ressembles the simple case with only one zone. The overall recurrence graph is a weighted sum of the zone recurrence graphs (proportional to the time spent in the zone). From these subgraphs, we apply the formula 6 to obtain the coupling.

To estimate whether the measured coupling indicator is a good indicator for the actual coupling between viewers, we plotted values of the measured coupling as well as the raw recurrence rate against the values of actual coupling. Figure 5 shows that the measured coupling (right panel) is partially neutralizing the high-level effects of sequentiality and sychronicity of the data streams and that it is a good estimator of the actual coupling between the viewers. On the contrary, we see that the raw recurrence rates on the left panel of the figure are sensitive to baseline variations due to the macro-structure of the recurrence plots and thus do not reflect the actual coupling.

Discussion and Conclusion

The coupling definition that we proposed in this contribution is a first step towards using cross-recurrence in everyday applications like shared groupware applications where the complexity of the stimulus and its availability for inspection is constantly changing over time.

This way of measuring coupling relies on the hypothesis that the objects in a zone are uniformly explored in a random sequence. In the program understanding

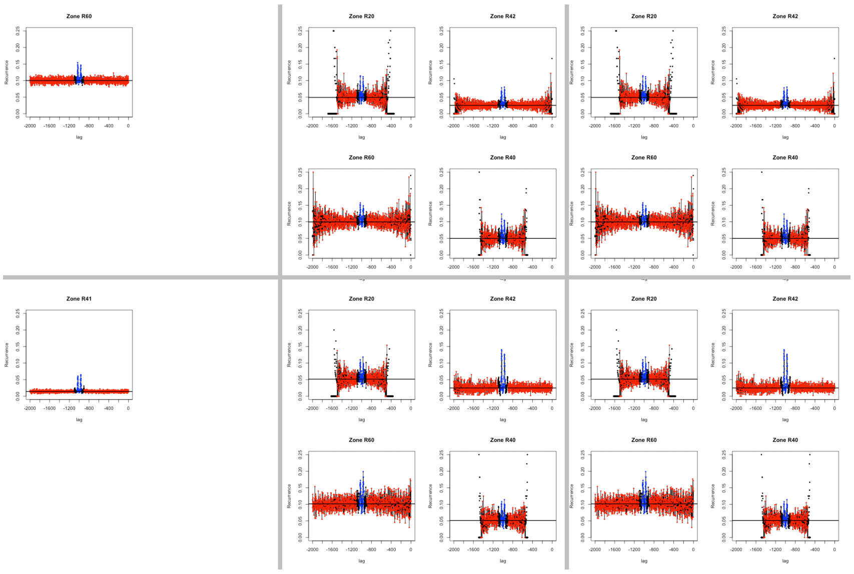

Figure 4. Decomposition of the recurrence into zones. Subplots correspond to the recurrence plots in figure 2.

domain, it is common that programmers "jump" from one statement to the other inside a method without necessarily following a clear sequence. In other domains like reading, the sequence of fixations is clearly not random as the viewers follow the words and lines in a paragraph in sequence. This phenomenon recreates attentional zones inside zones and reproduces the problems that we identified at a lower level of time granularity.

Moreover it appears that in actual data, the number of datapoints in each zone is often too small to have valid estimations of the corresponding baselines. A possible solution to solve this issue would be to define dynamic zones as clusters of objects that are explored "together" within a given timeframe or a given spatial criterion, and that contain enough observations. Another promising approach we are currently

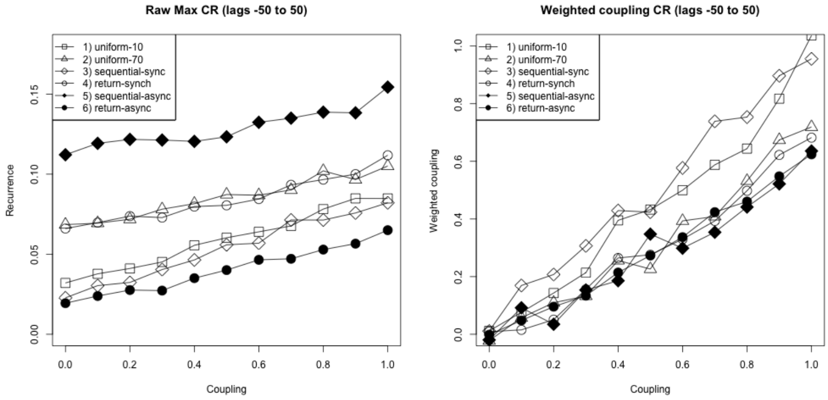

Figure 5. Recurrence rate and normalized coupling given a theoretical coupling from 0 to 1..

investigating consists of applying a low-pass filter to the raw recurrence curve in order to estimate a local dynamic baseline for each lag. The formula **??** can then be used to construct a coupling graph by using the filter value as a baseline.

The method we proposed in this contributions was used to estimate the gaze coupling among fourty pairs of programmers in a program understanding task (Jermann and Nüssli, sub). Pairs were using different types of text selection sharing and we have shown that the sharing of selection had a positive effect on coupling, especially when the selection was accompanied by dialogue.

Acknowledgments

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